Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/187-7.1.4-f-x-^m-d+e-x^2-^p-a+b-arcsinh-c-x-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [663]. This is test number [187].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (663)	0.00 (0)
Mathematica	99.85 (662)	0.15 (1)
Maple	70.74 (469)	29.26 (194)
Maxima	39.06 (259)	60.94 (404)
Fricas	36.35 (241)	63.65 (422)
Sympy	29.56 (196)	70.44 (467)
Mupad	20.06 (133)	79.94 (530)
Giac	14.33 (95)	85.67 (568)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

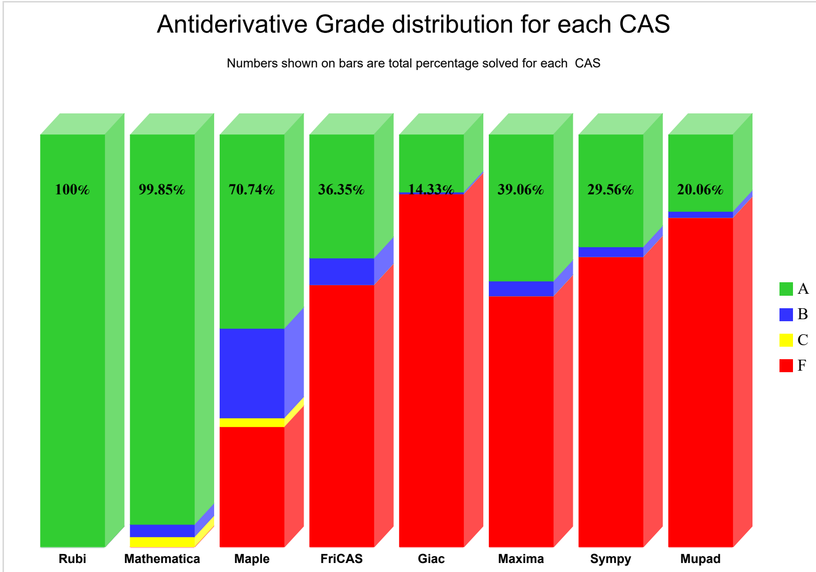
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

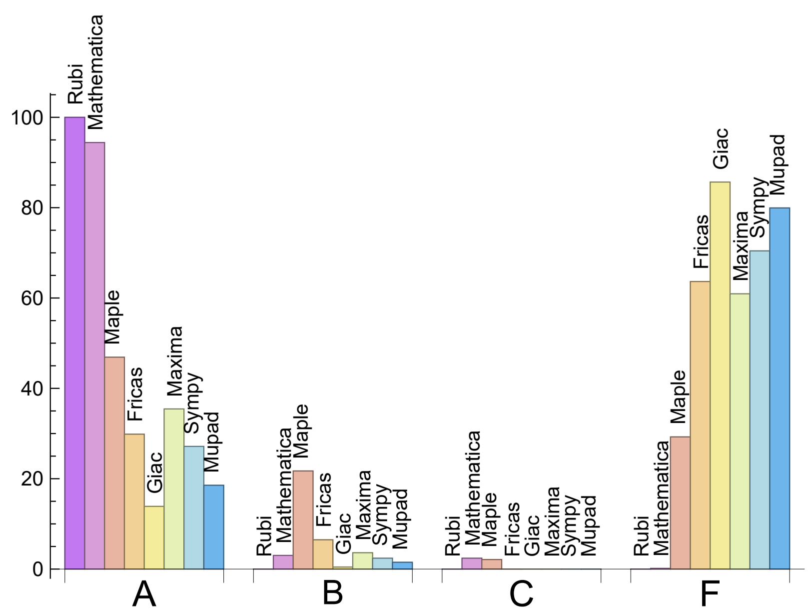
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	94.42	3.02	2.41	0.15
Maple	46.91	21.72	2.11	29.26
Maxima	35.44	3.62	0.00	60.94
Fricas	29.86	6.49	0.00	63.65
Sympy	27.15	2.41	0.00	70.44
Mupad	N/A	1.51	0.00	79.94
Giac	13.88	0.45	0.00	85.67

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	194	100.00 %	0.00 %	0.00 %
Fricas	422	84.83 %	0.00 %	15.17 %
Giac	568	48.42 %	0.35 %	51.23 %
Maxima	404	79.46 %	1.98 %	18.56 %
Sympy	467	83.94 %	7.07 %	8.99 %
Mupad	530	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

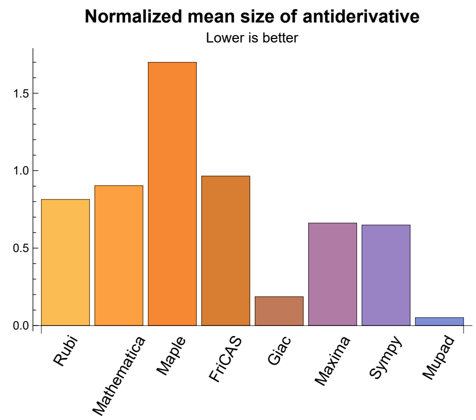
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

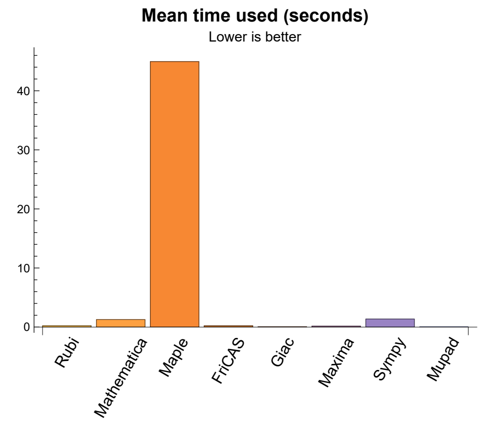
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	196.67	0.81	156.00	1.00
Mathematica	1.27	224.67	0.90	143.00	0.92
Maple	44.95	360.99	1.70	171.00	1.38
Maxima	0.16	108.46	0.66	18.00	0.71
Fricas	0.23	137.04	0.97	57.00	0.91
Sympy	1.36	100.08	0.65	0.00	0.00
Giac	0.05	8.65	0.19	0.00	0.00
Mupad	0.01	0.27	0.05	-1.00	-0.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{188, 189, 190, 321, 322, 323, 324, 325, 326, 327, 341, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 532, 533, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{575, 581, 587, 593, 594, 595, 596, 599, 600, 601, 602, 603, 604, 605, 634\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

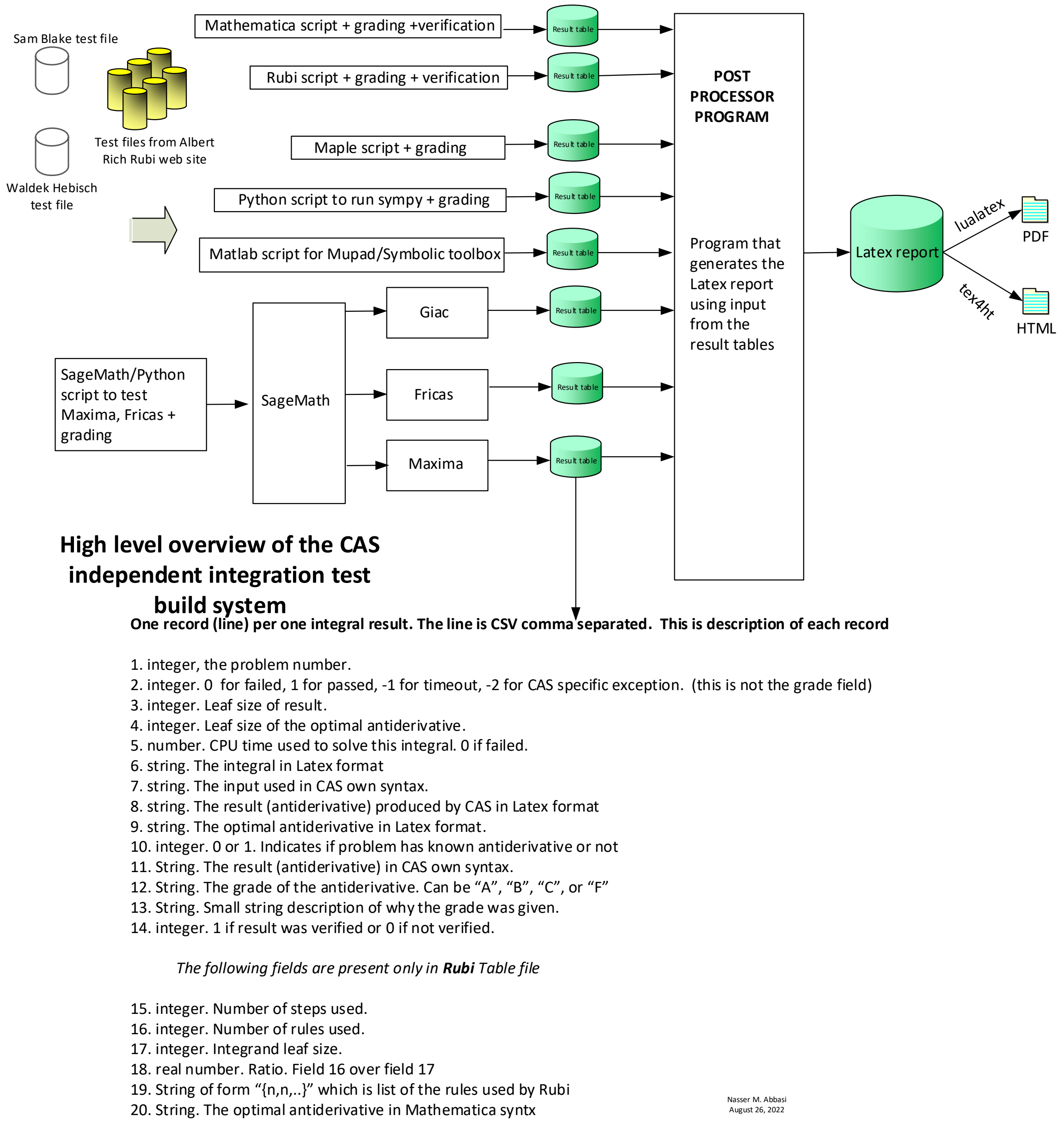
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 37, 39, 40, 41, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 231, 234, 236, 237, 238, 240, 242, 243, 244, 245, 246, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 588, 589, 590, 592, 593, 595, 596, 597, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 31, 33, 35, 42, 44, 228, 230, 233, 241, 331, 551, 564, 581, 586, 587, 591, 594, 598, 600, 601 }

C grade: { 38, 43, 45, 52, 54, 226, 232, 235, 239, 248, 250, 348, 612, 649, 650, 651 }

F grade: { 621 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 56, 58, 59, 61, 67, 69, 75, 77, 79, 83, 84, 86, 88, 96, 98, 107, 109, 112, 113, 114, 115, 116, 117, 119, 124, 126, 132, 134, 140, 142, 144, 150, 151, 153, 158, 161, 162, 163, 172, 174, 177, 178, 179, 180, 181, 182, 184, 188, 189, 190, 204, 206, 213, 215, 222, 224, 226, 228, 235, 254, 284, 285, 286, 287, 288, 289, 300, 302, 305, 320, 321, 322, 323, 324, 325, 326, 327, 334, 335, 336, 337, 338, 339, 340, 341, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 466, 471, 474, 475, 476, 479, 480, 483, 484, 487, 488, 489, 492, 493, 498, 499, 500, 504, 505, 506, 509, 510, 511, 515, 516, 520, 521, 525, 526, 527, 531, 532, 533, 606, 607, 608, 609, 610, 613, 614, 615, 616, 618, 619, 620, 621, 622, 623, 625, 626, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 42, 51, 53, 60, 62, 68, 70, 76, 78, 85, 87, 89, 91, 93, 95, 97, 99, 100, 102, 104, 106, 108, 110, 111, 118, 120, 121, 122, 123, 125, 127, 128, 129, 130, 131, 133, 135, 136, 137, 138, 139, 141, 143, 145, 146, 147, 148, 149, 152, 154, 156, 160, 164, 165, 167, 169, 171, 173, 175, 176, 183, 203, 205, 212, 214, 221, 223, 230, 232, 237, 239, 241, 244, 246, 248, 250, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 303, 304, 307, 309, 310, 311, 312, 313, 314, 315, 317, 319, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 624 }

C grade: { 90, 92, 94, 101, 103, 105, 155, 157, 159, 166, 168, 170, 611, 612 }

F grade: { 55, 57, 63, 64, 65, 66, 71, 72, 73, 74, 80, 81, 82, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 225, 227, 229, 231, 233, 234, 236, 238, 240, 242, 243, 245, 247, 249, 251, 252, 253, 282, 283, 306, 308, 316, 318, 328, 329, 330, 331, 332, 333, 342, 343, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 477, 478, 481, 482, 485, 486, 490, 491, 494, 495, 496, 497, 501, 502, 503, 507, 508, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 617, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 9, 11, 12, 14, 16, 18, 25, 27, 55, 57, 63, 65, 71, 73, 79, 80, 82, 84, 85, 89, 92, 95, 97, 103, 106, 110, 111, 112, 113, 114, 115, 116, 118, 120, 122, 127, 128, 130, 136, 138, 144, 145, 147, 149, 150, 152, 154, 157, 160, 162, 168, 169, 171, 175, 176, 177, 178, 179, 180, 181, 183, 188, 189, 190, 198, 200, 258, 260, 266, 268, 274, 276, 283, 285, 286, 290, 292, 294, 295, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 337, 341, 343, 345, 346, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 385, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 531, 532, 533, 539, 555, 556, 557, 561, 562, 563, 566, 567, 568, 569, 591, 606, 607, 608, 609, 610, 613, 614, 615, 616, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 648, 653, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 10, 13, 19, 20, 21, 22, 23, 62, 87, 104, 199, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 255 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 64, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 81, 83, 86, 88, 90, 91, 93, 94, 96, 98, 99, 100, 101, 102, 105, 107, 108, 109, 117, 119, 121, 123, 124, 125, 126, 129, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 146, 148, 151, 153, 155, 156, 158, 159, 161, 163, 164, 165, 166, 167, 170, 172, 173, 174, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 284, 287, 288, 289, 291, 293, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 338, 339, 340, 342, 344, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 558, 559, 560, 564, 565, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 647, 649, 650, 651, 652, 654 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 55, 63, 79, 80, 82, 90, 101, 112, 113, 114, 115, 118, 120, 122, 128, 130, 136, 138, 144, 145, 147, 149, 154, 155, 157, 159, 166, 168, 170, 177, 178, 179, 180, 183, 188, 189, 190, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 244, 258, 260, 266, 268, 274, 276, 282, 283, 284, 285, 290, 292, 294, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 341, 342, 343, 344, 345, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 487, 492, 504, 509, 515, 516, 520, 521, 525, 526, 527, 532, 533, 609, 610, 622, 623, 627, 628, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 7, 9, 16, 18, 57, 62, 65, 71, 73, 84, 87, 89, 92, 94, 103, 105, 116, 127, 152, 181, 237, 246, 286, 346, 385, 461, 531, 539, 556, 557, 561, 566, 567, 606, 607, 608, 613, 614, 615, 616, 649, 650, 651 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 64, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 81, 83, 85, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 102, 104, 106, 107, 108, 109, 110, 111, 117, 119, 121, 123, 124, 125, 126, 129, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 146, 148, 150, 151, 153, 156, 158, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 287, 288, 289, 291, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 510, 511, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 558, 559, 560, 562, 563, 564, 565, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 64, 66, 72, 74, 80, 81, 82, 83, 84, 112, 113, 114, 115, 116, 177, 178, 179, 180, 181, 188, 189, 190, 198, 199, 200, 202, 207, 208, 209, 211, 216, 217, 218, 220, 282, 283, 284, 285, 286, 323, 324, 325, 326, 327, 328, 329, 330, 341, 342, 343, 344, 345, 346, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 385, 386, 387, 394, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 466, 471, 475, 476, 480, 488, 489, 493, 499, 500, 505, 515, 516, 520, 527, 532, 533, 606, 607, 608, 609, 610, 613, 614, 615, 616, 622, 623, 627, 632, 633, 636, 637, 641, 642, 645, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 55, 57, 60, 63, 65, 73, 85, 201, 210, 219, 252, 253, 255, 393, 440, 531 }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 61, 62, 67, 68, 69, 70, 71, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 401, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 455, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 517, 518, 519, 521, 522, 523, 524, 525, 526, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 628, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 646, 649, 650, 651 }

2.1.7 Giac

A grade: { 111, 115, 176, 180, 188, 189, 190, 285, 324, 325, 326, 327, 341, 345, 353, 354, 361, 363, 369, 371, 377, 379, 386, 387, 395, 396, 398, 400, 404, 405, 409, 410, 416, 418, 424, 426, 432, 434, 442, 444, 446, 448, 450, 452, 454, 458, 459, 460, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 527, 531, 532, 533, 610, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 118, 183, 616 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 181, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 360, 362, 364, 365, 366, 367, 368, 370, 372, 373, 374, 375, 376, 378, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 394, 397, 399, 401, 402, 403, 406, 407, 408, 411, 412, 413, 414, 415, 417, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 433, 435, 436, 437, 438, 439, 440, 441, 443, 445, 447, 449, 451, 453, 455, 456, 457, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

2.1.8 Mupad

A grade: { 188, 189, 190, 321, 322, 323, 324, 325, 326, 327, 341, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 532, 533, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 116, 181, 286, 346, 385, 393, 440, 461, 531, 610 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	87	124	184	113	151	0	-1
N.S.	1	1.00	0.70	1.00	1.48	0.91	1.22	0.00	-0.01
time (sec)	N/A	0.074	0.067	0.964	0.260	0.372	0.764	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	88	115	166	109	138	0	-1
N.S.	1	1.00	0.73	0.96	1.38	0.91	1.15	0.00	-0.01
time (sec)	N/A	0.063	0.042	1.713	0.252	0.360	0.497	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	78	105	145	103	126	0	-1
N.S.	1	1.00	0.76	1.03	1.42	1.01	1.24	0.00	-0.01
time (sec)	N/A	0.068	0.060	0.964	0.258	0.383	0.336	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	85	127	98	117	0	-1
N.S.	1	1.00	0.89	0.98	1.46	1.13	1.34	0.00	-0.01
time (sec)	N/A	0.041	0.039	1.851	0.264	0.373	0.218	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	86	76	97	83	90	0	-1
N.S.	1	1.00	1.15	1.01	1.29	1.11	1.20	0.00	-0.01
time (sec)	N/A	0.038	0.032	0.483	0.264	0.371	0.156	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	162	0	0	0	0	-1
N.S.	1	1.00	1.02	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.045	2.976	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	74	69	64	156	0	0	-1
N.S.	1	1.00	1.12	1.05	0.97	2.36	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.021	0.484	0.267	0.374	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	111	166	0	0	0	0	-1
N.S.	1	1.00	0.87	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.043	4.835	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	87	91	169	0	0	-1
N.S.	1	1.00	1.16	1.09	1.14	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.026	0.490	0.259	0.379	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	119	167	319	165	230	0	-1
N.S.	1	1.00	0.66	0.92	1.76	0.91	1.27	0.00	-0.01
time (sec)	N/A	0.134	0.068	0.988	0.262	0.361	1.449	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	115	146	292	161	218	0	-1
N.S.	1	1.00	0.64	0.81	1.62	0.89	1.21	0.00	-0.01
time (sec)	N/A	0.114	0.058	1.746	0.286	0.365	1.065	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	111	148	261	153	202	0	-1
N.S.	1	1.00	0.71	0.94	1.66	0.97	1.29	0.00	-0.01
time (sec)	N/A	0.121	0.057	1.348	0.270	0.354	0.708	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	116	234	149	190	0	-1
N.S.	1	1.00	0.87	0.97	1.95	1.24	1.58	0.00	-0.01
time (sec)	N/A	0.045	0.076	2.084	0.269	0.421	0.508	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	119	194	133	165	0	-1
N.S.	1	1.00	0.74	0.93	1.52	1.04	1.29	0.00	-0.01
time (sec)	N/A	0.075	0.063	0.516	0.270	0.399	0.335	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	173	231	0	0	0	0	-1
N.S.	1	1.00	1.01	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.138	3.335	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	124	114	143	228	0	0	-1
N.S.	1	1.00	1.03	0.95	1.19	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.069	0.543	0.259	0.370	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	170	248	0	0	0	0	-1
N.S.	1	1.00	0.91	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.131	5.103	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	133	114	137	243	0	0	-1
N.S.	1	1.00	1.06	0.90	1.09	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.070	0.531	0.279	0.368	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	143	206	465	201	289	0	-1
N.S.	1	1.00	0.63	0.91	2.06	0.89	1.28	0.00	-0.00
time (sec)	N/A	0.191	0.084	1.013	0.271	0.354	3.247	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	139	173	429	197	280	0	-1
N.S.	1	1.00	0.70	0.87	2.16	0.99	1.41	0.00	-0.01
time (sec)	N/A	0.115	0.077	1.790	0.257	0.377	2.088	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	135	187	388	189	265	0	-1
N.S.	1	1.00	0.67	0.93	1.92	0.94	1.31	0.00	-0.00
time (sec)	N/A	0.166	0.071	1.023	0.259	0.376	1.474	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	143	352	185	253	0	-1
N.S.	1	1.00	0.88	0.99	2.43	1.28	1.74	0.00	-0.01
time (sec)	N/A	0.050	0.092	1.736	0.281	0.362	1.063	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	119	156	301	169	221	0	-1
N.S.	1	1.00	0.70	0.92	1.77	0.99	1.30	0.00	-0.01
time (sec)	N/A	0.110	0.072	0.520	0.270	0.341	0.726	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	189	284	0	0	0	0	-1
N.S.	1	1.00	0.86	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.095	3.605	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	163	151	231	276	0	0	-1
N.S.	1	1.00	1.02	0.94	1.44	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.082	0.641	0.263	0.411	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	210	299	0	0	0	0	-1
N.S.	1	1.00	0.84	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	0.167	6.023	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	171	155	208	289	0	0	-1
N.S.	1	1.00	0.98	0.89	1.20	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.087	0.528	0.262	0.394	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	170	245	0	0	0	0	-1
N.S.	1	1.00	1.09	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.179	3.457	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	181	148	0	0	0	0	-1
N.S.	1	1.00	1.34	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.142	3.027	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	121	194	0	0	0	0	-1
N.S.	1	1.00	1.12	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.112	2.563	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	167	90	0	0	0	0	-1
N.S.	1	1.00	2.29	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.047	1.780	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	135	157	0	0	0	0	-1
N.S.	1	1.00	1.93	2.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.066	2.540	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	207	71	0	0	0	0	-1
N.S.	1	1.00	3.39	1.16	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.082	0.065	2.648	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	182	203	0	0	0	0	-1
N.S.	1	1.00	1.80	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.104	0.701	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	240	251	0	0	0	0	-1
N.S.	1	1.00	2.12	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.182	2.963	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	247	252	0	0	0	0	-1
N.S.	1	1.00	1.58	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.137	0.678	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	268	260	0	0	0	0	-1
N.S.	1	1.00	1.57	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.170	2.517	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	241	187	0	0	0	0	-1
N.S.	1	1.00	1.66	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.079	4.882	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	221	219	0	0	0	0	-1
N.S.	1	1.00	1.74	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.115	3.116	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	74	61	0	65	0	0	-1
N.S.	1	1.00	1.35	1.11	0.00	1.18	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.038	0.553	0.000	0.354	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	216	219	0	0	0	0	-1
N.S.	1	1.00	1.74	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.071	0.642	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	234	283	0	0	0	0	-1
N.S.	1	1.00	2.13	2.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.297	2.869	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	253	263	0	0	0	0	-1
N.S.	1	1.00	1.51	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.390	0.667	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	326	293	0	0	0	0	-1
N.S.	1	1.00	2.23	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.322	3.701	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	311	316	0	0	0	0	-1
N.S.	1	1.00	1.30	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.417	2.530	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	341	292	0	0	0	0	-1
N.S.	1	1.00	1.83	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.382	0.658	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	79	108	0	99	0	0	-1
N.S.	1	1.00	0.81	1.11	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.061	0.526	0.000	0.349	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	340	292	0	0	0	0	-1
N.S.	1	1.00	1.85	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.136	0.662	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	56	76	0	98	0	0	-1
N.S.	1	1.00	0.70	0.95	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.044	0.527	0.000	0.375	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	341	284	0	0	0	0	-1
N.S.	1	1.00	1.92	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.104	0.666	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	289	451	0	0	0	0	-1
N.S.	1	1.00	1.82	2.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.433	4.458	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	298	352	0	0	0	0	-1
N.S.	1	1.00	1.34	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.737	3.417	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	353	549	0	0	0	0	-1
N.S.	1	1.00	1.52	2.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.573	4.838	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	380	406	0	0	0	0	-1
N.S.	1	1.00	1.29	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.644	2.581	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	106	0	134	158	221	0	-1
N.S.	1	1.00	0.97	0.00	1.23	1.45	2.03	0.00	-0.01
time (sec)	N/A	0.095	0.120	180.000	0.267	0.473	1.060	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	79	156	0	0	0	0	-1
N.S.	1	1.00	0.66	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.112	0.799	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	63	0	73	127	141	0	-1
N.S.	1	1.00	1.03	0.00	1.20	2.08	2.31	0.00	-0.02
time (sec)	N/A	0.044	0.073	180.000	0.266	0.384	0.299	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	100	0	0	0	0	-1
N.S.	1	1.00	1.03	1.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.088	1.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	131	171	0	0	0	0	-1
N.S.	1	1.00	1.47	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.126	4.371	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	155	0	0	110	0	-1
N.S.	1	1.00	1.23	2.54	0.00	0.00	1.80	0.00	-0.02
time (sec)	N/A	0.075	0.105	3.633	0.000	0.000	1.839	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	185	241	0	0	0	0	-1
N.S.	1	1.00	1.64	2.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	2.182	6.161	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	78	501	133	217	0	0	-1
N.S.	1	1.00	1.26	8.08	2.15	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.080	6.540	0.283	0.381	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	100	0	145	199	301	0	-1
N.S.	1	1.00	0.80	0.00	1.16	1.59	2.41	0.00	-0.01
time (sec)	N/A	0.113	0.103	180.000	0.267	0.390	11.904	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	154	0	0	0	262	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	1.59	0.00	-0.01
time (sec)	N/A	0.219	0.240	180.000	0.000	0.000	7.010	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	0	85	167	221	0	-1
N.S.	1	1.00	0.94	0.00	1.10	2.17	2.87	0.00	-0.01
time (sec)	N/A	0.052	0.080	180.000	0.302	0.383	3.962	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	0	0	0	185	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.67	0.00	-0.01
time (sec)	N/A	0.069	0.157	180.000	0.000	0.000	1.931	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	180	228	0	0	0	0	-1
N.S.	1	1.00	1.34	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.257	5.184	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	122	222	0	0	0	0	-1
N.S.	1	1.00	1.13	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.198	4.477	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	292	291	0	0	0	0	-1
N.S.	1	1.00	1.88	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	1.078	10.325	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	125	622	0	0	0	0	-1
N.S.	1	1.00	1.09	5.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.145	6.621	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	108	0	156	263	0	0	-1
N.S.	1	1.00	0.77	0.00	1.11	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.125	180.000	0.285	0.378	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	196	0	0	0	350	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.64	0.00	-0.00
time (sec)	N/A	0.298	0.407	180.000	0.000	0.000	63.796	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	96	225	299	0	-1
N.S.	1	1.00	0.86	0.00	1.03	2.42	3.22	0.00	-0.01
time (sec)	N/A	0.062	0.097	180.000	0.272	0.368	37.566	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	153	0	0	0	265	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	1.61	0.00	-0.01
time (sec)	N/A	0.126	0.261	180.000	0.000	0.000	21.669	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	257	284	0	0	0	0	-1
N.S.	1	1.00	1.44	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.241	5.789	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	168	283	0	0	0	0	-1
N.S.	1	1.00	1.07	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.272	6.400	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	349	348	0	0	0	0	-1
N.S.	1	1.00	1.70	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	1.265	11.025	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	179	692	0	0	0	0	-1
N.S.	1	1.00	1.08	4.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.254	7.279	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	28	40	0	0	-1
N.S.	1	1.00	0.88	0.81	0.88	1.25	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.010	1.544	0.521	0.360	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	108	0	174	161	182	0	-1
N.S.	1	1.00	0.72	0.00	1.17	1.08	1.22	0.00	-0.01
time (sec)	N/A	0.182	0.098	180.000	0.316	0.371	5.002	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	111	0	0	0	185	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.47	0.00	-0.01
time (sec)	N/A	0.147	0.150	180.000	0.000	0.000	5.314	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	82	0	117	132	122	0	-1
N.S.	1	1.00	0.84	0.00	1.19	1.35	1.24	0.00	-0.01
time (sec)	N/A	0.108	0.076	180.000	0.275	0.391	2.012	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	107	0	0	92	0	-1
N.S.	1	1.00	0.92	1.43	0.00	0.00	1.23	0.00	-0.01
time (sec)	N/A	0.082	0.115	1.209	0.000	0.000	2.657	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	49	72	55	96	60	0	-1
N.S.	1	1.00	1.17	1.71	1.31	2.29	1.43	0.00	-0.02
time (sec)	N/A	0.041	0.046	1.151	0.326	0.367	1.390	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	53	28	0	85	0	-1
N.S.	1	1.00	1.00	2.12	1.12	0.00	3.40	0.00	-0.04
time (sec)	N/A	0.020	0.013	0.666	0.297	0.000	1.502	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	96	72	0	0	0	0	-1
N.S.	1	1.00	1.71	1.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.085	0.102	1.098	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	84	101	132	0	0	-1
N.S.	1	1.00	1.02	2.05	2.46	3.22	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.065	2.050	0.307	0.427	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	185	226	0	0	0	0	-1
N.S.	1	1.00	1.61	1.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	1.804	4.291	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	99	373	121	222	0	0	-1
N.S.	1	1.00	1.02	3.85	1.25	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.099	3.881	0.296	0.402	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	229	0	196	0	0	-1
N.S.	1	1.00	0.96	1.67	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.115	5.189	0.000	0.432	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	147	269	0	0	0	0	-1
N.S.	1	1.00	1.12	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.264	5.089	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	87	159	119	165	0	0	-1
N.S.	1	1.00	1.01	1.85	1.38	1.92	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.094	5.150	0.528	0.417	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	196	0	0	0	0	-1
N.S.	1	1.00	0.98	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.207	3.333	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	103	0	127	0	0	-1
N.S.	1	1.00	1.16	2.29	0.00	2.82	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.066	2.377	0.000	0.455	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	66	132	58	0	0	0	-1
N.S.	1	1.00	1.29	2.59	1.14	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.057	1.602	0.290	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	143	157	0	0	0	0	-1
N.S.	1	1.00	1.52	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.228	3.539	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	69	181	119	0	0	0	-1
N.S.	1	1.00	0.74	1.95	1.28	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.104	2.494	0.304	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	269	233	0	0	0	0	-1
N.S.	1	1.00	1.66	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	2.760	4.223	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	127	604	0	0	0	0	-1
N.S.	1	1.00	0.83	3.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.136	5.362	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	202	970	0	0	0	0	-1
N.S.	1	1.00	1.05	5.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.355	8.031	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	132	237	0	218	0	0	-1
N.S.	1	1.00	0.90	1.62	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.123	9.845	0.000	0.445	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	166	897	0	0	0	0	-1
N.S.	1	1.00	1.19	6.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.254	6.900	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	176	138	187	0	0	-1
N.S.	1	1.00	0.89	1.68	1.31	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.107	8.245	0.491	0.419	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	88	729	137	0	0	0	-1
N.S.	1	1.00	1.10	9.11	1.71	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.102	6.525	0.287	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	124	0	165	0	0	-1
N.S.	1	1.00	0.96	1.65	0.00	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.081	3.728	0.000	0.435	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	100	619	126	0	0	0	-1
N.S.	1	1.00	0.93	5.73	1.17	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.085	3.280	0.293	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	209	225	0	0	0	0	-1
N.S.	1	1.00	1.41	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.541	4.177	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	123	778	0	0	0	0	-1
N.S.	1	1.00	0.82	5.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.129	5.126	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	331	314	0	0	0	0	-1
N.S.	1	1.00	1.34	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.312	4.020	4.676	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	142	1155	236	0	0	0	-1
N.S.	1	1.00	0.68	5.55	1.13	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	0.163	4.134	0.309	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	121	363	143	0	0	124	-1
N.S.	1	1.00	0.60	1.82	0.72	0.00	0.00	0.62	-0.00
time (sec)	N/A	0.089	0.119	2.543	0.315	0.000	0.000	0.437	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	63	66	83	83	82	0	-1
N.S.	1	1.00	0.73	0.77	0.97	0.97	0.95	0.00	-0.01
time (sec)	N/A	0.106	0.035	2.253	0.292	0.355	0.457	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	48	50	59	55	65	0	-1
N.S.	1	1.00	0.69	0.71	0.84	0.79	0.93	0.00	-0.01
time (sec)	N/A	0.071	0.029	2.997	0.294	0.399	0.331	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	40	55	62	42	0	-1
N.S.	1	1.00	0.86	0.82	1.12	1.27	0.86	0.00	-0.02
time (sec)	N/A	0.056	0.030	3.132	0.287	0.394	0.267	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	47	26	38	24	38	-1
N.S.	1	1.00	1.00	1.68	0.93	1.36	0.86	1.36	-0.04
time (sec)	N/A	0.031	0.022	2.420	0.286	0.381	0.208	0.403	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.015	0.005	0.246	0.304	0.374	0.177	0.000	0.106

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	42	0	0	0	0	-1
N.S.	1	1.00	1.68	1.24	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.057	0.072	2.244	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	56	25	39	0	71	-1
N.S.	1	1.00	1.07	2.07	0.93	1.44	0.00	2.63	-0.04
time (sec)	N/A	0.043	0.029	3.826	0.295	0.373	0.000	0.404	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	126	150	0	0	0	0	-1
N.S.	1	1.00	1.58	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.459	4.939	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	120	578	134	158	0	0	-1
N.S.	1	1.00	0.69	3.30	0.77	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.071	1.798	0.276	0.355	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	129	338	0	0	0	0	-1
N.S.	1	1.00	0.71	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.541	2.386	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	92	321	73	127	0	0	-1
N.S.	1	1.00	0.88	3.06	0.70	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.057	0.756	0.312	0.380	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	120	256	0	0	0	0	-1
N.S.	1	1.00	1.08	2.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.305	1.520	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	168	331	0	0	0	0	-1
N.S.	1	1.00	0.95	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.323	1.810	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	129	263	0	0	0	0	-1
N.S.	1	1.00	1.23	2.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.213	2.201	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	223	377	0	0	0	0	-1
N.S.	1	1.00	1.11	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.135	1.984	3.433	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	131	946	133	217	0	0	-1
N.S.	1	1.00	1.24	8.92	1.25	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.115	3.655	0.275	0.472	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	130	872	145	199	0	0	-1
N.S.	1	1.00	0.60	4.02	0.67	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.102	2.050	0.298	0.371	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	251	799	0	0	0	0	-1
N.S.	1	1.00	0.99	3.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.488	2.427	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	102	559	85	167	0	0	-1
N.S.	1	1.00	0.70	3.83	0.58	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.081	0.608	0.296	0.380	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	200	496	0	0	0	0	-1
N.S.	1	1.00	1.11	2.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.532	1.378	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	248	428	0	0	0	0	-1
N.S.	1	1.00	1.00	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.514	1.645	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	200	392	0	0	0	0	-1
N.S.	1	1.00	1.13	2.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.547	2.157	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	352	472	0	0	0	0	-1
N.S.	1	1.00	1.30	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	2.881	3.292	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	217	1107	0	0	0	0	-1
N.S.	1	1.00	1.18	6.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.370	3.668	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	140	996	156	263	0	0	-1
N.S.	1	1.00	0.53	3.74	0.59	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.118	0.123	1.583	0.298	0.364	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	388	1165	0	0	0	0	-1
N.S.	1	1.00	1.15	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.654	2.309	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	112	863	96	225	0	0	-1
N.S.	1	1.00	0.58	4.47	0.50	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.090	0.642	0.290	0.378	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	317	801	0	0	0	0	-1
N.S.	1	1.00	1.25	3.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.468	1.459	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	353	540	0	0	0	0	-1
N.S.	1	1.00	1.07	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.905	1.672	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	270	506	0	0	0	0	-1
N.S.	1	1.00	1.05	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.956	3.094	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	424	588	0	0	0	0	-1
N.S.	1	1.00	1.19	1.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	5.357	3.428	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	286	1316	0	0	0	0	-1
N.S.	1	1.00	1.08	4.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.668	3.704	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	28	40	0	0	-1
N.S.	1	1.00	0.88	0.81	0.88	1.25	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.010	0.895	0.483	0.378	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	119	625	174	161	0	0	-1
N.S.	1	1.00	0.55	2.91	0.81	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.177	0.101	3.269	0.314	0.354	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	151	519	0	0	0	0	-1
N.S.	1	1.00	0.79	2.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.403	4.062	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	93	358	117	132	0	0	-1
N.S.	1	1.00	0.65	2.52	0.82	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.084	2.651	0.292	0.397	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	121	273	0	0	0	0	-1
N.S.	1	1.00	1.02	2.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.502	3.652	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	74	148	55	96	0	0	-1
N.S.	1	1.00	1.16	2.31	0.86	1.50	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.061	1.087	0.333	0.367	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	48	77	28	0	0	0	-1
N.S.	1	1.00	1.02	1.64	0.60	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.021	0.348	0.292	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	129	234	0	0	0	0	-1
N.S.	1	1.00	1.06	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.239	1.894	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	67	183	101	132	0	0	-1
N.S.	1	1.00	1.06	2.90	1.60	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.063	0.102	1.972	0.347	0.379	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	229	380	0	0	0	0	-1
N.S.	1	1.00	1.13	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.164	2.001	3.494	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	135	792	121	222	0	0	-1
N.S.	1	1.00	0.96	5.62	0.86	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.132	4.648	0.285	0.425	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	148	367	0	197	0	0	-1
N.S.	1	1.00	0.70	1.73	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.160	3.166	0.000	0.447	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	161	366	0	0	0	0	-1
N.S.	1	1.00	0.78	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.304	4.094	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	143	261	119	166	0	0	-1
N.S.	1	1.00	1.05	1.92	0.88	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.148	2.608	0.541	0.408	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	146	232	0	0	0	0	-1
N.S.	1	1.00	1.12	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.123	3.556	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	82	164	0	128	0	0	-1
N.S.	1	1.00	1.17	2.34	0.00	1.83	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.101	0.915	0.000	0.447	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	100	143	58	0	0	0	-1
N.S.	1	1.00	1.32	1.88	0.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.105	1.385	0.289	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	231	274	0	0	0	0	-1
N.S.	1	1.00	1.19	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.616	1.653	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	163	206	119	0	0	0	-1
N.S.	1	1.00	1.14	1.44	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.187	1.904	0.295	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	369	389	0	0	0	0	-1
N.S.	1	1.00	1.29	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	5.659	3.234	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	216	968	0	0	0	0	-1
N.S.	1	1.00	0.95	4.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.125	0.209	3.333	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	222	1607	0	0	0	0	-1
N.S.	1	1.00	0.79	5.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	0.646	4.677	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	154	400	0	219	0	0	-1
N.S.	1	1.00	0.73	1.90	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.134	0.155	3.140	0.000	0.441	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	191	1430	0	0	0	0	-1
N.S.	1	1.00	0.94	7.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.351	4.049	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	151	263	138	188	0	0	-1
N.S.	1	1.00	1.05	1.83	0.96	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.150	3.368	0.505	0.419	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	1175	137	0	0	0	-1
N.S.	1	1.00	0.99	9.87	1.15	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.122	3.408	0.290	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	130	198	0	166	0	0	-1
N.S.	1	1.00	1.14	1.74	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.118	1.014	0.000	0.437	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	143	1006	126	0	0	0	-1
N.S.	1	1.00	0.97	6.84	0.86	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.129	1.568	0.298	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	247	364	0	0	0	0	-1
N.S.	1	1.00	0.94	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.786	1.744	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	227	1257	0	0	0	0	-1
N.S.	1	1.00	1.06	5.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.227	2.024	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	409	546	0	0	0	0	-1
N.S.	1	1.00	1.02	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	4.690	3.319	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	267	1792	236	0	0	0	-1
N.S.	1	1.00	0.90	6.03	0.79	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.243	3.365	0.312	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	121	363	143	0	0	124	-1
N.S.	1	1.00	0.60	1.82	0.72	0.00	0.00	0.62	-0.00
time (sec)	N/A	0.087	0.104	2.586	0.318	0.000	0.000	0.452	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	63	68	83	83	82	0	-1
N.S.	1	1.00	0.73	0.79	0.97	0.97	0.95	0.00	-0.01
time (sec)	N/A	0.099	0.034	2.478	0.279	0.386	0.444	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	48	50	59	55	65	0	-1
N.S.	1	1.00	0.69	0.71	0.84	0.79	0.93	0.00	-0.01
time (sec)	N/A	0.069	0.029	3.240	0.298	0.384	0.344	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	40	55	62	42	0	-1
N.S.	1	1.00	0.86	0.82	1.12	1.27	0.86	0.00	-0.02
time (sec)	N/A	0.055	0.029	1.595	0.309	0.357	0.270	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	47	26	38	24	38	-1
N.S.	1	1.00	1.00	1.68	0.93	1.36	0.86	1.36	-0.04
time (sec)	N/A	0.028	0.022	2.093	0.321	0.354	0.205	0.427	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.014	0.005	0.293	0.313	0.358	0.179	0.000	0.096

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	42	0	0	0	0	-1
N.S.	1	1.00	1.68	1.24	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	0.081	1.317	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	56	25	39	0	71	-1
N.S.	1	1.00	1.07	2.07	0.93	1.44	0.00	2.63	-0.04
time (sec)	N/A	0.040	0.031	3.900	0.289	0.368	0.000	0.407	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	126	150	0	0	0	0	-1
N.S.	1	1.00	1.58	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.104	5.020	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	257	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.311	0.328	180.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	188	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.012	180.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.042	4.564	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	2.633	180.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	3.892	180.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	4.138	180.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	332	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	0.838	180.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	233	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.233	0.359	180.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	179	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.133	0.048	180.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	129	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.041	180.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	206	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.157	0.161	180.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	286	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.292	180.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	97	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.024	180.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	201	0	441	260	388	0	-1
N.S.	1	1.00	0.71	0.00	1.56	0.92	1.37	0.00	-0.00
time (sec)	N/A	0.298	0.167	180.000	0.309	0.376	1.112	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	186	0	442	240	332	0	-1
N.S.	1	1.00	0.94	0.00	2.23	1.21	1.68	0.00	-0.01
time (sec)	N/A	0.359	0.160	180.000	0.344	0.390	0.803	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	177	0	346	225	313	0	-1
N.S.	1	1.00	0.86	0.00	1.68	1.09	1.52	0.00	-0.00
time (sec)	N/A	0.234	0.151	180.000	0.305	0.371	0.540	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	155	0	347	204	269	0	-1
N.S.	1	1.00	1.15	0.00	2.57	1.51	1.99	0.00	-0.01
time (sec)	N/A	0.095	0.131	180.000	0.331	0.354	0.378	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	135	0	230	178	224	0	-1
N.S.	1	1.00	1.08	0.00	1.84	1.42	1.79	0.00	-0.01
time (sec)	N/A	0.103	0.107	180.000	0.294	0.356	0.241	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	222	425	0	0	0	0	-1
N.S.	1	1.00	1.34	2.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.271	3.691	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	192	239	0	0	0	0	-1
N.S.	1	1.00	1.47	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	0.275	4.108	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	212	478	0	0	0	0	-1
N.S.	1	1.00	1.18	2.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.212	6.482	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	245	272	0	0	0	0	-1
N.S.	1	1.00	1.55	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.493	5.942	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	251	0	760	368	563	0	-1
N.S.	1	1.00	0.65	0.00	1.97	0.95	1.46	0.00	-0.00
time (sec)	N/A	0.511	0.222	180.000	0.306	0.418	2.149	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	237	0	762	348	515	0	-1
N.S.	1	1.00	0.80	0.00	2.57	1.18	1.74	0.00	-0.00
time (sec)	N/A	0.697	0.215	180.000	0.357	0.356	2.020	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	227	0	619	327	483	0	-1
N.S.	1	1.00	0.75	0.00	2.04	1.08	1.59	0.00	-0.00
time (sec)	N/A	0.413	0.218	180.000	0.323	0.354	1.132	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	208	0	621	307	430	0	-1
N.S.	1	1.00	1.02	0.00	3.04	1.50	2.11	0.00	-0.00
time (sec)	N/A	0.138	0.182	180.000	0.331	0.355	0.810	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	191	0	457	278	389	0	-1
N.S.	1	1.00	0.89	0.00	2.14	1.30	1.82	0.00	-0.00
time (sec)	N/A	0.184	0.157	180.000	0.313	0.382	0.557	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	336	586	0	0	0	0	-1
N.S.	1	1.00	1.31	2.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.248	5.386	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	306	364	0	0	0	0	-1
N.S.	1	1.00	1.34	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	0.690	4.280	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	319	669	0	0	0	0	-1
N.S.	1	1.00	1.17	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	0.594	6.553	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	357	363	0	0	0	0	-1
N.S.	1	1.00	1.44	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.473	0.567	6.102	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	299	0	1109	444	702	0	-1
N.S.	1	1.00	0.64	0.00	2.38	0.95	1.51	0.00	-0.00
time (sec)	N/A	0.678	0.265	180.000	0.301	0.426	4.356	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	285	0	1112	424	654	0	-1
N.S.	1	1.00	0.76	0.00	2.96	1.13	1.74	0.00	-0.00
time (sec)	N/A	1.075	0.277	180.000	0.342	0.381	3.105	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	275	0	922	403	626	0	-1
N.S.	1	1.00	0.72	0.00	2.41	1.05	1.64	0.00	-0.00
time (sec)	N/A	0.593	0.275	180.000	0.345	0.435	2.166	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	256	0	925	383	573	0	-1
N.S.	1	1.00	0.98	0.00	3.54	1.47	2.20	0.00	-0.00
time (sec)	N/A	0.192	0.226	180.000	0.332	0.390	1.653	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	239	0	712	354	524	0	-1
N.S.	1	1.00	0.82	0.00	2.45	1.22	1.80	0.00	-0.00
time (sec)	N/A	0.259	0.173	180.000	0.306	0.401	1.140	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	429	706	0	0	0	0	-1
N.S.	1	1.00	1.27	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	0.459	4.635	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	466	463	0	0	0	0	-1
N.S.	1	1.00	1.52	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.466	0.858	4.655	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	512	788	0	0	0	0	-1
N.S.	1	1.00	1.45	2.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.578	0.387	8.818	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	461	468	0	0	0	0	-1
N.S.	1	1.00	1.41	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.645	0.678	6.470	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	365	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	0.846	0.102	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	292	347	0	0	0	0	-1
N.S.	1	1.00	1.47	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.316	3.805	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	293	0	0	0	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.402	0.104	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	281	203	0	0	0	0	-1
N.S.	1	1.00	2.68	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.184	2.089	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	274	0	0	0	0	0	-1
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.170	180.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	400	350	0	0	0	0	-1
N.S.	1	1.00	3.45	3.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.259	3.112	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	363	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.737	0.099	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	419	670	0	0	0	0	-1
N.S.	1	1.00	2.16	3.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.726	3.885	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	602	0	0	0	0	0	-1
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.458	7.292	0.103	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	482	0	0	0	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	1.385	0.102	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	320	454	0	0	0	0	-1
N.S.	1	1.00	1.50	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.608	7.227	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	385	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	1.095	0.109	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	145	206	0	185	0	0	-1
N.S.	1	1.00	1.71	2.42	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.122	3.421	0.000	0.378	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	403	0	0	0	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.178	1.021	0.106	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	428	724	0	0	0	0	-1
N.S.	1	1.00	2.22	3.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	1.296	3.898	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	549	0	0	0	0	0	-1
N.S.	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	6.825	0.102	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	594	747	0	0	0	0	-1
N.S.	1	1.00	2.35	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.634	4.388	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	764	0	0	0	0	0	-1
N.S.	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.667	7.941	0.106	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	552	0	0	0	0	0	-1
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	2.141	0.108	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	186	481	0	283	0	0	-1
N.S.	1	1.00	1.11	2.88	0.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.201	6.334	0.000	0.388	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	550	0	0	0	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	1.713	0.104	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	152	410	0	273	0	0	-1
N.S.	1	1.00	1.05	2.83	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.130	4.233	0.000	0.459	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	546	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	1.641	0.107	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	560	1129	0	0	0	0	-1
N.S.	1	1.00	2.04	4.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	2.621	5.570	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	716	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.527	7.091	0.102	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	688	1365	0	0	0	0	-1
N.S.	1	1.00	1.81	3.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	7.333	6.078	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	937	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.942	8.796	0.103	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	284	0	0	0	583	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	1.94	0.00	-0.00
time (sec)	N/A	0.247	0.619	180.000	0.000	0.000	36.578	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	202	0	0	0	405	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.93	0.00	-0.00
time (sec)	N/A	0.153	0.388	180.000	0.000	0.000	3.704	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	124	179	0	0	0	0	-1
N.S.	1	1.00	1.02	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.206	1.206	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	72	47	0	88	0	-1
N.S.	1	1.00	1.00	2.88	1.88	0.00	3.52	0.00	-0.04
time (sec)	N/A	0.037	0.020	0.807	0.291	0.000	1.619	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	153	306	0	0	0	0	-1
N.S.	1	1.00	1.47	2.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.241	2.394	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	293	1729	0	0	0	0	-1
N.S.	1	1.00	1.44	8.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.409	6.099	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	222	1162	302	316	0	0	-1
N.S.	1	1.00	0.62	3.25	0.84	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.354	0.169	2.328	0.355	0.391	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	207	618	0	0	0	0	-1
N.S.	1	1.00	0.71	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.890	2.701	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	166	657	183	249	0	0	-1
N.S.	1	1.00	0.92	3.65	1.02	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.153	0.981	0.294	0.422	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	200	480	0	0	0	0	-1
N.S.	1	1.00	1.09	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.740	1.673	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	352	823	0	0	0	0	-1
N.S.	1	1.00	1.04	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.791	2.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	232	625	0	0	0	0	-1
N.S.	1	1.00	1.11	2.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.765	2.480	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	446	870	0	0	0	0	-1
N.S.	1	1.00	1.25	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	3.391	3.905	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	240	2557	0	0	0	0	-1
N.S.	1	1.00	0.82	8.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.535	4.069	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	251	1766	346	402	0	0	-1
N.S.	1	1.00	0.52	3.66	0.72	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.533	0.250	2.203	0.344	0.475	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	508	1552	0	0	0	0	-1
N.S.	1	1.00	1.25	3.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.843	2.705	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	198	1149	230	332	0	0	-1
N.S.	1	1.00	0.74	4.30	0.86	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.216	0.949	0.328	0.367	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	329	959	0	0	0	0	-1
N.S.	1	1.00	1.12	3.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.178	1.362	1.542	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	520	1053	0	0	0	0	-1
N.S.	1	1.00	1.04	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	1.604	2.511	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	369	954	0	0	0	0	-1
N.S.	1	1.00	0.93	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	2.056	2.447	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	771	1131	0	0	0	0	-1
N.S.	1	1.00	1.43	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	7.264	3.686	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	458	2796	0	0	0	0	-1
N.S.	1	1.00	1.21	7.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	0.947	4.060	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	277	2014	390	525	0	0	-1
N.S.	1	1.00	0.44	3.22	0.62	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.850	0.291	2.221	0.304	0.403	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	619	2280	0	0	0	0	-1
N.S.	1	1.00	1.15	4.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.686	1.412	2.770	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	224	1773	274	446	0	0	-1
N.S.	1	1.00	0.61	4.84	0.75	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.251	0.947	0.286	0.381	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	499	1568	0	0	0	0	-1
N.S.	1	1.00	1.19	3.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	1.044	1.612	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	710	1321	0	0	0	0	-1
N.S.	1	1.00	1.12	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.582	2.940	1.926	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	550	1223	0	0	0	0	-1
N.S.	1	1.00	1.04	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	1.529	2.452	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	990	1404	0	0	0	0	-1
N.S.	1	1.00	1.44	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.630	7.348	3.793	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	616	3311	0	0	0	0	-1
N.S.	1	1.00	1.10	5.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.626	1.610	4.114	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	0	0	131	146	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.86	0.95	0.00	-0.01
time (sec)	N/A	0.201	0.053	180.000	0.000	0.380	0.858	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	79	0	101	98	121	0	-1
N.S.	1	1.00	0.65	0.00	0.83	0.80	0.99	0.00	-0.01
time (sec)	N/A	0.142	0.043	180.000	0.281	0.402	0.568	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	72	69	0	102	78	0	-1
N.S.	1	1.00	0.83	0.79	0.00	1.17	0.90	0.00	-0.01
time (sec)	N/A	0.106	0.032	2.468	0.000	0.349	0.334	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	48	70	49	74	-1
N.S.	1	1.00	0.92	1.23	0.92	1.35	0.94	1.42	-0.02
time (sec)	N/A	0.055	0.023	2.289	0.267	0.419	0.256	0.411	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.022	0.006	0.254	0.263	0.356	0.204	0.000	0.093

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	100	144	0	0	0	0	-1
N.S.	1	1.00	1.47	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.078	2.603	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	132	0	0	0	0	-1
N.S.	1	1.00	0.98	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.118	0.231	4.243	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	188	233	0	0	0	0	-1
N.S.	1	1.00	1.39	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.955	4.990	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	230	1227	353	319	0	0	-1
N.S.	1	1.00	0.60	3.20	0.92	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.231	3.994	0.285	0.376	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	268	992	0	0	0	0	-1
N.S.	1	1.00	0.83	3.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.553	4.651	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	176	706	243	254	0	0	-1
N.S.	1	1.00	0.66	2.66	0.92	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.170	3.434	0.287	0.390	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	198	506	0	0	0	0	-1
N.S.	1	1.00	0.97	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.594	4.060	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	127	296	125	179	0	0	-1
N.S.	1	1.00	0.92	2.14	0.91	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.146	1.306	0.282	0.399	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	62	120	47	0	0	0	-1
N.S.	1	1.00	1.32	2.55	1.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.057	0.566	0.291	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	266	564	0	0	0	0	-1
N.S.	1	1.00	1.19	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.574	2.570	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	168	526	0	0	0	0	-1
N.S.	1	1.00	1.01	3.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.296	2.129	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	455	901	0	0	0	0	-1
N.S.	1	1.00	1.26	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	3.849	3.912	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	278	2146	0	0	0	0	-1
N.S.	1	1.00	0.93	7.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	0.483	3.976	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	427	934	0	0	0	0	-1
N.S.	1	1.00	0.83	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.512	0.379	3.964	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	288	816	0	0	0	0	-1
N.S.	1	1.00	0.72	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	1.333	4.646	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	318	702	0	0	0	0	-1
N.S.	1	1.00	0.83	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.292	3.396	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	215	478	0	0	0	0	-1
N.S.	1	1.00	0.92	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.233	0.648	4.111	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	217	446	0	0	0	0	-1
N.S.	1	1.00	1.15	2.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.275	1.148	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	152	343	0	0	0	0	-1
N.S.	1	1.00	0.85	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.264	1.602	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	568	0	0	0	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	1.078	0.099	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	296	659	0	0	0	0	-1
N.S.	1	1.00	0.97	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.643	2.079	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	884	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	7.000	0.104	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	438	2608	0	0	0	0	-1
N.S.	1	1.00	0.97	5.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	0.588	3.952	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	333	1042	0	0	0	0	-1
N.S.	1	1.00	0.65	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.603	1.158	4.110	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	359	3705	0	0	0	0	-1
N.S.	1	1.00	0.90	9.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.487	0.831	4.772	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	301	704	0	0	0	0	-1
N.S.	1	1.00	0.98	2.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.721	3.612	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	280	3132	0	0	0	0	-1
N.S.	1	1.00	0.90	10.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.610	4.188	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	254	591	0	0	0	0	-1
N.S.	1	1.00	0.94	2.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.639	1.441	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	236	2728	0	0	0	0	-1
N.S.	1	1.00	0.81	9.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.859	1.817	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	547	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	2.728	0.100	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	408	3513	0	0	0	0	-1
N.S.	1	1.00	0.97	8.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	1.243	2.263	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	983	0	0	0	0	0	-1
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.790	7.339	0.106	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	417	4951	0	0	0	0	-1
N.S.	1	1.00	0.82	9.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.753	2.089	5.002	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	178	570	0	0	0	0	-1
N.S.	1	1.00	0.49	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	0.540	2.742	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	936	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.881	1.963	180.000	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	0.204	180.000	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.136	180.000	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.097	1.915	180.000	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.104	2.954	180.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.112	2.954	180.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	0.359	180.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	169	0	276	248	355	0	-1
N.S.	1	1.00	0.47	0.00	0.77	0.69	0.99	0.00	-0.00
time (sec)	N/A	0.517	0.114	180.000	0.277	0.398	1.479	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	137	0	210	204	262	0	-1
N.S.	1	1.00	0.52	0.00	0.79	0.77	0.99	0.00	-0.00
time (sec)	N/A	0.299	0.080	180.000	0.264	0.407	0.707	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	99	0	124	140	150	0	-1
N.S.	1	1.00	0.65	0.00	0.81	0.92	0.98	0.00	-0.01
time (sec)	N/A	0.150	0.067	180.000	0.283	0.417	0.314	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	454	0	0	0	0	0	-1
N.S.	1	1.00	2.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.170	180.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	568	0	0	0	0	0	-1
N.S.	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	1.598	2.048	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	654	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	4.663	2.509	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	177	802	0	0	0	0	-1
N.S.	1	1.00	0.35	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.402	0.502	2.372	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	136	484	0	0	0	0	-1
N.S.	1	1.00	0.39	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.203	2.267	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	86	231	0	0	0	0	-1
N.S.	1	1.00	0.42	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.104	2.459	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	14	0	0	0	-1
N.S.	1	1.00	1.00	0.98	0.35	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.027	1.051	0.278	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	133	262	0	0	0	0	-1
N.S.	1	1.00	0.61	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	0.181	2.585	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	195	550	0	0	0	0	-1
N.S.	1	1.00	0.54	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	0.372	2.873	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	297	888	0	0	0	0	-1
N.S.	1	1.00	0.58	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.453	2.852	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	0.347	180.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	121	0	0	166	185	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.89	0.99	0.00	-0.01
time (sec)	N/A	0.325	0.057	180.000	0.000	0.406	0.911	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	0	127	128	148	0	-1
N.S.	1	1.00	0.64	0.00	0.83	0.84	0.97	0.00	-0.01
time (sec)	N/A	0.222	0.050	180.000	0.263	0.385	0.603	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	83	84	0	128	100	0	-1
N.S.	1	1.00	0.79	0.80	0.00	1.22	0.95	0.00	-0.01
time (sec)	N/A	0.144	0.046	2.591	0.000	0.409	0.468	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	90	61	92	61	101	-1
N.S.	1	1.00	0.91	1.41	0.95	1.44	0.95	1.58	-0.02
time (sec)	N/A	0.088	0.023	2.458	0.258	0.363	0.335	0.432	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.022	0.006	0.288	0.268	0.372	0.249	0.000	0.099

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	146	197	0	0	0	0	-1
N.S.	1	1.00	1.43	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.094	2.767	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	97	187	0	0	0	0	-1
N.S.	1	1.00	1.10	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.150	4.523	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	304	377	0	0	0	0	-1
N.S.	1	1.00	1.45	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	2.950	5.102	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	42	0	0	0	0	-1
N.S.	1	1.00	0.64	0.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.067	2.279	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	33	0	0	0	0	-1
N.S.	1	1.00	0.68	0.66	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.048	1.782	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	22	0	0	0	0	-1
N.S.	1	1.00	0.79	0.76	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.019	1.638	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.252	180.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	1.121	180.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	152	199	0	0	0	0	-1
N.S.	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.265	9.334	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	135	178	0	0	0	0	-1
N.S.	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	0.215	6.246	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	79	0	0	0	0	-1
N.S.	1	1.00	0.79	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.132	4.717	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	91	118	0	0	0	0	-1
N.S.	1	1.00	0.75	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.153	4.187	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	63	79	0	0	0	0	-1
N.S.	1	1.00	0.77	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.099	4.983	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	1.375	180.000	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.199	0.837	180.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	3.198	180.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	0.589	180.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	179	238	0	0	0	0	-1
N.S.	1	1.00	0.73	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.480	7.875	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	152	199	0	0	0	0	-1
N.S.	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.350	8.975	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	136	178	0	0	0	0	-1
N.S.	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.324	6.117	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	109	139	0	0	0	0	-1
N.S.	1	1.00	0.76	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.190	9.432	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.495	1.399	180.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	1.025	180.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.090	3.250	180.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	0.598	180.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	180	238	0	0	0	0	-1
N.S.	1	1.00	0.73	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.716	6.161	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	197	259	0	0	0	0	-1
N.S.	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.630	12.273	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	180	238	0	0	0	0	-1
N.S.	1	1.00	0.73	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.589	7.036	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	153	199	0	0	0	0	-1
N.S.	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.410	12.635	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.777	1.413	180.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.589	0.966	180.000	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	3.239	180.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.093	0.659	180.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	30	0	0	0	0	-1
N.S.	1	1.00	0.76	0.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.102	0.057	2.951	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	23	0	0	0	0	-1
N.S.	1	1.00	0.81	0.85	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.101	0.062	2.793	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	21	0	0	0	0	-1
N.S.	1	1.00	0.81	0.78	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.091	0.057	2.565	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	21	0	0	0	0	-1
N.S.	1	1.00	0.81	0.78	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.093	0.021	1.618	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	-1
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.055	0.055	2.368	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	21	7	0	9
N.S.	1	1.00	1.00	1.11	1.00	2.33	0.78	0.00	1.00
time (sec)	N/A	0.024	0.016	0.284	0.268	0.343	0.245	0.000	0.104

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	0.760	180.000	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	0.105	180.000	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	136	178	0	0	0	0	-1
N.S.	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.250	5.888	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	109	139	0	0	0	0	-1
N.S.	1	1.00	0.76	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.163	10.382	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	92	118	0	0	0	0	-1
N.S.	1	1.00	0.76	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.159	6.100	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	79	0	0	0	0	-1
N.S.	1	1.00	0.79	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.145	5.443	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	58	0	0	0	0	-1
N.S.	1	1.00	0.85	1.07	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.120	0.089	2.803	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	28	26	0	16
N.S.	1	1.00	1.00	1.06	1.00	1.75	1.62	0.00	1.00
time (sec)	N/A	0.036	0.026	0.278	0.273	0.336	0.824	0.000	0.137

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	1.245	180.000	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.099	0.859	180.000	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.118	1.511	180.000	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	0.780	180.000	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.052	180.000	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.099	1.082	180.000	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.094	0.821	180.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.102	0.661	180.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.093	0.373	180.000	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	0.088	180.000	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	0.254	180.000	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	0.445	180.000	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	82	106	0	0	0	0	-1
N.S.	1	1.00	0.87	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.250	3.356	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	69	84	0	0	0	0	-1
N.S.	1	1.00	0.90	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.173	3.426	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	60	0	0	0	0	-1
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.104	2.394	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	1.062	180.000	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.079	2.583	180.000	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	175	633	0	0	0	0	-1
N.S.	1	1.00	0.82	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	0.481	6.760	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	248	0	0	0	0	-1
N.S.	1	1.00	0.88	2.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.319	0.232	5.500	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	126	364	0	0	0	0	-1
N.S.	1	1.00	0.85	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.269	5.336	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	192	0	0	0	0	-1
N.S.	1	1.00	0.86	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.126	5.487	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	7.358	180.000	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.096	1.943	180.000	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	12.815	180.000	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	2.490	180.000	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	399	958	0	0	0	0	-1
N.S.	1	1.00	1.44	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.589	0.582	7.755	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	306	704	0	0	0	0	-1
N.S.	1	1.00	1.40	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	0.532	9.245	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	295	633	0	0	0	0	-1
N.S.	1	1.00	1.38	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	0.448	6.464	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	122	420	0	0	0	0	-1
N.S.	1	1.00	0.82	2.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.354	8.665	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	5.218	180.000	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	2.952	180.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.093	9.748	180.000	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	1.878	180.000	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	408	1070	0	0	0	0	-1
N.S.	1	1.00	1.47	3.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.760	0.963	7.882	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	413	1044	0	0	0	0	-1
N.S.	1	1.00	1.47	3.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.620	0.828	12.356	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	404	958	0	0	0	0	-1
N.S.	1	1.00	1.47	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	0.732	7.583	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	311	704	0	0	0	0	-1
N.S.	1	1.00	1.44	3.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.521	11.487	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	6.373	180.000	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	2.320	180.000	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.090	9.754	180.000	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	2.057	180.000	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	158	633	0	0	0	0	-1
N.S.	1	1.00	0.77	3.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	0.250	7.224	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	117	420	0	0	0	0	-1
N.S.	1	1.00	0.83	2.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.207	10.138	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	113	364	0	0	0	0	-1
N.S.	1	1.00	0.80	2.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.195	6.974	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	70	192	0	0	0	0	-1
N.S.	1	1.00	0.89	2.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.274	5.739	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	151	0	0	0	0	-1
N.S.	1	1.00	0.82	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.096	3.511	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	30	36	0	18
N.S.	1	1.00	1.00	1.06	1.00	1.67	2.00	0.00	1.00
time (sec)	N/A	0.032	0.011	0.277	0.262	0.382	1.788	0.000	0.139

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	3.673	180.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.876	180.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	5.369	180.000	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	2.447	180.000	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	3.556	180.000	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	1.672	180.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	5.245	180.000	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	13.330	180.000	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	10.276	180.000	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.096	3.950	180.000	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.063	7.392	180.000	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	1.936	180.000	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.087	8.932	180.000	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	7.815	180.000	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	0.702	180.000	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	0.404	180.000	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.077	0.093	180.000	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.101	0.257	180.000	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	0.464	180.000	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	0.626	180.000	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	12	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.92	0.00	0.85
time (sec)	N/A	0.022	0.008	0.278	0.264	0.359	0.401	0.000	0.094

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	271	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.831	1.410	180.000	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	436	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.863	0.986	180.000	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	321	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	2.164	180.000	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	295	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.302	0.534	180.000	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.480	2.739	180.000	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	490	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.992	2.210	180.000	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	577	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.181	1.755	180.000	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	475	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.837	3.325	180.000	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	440	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	1.156	180.000	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	375	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.869	1.996	180.000	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	142	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.131	180.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	104	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.068	180.000	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0	-1
N.S.	1	1.00	1.00	0.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.027	1.260	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.423	5.069	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.951	5.049	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	186	0	0	0	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	0.218	180.000	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	126	0	0	0	0	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.187	180.000	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0	-1
N.S.	1	1.00	1.00	0.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.031	1.744	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.460	5.125	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	201	0	0	0	0	0	-1
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	0.237	180.000	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	135	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.202	180.000	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0	-1
N.S.	1	1.00	1.00	0.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.026	1.225	0.000	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.440	4.972	0.000	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	156	0	0	0	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.122	180.000	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	110	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.060	180.000	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	20	0	0	-1
N.S.	1	1.00	1.00	0.87	0.00	0.51	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.019	1.930	0.000	0.361	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.240	5.414	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.622	7.059	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	210	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.205	180.000	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	133	0	0	0	0	0	-1
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.170	180.000	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	20	0	0	-1
N.S.	1	1.00	1.00	0.87	0.00	0.51	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.021	1.841	0.000	0.362	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.264	4.874	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	34	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.063	0.047	180.000	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	197	0	0	0	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.214	180.000	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	141	0	0	0	0	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.129	0.127	180.000	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	101	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.069	180.000	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	0	0	0	-1
N.S.	1	1.00	1.00	0.90	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.028	1.271	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.510	5.108	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	1.084	5.242	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	399	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.711	180.000	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	225	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.252	180.000	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	115	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.126	180.000	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	57	0	0	-1
N.S.	1	1.00	1.00	0.90	0.00	1.42	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.029	1.232	0.000	0.337	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.514	4.888	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	1.022	5.077	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	262	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.279	180.000	0.000	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	122	0	0	0	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.107	180.000	0.000	0.000	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	57	0	0	-1
N.S.	1	1.00	1.00	0.86	0.00	1.36	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.028	1.724	0.000	0.363	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.515	4.944	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	1.020	5.207	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	170	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.684	180.000	0.000	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	229	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.685	180.000	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	160	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.427	180.000	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.365	0.159	180.000	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	0.160	180.000	0.000	0.000	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	616	616	429	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.462	2.162	180.000	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	390	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.355	1.236	180.000	0.000	0.000	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	287	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	1.004	180.000	0.000	0.000	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.685	0.181	180.000	0.000	0.000	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.498	0.455	180.000	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	816	816	667	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	5.188	180.000	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	685	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	2.000	180.000	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	529	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	3.734	180.000	0.000	0.000	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	756	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.088	0.201	180.000	0.000	0.000	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	455	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	0.454	180.000	0.000	0.000	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	0.344	180.000	0.000	0.000	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.143	180.000	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.147	180.000	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.056	180.000	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	83	34	29	33
N.S.	1	1.00	1.00	1.06	1.00	4.88	2.00	1.71	1.94
time (sec)	N/A	0.027	0.008	0.295	0.283	0.376	0.327	0.400	0.285

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.066	3.964	180.000	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	1.307	180.000	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	361	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	0.892	180.000	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	273	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.999	180.000	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	233	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.313	180.000	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	227	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.297	180.000	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	283	0	0	0	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.938	180.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	141	0	219	548	0	0	-1
N.S.	1	1.00	0.75	0.00	1.17	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.271	180.000	0.279	0.455	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	683	0	0	0	0	0	-1
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	1.076	180.000	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	352	0	0	0	0	0	-1
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.533	180.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	273	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.975	180.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	344	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.603	180.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	514	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	1.475	180.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	706	0	0	0	0	0	-1
N.S.	1	1.00	1.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.270	3.827	180.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	481	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.750	180.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	683	0	0	0	0	0	-1
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	1.029	180.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	565	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.730	180.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	465	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.426	0.994	180.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	779	0	0	0	0	0	-1
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	2.517	180.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	1005	0	0	0	0	0	-1
N.S.	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	4.914	180.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	465	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	1.006	180.000	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	344	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.601	180.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	227	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.302	180.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	113	0	32	0	0	0	-1
N.S.	1	1.00	1.92	0.00	0.54	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.256	180.000	0.279	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	98	443	0	0	-1
N.S.	1	1.00	1.02	0.00	0.88	3.99	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.221	180.000	0.495	0.455	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	143	0	233	576	0	0	-1
N.S.	1	1.00	0.48	0.00	0.79	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.293	180.000	0.493	0.488	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	781	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	2.581	180.000	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	515	0	0	0	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	1.550	180.000	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	285	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	0.781	180.000	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	94	0	98	443	0	0	-1
N.S.	1	1.00	0.84	0.00	0.88	3.96	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.234	180.000	0.492	0.435	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	118	0	82	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.80	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.267	180.000	0.267	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	201	0	237	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.375	180.000	0.287	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	1083	0	0	0	0	0	-1
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	6.142	180.000	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	706	0	0	0	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	3.838	180.000	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	131	0	220	548	0	0	-1
N.S.	1	1.00	0.71	0.00	1.19	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.263	180.000	0.289	0.456	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	139	0	232	576	0	0	-1
N.S.	1	1.00	0.47	0.00	0.79	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.250	180.000	0.496	0.459	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	202	0	237	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.362	180.000	0.297	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	193	0	159	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.78	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.338	180.000	0.277	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	890	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.700	1.539	180.000	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	705	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	1.158	180.000	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	352	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.647	180.000	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	315	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.604	180.000	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	594	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.654	2.685	180.000	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	783	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.758	7.296	180.000	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	1084	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.538	2.110	180.000	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	524	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	1.106	180.000	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	705	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	1.135	180.000	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	532	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	1.349	180.000	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	752	752	1174	0	0	0	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	5.865	180.000	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	1609	0	0	0	0	0	-1
N.S.	1	1.00	2.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.804	8.679	180.000	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	735	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.386	1.462	180.000	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	1084	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	2.037	180.000	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	890	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	1.475	180.000	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	723	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	2.112	180.000	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	972	972	2492	0	0	0	0	0	-1
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.895	9.616	180.000	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	790	790	2622	0	0	0	0	0	-1
N.S.	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.940	10.764	180.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	723	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	2.013	180.000	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	529	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	1.240	180.000	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	315	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	0.582	180.000	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	168	0	53	0	0	0	-1
N.S.	1	1.00	2.85	0.00	0.90	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.176	0.457	180.000	0.289	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	508	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	1.425	180.000	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	942	942	524	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.872	4.616	180.000	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	972	972	2143	0	0	0	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.901	11.568	180.000	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	752	752	1084	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.784	8.853	180.000	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	530	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.641	3.582	180.000	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	511	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	1.455	180.000	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	488	0	0	0	0	0	-1
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.824	180.000	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	743	743	754	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	7.512	180.000	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	794	794	2552	0	0	0	0	0	-1
N.S.	1	1.00	3.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.926	11.049	180.000	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	584	584	1617	0	0	0	0	0	-1
N.S.	1	1.00	2.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	8.613	180.000	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	788	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.765	7.364	180.000	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	942	942	528	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	4.670	180.000	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	743	743	757	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.603	7.548	180.000	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	642	0	0	0	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	5.222	180.000	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	260	451	411	1039	593	0	-1
N.S.	1	1.00	0.83	1.45	1.32	3.33	1.90	0.00	-0.00
time (sec)	N/A	0.240	0.190	0.642	0.287	0.382	1.995	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	187	316	285	611	389	0	-1
N.S.	1	1.00	0.85	1.43	1.29	2.76	1.76	0.00	-0.00
time (sec)	N/A	0.184	0.175	0.647	0.268	0.351	0.756	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	204	180	321	240	0	-1
N.S.	1	1.00	0.85	1.39	1.22	2.18	1.63	0.00	-0.01
time (sec)	N/A	0.101	0.099	0.645	0.265	0.398	0.356	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	71	109	93	134	109	0	-1
N.S.	1	1.00	0.88	1.35	1.15	1.65	1.35	0.00	-0.01
time (sec)	N/A	0.050	0.052	0.598	0.258	0.351	0.150	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	30	43	26	41	28
N.S.	1	1.00	1.00	1.03	1.00	1.43	0.87	1.37	0.93
time (sec)	N/A	0.010	0.007	0.579	0.261	0.379	0.056	0.394	0.244

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	434	233	0	0	0	0	-1
N.S.	1	1.00	0.89	0.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.587	0.310	11.789	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	707	707	622	1766	0	0	0	0	-1
N.S.	1	1.00	0.88	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.780	1.215	22.507	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	559	559	443	752	680	1528	989	0	-1
N.S.	1	1.00	0.79	1.35	1.22	2.73	1.77	0.00	-0.00
time (sec)	N/A	0.629	0.303	2.242	0.292	0.389	1.222	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	289	438	429	767	595	0	-1
N.S.	1	1.00	0.88	1.33	1.30	2.33	1.81	0.00	-0.00
time (sec)	N/A	0.409	0.235	1.752	0.274	0.409	0.579	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	164	206	222	304	279	0	-1
N.S.	1	1.00	1.07	1.35	1.45	1.99	1.82	0.00	-0.01
time (sec)	N/A	0.204	0.131	0.911	0.272	0.392	0.264	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	72	72	96	82	111	-1
N.S.	1	1.00	1.61	1.57	1.57	2.09	1.78	2.41	-0.02
time (sec)	N/A	0.045	0.043	1.838	0.265	0.373	0.090	0.462	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	985	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.977	0.536	180.000	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	670	670	444	654	0	0	0	0	-1
N.S.	1	1.00	0.66	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.911	0.643	11.072	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	253	380	0	0	0	0	-1
N.S.	1	1.00	0.65	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	0.360	8.881	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	126	178	0	0	0	0	-1
N.S.	1	1.00	0.70	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.161	6.938	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	0	56	0	0	0	0	-1
N.S.	1	1.00	0.00	1.04	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.018	3.041	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.511	180.000	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	2.278	180.000	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	356	1036	0	0	0	0	-1
N.S.	1	1.00	0.72	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	1.353	9.452	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	190	438	0	0	0	0	-1
N.S.	1	1.00	0.77	1.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.593	7.012	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	118	0	0	0	0	-1
N.S.	1	1.00	0.84	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.110	3.171	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	7.661	180.000	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	19.995	180.000	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	672	672	535	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.195	4.168	180.000	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	319	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.606	1.974	180.000	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.092	0.116	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	5.135	180.000	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	10.911	180.000	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	770	0	0	0	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.811	2.910	180.000	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	251	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.374	0.111	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	1.910	180.000	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	5.876	180.000	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	530	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.803	0.761	180.000	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	218	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.429	180.000	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	101	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.081	0.120	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	0.105	180.000	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.054	180.000	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	303	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	0.937	180.000	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.171	0.121	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.107	180.000	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	0.060	180.000	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	3.076	180.000	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	2.717	180.000	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	75	0	0	329	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	4.70	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.078	180.000	0.000	0.363	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	139	0	0	970	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	6.64	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.223	180.000	0.000	0.429	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	191	0	0	2521	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	11.11	0.00	0.00	-0.00
time (sec)	N/A	0.551	0.274	180.000	0.000	0.477	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	8.310	180.000	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	7.630	180.000	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	2.127	180.000	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	4.397	180.000	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	0.726	180.000	0.000	0.000	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	0.738	180.000	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	1.034	180.000	0.000	0.000	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	2.519	180.000	0.000	0.000	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	2.941	180.000	0.000	0.000	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	4.914	180.000	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	12.501	180.000	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	20.573	180.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [570] had the largest ratio of [37]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	22	0.227
2	A	6	6	1.00	22	0.273
3	A	5	5	1.00	22	0.227
4	A	4	3	1.00	20	0.150
5	A	5	4	1.00	19	0.210
6	A	8	8	1.00	22	0.364
7	A	6	7	1.00	22	0.318
8	A	8	8	1.00	22	0.364
9	A	6	7	1.00	22	0.318
10	A	6	6	1.00	24	0.250
11	A	7	8	1.00	24	0.333
12	A	5	5	1.00	24	0.208
13	A	5	3	1.00	22	0.136
14	A	5	5	1.00	21	0.238
15	A	12	8	1.00	24	0.333
16	A	7	7	1.00	24	0.292
17	A	12	10	1.00	24	0.417
18	A	7	8	1.00	24	0.333
19	A	5	5	1.00	24	0.208
20	A	8	7	1.00	24	0.292
21	A	5	5	1.00	24	0.208
22	A	6	3	1.00	22	0.136
23	A	5	5	1.00	21	0.238
24	A	17	8	1.00	24	0.333
25	A	7	7	1.00	24	0.292

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	17	10	1.00	24	0.417
27	A	8	8	1.00	24	0.333
28	A	12	8	1.00	24	0.333
29	A	8	8	1.00	24	0.333
30	A	8	6	1.00	24	0.250
31	A	5	5	1.00	22	0.227
32	A	6	4	1.00	21	0.190
33	A	7	5	1.00	24	0.208
34	A	10	8	1.00	24	0.333
35	A	9	7	1.00	24	0.292
36	A	15	9	1.00	24	0.375
37	A	12	9	1.00	24	0.375
38	A	8	8	1.00	24	0.333
39	A	8	6	1.00	24	0.250
40	A	2	2	1.00	22	0.091
41	A	8	6	1.00	21	0.286
42	A	9	7	1.00	24	0.292
43	A	13	11	1.00	24	0.458
44	A	12	9	1.00	24	0.375
45	A	19	12	1.00	24	0.500
46	A	12	8	1.00	24	0.333
47	A	4	3	1.00	24	0.125
48	A	10	7	1.00	24	0.292
49	A	3	3	1.00	22	0.136
50	A	10	6	1.00	21	0.286
51	A	12	8	1.00	24	0.333
52	A	16	11	1.00	24	0.458
53	A	16	10	1.00	24	0.417
54	A	23	12	1.00	24	0.500
55	A	3	4	1.00	26	0.154
56	A	5	4	1.00	26	0.154
57	A	2	1	1.00	24	0.042
58	A	3	3	1.00	23	0.130
59	A	8	6	1.00	26	0.231
60	A	3	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	6	1.00	26	0.231
62	A	3	2	1.00	26	0.077
63	A	4	5	1.00	26	0.192
64	A	8	6	1.00	26	0.231
65	A	3	2	1.00	24	0.083
66	A	6	5	1.00	23	0.217
67	A	10	7	1.00	26	0.269
68	A	6	5	1.00	26	0.192
69	A	11	8	1.00	26	0.308
70	A	6	5	1.00	26	0.192
71	A	4	5	1.00	26	0.192
72	A	12	8	1.00	26	0.308
73	A	3	2	1.00	24	0.083
74	A	8	6	1.00	23	0.261
75	A	13	8	1.00	26	0.308
76	A	10	8	1.00	26	0.308
77	A	13	9	1.00	26	0.346
78	A	10	7	1.00	26	0.269
79	A	3	3	1.00	12	0.250
80	A	6	4	1.00	26	0.154
81	A	5	3	1.00	26	0.115
82	A	4	4	1.00	26	0.154
83	A	3	3	1.00	26	0.115
84	A	2	2	1.00	24	0.083
85	A	1	1	1.00	23	0.043
86	A	6	4	1.00	26	0.154
87	A	2	2	1.00	26	0.077
88	A	8	6	1.00	26	0.231
89	A	4	4	1.00	26	0.154
90	A	5	6	1.00	26	0.231
91	A	7	6	1.00	26	0.231
92	A	4	6	1.00	26	0.231
93	A	3	3	1.00	26	0.115
94	A	2	2	1.00	24	0.083
95	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	8	6	1.00	26	0.231
97	A	5	6	1.00	26	0.231
98	A	11	8	1.00	26	0.308
99	A	5	6	1.00	26	0.231
100	A	11	6	1.00	26	0.231
101	A	5	7	1.00	26	0.269
102	A	7	5	1.00	26	0.192
103	A	4	6	1.00	26	0.231
104	A	4	3	1.00	26	0.115
105	A	3	3	1.00	24	0.125
106	A	4	4	1.00	23	0.174
107	A	11	7	1.00	26	0.269
108	A	5	7	1.00	26	0.269
109	A	15	10	1.00	26	0.385
110	A	5	7	1.00	26	0.269
111	A	6	4	1.00	19	0.210
112	A	5	3	1.00	21	0.143
113	A	4	4	1.00	21	0.190
114	A	3	3	1.00	21	0.143
115	A	2	2	1.00	19	0.105
116	A	1	1	1.00	18	0.056
117	A	6	4	1.00	21	0.190
118	A	2	2	1.00	21	0.095
119	A	8	6	1.00	21	0.286
120	A	3	4	1.00	26	0.154
121	A	5	4	1.00	26	0.154
122	A	2	1	1.00	24	0.042
123	A	3	3	1.00	23	0.130
124	A	8	6	1.00	26	0.231
125	A	3	3	1.00	26	0.115
126	A	8	6	1.00	26	0.231
127	A	3	2	1.00	26	0.077
128	A	4	5	1.00	26	0.192
129	A	8	6	1.00	26	0.231
130	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	6	5	1.00	23	0.217
132	A	10	7	1.00	26	0.269
133	A	6	5	1.00	26	0.192
134	A	11	8	1.00	26	0.308
135	A	6	5	1.00	26	0.192
136	A	4	5	1.00	26	0.192
137	A	12	8	1.00	26	0.308
138	A	3	2	1.00	24	0.083
139	A	8	6	1.00	23	0.261
140	A	13	8	1.00	26	0.308
141	A	10	8	1.00	26	0.308
142	A	13	9	1.00	26	0.346
143	A	10	7	1.00	26	0.269
144	A	3	3	1.00	12	0.250
145	A	6	4	1.00	26	0.154
146	A	5	3	1.00	26	0.115
147	A	4	4	1.00	26	0.154
148	A	3	3	1.00	26	0.115
149	A	2	2	1.00	24	0.083
150	A	1	1	1.00	23	0.043
151	A	6	4	1.00	26	0.154
152	A	2	2	1.00	26	0.077
153	A	8	6	1.00	26	0.231
154	A	4	4	1.00	26	0.154
155	A	5	6	1.00	26	0.231
156	A	7	6	1.00	26	0.231
157	A	4	6	1.00	26	0.231
158	A	3	3	1.00	26	0.115
159	A	2	2	1.00	24	0.083
160	A	2	2	1.00	23	0.087
161	A	8	6	1.00	26	0.231
162	A	5	6	1.00	26	0.231
163	A	11	8	1.00	26	0.308
164	A	5	6	1.00	26	0.231
165	A	11	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	5	7	1.00	26	0.269
167	A	7	5	1.00	26	0.192
168	A	4	6	1.00	26	0.231
169	A	4	3	1.00	26	0.115
170	A	3	3	1.00	24	0.125
171	A	4	4	1.00	23	0.174
172	A	11	7	1.00	26	0.269
173	A	5	7	1.00	26	0.269
174	A	15	10	1.00	26	0.385
175	A	5	7	1.00	26	0.269
176	A	6	4	1.00	19	0.210
177	A	5	3	1.00	21	0.143
178	A	4	4	1.00	21	0.190
179	A	3	3	1.00	21	0.143
180	A	2	2	1.00	19	0.105
181	A	1	1	1.00	18	0.056
182	A	6	4	1.00	21	0.190
183	A	2	2	1.00	21	0.095
184	A	8	6	1.00	21	0.286
185	A	6	7	1.00	24	0.292
186	A	5	6	1.00	24	0.250
187	A	4	5	1.00	22	0.227
188	A	0	0	0.00	0	0.000
189	A	0	0	0.00	0	0.000
190	A	0	0	0.00	0	0.000
191	A	9	6	1.00	26	0.231
192	A	6	5	1.00	26	0.192
193	A	3	3	1.00	26	0.115
194	A	1	1	1.00	26	0.038
195	A	3	3	1.00	26	0.115
196	A	5	3	1.00	26	0.115
197	A	1	1	1.00	21	0.048
198	A	11	10	1.00	24	0.417
199	A	14	6	1.00	24	0.250
200	A	9	10	1.00	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	7	6	1.00	22	0.273
202	A	6	4	1.00	21	0.190
203	A	10	10	1.00	24	0.417
204	A	12	9	1.00	24	0.375
205	A	10	10	1.00	24	0.417
206	A	16	8	1.00	24	0.333
207	A	16	11	1.00	26	0.423
208	A	25	7	1.00	26	0.269
209	A	14	11	1.00	26	0.423
210	A	9	7	1.00	24	0.292
211	A	10	5	1.00	23	0.217
212	A	17	12	1.00	26	0.462
213	A	17	11	1.00	26	0.423
214	A	17	12	1.00	26	0.462
215	A	24	10	1.00	26	0.385
216	A	21	11	1.00	26	0.423
217	A	40	9	1.00	26	0.346
218	A	19	11	1.00	26	0.423
219	A	11	7	1.00	24	0.292
220	A	14	5	1.00	23	0.217
221	A	26	13	1.00	26	0.500
222	A	24	12	1.00	26	0.462
223	A	28	15	1.00	26	0.577
224	A	31	12	1.00	26	0.462
225	A	16	9	1.00	26	0.346
226	A	10	9	1.00	26	0.346
227	A	11	8	1.00	26	0.308
228	A	6	6	1.00	24	0.250
229	A	8	5	1.00	23	0.217
230	A	9	6	1.00	26	0.231
231	A	15	10	1.00	26	0.385
232	A	12	9	1.00	26	0.346
233	A	24	11	1.00	26	0.423
234	A	15	14	1.00	26	0.538
235	A	10	9	1.00	26	0.346

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	11	8	1.00	26	0.308
237	A	3	3	1.00	24	0.125
238	A	11	8	1.00	23	0.348
239	A	12	9	1.00	26	0.346
240	A	20	14	1.00	26	0.538
241	A	17	15	1.00	26	0.577
242	A	32	15	1.00	26	0.577
243	A	16	13	1.00	26	0.500
244	A	8	6	1.00	26	0.231
245	A	15	10	1.00	26	0.385
246	A	5	5	1.00	24	0.208
247	A	15	9	1.00	23	0.391
248	A	17	11	1.00	26	0.423
249	A	27	15	1.00	26	0.577
250	A	23	19	1.00	26	0.731
251	A	43	17	1.00	26	0.654
252	A	16	8	1.00	25	0.320
253	A	10	8	1.00	25	0.320
254	A	5	5	1.00	25	0.200
255	A	1	1	1.00	25	0.040
256	A	6	6	1.00	25	0.240
257	A	9	9	1.00	25	0.360
258	A	14	8	1.00	28	0.286
259	A	10	6	1.00	28	0.214
260	A	5	4	1.00	26	0.154
261	A	5	5	1.00	25	0.200
262	A	12	8	1.00	28	0.286
263	A	7	7	1.00	28	0.250
264	A	13	10	1.00	28	0.357
265	A	9	9	1.00	28	0.321
266	A	20	14	1.00	28	0.500
267	A	17	11	1.00	28	0.393
268	A	6	6	1.00	26	0.231
269	A	10	8	1.00	25	0.320
270	A	17	12	1.00	28	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	14	13	1.00	28	0.464
272	A	18	15	1.00	28	0.536
273	A	16	11	1.00	28	0.393
274	A	27	18	1.00	28	0.643
275	A	25	14	1.00	28	0.500
276	A	6	6	1.00	26	0.231
277	A	16	8	1.00	25	0.320
278	A	23	16	1.00	28	0.571
279	A	23	15	1.00	28	0.536
280	A	25	20	1.00	28	0.714
281	A	27	15	1.00	28	0.536
282	A	10	5	1.00	23	0.217
283	A	8	7	1.00	23	0.304
284	A	5	5	1.00	23	0.217
285	A	3	3	1.00	21	0.143
286	A	1	1	1.00	20	0.050
287	A	8	5	1.00	23	0.217
288	A	6	6	1.00	23	0.261
289	A	13	10	1.00	23	0.435
290	A	14	7	1.00	28	0.250
291	A	10	5	1.00	28	0.179
292	A	9	7	1.00	28	0.250
293	A	5	5	1.00	28	0.179
294	A	4	3	1.00	26	0.115
295	A	1	1	1.00	25	0.040
296	A	8	5	1.00	28	0.179
297	A	6	6	1.00	28	0.214
298	A	13	10	1.00	28	0.357
299	A	9	9	1.00	28	0.321
300	A	22	12	1.00	28	0.429
301	A	14	11	1.00	28	0.393
302	A	13	9	1.00	28	0.321
303	A	7	7	1.00	28	0.250
304	A	7	5	1.00	26	0.192
305	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	15	10	1.00	28	0.357
307	A	14	10	1.00	28	0.357
308	A	26	14	1.00	28	0.500
309	A	24	11	1.00	28	0.393
310	A	26	11	1.00	28	0.393
311	A	16	9	1.00	28	0.321
312	A	16	7	1.00	28	0.250
313	A	9	9	1.00	28	0.321
314	A	9	7	1.00	26	0.269
315	A	9	9	1.00	25	0.360
316	A	24	12	1.00	28	0.429
317	A	19	14	1.00	28	0.500
318	A	38	17	1.00	28	0.607
319	A	32	15	1.00	28	0.536
320	A	13	10	1.00	21	0.476
321	A	0	0	0.00	0	0.000
322	A	0	0	0.00	0	0.000
323	A	0	0	0.00	0	0.000
324	A	0	0	0.00	0	0.000
325	A	0	0	0.00	0	0.000
326	A	0	0	0.00	0	0.000
327	A	0	0	0.00	0	0.000
328	A	24	13	1.00	19	0.684
329	A	17	11	1.00	19	0.579
330	A	10	7	1.00	17	0.412
331	A	10	6	1.00	19	0.316
332	A	18	10	1.00	19	0.526
333	A	28	11	1.00	19	0.579
334	A	24	9	1.00	21	0.429
335	A	14	8	1.00	21	0.381
336	A	6	5	1.00	21	0.238
337	A	1	1	1.00	21	0.048
338	A	7	7	1.00	21	0.333
339	A	11	10	1.00	21	0.476
340	A	17	11	1.00	21	0.524

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	0	0	0.00	0	0.000
342	A	13	4	1.00	23	0.174
343	A	10	6	1.00	23	0.261
344	A	6	4	1.00	23	0.174
345	A	4	3	1.00	21	0.143
346	A	1	1	1.00	20	0.050
347	A	10	6	1.00	23	0.261
348	A	7	7	1.00	23	0.304
349	A	18	10	1.00	23	0.435
350	A	7	3	1.00	19	0.158
351	A	6	3	1.00	19	0.158
352	A	5	3	1.00	17	0.176
353	A	0	0	0.00	0	0.000
354	A	0	0	0.00	0	0.000
355	A	12	5	1.00	27	0.185
356	A	12	5	1.00	27	0.185
357	A	6	5	1.00	27	0.185
358	A	9	5	1.00	25	0.200
359	A	6	5	1.00	24	0.208
360	A	0	0	0.00	0	0.000
361	A	0	0	0.00	0	0.000
362	A	0	0	0.00	0	0.000
363	A	0	0	0.00	0	0.000
364	A	15	5	1.00	27	0.185
365	A	12	5	1.00	27	0.185
366	A	12	5	1.00	25	0.200
367	A	9	5	1.00	24	0.208
368	A	0	0	0.00	0	0.000
369	A	0	0	0.00	0	0.000
370	A	0	0	0.00	0	0.000
371	A	0	0	0.00	0	0.000
372	A	15	5	1.00	27	0.185
373	A	15	5	1.00	27	0.185
374	A	15	5	1.00	25	0.200
375	A	12	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	0	0	0.00	0	0.000
377	A	0	0	0.00	0	0.000
378	A	0	0	0.00	0	0.000
379	A	0	0	0.00	0	0.000
380	A	5	3	1.00	23	0.130
381	A	5	3	1.00	23	0.130
382	A	4	3	1.00	23	0.130
383	A	4	3	1.00	23	0.130
384	A	2	2	1.00	21	0.095
385	A	1	1	1.00	20	0.050
386	A	0	0	0.00	0	0.000
387	A	0	0	0.00	0	0.000
388	A	12	5	1.00	27	0.185
389	A	9	5	1.00	27	0.185
390	A	9	5	1.00	27	0.185
391	A	6	5	1.00	27	0.185
392	A	4	4	1.00	25	0.160
393	A	1	1	1.00	24	0.042
394	A	0	0	0.00	0	0.000
395	A	0	0	0.00	0	0.000
396	A	0	0	0.00	0	0.000
397	A	0	0	0.00	0	0.000
398	A	0	0	0.00	0	0.000
399	A	0	0	0.00	0	0.000
400	A	0	0	0.00	0	0.000
401	A	0	0	0.00	0	0.000
402	A	0	0	0.00	0	0.000
403	A	0	0	0.00	0	0.000
404	A	0	0	0.00	0	0.000
405	A	0	0	0.00	0	0.000
406	A	8	4	1.00	19	0.210
407	A	7	4	1.00	19	0.210
408	A	6	4	1.00	17	0.235
409	A	0	0	0.00	0	0.000
410	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	22	6	1.00	27	0.222
412	A	16	7	1.00	27	0.259
413	A	14	7	1.00	25	0.280
414	A	7	7	1.00	24	0.292
415	A	0	0	0.00	0	0.000
416	A	0	0	0.00	0	0.000
417	A	0	0	0.00	0	0.000
418	A	0	0	0.00	0	0.000
419	A	28	6	1.00	27	0.222
420	A	19	6	1.00	27	0.222
421	A	22	8	1.00	25	0.320
422	A	10	6	1.00	24	0.250
423	A	0	0	0.00	0	0.000
424	A	0	0	0.00	0	0.000
425	A	0	0	0.00	0	0.000
426	A	0	0	0.00	0	0.000
427	A	34	6	1.00	27	0.222
428	A	28	6	1.00	27	0.222
429	A	28	8	1.00	25	0.320
430	A	13	6	1.00	24	0.250
431	A	0	0	0.00	0	0.000
432	A	0	0	0.00	0	0.000
433	A	0	0	0.00	0	0.000
434	A	0	0	0.00	0	0.000
435	A	13	6	1.00	27	0.222
436	A	10	6	1.00	27	0.222
437	A	10	6	1.00	27	0.222
438	A	7	7	1.00	27	0.259
439	A	5	5	1.00	25	0.200
440	A	1	1	1.00	24	0.042
441	A	0	0	0.00	0	0.000
442	A	0	0	0.00	0	0.000
443	A	0	0	0.00	0	0.000
444	A	0	0	0.00	0	0.000
445	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	0	0	0.00	0	0.000
447	A	0	0	0.00	0	0.000
448	A	0	0	0.00	0	0.000
449	A	0	0	0.00	0	0.000
450	A	0	0	0.00	0	0.000
451	A	0	0	0.00	0	0.000
452	A	0	0	0.00	0	0.000
453	A	0	0	0.00	0	0.000
454	A	0	0	0.00	0	0.000
455	A	0	0	0.00	0	0.000
456	A	0	0	0.00	0	0.000
457	A	0	0	0.00	0	0.000
458	A	0	0	0.00	0	0.000
459	A	0	0	0.00	0	0.000
460	A	0	0	0.00	0	0.000
461	A	1	1	1.00	20	0.050
462	A	27	7	1.00	26	0.269
463	A	32	7	1.00	26	0.269
464	A	17	9	1.00	24	0.375
465	A	14	7	1.00	23	0.304
466	A	0	0	0.00	0	0.000
467	A	32	7	1.00	28	0.250
468	A	42	7	1.00	28	0.250
469	A	32	9	1.00	26	0.346
470	A	19	7	1.00	25	0.280
471	A	0	0	0.00	0	0.000
472	A	24	11	1.00	23	0.478
473	A	10	9	1.00	23	0.391
474	A	1	1	1.00	23	0.043
475	A	0	0	0.00	0	0.000
476	A	0	0	0.00	0	0.000
477	A	26	12	1.00	23	0.522
478	A	11	9	1.00	23	0.391
479	A	1	1	1.00	23	0.043
480	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	39	14	1.00	23	0.609
482	A	13	11	1.00	23	0.478
483	A	1	1	1.00	23	0.043
484	A	0	0	0.00	0	0.000
485	A	24	11	1.00	22	0.500
486	A	10	9	1.00	22	0.409
487	A	1	1	1.00	22	0.045
488	A	0	0	0.00	0	0.000
489	A	0	0	0.00	0	0.000
490	A	26	12	1.00	22	0.546
491	A	11	9	1.00	22	0.409
492	A	1	1	1.00	22	0.045
493	A	0	0	0.00	0	0.000
494	A	6	5	1.00	17	0.294
495	A	18	6	1.00	23	0.261
496	A	13	6	1.00	23	0.261
497	A	8	6	1.00	23	0.261
498	A	1	1	1.00	23	0.043
499	A	0	0	0.00	0	0.000
500	A	0	0	0.00	0	0.000
501	A	19	7	1.00	23	0.304
502	A	14	7	1.00	23	0.304
503	A	9	8	1.00	23	0.348
504	A	1	1	1.00	23	0.043
505	A	0	0	0.00	0	0.000
506	A	0	0	0.00	0	0.000
507	A	18	10	1.00	23	0.435
508	A	7	6	1.00	23	0.261
509	A	1	1	1.00	23	0.043
510	A	0	0	0.00	0	0.000
511	A	0	0	0.00	0	0.000
512	A	6	4	1.00	28	0.143
513	A	9	4	1.00	26	0.154
514	A	6	4	1.00	25	0.160
515	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	0	0	0.00	0	0.000
517	A	12	4	1.00	28	0.143
518	A	12	4	1.00	26	0.154
519	A	9	4	1.00	25	0.160
520	A	0	0	0.00	0	0.000
521	A	0	0	0.00	0	0.000
522	A	15	4	1.00	28	0.143
523	A	15	4	1.00	26	0.154
524	A	12	4	1.00	25	0.160
525	A	0	0	0.00	0	0.000
526	A	0	0	0.00	0	0.000
527	A	0	0	0.00	0	0.000
528	A	9	4	1.00	23	0.174
529	A	6	4	1.00	23	0.174
530	A	4	3	1.00	21	0.143
531	A	1	1	1.00	20	0.050
532	A	0	0	0.00	0	0.000
533	A	0	0	0.00	0	0.000
534	A	13	8	1.00	35	0.229
535	A	8	6	1.00	35	0.171
536	A	4	4	1.00	35	0.114
537	A	6	5	1.00	35	0.143
538	A	8	8	1.00	35	0.229
539	A	6	6	1.00	35	0.171
540	A	12	9	1.00	35	0.257
541	A	7	6	1.00	35	0.171
542	A	8	6	1.00	35	0.171
543	A	9	7	1.00	35	0.200
544	A	10	10	1.00	35	0.286
545	A	9	9	1.00	35	0.257
546	A	9	7	1.00	35	0.200
547	A	12	9	1.00	35	0.257
548	A	13	8	1.00	35	0.229
549	A	13	7	1.00	35	0.200
550	A	7	9	1.00	35	0.257

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	10	8	1.00	35	0.229
552	A	13	7	1.00	35	0.200
553	A	9	7	1.00	35	0.200
554	A	6	5	1.00	35	0.143
555	A	2	2	1.00	35	0.057
556	A	5	6	1.00	35	0.171
557	A	8	8	1.00	35	0.229
558	A	7	9	1.00	35	0.257
559	A	10	10	1.00	35	0.286
560	A	8	8	1.00	35	0.229
561	A	5	6	1.00	35	0.171
562	A	3	3	1.00	35	0.086
563	A	8	8	1.00	35	0.229
564	A	10	8	1.00	35	0.229
565	A	9	9	1.00	35	0.257
566	A	6	6	1.00	35	0.171
567	A	8	8	1.00	35	0.229
568	A	8	8	1.00	35	0.229
569	A	5	5	1.00	35	0.143
570	A	23	13	1.00	37	0.351
571	A	13	11	1.00	37	0.297
572	A	6	6	1.00	37	0.162
573	A	8	6	1.00	37	0.162
574	A	19	13	1.00	37	0.351
575	A	20	12	1.00	37	0.324
576	A	19	15	1.00	37	0.405
577	A	11	9	1.00	37	0.243
578	A	13	11	1.00	37	0.297
579	A	11	9	1.00	37	0.243
580	A	23	15	1.00	37	0.405
581	A	21	13	1.00	37	0.351
582	A	17	9	1.00	37	0.243
583	A	19	15	1.00	37	0.405
584	A	23	13	1.00	37	0.351
585	A	17	10	1.00	37	0.270

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	28	19	1.00	37	0.514
587	A	25	16	1.00	37	0.432
588	A	17	10	1.00	37	0.270
589	A	11	9	1.00	37	0.243
590	A	8	6	1.00	37	0.162
591	A	2	2	1.00	37	0.054
592	A	16	11	1.00	37	0.297
593	A	30	18	1.00	37	0.486
594	A	28	19	1.00	37	0.514
595	A	23	15	1.00	37	0.405
596	A	19	13	1.00	37	0.351
597	A	16	11	1.00	37	0.297
598	A	7	7	1.00	37	0.189
599	A	21	14	1.00	37	0.378
600	A	25	16	1.00	37	0.432
601	A	21	13	1.00	37	0.351
602	A	20	12	1.00	37	0.324
603	A	30	18	1.00	37	0.486
604	A	21	14	1.00	37	0.378
605	A	10	10	1.00	37	0.270
606	A	5	5	1.00	18	0.278
607	A	5	5	1.00	18	0.278
608	A	5	5	1.00	18	0.278
609	A	4	3	1.00	16	0.188
610	A	3	2	1.00	8	0.250
611	A	18	6	1.00	18	0.333
612	A	26	9	1.00	18	0.500
613	A	26	7	1.00	20	0.350
614	A	17	7	1.00	20	0.350
615	A	10	7	1.00	18	0.389
616	A	3	3	1.00	10	0.300
617	A	22	7	1.00	20	0.350
618	A	42	7	1.00	20	0.350
619	A	27	7	1.00	20	0.350
620	A	15	7	1.00	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	4	4	1.00	10	0.400
622	A	0	0	0.00	0	0.000
623	A	0	0	0.00	0	0.000
624	A	26	7	1.00	20	0.350
625	A	15	7	1.00	18	0.389
626	A	5	5	1.00	10	0.500
627	A	0	0	0.00	0	0.000
628	A	0	0	0.00	0	0.000
629	A	42	9	1.00	22	0.409
630	A	23	9	1.00	20	0.450
631	A	7	6	1.00	12	0.500
632	A	0	0	0.00	0	0.000
633	A	0	0	0.00	0	0.000
634	A	32	12	1.00	20	0.600
635	A	8	7	1.00	12	0.583
636	A	0	0	0.00	0	0.000
637	A	0	0	0.00	0	0.000
638	A	39	8	1.00	22	0.364
639	A	21	8	1.00	20	0.400
640	A	6	5	1.00	12	0.417
641	A	0	0	0.00	0	0.000
642	A	0	0	0.00	0	0.000
643	A	21	8	1.00	20	0.400
644	A	7	6	1.00	12	0.500
645	A	0	0	0.00	0	0.000
646	A	0	0	0.00	0	0.000
647	A	0	0	0.00	0	0.000
648	A	0	0	0.00	0	0.000
649	A	6	7	1.00	20	0.350
650	A	7	9	1.00	20	0.450
651	A	8	10	1.00	20	0.500
652	A	0	0	0.00	0	0.000
653	A	0	0	0.00	0	0.000
654	A	0	0	0.00	0	0.000
655	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	0	0	0.00	0	0.000
657	A	0	0	0.00	0	0.000
658	A	0	0	0.00	0	0.000
659	A	0	0	0.00	0	0.000
660	A	0	0	0.00	0	0.000
661	A	0	0	0.00	0	0.000
662	A	0	0	0.00	0	0.000
663	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^4(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=124

$$-\frac{2bd\sqrt{1+c^2x^2}}{35c^5}-\frac{bd(1+c^2x^2)^{3/2}}{105c^5}+\frac{8bd(1+c^2x^2)^{5/2}}{175c^5}-\frac{bd(1+c^2x^2)^{7/2}}{49c^5}+\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+\right.$$

```
[Out] -1/105*b*d*(c^2*x^2+1)^(3/2)/c^5+8/175*b*d*(c^2*x^2+1)^(5/2)/c^5-1/49*b*d*(
c^2*x^2+1)^(7/2)/c^5+1/5*d*x^5*(a+b*arcsinh(c*x))+1/7*c^2*d*x^7*(a+b*arcsin
h(c*x))-2/35*b*d*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {14, 5803, 12, 457, 78}

$$\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd(c^2x^2+1)^{7/2}}{49c^5}+\frac{8bd(c^2x^2+1)^{5/2}}{175c^5}-\frac{bd(c^2x^2+1)^{3/2}}{105c^5}-\frac{2bd\sqrt{c^2x^2+1}}{35c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(35*c^5) - (b*d*(1 + c^2*x^2)^(3/2))/(105*c^5) +
 (8*b*d*(1 + c^2*x^2)^(5/2))/(175*c^5) - (b*d*(1 + c^2*x^2)^(7/2))/(49*c^5)
 + (d*x^5*(a + b*ArcSinh[c*x]))/5 + (c^2*d*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx &= \frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{dx}{35}\\ &= \frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{35}(bcd)\int\\ &= \frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{70}(bcd)\mathrm{Su}\\ &= \frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{70}(bcd)\mathrm{Su}\\ &= -\frac{2bd\sqrt{1+c^2x^2}}{35c^5}-\frac{bd(1+c^2x^2)^{3/2}}{105c^5}+\frac{8bd(1+c^2x^2)^{5/2}}{175c^5}-\frac{bd(1+}{4}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 87, normalized size = 0.70

$$\frac{d\left(105ax^5(7+5c^2x^2)-\frac{b\sqrt{1+c^2x^2}\,(152-76c^2x^2+57c^4x^4+75c^6x^6)}{c^5}+105bx^5(7+5c^2x^2)\sinh^{-1}(cx)\right)}{3675}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(105*a*x^5*(7 + 5*c^2*x^2) - (b*Sqrt[1 + c^2*x^2]*(152 - 76*c^2*x^2 + 57
*c^4*x^4 + 75*c^6*x^6))/c^5 + 105*b*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]))/3675
```

Maple [A]
time = 0.96, size = 124, normalized size = 1.00

method	result
derivativedivides	$\frac{da\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+bd\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{19c^4x^4\sqrt{c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{c^2x^2}}{3675}\right.}{c^5}$
default	$\frac{da\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+bd\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{19c^4x^4\sqrt{c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{c^2x^2}}{3675}\right.}{c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(d*a*(1/7*c^7*x^7+1/5*c^5*x^5)+b*d*(1/7*arcsinh(c*x)*c^7*x^7+1/5*arcs
inh(c*x)*c^5*x^5-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-19/1225*c^4*x^4*(c^2*x^2+1)
^(1/2)+76/3675*c^2*x^2*(c^2*x^2+1)^(1/2)-152/3675*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.26, size = 184, normalized size = 1.48

$$\frac{1}{7}ac^2dx^7+\frac{1}{5}adx^5+\frac{1}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d+\frac{1}{75}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^
2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6
- 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arcsinh(c*x) - (3*sqr
t(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/
c^6)*c)*b*d
```

Fricas [A]
time = 0.37, size = 113, normalized size = 0.91

$$\frac{525\,ac^7dx^7+735\,ac^5dx^5+105\,(5\,bc^7dx^7+7\,bc^5dx^5)\log\left(cx+\sqrt{c^2x^2+1}\right)-(75\,bc^6dx^6+57\,bc^4dx^4-76\,bc^2dx^2+152\,bd)\sqrt{c^2x^2+1}}{3675\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*d*x^7 + 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 + 7*b*c^5*d*
x^5)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d*x^6 + 57*b*c^4*d*x^4 - 76*b
*c^2*d*x^2 + 152*b*d)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A]
time = 0.76, size = 151, normalized size = 1.22

$$\begin{cases}\frac{ac^2dx^7}{7}+\frac{adx^5}{5}+\frac{bc^2dx^7\operatorname{asinh}(cx)}{7}-\frac{bcdx^6\sqrt{c^2x^2+1}}{49}+\frac{bdx^5\operatorname{asinh}(cx)}{5}-\frac{19bdx^4\sqrt{c^2x^2+1}}{1225c}+\frac{76bdx^2\sqrt{c^2x^2+1}}{3675c^3}-\frac{152bd\sqrt{c^2x^2+1}}{3675c^5} & \text{for } c\neq 0\\ \frac{adx^5}{5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**7/7 + a*d*x**5/5 + b*c**2*d*x**7*asinh(c*x)/7 - b*c*
d*x**6*sqrt(c**2*x**2 + 1)/49 + b*d*x**5*asinh(c*x)/5 - 19*b*d*x**4*sqrt(c*
*2*x**2 + 1)/(1225*c) + 76*b*d*x**2*sqrt(c**2*x**2 + 1)/(3675*c**3) - 152*b
*d*sqrt(c**2*x**2 + 1)/(3675*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.2 $\int x^3(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=120

$$\frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}-\frac{bd\sinh^{-1}(cx)}{24c^4}+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+\right.$$

```
[Out] -1/24*b*d*arcsinh(c*x)/c^4+1/4*d*x^4*(a+b*arcsinh(c*x))+1/6*c^2*d*x^6*(a+b*
arcsinh(c*x))+1/24*b*d*x*(c^2*x^2+1)^(1/2)/c^3-1/36*b*d*x^3*(c^2*x^2+1)^(1/
2)/c-1/36*b*c*d*x^5*(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {14, 5803, 12, 470, 327, 221}

$$\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)-\frac{bd\sinh^{-1}(cx)}{24c^4}-\frac{1}{36}bcdx^5\sqrt{c^2x^2+1}-\frac{bdx^3\sqrt{c^2x^2+1}}{36c}+\frac{bdx\sqrt{c^2x^2+1}}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (b*d*x*Sqrt[1 + c^2*x^2])/(24*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2])/(36*c) - (
b*c*d*x^5*Sqrt[1 + c^2*x^2])/36 - (b*d*ArcSinh[c*x])/(24*c^4) + (d*x^4*(a +
 b*ArcSinh[c*x]))/4 + (c^2*d*x^6*(a + b*ArcSinh[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{dx}{12}$$

$$=\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int$$

$$=-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\right.$$

$$=-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx\right.$$

$$=\frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4$$

$$=\frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}-\frac{bd\sin}{}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 0.73

$$\frac{d\left(6ac^4x^4(3+2c^2x^2)+bcx\sqrt{1+c^2x^2}\,(3-2c^2x^2-2c^4x^4)+3b(-1+6c^4x^4+4c^6x^6)\sinh^{-1}(cx)\right)}{72c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(6*a*c^4*x^4*(3 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 - 2*c^2*x^2 -
2*c^4*x^4) + 3*b*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]))/(72*c^4)
```

Maple [A]
time = 1.71, size = 115, normalized size = 0.96

method	result
derivativedivides	$\frac{da\left(\frac{(c^2x^2+1)^3}{6}-\frac{(c^2x^2+1)^2}{4}\right)+bd\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{\operatorname{arcsinh}(cx)}{24}-\frac{cx(c^2x^2+1)^{\frac{5}{2}}}{36}+\frac{cx(c^2x^2+1)^{\frac{3}{2}}}{36}+\sqrt{c^2x}\right.}{c^4}$
default	$\frac{da\left(\frac{(c^2x^2+1)^3}{6}-\frac{(c^2x^2+1)^2}{4}\right)+bd\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{\operatorname{arcsinh}(cx)}{24}-\frac{cx(c^2x^2+1)^{\frac{5}{2}}}{36}+\frac{cx(c^2x^2+1)^{\frac{3}{2}}}{36}+\sqrt{c^2x}\right.}{c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d*a*(1/6*(c^2*x^2+1)^3-1/4*(c^2*x^2+1)^2)+b*d*(1/6*arcsinh(c*x)*c^6*
x^6+1/4*arcsinh(c*x)*c^4*x^4-1/24*arcsinh(c*x)-1/36*c*x*(c^2*x^2+1)^(5/2)+1
/36*c*x*(c^2*x^2+1)^(3/2)+1/24*(c^2*x^2+1)^(1/2)*c*x))
```

Maxima [A]
time = 0.25, size = 166, normalized size = 1.38

$$\frac{1}{6}ac^2dx^6+\frac{1}{4}adx^4+\frac{1}{288}\left(48x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)bc^2d+\frac{1}{32}\left(8x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^
2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6
- 15*arcsinh(c*x)/c^7)*c)*b*c^2*d + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*
x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*d
```

Fricas [A]
time = 0.36, size = 109, normalized size = 0.91

$$\frac{12\,ac^6dx^6+18\,ac^4dx^4+3\,(4\,bc^6dx^6+6\,bc^4dx^4-bd)\log\left(cx+\sqrt{c^2x^2+1}\right)-(2\,bc^5dx^5+2\,bc^3dx^3-3\,bcdx)\sqrt{c^2x^2+1}}{72\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/72*(12*a*c^6*d*x^6 + 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 + 6*b*c^4*d*x^4 -
b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^5*d*x^5 + 2*b*c^3*d*x^3 - 3*b*c*
d*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A]
time = 0.50, size = 138, normalized size = 1.15

$$\begin{cases} \frac{ac^2dx^6}{6} + \frac{adx^4}{4} + \frac{bc^2dx^6\operatorname{asinh}(cx)}{6} - \frac{bcdx^5\sqrt{c^2x^2+1}}{36} + \frac{bdx^4\operatorname{asinh}(cx)}{4} - \frac{bdx^3\sqrt{c^2x^2+1}}{36c} + \frac{bdx\sqrt{c^2x^2+1}}{24c^3} - \frac{bd\operatorname{asinh}(cx)}{24c^4} & \text{for } c \neq 0 \\ \frac{adx^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**6/6 + a*d*x**4/4 + b*c**2*d*x**6*asinh(c*x)/6 - b*c*
d*x**5*sqrt(c**2*x**2 + 1)/36 + b*d*x**4*asinh(c*x)/4 - b*d*x**3*sqrt(c**2*
x**2 + 1)/(36*c) + b*d*x*sqrt(c**2*x**2 + 1)/(24*c**3) - b*d*asinh(c*x)/(24
*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.3 $\int x^2(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=102

$$\frac{2bd\sqrt{1+c^2x^2}}{15c^3}+\frac{bd(1+c^2x^2)^{3/2}}{45c^3}-\frac{bd(1+c^2x^2)^{5/2}}{25c^3}+\frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] 1/45*b*d*(c^2*x^2+1)^(3/2)/c^3-1/25*b*d*(c^2*x^2+1)^(5/2)/c^3+1/3*d*x^3*(a+
b*arcsinh(c*x))+1/5*c^2*d*x^5*(a+b*arcsinh(c*x))+2/15*b*d*(c^2*x^2+1)^(1/2)
/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {14, 5803, 12, 457, 78}

$$\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd(c^2x^2+1)^{5/2}}{25c^3}+\frac{bd(c^2x^2+1)^{3/2}}{45c^3}+\frac{2bd\sqrt{c^2x^2+1}}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d*Sqrt[1 + c^2*x^2])/(15*c^3) + (b*d*(1 + c^2*x^2)^(3/2))/(45*c^3) - (
b*d*(1 + c^2*x^2)^(5/2))/(25*c^3) + (d*x^3*(a + b*ArcSinh[c*x]))/3 + (c^2*d
*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx &= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{dx}{15}\\ &= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{15}(bcd)\int\\ &= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{30}(bcd)\mathrm{Su}\\ &= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{30}(bcd)\mathrm{Su}\\ &= \frac{2bd\sqrt{1+c^2x^2}}{15c^3}+\frac{bd(1+c^2x^2)^{3/2}}{45c^3}-\frac{bd(1+c^2x^2)^{5/2}}{25c^3}+\frac{1}{3}dx^3(a+\end{aligned}$$

Mathematica [A]
time = 0.06, size = 78, normalized size = 0.76

$$\frac{1}{225}d\left(15ax^3\left(5+3c^2x^2\right)+\frac{b\sqrt{1+c^2x^2}\left(26-13c^2x^2-9c^4x^4\right)}{c^3}+15bx^3\left(5+3c^2x^2\right)\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(15*a*x^3*(5 + 3*c^2*x^2) + (b*Sqrt[1 + c^2*x^2]*(26 - 13*c^2*x^2 - 9*c^
4*x^4))/c^3 + 15*b*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]))/225
```

Maple [A]
time = 0.96, size = 105, normalized size = 1.03

method	result
derivativedivides	$\frac{da\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+bd\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{13c^2x^2\sqrt{c^2x^2+1}}{225}+\frac{26\sqrt{c^2x^2+}}{225}\right.}{c^3}$
default	$\frac{da\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+bd\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{13c^2x^2\sqrt{c^2x^2+1}}{225}+\frac{26\sqrt{c^2x^2+}}{225}\right.}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c^3*(d*a*(1/5*c^5*x^5+1/3*c^3*x^3)+b*d*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcs
inh(c*x)*c^3*x^3-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^
(1/2)+26/225*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.26, size = 145, normalized size = 1.42

$$\frac{1}{5}ac^2dx^5+\frac{1}{75}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d+\frac{1}{3}adx^3+\frac{1}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] 1/5*a*c^2*d*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2
- 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^2*d + 1/3*a
*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^
2*x^2 + 1)/c^4))*b*d
```

Fricas [A]
time = 0.38, size = 103, normalized size = 1.01

$$\frac{45\,ac^5dx^5+75\,ac^3dx^3+15\,(3\,bc^5dx^5+5\,bc^3dx^3)\log\left(cx+\sqrt{c^2x^2+1}\right)-(9\,bc^4dx^4+13\,bc^2dx^2-26\,bd)\sqrt{c^2x^2+1}}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] 1/225*(45*a*c^5*d*x^5 + 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 + 5*b*c^3*d*x^3)
*log(c*x + sqrt(c^2*x^2 + 1)) - (9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 26*b*d)*s
qrt(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.34, size = 126, normalized size = 1.24

$$\begin{cases}\frac{ac^2dx^5}{5}+\frac{adx^3}{3}+\frac{bc^2dx^5\operatorname{asinh}(cx)}{5}-\frac{bcdx^4\sqrt{c^2x^2+1}}{25}+\frac{bdx^3\operatorname{asinh}(cx)}{3}-\frac{13bdx^2\sqrt{c^2x^2+1}}{225c}+\frac{26bd\sqrt{c^2x^2+1}}{225c^3} & \text{for } c\neq 0\\ \frac{adx^3}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**5/5 + a*d*x**3/3 + b*c**2*d*x**5*asinh(c*x)/5 - b*c*
d*x**4*sqrt(c**2*x**2 + 1)/25 + b*d*x**3*asinh(c*x)/3 - 13*b*d*x**2*sqrt(c*
*2*x**2 + 1)/(225*c) + 26*b*d*sqrt(c**2*x**2 + 1)/(225*c**3), Ne(c, 0)), (a
*d*x**3/3, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.4 $\int x(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=87

$$-\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx(1+c^2x^2)^{3/2}}{16c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}+\frac{d(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}$$

```
[Out] -1/16*b*d*x*(c^2*x^2+1)^(3/2)/c-3/32*b*d*arcsinh(c*x)/c^2+1/4*d*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))/c^2-3/32*b*d*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {5798, 201, 221}

$$\frac{d(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{bdx(c^2x^2+1)^{3/2}}{16c}-\frac{3bdx\sqrt{c^2x^2+1}}{32c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*x*Sqrt[1 + c^2*x^2])/(32*c) - (b*d*x*(1 + c^2*x^2)^(3/2))/(16*c) -
(3*b*d*ArcSinh[c*x])/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4
*c^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{d(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{(bd)\int(1+c^2x^2)^{3/2}\,dx}{4c}$$
$$=-\frac{bdx(1+c^2x^2)^{3/2}}{16c}+\frac{d(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{(3bd)\int\sqrt{}$$
$$=-\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx(1+c^2x^2)^{3/2}}{16c}+\frac{d(1+c^2x^2)^2\left(a+b\sinh^{-}\right.}{4c^2}$$
$$=-\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx(1+c^2x^2)^{3/2}}{16c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}+\frac{d(1+}{}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.89

$$\frac{d\left(cx\left(8acx(2+c^2x^2)-b\sqrt{1+c^2x^2}\,(5+2c^2x^2)\right)+b(5+16c^2x^2+8c^4x^4)\sinh^{-1}(cx)\right)}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(c*x*(8*a*c*x*(2 + c^2*x^2) - b*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)) + b*(
5 + 16*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x]))/(32*c^2)
```

Maple [A]
time = 1.85, size = 85, normalized size = 0.98

method	result	size
derivativedivides	$\frac{\frac{d\left(c^2x^2+1\right)^2a}{4}+bd\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{5\operatorname{arcsinh}(cx)}{32}-\frac{cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{16}-\frac{3\sqrt{c^2x^2+1}\,cx}{32}\right)}{c^2}$	85
default	$\frac{\frac{d\left(c^2x^2+1\right)^2a}{4}+bd\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{5\operatorname{arcsinh}(cx)}{32}-\frac{cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{16}-\frac{3\sqrt{c^2x^2+1}\,cx}{32}\right)}{c^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*d*(c^2*x^2+1)^2*a+b*d*(1/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)
*c^2*x^2+5/32*arcsinh(c*x)-1/16*c*x*(c^2*x^2+1)^(3/2)-3/32*(c^2*x^2+1)^(1/2
)*c*x))
```

Maxima [A]
time = 0.26, size = 127, normalized size = 1.46

$$\frac{1}{4}ac^2dx^4+\frac{1}{32}\left(8x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)bc^2d+\frac{1}{2}adx^2+\frac{1}{4}\left(2x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^2*d*x^4 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 -
 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*c^2*d + 1/2*a*d*x^2 +
 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*
b*d
```

Fricas [A]
time = 0.37, size = 98, normalized size = 1.13

$$\frac{8\,ac^4dx^4+16\,ac^2dx^2+(8\,bc^4dx^4+16\,bc^2dx^2+5\,bd)\log\left(cx+\sqrt{c^2x^2+1}\right)-(2\,bc^3dx^3+5\,bcdx)\sqrt{c^2x^2+1}}{32\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*d*x^4 + 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 + 16*b*c^2*d*x^2 + 5*
b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(c^2*x^
2 + 1))/c^2
```

Sympy [A]
time = 0.22, size = 117, normalized size = 1.34

$$\begin{cases}\frac{ac^2dx^4}{4}+\frac{adx^2}{2}+\frac{bc^2dx^4\operatorname{asinh}(cx)}{4}-\frac{bcdx^3\sqrt{c^2x^2+1}}{16}+\frac{bdx^2\operatorname{asinh}(cx)}{2}-\frac{5bdx\sqrt{c^2x^2+1}}{32c}+\frac{5bd\operatorname{asinh}(cx)}{32c^2} & \text{for } c\neq 0\\ \frac{adx^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**4/4 + a*d*x**2/2 + b*c**2*d*x**4*asinh(c*x)/4 - b*c*
d*x**3*sqrt(c**2*x**2 + 1)/16 + b*d*x**2*asinh(c*x)/2 - 5*b*d*x*sqrt(c**2*x
**2 + 1)/(32*c) + 5*b*d*asinh(c*x)/(32*c**2), Ne(c, 0)), (a*d*x**2/2, True)
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.5 $\int \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(c x)\right) \, dx$

Optimal. Leaf size=75

$$-\frac{2 b d \sqrt{1+c^2 x^2}}{3 c}-\frac{b d \left(1+c^2 x^2\right)^{3/2}}{9 c}+d x \left(a+b \sinh^{-1}(c x)\right)+\frac{1}{3} c^2 d x^3 \left(a+b \sinh^{-1}(c x)\right)$$

```
[Out] -1/9*b*d*(c^2*x^2+1)^(3/2)/c+d*x*(a+b*arcsinh(c*x))+1/3*c^2*d*x^3*(a+b*arcs
inh(c*x))-2/3*b*d*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5784, 12, 455, 45}

$$\frac{1}{3} c^2 d x^3 \left(a+b \sinh^{-1}(c x)\right)+d x \left(a+b \sinh^{-1}(c x)\right)-\frac{b d \left(c^2 x^2+1\right)^{3/2}}{9 c}-\frac{2 b d \sqrt{c^2 x^2+1}}{3 c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(3*c) - (b*d*(1 + c^2*x^2)^(3/2))/(9*c) + d*x*(a
 + b*ArcSinh[c*x]) + (c^2*d*x^3*(a + b*ArcSinh[c*x]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
```

```
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx = dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-(bc)\int \frac{dx(3+}{3\sqrt{1+}}$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{3}(bcd)\int \frac{x(3-}{\sqrt{1-}}$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{6}(bcd)\text{Subst}\Big($$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{6}(bcd)\text{Subst}\Big($$

$$= -\frac{2bd\sqrt{1+c^2x^2}}{3c}-\frac{bd\left(1+c^2x^2\right)^{3/2}}{9c}+dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d$$

Mathematica [**A**]
time = 0.03, size = 86, normalized size = 1.15

$$adx+\frac{1}{3}ac^2dx^3-\frac{7bd\sqrt{1+c^2x^2}}{9c}-\frac{1}{9}bcdx^2\sqrt{1+c^2x^2}+bdx\sinh^{-1}(cx)+\frac{1}{3}bc^2dx^3\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*d*x + (a*c^2*d*x^3)/3 - (7*b*d*Sqrt[1 + c^2*x^2])/(9*c) - (b*c*d*x^2*Sqrt
[1 + c^2*x^2])/9 + b*d*x*ArcSinh[c*x] + (b*c^2*d*x^3*ArcSinh[c*x])/3
```

Maple [**A**]
time = 0.48, size = 76, normalized size = 1.01

method	result	size
derivativedivides	$\frac{da\left(\frac{1}{3}c^3x^3+cx\right)+bd\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$	76
default	$\frac{da\left(\frac{1}{3}c^3x^3+cx\right)+bd\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(d*a*(1/3*c^3*x^3+c*x)+b*d*(1/3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1
/9*c^2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.26, size = 97, normalized size = 1.29

$$\frac{1}{3}\,ac^2dx^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d+adx+\frac{\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^2x^2+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c^2*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^
2 + 1))*b*d/c
```

Fricas [A]
time = 0.37, size = 83, normalized size = 1.11

$$\frac{3\,ac^3dx^3+9\,acdx+3\,(bc^3dx^3+3\,bcdx)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bc^2dx^2+7\,bd)\sqrt{c^2x^2+1}}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*d*x^3 + 9*a*c*d*x + 3*(b*c^3*d*x^3 + 3*b*c*d*x)*log(c*x + sqrt
(c^2*x^2 + 1)) - (b*c^2*d*x^2 + 7*b*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A]
time = 0.16, size = 90, normalized size = 1.20

$$\begin{cases}\frac{ac^2dx^3}{3}+adx+\frac{bc^2dx^3\operatorname{asinh}\left(cx\right)}{3}-\frac{bcdx^2\sqrt{c^2x^2+1}}{9}+bdx\operatorname{asinh}\left(cx\right)-\frac{7bd\sqrt{c^2x^2+1}}{9c} & \text{for}\: c\neq 0\\ adx & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**3/3 + a*d*x + b*c**2*d*x**3*asinh(c*x)/3 - b*c*d*x**
2*sqrt(c**2*x**2 + 1)/9 + b*d*x*asinh(c*x) - 7*b*d*sqrt(c**2*x**2 + 1)/(9*c
), Ne(c, 0)), (a*d*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.6 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=111

$$-\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d(1+c^2x^2)\,(a+b\sinh^{-1}(cx))+\frac{d(a+b\sinh^{-1}(cx))^2}{2b}+d(a+b\sinh^{-}$$

```
[Out] -1/4*b*d*arcsinh(c*x)+1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/2*d*(a+b*arcsi
nh(c*x))^2/b+d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*d
*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5801, 5775, 3797, 2221, 2317, 2438, 201, 221}

$$\frac{1}{2}d(c^2x^2+1)\,(a+b\sinh^{-1}(cx))+\frac{d(a+b\sinh^{-1}(cx))^2}{2b}+d\log\left(1-e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))-\frac{1}{4}bcdx\sqrt{c^2x^2+1}-\frac{1}{2}bd\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)-\frac{1}{4}bd\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -1/4*(b*c*d*x*Sqrt[1 + c^2*x^2]) - (b*d*ArcSinh[c*x])/4 + (d*(1 + c^2*x^2)*
(a + b*ArcSinh[c*x]))/2 + (d*(a + b*ArcSinh[c*x])^2)/(2*b) + d*(a + b*ArcSi
nh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*d*PolyLog[2, E^(-2*ArcSinh[c*x])
])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)\,(a+b\sinh^{-1}(cx))}{x}\,dx = \frac{1}{2}d(1+c^2x^2)\,(a+b\sinh^{-1}(cx)) + d\int \frac{a+b\sinh^{-1}(cx)}{x}\,dx - \frac{1}{2}(bcd)$$
$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2} + \frac{1}{2}d(1+c^2x^2)\,(a+b\sinh^{-1}(cx)) + d\text{Subst}\Big(\int$$
$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2} - \frac{1}{4}bd\sinh^{-1}(cx) + \frac{1}{2}d(1+c^2x^2)\,(a+b\sinh^{-1}(c$$
$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2} - \frac{1}{4}bd\sinh^{-1}(cx) + \frac{1}{2}d(1+c^2x^2)\,(a+b\sinh^{-1}(c$$
$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2} - \frac{1}{4}bd\sinh^{-1}(cx) + \frac{1}{2}d(1+c^2x^2)\,(a+b\sinh^{-1}(c$$
$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2} - \frac{1}{4}bd\sinh^{-1}(cx) + \frac{1}{2}d(1+c^2x^2)\,(a+b\sinh^{-1}(c$$

Mathematica [A]
time = 0.04, size = 113, normalized size = 1.02

$$\frac{1}{2}ac^2dx^2 - \frac{1}{4}bcdx\sqrt{1+c^2x^2} + \frac{1}{4}bd\sinh^{-1}(cx) + \frac{1}{2}bc^2dx^2\sinh^{-1}(cx) - \frac{1}{2}bd\sinh^{-1}(cx)^2 + bd\sinh^{-1}(cx)\log\left(1-e^{2\sinh^{-1}(cx)}\right) + ad\log(x) + \frac{1}{2}bd\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (a*c^2*d*x^2)/2 - (b*c*d*x*Sqrt[1 + c^2*x^2])/4 + (b*d*ArcSinh[c*x])/4 + (b
*c^2*d*x^2*ArcSinh[c*x])/2 - (b*d*ArcSinh[c*x]^2)/2 + b*d*ArcSinh[c*x]*Log[
1 - E^(2*ArcSinh[c*x])] + a*d*Log[x] + (b*d*PolyLog[2, E^(2*ArcSinh[c*x])])
/2
```

Maple [A]
time = 2.98, size = 162, normalized size = 1.46

method	result
derivativedivides	$\frac{a\,c^2d\,x^2}{2} + ad\ln(cx) - \frac{bd\operatorname{arcsinh}(cx)^2}{2} + \frac{bd\operatorname{arcsinh}(cx)c^2x^2}{2} - \frac{bcdx\sqrt{c^2x^2+1}}{4} + \frac{bd\operatorname{arcsinh}(cx)}{4} + bd$
default	$\frac{a\,c^2d\,x^2}{2} + ad\ln(cx) - \frac{bd\operatorname{arcsinh}(cx)^2}{2} + \frac{bd\operatorname{arcsinh}(cx)c^2x^2}{2} - \frac{bcdx\sqrt{c^2x^2+1}}{4} + \frac{bd\operatorname{arcsinh}(cx)}{4} + bd$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*c^2*d*x^2+a*d*ln(c*x)-1/2*b*d*arcsinh(c*x)^2+1/2*b*d*arcsinh(c*x)*c^2
*x^2-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)+1/4*b*d*arcsinh(c*x)+b*d*arcsinh(c*x)*ln
```

```
(1+c*x+(c^2*x^2+1)^(1/2))+b*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*d*arcsinh
(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
[Out] 1/2*a*c^2*d*x^2 + a*d*log(x) + integrate(b*c^2*d*x*log(c*x + sqrt(c^2*x^2 +
 1)) + b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a}{x}\,dx + \int ac^2x\,dx + \int \frac{b\operatorname{asinh}(cx)}{x}\,dx + \int bc^2x\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x,x)
[Out] d*(Integral(a/x, x) + Integral(a*c**2*x, x) + Integral(b*asinh(c*x)/x, x) +
 Integral(b*c**2*x*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x, x)
```

3.7 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=66

$$-bcd\sqrt{1+c^2x^2}-\frac{d(a+b\sinh^{-1}(cx))}{x}+c^2dx(a+b\sinh^{-1}(cx))-bcd\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)$$

```
[Out] -d*(a+b*arcsinh(c*x))/x+c^2*d*x*(a+b*arcsinh(c*x))-b*c*d*arctanh((c^2*x^2+1
)^(1/2))-b*c*d*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {14, 5803, 12, 457, 81, 65, 214}

$$c^2dx(a+b\sinh^{-1}(cx))-\frac{d(a+b\sinh^{-1}(cx))}{x}-bcd\sqrt{c^2x^2+1}-bcd\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2]) - (d*(a + b*ArcSinh[c*x]))/x + c^2*d*x*(a + b*Ar
cSinh[c*x]) - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)-(bc)\int \frac{d(-1+c^2x}{x\sqrt{1+c^2x}}$$

$$= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)-(bcd)\int \frac{-1+c^2}{x\sqrt{1+c^2}}$$

$$= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}(bcd)\text{Subst}\Bigg(\int \frac{}{x}$$

$$= -bcd\sqrt{1+c^2x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}$$

$$= -bcd\sqrt{1+c^2x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -bcd\sqrt{1+c^2x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)-b$$

Mathematica [**A**]

time = 0.02, size = 74, normalized size = 1.12

$$-\frac{ad}{x}+ac^2dx-bcd\sqrt{1+c^2x^2}-\frac{bd\sinh^{-1}(cx)}{x}+bc^2dx\sinh^{-1}(cx)-bcd\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*c^2*d*x - b*c*d*Sqrt[1 + c^2*x^2] - (b*d*ArcSinh[c*x])/x + b
*c^2*d*x*ArcSinh[c*x] - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Maple [A]
time = 0.48, size = 69, normalized size = 1.05

method	result
derivativedivides	$c\left(ad\left(cx-\frac{1}{cx}\right)+bd\left(\operatorname{arcsinh}\left(cx\right)cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right.\right.\right.$
default	$c\left(ad\left(cx-\frac{1}{cx}\right)+bd\left(\operatorname{arcsinh}\left(cx\right)cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a*d*(c*x-1/c/x)+b*d*(arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)
-arctanh(1/(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.97

$$ac^2dx+\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^2x^2+1}\right)bcd-\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}\left(cx\right)}{x}\right)bd-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] a*c^2*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c*d - (c*arcsinh(1/(c*
abs(x))) + arcsinh(c*x)/x)*b*d - a*d/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(62) = 124.
time = 0.37, size = 156, normalized size = 2.36

$$\frac{ac^2dx^2-bcdx\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+bcdx\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-\sqrt{c^2x^2+1}\,bcdx-(bc^2-b)dx\log\left(-cx+\sqrt{c^2x^2+1}\right)-ad+(bc^2dx^2-(bc^2-b)dx-bd)\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (a*c^2*d*x^2 - b*c*d*x*log(-c*x + sqrt(c^2*x^2 + 1) + 1) + b*c*d*x*log(-c*x
 + sqrt(c^2*x^2 + 1) - 1) - sqrt(c^2*x^2 + 1)*b*c*d*x - (b*c^2 - b)*d*x*log
(-c*x + sqrt(c^2*x^2 + 1)) - a*d + (b*c^2*d*x^2 - (b*c^2 - b)*d*x - b*d)*lo
g(c*x + sqrt(c^2*x^2 + 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int ac^2\,dx+\int\frac{a}{x^2}\,dx+\int bc^2\operatorname{asinh}(cx)\,dx+\int\frac{b\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(b*c**2*asinh(c*x),
x) + Integral(b*asinh(c*x)/x**2, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^2, x)
```

3.8 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=128

$$-\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+c^2d\big(a+b\sin$$

```
[Out] 1/2*b*c^2*d*arcsinh(c*x)-1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))/x^2+1/2*c^2*d
*(a+b*arcsinh(c*x))^2/b+c^2*d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1
/2))^2)-1/2*b*c^2*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*c*d*(c^2*x
^2+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5802, 283, 221, 5775, 3797, 2221, 2317, 2438}

$$-\frac{d(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+c^2d\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}bc^2d\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)-\frac{bcd\sqrt{c^2x^2+1}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d*Sqrt[1 + c^2*x^2])/x + (b*c^2*d*ArcSinh[c*x])/2 - (d*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x]))/(2*x^2) + (c^2*d*(a + b*ArcSinh[c*x])^2)/(2*b) +
c^2*d*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*c^2*d*PolyLog[
2, E^(-2*ArcSinh[c*x])])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)\,(a+b\sinh^{-1}(cx))}{x^3}\,dx = -\frac{d(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{2x^2}+\frac{1}{2}(bcd)\int \frac{\sqrt{1+c^2x^2}}{x^2}\,dx+(c^2d$$
$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}-\frac{d(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{2x^2}+(c^2d)\,\mathrm{Subst}\Big($$
$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d(1+c^2x^2)\,(a+b\sinh^{-1}(cx}{2x^2}$$
$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d(1+c^2x^2)\,(a+b\sinh^{-1}(cx}{2x^2}$$
$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d(1+c^2x^2)\,(a+b\sinh^{-1}(cx}{2x^2}$$
$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d(1+c^2x^2)\,(a+b\sinh^{-1}(cx}{2x^2}$$

Mathematica [**A**]
time = 0.04, size = 111, normalized size = 0.87

$$-\frac{ad}{2x^2}-\frac{bcd\sqrt{1+c^2x^2}}{2x}-\frac{bd\sinh^{-1}(cx)}{2x^2}-\frac{1}{2}bc^2d\sinh^{-1}(cx)^2+bc^2d\sinh^{-1}(cx)\log\left(1-e^{2\sinh^{-1}(cx)}\right)+ac^2d\log(x)+\frac{1}{2}bc^2d\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*d)/x^2 - (b*c*d*Sqrt[1 + c^2*x^2])/(2*x) - (b*d*ArcSinh[c*x])/(2*x^
2) - (b*c^2*d*ArcSinh[c*x]^2)/2 + b*c^2*d*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh
[c*x])] + a*c^2*d*Log[x] + (b*c^2*d*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

Maple [**A**]
time = 4.84, size = 166, normalized size = 1.30

method	result
derivativedivides	$c^2\Bigg(ad\ln(cx)-\frac{ad}{2c^2x^2}-\frac{bd\,\mathrm{arcsinh}(cx)^2}{2}-\frac{bd\sqrt{c^2x^2+1}}{2cx}+\frac{bd}{2}-\frac{bd\,\mathrm{arcsinh}(cx)}{2c^2x^2}+bd\,\mathrm{arcsinh}\,(cx$
default	$c^2\Bigg(ad\ln(cx)-\frac{ad}{2c^2x^2}-\frac{bd\,\mathrm{arcsinh}(cx)^2}{2}-\frac{bd\sqrt{c^2x^2+1}}{2cx}+\frac{bd}{2}-\frac{bd\,\mathrm{arcsinh}(cx)}{2c^2x^2}+bd\,\mathrm{arcsinh}\,(cx$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(a*d*ln(c*x)-1/2*a*d/c^2/x^2-1/2*b*d*arcsinh(c*x)^2-1/2*b*d/c/x*(c^2*x^
2+1)^(1/2)+1/2*b*d-1/2*b*d*arcsinh(c*x)/c^2/x^2+b*d*arcsinh(c*x)*ln(1+c*x+(
```

```
c^2*x^2+1)^(1/2))+b*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*d*arcsinh(c*x)*ln
(1-c*x-(c^2*x^2+1)^(1/2))+b*d*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
[Out] b*c^2*d*integrate(log(c*x + sqrt(c^2*x^2 + 1))/x, x) + a*c^2*d*log(x) - 1/2
*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a}{x^3}\,dx + \int \frac{ac^2}{x}\,dx + \int \frac{b \operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{bc^2 \operatorname{asinh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**3,x)
[Out] d*(Integral(a/x**3, x) + Integral(a*c**2/x, x) + Integral(b*asinh(c*x)/x**3
, x) + Integral(b*c**2*asinh(c*x)/x, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^3,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^3, x)
```

3.9 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=80

$$-\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{d(a+b\sinh^{-1}(cx))}{3x^3}-\frac{c^2d(a+b\sinh^{-1}(cx))}{x}-\frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)$$

```
[Out] -1/3*d*(a+b*arcsinh(c*x))/x^3-c^2*d*(a+b*arcsinh(c*x))/x-5/6*b*c^3*d*arctan
h((c^2*x^2+1)^(1/2))-1/6*b*c*d*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {14, 5803, 12, 457, 79, 65, 214}

$$-\frac{c^2d(a+b\sinh^{-1}(cx))}{x}-\frac{d(a+b\sinh^{-1}(cx))}{3x^3}-\frac{bcd\sqrt{c^2x^2+1}}{6x^2}-\frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d*Sqrt[1 + c^2*x^2])/x^2 - (d*(a + b*ArcSinh[c*x]))/(3*x^3) - (c^
2*d*(a + b*ArcSinh[c*x]))/x - (5*b*c^3*d*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-(bc)\int \frac{d(-1-3c}{3x^3\sqrt{1+c}} \\
&= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{3}(bcd)\int \frac{-1-3}{x^3\sqrt{1+}} \\
&= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{6}(bcd)\mathrm{Subst}\bigg(\int \\
&= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}+ \\
&= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}+ \\
&= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 93, normalized size = 1.16

$$-\frac{ad}{3x^3}-\frac{ac^2d}{x}-\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{bd\sinh^{-1}(cx)}{3x^3}-\frac{bc^2d\sinh^{-1}(cx)}{x}-\frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/3*(a*d)/x^3 - (a*c^2*d)/x - (b*c*d*Sqrt[1 + c^2*x^2])/(6*x^2) - (b*d*Arc
Sinh[c*x])/(3*x^3) - (b*c^2*d*ArcSinh[c*x])/x - (5*b*c^3*d*ArcTanh[Sqrt[1 +
 c^2*x^2]])/6
```

Maple [A]
time = 0.49, size = 87, normalized size = 1.09

method	result
derivativedivides	$c^3\left(ad\left(-\frac{1}{cx}-\frac{1}{3c^3x^3}\right)+bd\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{5\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{6}-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}\right.\right.$
default	$c^3\left(ad\left(-\frac{1}{cx}-\frac{1}{3c^3x^3}\right)+bd\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{5\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{6}-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a*d*(-1/c/x-1/3/c^3/x^3)+b*d*(-arcsinh(c*x)/c/x-1/3*arcsinh(c*x)/c^3/x
^3-5/6*arctanh(1/(c^2*x^2+1)^(1/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.26, size = 91, normalized size = 1.14

$$-\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d+\frac{1}{6}\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)c-\frac{2\operatorname{arsinh}(cx)}{x^3}\right)bd-\frac{ac^2d}{x}-\frac{ad}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*c^2*d + 1/6*((c^2*arcsinh(1/(
c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d - a*c^2*d/x
 - 1/3*a*d/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(70) = 140$.
time = 0.38, size = 169, normalized size = 2.11

$$-\frac{5\,bc^3dx^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)-5\,bc^3dx^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)+6\,ac^2dx^2-2\,(3\,bc^2+b)dx^3\log\left(-cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2x^2+1}\,bcdx+2\,ad+2\,(3\,bc^2dx^2-(3\,bc^2+b)dx^3+bd)\log\left(cx+\sqrt{c^2x^2+1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(5*b*c^3*d*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) - 5*b*c^3*d*x^3*log(-
c*x + sqrt(c^2*x^2 + 1) - 1) + 6*a*c^2*d*x^2 - 2*(3*b*c^2 + b)*d*x^3*log(-c
*x + sqrt(c^2*x^2 + 1)) + sqrt(c^2*x^2 + 1)*b*c*d*x + 2*a*d + 2*(3*b*c^2*d*
x^2 - (3*b*c^2 + b)*d*x^3 + b*d)*log(c*x + sqrt(c^2*x^2 + 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a}{x^4}\,dx+\int\frac{ac^2}{x^2}\,dx+\int\frac{b\operatorname{asinh}(cx)}{x^4}\,dx+\int\frac{bc^2\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d*(Integral(a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(b*asinh(c*x)/x
**4, x) + Integral(b*c**2*asinh(c*x)/x**2, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\operatorname{asinh}(c\,x))\,(d\,c^2\,x^2+d)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^4, x)
```

3.10 $\int x^4(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=181

$$-\frac{8bd^2\sqrt{1+c^2x^2}}{315c^5}-\frac{4bd^2(1+c^2x^2)^{3/2}}{945c^5}-\frac{bd^2(1+c^2x^2)^{5/2}}{525c^5}+\frac{10bd^2(1+c^2x^2)^{7/2}}{441c^5}-\frac{bd^2(1+c^2x^2)^{9/2}}{81c^5}+\frac{1}{5}d^2x^5(a+b$$

```
[Out] -4/945*b*d^2*(c^2*x^2+1)^(3/2)/c^5-1/525*b*d^2*(c^2*x^2+1)^(5/2)/c^5+10/441
*b*d^2*(c^2*x^2+1)^(7/2)/c^5-1/81*b*d^2*(c^2*x^2+1)^(9/2)/c^5+1/5*d^2*x^5*(
a+b*arcsinh(c*x))+2/7*c^2*d^2*x^7*(a+b*arcsinh(c*x))+1/9*c^4*d^2*x^9*(a+b*a
rcsinh(c*x))-8/315*b*d^2*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {276, 5803, 12, 1265, 911, 1167}

$$\frac{1}{9}c^4d^2x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2(c^2x^2+1)^{9/2}}{81c^5}+\frac{10bd^2(c^2x^2+1)^{7/2}}{441c^5}-\frac{bd^2(c^2x^2+1)^{5/2}}{525c^5}-\frac{4bd^2(c^2x^2+1)^{3/2}}{945c^5}-\frac{8bd^2\sqrt{c^2x^2+1}}{315c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(315*c^5) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(945
*c^5) - (b*d^2*(1 + c^2*x^2)^(5/2))/(525*c^5) + (10*b*d^2*(1 + c^2*x^2)^(7/
2))/(441*c^5) - (b*d^2*(1 + c^2*x^2)^(9/2))/(81*c^5) + (d^2*x^5*(a + b*ArcS
inh[c*x]))/5 + (2*c^2*d^2*x^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^2*x^9*(a + b
*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^4\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$=-\frac{8bd^2\sqrt{1+c^2x^2}}{315c^5}-\frac{4bd^2(1+c^2x^2)^{3/2}}{945c^5}-\frac{bd^2(1+c^2x^2)^{5/2}}{525c^5}+\frac{10b}{}$$

Mathematica [A]
time = 0.07, size = 119, normalized size = 0.66

$$\frac{d^2\left(315ac^5x^5(63+90c^2x^2+35c^4x^4)-b\sqrt{1+c^2x^2}\,(2104-1052c^2x^2+789c^4x^4+2650c^6x^6+1225c^8x^8)+315bc^5x^5(63+90c^2x^2+35c^4x^4)\sinh^{-1}(cx)\right)}{99225c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(2
104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) + 315*b*c^5
*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSinh[c*x]))/(99225*c^5)
```

Maple [A]
time = 0.99, size = 167, normalized size = 0.92

method	result
derivativedivides	$\frac{d^2a\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{2\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-\frac{106c^6x^6\sqrt{}}{3}\right.}{c^5}$
default	$\frac{d^2a\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{2\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-\frac{106c^6x^6\sqrt{}}{3}\right.}{c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(d^2*a*(1/9*c^9*x^9+2/7*c^7*x^7+1/5*c^5*x^5)+d^2*b*(1/9*arcsinh(c*x)*
c^9*x^9+2/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/81*c^8*x^8*(c^2
*x^2+1)^(1/2)-106/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-263/33075*c^4*x^4*(c^2*x^2
+1)^(1/2)+1052/99225*c^2*x^2*(c^2*x^2+1)^(1/2)-2104/99225*(c^2*x^2+1)^(1/2)
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(155) = 310.
time = 0.26, size = 319, normalized size = 1.76

$$\frac{1}{9}ac^4d^2x^9+\frac{2}{7}ac^2d^2x^7+\frac{1}{2835}\left(315\,x^9\operatorname{arsinh}(cx)-\left(\frac{35\sqrt{c^2x^2+1}\,x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}\,x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2+1}}{c^{10}}\right)c\right)bc^4d^2+\frac{1}{5}ad^2x^5+\frac{2}{245}\left(35\,x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d^2+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^4*d^2*x^9 + 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
+ 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c
)*b*c^4*d^2 + 1/5*a*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2
```

```
+ 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 -
16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqr
t(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/
c^6)*c)*b*d^2
```

Fricas [A]
time = 0.36, size = 165, normalized size = 0.91

$$\frac{11025\,ac^9d^2x^9 + 28350\,ac^7d^2x^7 + 19845\,ac^5d^2x^5 + 315\,(35\,bc^9d^2x^9 + 90\,bc^7d^2x^7 + 63\,bc^5d^2x^5)\log\left(cx + \sqrt{c^2x^2 + 1}\right) - (1225\,bc^8d^2x^8 + 2650\,bc^6d^2x^6 + 789\,bc^4d^2x^4 - 1052\,bc^2d^2x^2 + 2104\,bd^2)\sqrt{c^2x^2 + 1}}{99225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^2*x^9 + 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
315*(35*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sqrt
(c^2*x^2 + 1)) - (1225*b*c^8*d^2*x^8 + 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x
^4 - 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A]
time = 1.45, size = 230, normalized size = 1.27

$$\begin{cases} \frac{ac^4d^2x^9}{9} + \frac{2ac^2d^2x^7}{7} + \frac{ad^2x^5}{5} + \frac{bc^4d^2x^9\operatorname{asinh}(cx)}{9} - \frac{bc^3d^2x^8\sqrt{c^2x^2+1}}{81} + \frac{2bc^2d^2x^7\operatorname{asinh}(cx)}{7} - \frac{106bcd^2x^6\sqrt{c^2x^2+1}}{3969} + \frac{bd^2x^5\operatorname{asinh}(cx)}{5} - \frac{263bd^2x^4\sqrt{c^2x^2+1}}{33075c} + \frac{1052bd^2x^2\sqrt{c^2x^2+1}}{99225c^3} - \frac{2104bd^2\sqrt{c^2x^2+1}}{99225c^5} & \text{for}\ c \neq 0 \\ \frac{ad^2x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**9/9 + 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c*
*4*d**2*x**9*asinh(c*x)/9 - b*c**3*d**2*x**8*sqrt(c**2*x**2 + 1)/81 + 2*b*c
**2*d**2*x**7*asinh(c*x)/7 - 106*b*c*d**2*x**6*sqrt(c**2*x**2 + 1)/3969 + b
*d**2*x**5*asinh(c*x)/5 - 263*b*d**2*x**4*sqrt(c**2*x**2 + 1)/(33075*c) + 1
052*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**3) - 2104*b*d**2*sqrt(c**2*x*
*2 + 1)/(99225*c**5), Ne(c, 0)), (a*d**2*x**5/5, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.11 $\int x^3(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=180

$$\frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3}-\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c}-\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}-\frac{73bd^2\sinh^{-1}(cx)}{3072c^4}+$$

```
[Out] -73/3072*b*d^2*arcsinh(c*x)/c^4+1/4*d^2*x^4*(a+b*arcsinh(c*x))+1/3*c^2*d^2*
x^6*(a+b*arcsinh(c*x))+1/8*c^4*d^2*x^8*(a+b*arcsinh(c*x))+73/3072*b*d^2*x*(
c^2*x^2+1)^(1/2)/c^3-73/4608*b*d^2*x^3*(c^2*x^2+1)^(1/2)/c-43/1152*b*c*d^2*
x^5*(c^2*x^2+1)^(1/2)-1/64*b*c^3*d^2*x^7*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 45, 5803, 12, 1281, 470, 327, 221}

$$\frac{1}{8}c^4d^2x^8\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)-\frac{73bd^2\sinh^{-1}(cx)}{3072c^4}-\frac{43bcd^2x^5\sqrt{c^2x^2+1}}{1152}-\frac{73bd^2x^3\sqrt{c^2x^2+1}}{4608c}+\frac{73bd^2x\sqrt{c^2x^2+1}}{3072c^3}-\frac{1}{64}bc^3d^2x^7\sqrt{c^2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (73*b*d^2*x*Sqrt[1 + c^2*x^2])/(3072*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]
)/(4608*c) - (43*b*c*d^2*x^5*Sqrt[1 + c^2*x^2])/1152 - (b*c^3*d^2*x^7*Sqrt[
1 + c^2*x^2])/64 - (73*b*d^2*ArcSinh[c*x])/(3072*c^4) + (d^2*x^4*(a + b*Arc
Sinh[c*x]))/4 + (c^2*d^2*x^6*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^8*(a + b*
ArcSinh[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^3 (d+c^2dx^2)^2 (a+b\sinh^{-1}(cx)) \, dx = \frac{1}{4}d^2x^4(a+b\sinh^{-1}(cx)) + \frac{1}{3}c^2d^2x^6(a+b\sinh^{-1}(cx)) + \frac{1}{8}c^4d^2x$$

$$= \frac{1}{4}d^2x^4(a+b\sinh^{-1}(cx)) + \frac{1}{3}c^2d^2x^6(a+b\sinh^{-1}(cx)) + \frac{1}{8}c^4d^2x$$

$$= -\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2} + \frac{1}{4}d^2x^4(a+b\sinh^{-1}(cx)) + \frac{1}{3}c^2d^2x^6(a$$

$$= -\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152} - \frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2} + \frac{1}{4}d^2x^4(a+b\,\mathrm{si}$$

$$= -\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c} - \frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152} - \frac{1}{64}bc^3d^2x^7\sqrt{1+}$$

$$= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3} - \frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c} - \frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}$$

$$= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3} - \frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c} - \frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}$$

Mathematica [A]
time = 0.06, size = 115, normalized size = 0.64

$$\frac{d^2\left(384ac^4x^4(6+8c^2x^2+3c^4x^4) - bcx\sqrt{1+c^2x^2}(-219+146c^2x^2+344c^4x^4+144c^6x^6) + 3b(-73+768c^4x^4+1024c^6x^6+384c^8x^8)\sinh^{-1}(cx)\right)}{9216c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
[Out] (d^2*(384*a*c^4*x^4*(6 + 8*c^2*x^2 + 3*c^4*x^4) - b*c*x*Sqrt[1 + c^2*x^2]*(
-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6) + 3*b*(-73 + 768*c^4*x^4 +
1024*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]))/(9216*c^4)
```

Maple [A]
time = 1.75, size = 146, normalized size = 0.81

method	result
derivativedivides	$\frac{d^2a\left(\frac{(c^2x^2+1)^4}{8}-\frac{(c^2x^2+1)^3}{6}\right)+d^2b\left(\frac{\mathrm{arcsinh}(cx)c^8x^8}{8}+\frac{\mathrm{arcsinh}(cx)c^6x^6}{3}+\frac{\mathrm{arcsinh}(cx)c^4x^4}{4}-\frac{73\,\mathrm{arcsinh}(cx)}{3072}-\frac{cx(c^2x^2+1)^{\frac{7}{2}}}{64}\right.}{c^4}$
default	$\frac{d^2a\left(\frac{(c^2x^2+1)^4}{8}-\frac{(c^2x^2+1)^3}{6}\right)+d^2b\left(\frac{\mathrm{arcsinh}(cx)c^8x^8}{8}+\frac{\mathrm{arcsinh}(cx)c^6x^6}{3}+\frac{\mathrm{arcsinh}(cx)c^4x^4}{4}-\frac{73\,\mathrm{arcsinh}(cx)}{3072}-\frac{cx(c^2x^2+1)^{\frac{7}{2}}}{64}\right.}{c^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/c^4*(d^2*a*(1/8*(c^2*x^2+1)^4-1/6*(c^2*x^2+1)^3)+d^2*b*(1/8*arcsinh(c*x)*c^8*x^8+1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-73/3072*arcsinh(c*x)-1/64*c*x*(c^2*x^2+1)^(7/2)+11/1152*c*x*(c^2*x^2+1)^(5/2)+55/4608*c*x*(c^2*x^2+1)^(3/2)+55/3072*(c^2*x^2+1)^(1/2)*c*x))

Maxima [A]
time = 0.29, size = 292, normalized size = 1.62

$$\frac{1}{8}ac^4d^2x^8+\frac{1}{3}ac^2d^2x^6+\frac{1}{3072}\left(384\,x^8\operatorname{arsinh}(cx)-\left(\frac{48\sqrt{c^2x^2+1}\,x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}\,x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}\,x^3}{c^6}-\frac{105\sqrt{c^2x^2+1}\,x}{c^8}+\frac{105\operatorname{arsinh}(cx)}{c^9}\right)c\right)bc^4d^2+\frac{1}{4}ad^2x^4+\frac{1}{144}\left(48\,x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)bc^2d^2+\frac{1}{32}\left(8\,x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] 1/8*a*c^4*d^2*x^8 + 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*b*c^4*d^2 + 1/4*a*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^2*d^2 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*d^2

Fricas [A]
time = 0.37, size = 161, normalized size = 0.89

$$\frac{1152\,ac^8d^2x^8+3072\,ac^6d^2x^6+2304\,ac^4d^2x^4+3\,(384\,bc^8d^2x^8+1024\,bc^6d^2x^6+768\,bc^4d^2x^4-73\,bd^2)\log\left(cx+\sqrt{c^2x^2+1}\right)-(144\,bc^7d^2x^7+344\,bc^5d^2x^5+146\,bc^3d^2x^3-219\,bcd^2x)\sqrt{c^2x^2+1}}{9216\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] 1/9216*(1152*a*c^8*d^2*x^8 + 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(384*b*c^8*d^2*x^8 + 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log(c*x + sqrt(c^2*x^2 + 1)) - (144*b*c^7*d^2*x^7 + 344*b*c^5*d^2*x^5 + 146*b*c^3*d^2*x^3 - 219*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4

Sympy [A]
time = 1.07, size = 218, normalized size = 1.21

$$\begin{cases}\frac{ac^4d^2x^8}{8}+\frac{ac^2d^2x^6}{3}+\frac{ad^2x^4}{4}+\frac{bc^4d^2x^8\operatorname{asinh}(cx)}{8}-\frac{bc^3d^2x^7\sqrt{c^2x^2+1}}{64}+\frac{bc^2d^2x^6\operatorname{asinh}(cx)}{3}-\frac{43bcd^2x^5\sqrt{c^2x^2+1}}{1152}+\frac{bd^2x^4\operatorname{asinh}(cx)}{4}-\frac{73bd^2x^3\sqrt{c^2x^2+1}}{4608c}+\frac{73bd^2x\sqrt{c^2x^2+1}}{3072c^3}-\frac{73bd^2\operatorname{asinh}(cx)}{3072c^4} & \text{for}\ c\neq 0\\ \frac{ad^2x^4}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)

[Out] Piecewise((a*c**4*d**2*x**8/8 + a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**4*d**2*x**8*asinh(c*x)/8 - b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)/64 + b*c**2*

```
d**2*x**6*asinh(c*x)/3 - 43*b*c*d**2*x**5*sqrt(c**2*x**2 + 1)/1152 + b*d**2
*x**4*asinh(c*x)/4 - 73*b*d**2*x**3*sqrt(c**2*x**2 + 1)/(4608*c) + 73*b*d**
2*x*sqrt(c**2*x**2 + 1)/(3072*c**3) - 73*b*d**2*asinh(c*x)/(3072*c**4), Ne(
c, 0)), (a*d**2*x**4/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.12 $\int x^2(d+c^2dx^2)^2 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=157

$$\frac{8bd^2\sqrt{1+c^2x^2}}{105c^3}+\frac{4bd^2(1+c^2x^2)^{3/2}}{315c^3}+\frac{bd^2(1+c^2x^2)^{5/2}}{175c^3}-\frac{bd^2(1+c^2x^2)^{7/2}}{49c^3}+\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5$$

```
[Out] 4/315*b*d^2*(c^2*x^2+1)^(3/2)/c^3+1/175*b*d^2*(c^2*x^2+1)^(5/2)/c^3-1/49*b*
d^2*(c^2*x^2+1)^(7/2)/c^3+1/3*d^2*x^3*(a+b*arcsinh(c*x))+2/5*c^2*d^2*x^5*(a
+b*arcsinh(c*x))+1/7*c^4*d^2*x^7*(a+b*arcsinh(c*x))+8/105*b*d^2*(c^2*x^2+1)
^(1/2)/c^3
```

Rubi [A]
time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {276, 5803, 12, 1265, 785}

$$\frac{1}{7}c^4d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2(c^2x^2+1)^{7/2}}{49c^3}+\frac{bd^2(c^2x^2+1)^{5/2}}{175c^3}+\frac{4bd^2(c^2x^2+1)^{3/2}}{315c^3}+\frac{8bd^2\sqrt{c^2x^2+1}}{105c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (8*b*d^2*Sqrt[1 + c^2*x^2])/(105*c^3) + (4*b*d^2*(1 + c^2*x^2)^(3/2))/(315*
c^3) + (b*d^2*(1 + c^2*x^2)^(5/2))/(175*c^3) - (b*d^2*(1 + c^2*x^2)^(7/2))/
(49*c^3) + (d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (2*c^2*d^2*x^5*(a + b*ArcSinh
[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x$$

$$=\frac{8bd^2\sqrt{1+c^2x^2}}{105c^3}+\frac{4bd^2(1+c^2x^2)^{3/2}}{315c^3}+\frac{bd^2(1+c^2x^2)^{5/2}}{175c^3}-\frac{bd^2(1}{}$$

Mathematica [A]
time = 0.06, size = 111, normalized size = 0.71

$$\frac{d^2\left(105ac^3x^3(35+42c^2x^2+15c^4x^4)-b\sqrt{1+c^2x^2}\,(-818+409c^2x^2+612c^4x^4+225c^6x^6)+105bc^3x^3(35+42c^2x^2+15c^4x^4)\sinh^{-1}(cx)\right)}{11025c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(105*a*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(-
818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(35 + 42*c^2
*x^2 + 15*c^4*x^4)*ArcSinh[c*x]))/(11025*c^3)
```

Maple [A]
time = 1.35, size = 148, normalized size = 0.94

method	result
derivativedivides	$\frac{d^2 a\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x^4\sqrt{c}}{12}\right.}{c^3}$
default	$\frac{d^2 a\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x^4\sqrt{c}}{12}\right.}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^2*a*(1/7*c^7*x^7+2/5*c^5*x^5+1/3*c^3*x^3)+d^2*b*(1/7*arcsinh(c*x)*
c^7*x^7+2/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/49*c^6*x^6*(c^2
*x^2+1)^(1/2)-68/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-409/11025*c^2*x^2*(c^2*x^2+
1)^(1/2)+818/11025*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.27, size = 261, normalized size = 1.66

$$\frac{1}{7}\,ac^4d^2x^7+\frac{2}{5}\,ac^2d^2x^5+\frac{1}{245}\left(35\,x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^4d^2+\frac{2}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d^2+\frac{1}{3}\,ad^2x^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^2*x^7 + 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^2 + 2/75*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2
```

Fricas [A]
time = 0.35, size = 153, normalized size = 0.97

$$\frac{1575\,ac^7d^2x^7+4410\,ac^5d^2x^5+3675\,ac^3d^2x^3+105\,(15\,bc^7d^2x^7+42\,bc^5d^2x^5+35\,bc^3d^2x^3)\log\left(cx+\sqrt{c^2x^2+1}\right)-(225\,bc^6d^2x^6+612\,bc^4d^2x^4+409\,bc^2d^2x^2-818\,bd^2)\sqrt{c^2x^2+1}}{11025\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*d^2*x^7 + 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 105
*(15*b*c^7*d^2*x^7 + 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(c^
2*x^2 + 1)) - (225*b*c^6*d^2*x^6 + 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 -
818*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.71, size = 202, normalized size = 1.29

$$\begin{cases} \frac{ac^4d^2x^7}{7} + \frac{2ac^2d^2x^5}{5} + \frac{ad^2x^3}{3} + \frac{bc^4d^2x^7\operatorname{asinh}(cx)}{7} - \frac{bc^3d^2x^6\sqrt{c^2x^2+1}}{49} + \frac{2bc^2d^2x^5\operatorname{asinh}(cx)}{5} - \frac{68bcd^2x^4\sqrt{c^2x^2+1}}{1225} + \frac{bd^2x^3\operatorname{asinh}(cx)}{3} - \frac{409bd^2x^2\sqrt{c^2x^2+1}}{11025c} + \frac{818bd^2\sqrt{c^2x^2+1}}{11025c^3} & \text{for } c \neq 0 \\ \frac{ad^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**7/7 + 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c*
*4*d**2*x**7*asinh(c*x)/7 - b*c**3*d**2*x**6*sqrt(c**2*x**2 + 1)/49 + 2*b*c
**2*d**2*x**5*asinh(c*x)/5 - 68*b*c*d**2*x**4*sqrt(c**2*x**2 + 1)/1225 + b*
d**2*x**3*asinh(c*x)/3 - 409*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(11025*c) + 81
8*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3), Ne(c, 0)), (a*d**2*x**3/3, True)
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.13 $\int x(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=120

$$-\frac{5bd^2x\sqrt{1+c^2x^2}}{96c}-\frac{5bd^2x(1+c^2x^2)^{3/2}}{144c}-\frac{bd^2x(1+c^2x^2)^{5/2}}{36c}-\frac{5bd^2\sinh^{-1}(cx)}{96c^2}+\frac{d^2(1+c^2x^2)^3\left(a+b\sinh^{-1}(c\right.}{6c^2}$$

```
[Out] -5/144*b*d^2*x*(c^2*x^2+1)^(3/2)/c-1/36*b*d^2*x*(c^2*x^2+1)^(5/2)/c-5/96*b*
d^2*arcsinh(c*x)/c^2+1/6*d^2*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c^2-5/96*b*d^
2*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 201, 221}

$$\frac{d^2(c^2x^2+1)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2}-\frac{bd^2x(c^2x^2+1)^{5/2}}{36c}-\frac{5bd^2x(c^2x^2+1)^{3/2}}{144c}-\frac{5bd^2x\sqrt{c^2x^2+1}}{96c}-\frac{5bd^2\sinh^{-1}(cx)}{96c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*d^2*x*Sqrt[1 + c^2*x^2])/(96*c) - (5*b*d^2*x*(1 + c^2*x^2)^(3/2))/(14
4*c) - (b*d^2*x*(1 + c^2*x^2)^(5/2))/(36*c) - (5*b*d^2*ArcSinh[c*x])/(96*c^
2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{d^2(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2}-\frac{(bd^2)\int(1+c^2x^2)^{5/2}\,dx}{6c}$$
$$=-\frac{bd^2x(1+c^2x^2)^{5/2}}{36c}+\frac{d^2(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2}-\frac{(5bd^2)}{}$$
$$=-\frac{5bd^2x(1+c^2x^2)^{3/2}}{144c}-\frac{bd^2x(1+c^2x^2)^{5/2}}{36c}+\frac{d^2(1+c^2x^2)^3\left(a+b\right.}{6c^2}$$
$$=-\frac{5bd^2x\sqrt{1+c^2x^2}}{96c}-\frac{5bd^2x(1+c^2x^2)^{3/2}}{144c}-\frac{bd^2x(1+c^2x^2)^{5/2}}{36c}+$$
$$=-\frac{5bd^2x\sqrt{1+c^2x^2}}{96c}-\frac{5bd^2x(1+c^2x^2)^{3/2}}{144c}-\frac{bd^2x(1+c^2x^2)^{5/2}}{36c}-$$

Mathematica [A]
time = 0.08, size = 104, normalized size = 0.87

$$\frac{d^2\left(cx\left(48acx(3+3c^2x^2+c^4x^4)-b\sqrt{1+c^2x^2}(33+26c^2x^2+8c^4x^4)\right)+3b(11+48c^2x^2+48c^4x^4+16c^6x^6)\sinh^{-1}(cx)\right)}{288c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(c*x*(48*a*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(33 + 2
6*c^2*x^2 + 8*c^4*x^4)) + 3*b*(11 + 48*c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6)*A
rcSinh[c*x]))/(288*c^2)
```

Maple [A]
time = 2.08, size = 116, normalized size = 0.97

method	result
derivativedivides	$\frac{\frac{d^2(c^2x^2+1)^3a}{6}+d^2b\left(\frac{\text{arcsinh}(cx)c^6x^6}{6}+\frac{\text{arcsinh}(cx)c^4x^4}{2}+\frac{\text{arcsinh}(cx)c^2x^2}{2}+\frac{11\,\text{arcsinh}(cx)}{96}-\frac{cx(c^2x^2+1)^{\frac{5}{2}}}{36}-\frac{5cx(c^2x^2+1)^{\frac{3}{2}}}{144}-\right.}{c^2}$
default	$\frac{\frac{d^2(c^2x^2+1)^3a}{6}+d^2b\left(\frac{\text{arcsinh}(cx)c^6x^6}{6}+\frac{\text{arcsinh}(cx)c^4x^4}{2}+\frac{\text{arcsinh}(cx)c^2x^2}{2}+\frac{11\,\text{arcsinh}(cx)}{96}-\frac{cx(c^2x^2+1)^{\frac{5}{2}}}{36}-\frac{5cx(c^2x^2+1)^{\frac{3}{2}}}{144}-\right.}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/6*d^2*(c^2*x^2+1)^3*a+d^2*b*(1/6*arcsinh(c*x)*c^6*x^6+1/2*arcsinh(
c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+11/96*arcsinh(c*x)-1/36*c*x*(c^2*x^2+
1)^(5/2)-5/144*c*x*(c^2*x^2+1)^(3/2)-5/96*(c^2*x^2+1)^(1/2)*c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(104) = 208$.
time = 0.27, size = 234, normalized size = 1.95

$$\frac{1}{6}\,ac^4d^2x^6+\frac{1}{2}\,ac^2d^2x^4+\frac{1}{288}\left(48\,x^6\operatorname{arsinh}\left(cx\right)-\left(\frac{8\,\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\,\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\,\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\,\operatorname{arsinh}\left(cx\right)}{c^7}\right)c\right)bc^4d^2+\frac{1}{16}\left(8\,x^4\operatorname{arsinh}\left(cx\right)-\left(\frac{2\,\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\,\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\,\operatorname{arsinh}\left(cx\right)}{c^5}\right)c\right)bc^2d^2+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\left(2\,x^2\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\operatorname{arsinh}\left(cx\right)}{c^3}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^2*x^6 + 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqr
t(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1
)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^4*d^2 + 1/16*(8*x^4*arcsinh(c*x) - (2
*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5
)*c)*b*c^2*d^2 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2
```

Fricas [A]
time = 0.42, size = 149, normalized size = 1.24

$$\frac{48\,ac^6d^2x^6+144\,ac^4d^2x^4+144\,ac^2d^2x^2+3\left(16\,bc^6d^2x^6+48\,bc^4d^2x^4+48\,bc^2d^2x^2+11\,bd^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(8\,bc^5d^2x^5+26\,bc^3d^2x^3+33\,bcd^2x\right)\sqrt{c^2x^2+1}}{288\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*d^2*x^6 + 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*c
^6*d^2*x^6 + 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 + 11*b*d^2)*log(c*x + sqrt
(c^2*x^2 + 1)) - (8*b*c^5*d^2*x^5 + 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqrt(c
^2*x^2 + 1))/c^2
```

Sympy [A]
time = 0.51, size = 190, normalized size = 1.58

$$\begin{cases} \frac{ac^4d^2x^6}{6}+\frac{ac^2d^2x^4}{2}+\frac{ad^2x^2}{2}+\frac{bc^4d^2x^6\operatorname{asinh}\left(cx\right)}{6}-\frac{bc^3d^2x^5\sqrt{c^2x^2+1}}{36}+\frac{bc^2d^2x^4\operatorname{asinh}\left(cx\right)}{2}-\frac{13bcd^2x^3\sqrt{c^2x^2+1}}{144}+\frac{bd^2x^2\operatorname{asinh}\left(cx\right)}{2}-\frac{11bd^2x\sqrt{c^2x^2+1}}{96c}+\frac{11bd^2\operatorname{asinh}\left(cx\right)}{96c^2} & \text{for}\: c\neq 0 \\ \frac{ad^2x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**6/6 + a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**4
*d**2*x**6*asinh(c*x)/6 - b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)/36 + b*c**2*
d**2*x**4*asinh(c*x)/2 - 13*b*c*d**2*x**3*sqrt(c**2*x**2 + 1)/144 + b*d**2*
x**2*asinh(c*x)/2 - 11*b*d**2*x*sqrt(c**2*x**2 + 1)/(96*c) + 11*b*d**2*asin
h(c*x)/(96*c**2), Ne(c, 0)), (a*d**2*x**2/2, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.14 $\int \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=128

$$-\frac{8bd^2\sqrt{1+c^2x^2}}{15c}-\frac{4bd^2\left(1+c^2x^2\right)^{3/2}}{45c}-\frac{bd^2\left(1+c^2x^2\right)^{5/2}}{25c}+d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] -4/45*b*d^2*(c^2*x^2+1)^(3/2)/c-1/25*b*d^2*(c^2*x^2+1)^(5/2)/c+d^2*x*(a+b*a
rcsinh(c*x))+2/3*c^2*d^2*x^3*(a+b*arcsinh(c*x))+1/5*c^4*d^2*x^5*(a+b*arcsin
h(c*x))-8/15*b*d^2*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {200, 5784, 12, 1261, 712}

$$\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{5/2}}{25c}-\frac{4bd^2\left(c^2x^2+1\right)^{3/2}}{45c}-\frac{8bd^2\sqrt{c^2x^2+1}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(15*c) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(45*c)
- (b*d^2*(1 + c^2*x^2)^(5/2))/(25*c) + d^2*x*(a + b*ArcSinh[c*x]) + (2*c^2*
d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)\,dx &= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5(a+\\ &= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5(a+\\ &= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5(a+\\ &= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5(a+\\ &= -\frac{8bd^2\sqrt{1+c^2x^2}}{15c}-\frac{4bd^2(1+c^2x^2)^{3/2}}{45c}-\frac{bd^2(1+c^2x^2)^{5/2}}{25c}+d^2x(a\end{aligned}$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 0.74

$$\frac{d^2\left(15acx(15+10c^2x^2+3c^4x^4)-b\sqrt{1+c^2x^2}\,(149+38c^2x^2+9c^4x^4)+15bcx(15+10c^2x^2+3c^4x^4)\sinh^{-1}(cx)\right)}{225c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(149 + 3
8*c^2*x^2 + 9*c^4*x^4) + 15*b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x
]))/(225*c)
```

Maple [A]
time = 0.52, size = 119, normalized size = 0.93

method	result
derivativedivides	$\frac{d^2a\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{38c^2x^2\sqrt{c^2x}}{225}\right.}{c}$

default	$\frac{d^2 a\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{38c^2x^2\sqrt{c^2x^2}}{225}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(d^2*a*(1/5*c^5*x^5+2/3*c^3*x^3+c*x)+d^2*b*(1/5*arcsinh(c*x)*c^5*x^5+2/
3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-38/2
25*c^2*x^2*(c^2*x^2+1)^(1/2)-149/225*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.27, size = 194, normalized size = 1.52

$$\frac{1}{5}\,ac^4d^2x^5+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^2+\frac{2}{3}\,ac^2d^2x^3+\frac{2}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d^2+ad^2x+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^
2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2 + 2
/3*a*c^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c
^2*x^2 + 1))*b*d^2/c
```

Fricas [A]
time = 0.40, size = 133, normalized size = 1.04

$$\frac{45\,ac^5d^2x^5+150\,ac^3d^2x^3+225\,acd^2x+15\left(3\,bc^5d^2x^5+10\,bc^3d^2x^3+15\,bcd^2x\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(9\,bc^4d^2x^4+38\,bc^2d^2x^2+149\,bd^2\right)\sqrt{c^2x^2+1}}{225\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d^2*x^5 + 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*d
^2*x^5 + 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (9
*b*c^4*d^2*x^4 + 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A]
time = 0.34, size = 165, normalized size = 1.29

$$\begin{cases} \frac{ac^4d^2x^5}{5}+\frac{2ac^2d^2x^3}{3}+ad^2x+\frac{bc^4d^2x^5\operatorname{asinh}(cx)}{5}-\frac{bc^3d^2x^4\sqrt{c^2x^2+1}}{25}+\frac{2bc^2d^2x^3\operatorname{asinh}(cx)}{3}-\frac{38bcd^2x^2\sqrt{c^2x^2+1}}{225}+bd^2x\operatorname{asinh}(cx)-\frac{149bd^2\sqrt{c^2x^2+1}}{225c} & \text{for } c\neq 0 \\ ad^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**5/5 + 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d*
*2*x**5*asinh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)/25 + 2*b*c**2*d
**2*x**3*asinh(c*x)/3 - 38*b*c*d**2*x**2*sqrt(c**2*x**2 + 1)/225 + b*d**2*x
*asinh(c*x) - 149*b*d**2*sqrt(c**2*x**2 + 1)/(225*c), Ne(c, 0)), (a*d**2*x,
 True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.15 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=172

$$-\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^2(1+$$

```
[Out] -1/16*b*c*d^2*x*(c^2*x^2+1)^(3/2)-11/32*b*d^2*arcsinh(c*x)+1/2*d^2*(c^2*x^2
+1)*(a+b*arcsinh(c*x))+1/4*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))+1/2*d^2*(a+
b*arcsinh(c*x))^2/b+d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2
)-1/2*b*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-11/32*b*c*d^2*x*(c^2*x^2
+1)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5801, 5775, 3797, 2221, 2317, 2438, 201, 221}

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+d^2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{16}bcd^2x\left(c^2x^2+1\right)^{3/2}-\frac{11}{32}bcd^2x\sqrt{c^2x^2+1}-\frac{1}{2}bd^2\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)-\frac{11}{32}bd^2\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-11*b*c*d^2*x*Sqrt[1 + c^2*x^2])/32 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2))/16 -
 (11*b*d^2*ArcSinh[c*x])/32 + (d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 +
(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 + (d^2*(a + b*ArcSinh[c*x])^2)
/(2*b) + d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*d^2*Pol
yLog[2, E^(-2*ArcSinh[c*x])])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{4}d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(\right.}{x}$$
$$= -\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}+\frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^2(1$$
$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}+\frac{1}{2}d^2\left(1+c^2x^2\right)(a+$$
$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+$$
$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+$$
$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+$$
$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+$$

Mathematica [A]
time = 0.14, size = 173, normalized size = 1.01

$$\frac{d^2\left(-16a^2+24ab+32abc^2x^2+8abc^4x^4-13b^2cx\sqrt{1+c^2x^2}-2b^2c^3x^3\sqrt{1+c^2x^2}-16b^2\sinh^{-1}(cx)^2+32ab\log\left(1-e^{2\sinh^{-1}(cx)}\right)+b\sinh^{-1}(cx)\left(-32a+b(13+32c^2x^2+8c^4x^4)+32b\log\left(1-e^{2\sinh^{-1}(cx)}\right)\right)+16b^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^2*(-16*a^2 + 24*a*b + 32*a*b*c^2*x^2 + 8*a*b*c^4*x^4 - 13*b^2*c*x*Sqrt[1
 + c^2*x^2] - 2*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 16*b^2*ArcSinh[c*x]^2 + 32*
a*b*Log[1 - E^(2*ArcSinh[c*x])] + b*ArcSinh[c*x]*(-32*a + b*(13 + 32*c^2*x^
2 + 8*c^4*x^4) + 32*b*Log[1 - E^(2*ArcSinh[c*x])]) + 16*b^2*PolyLog[2, E^(2
*ArcSinh[c*x])]))/(32*b)
```

Maple [A]
time = 3.34, size = 231, normalized size = 1.34

method	result
derivativedivides	$\frac{a\,d^2c^4x^4}{4}+a\,d^2c^2x^2+a\,d^2\ln(cx)+\frac{13b\,d^2\,\mathrm{arcsinh}(cx)}{32}-\frac{d^2b\,\mathrm{arcsinh}(cx)^2}{2}+d^2b\,\mathrm{polylog}\left(2,-cx-\right.$
default	$\frac{a\,d^2c^4x^4}{4}+a\,d^2c^2x^2+a\,d^2\ln(cx)+\frac{13b\,d^2\,\mathrm{arcsinh}(cx)}{32}-\frac{d^2b\,\mathrm{arcsinh}(cx)^2}{2}+d^2b\,\mathrm{polylog}\left(2,-cx-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*d^2*c^4*x^4+a*d^2*c^2*x^2+a*d^2*ln(c*x)+13/32*b*d^2*arcsinh(c*x)-1/2*
d^2*b*arcsinh(c*x)^2+d^2*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^2*b*polylog(
2,c*x+(c^2*x^2+1)^(1/2))+d^2*b*arcsinh(c*x)*c^2*x^2+1/4*d^2*b*arcsinh(c*x)*
c^4*x^4-1/16*d^2*b*c^3*x^3*(c^2*x^2+1)^(1/2)-13/32*b*c*d^2*x*(c^2*x^2+1)^(1
/2)+d^2*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+d^2*b*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^2*x^4 + a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*b*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) +
 b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int \frac{a}{x}\, dx + \int 2ac^{2}x\, dx + \int ac^{4}x^{3}\, dx + \int \frac{b\operatorname{asinh}{\left(cx \right)}}{x}\, dx + \int 2bc^{2}x\operatorname{asinh}{\left(cx \right)}\, dx + \int bc^{4}x^{3}\operatorname{asinh}{\left(cx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x,x)
```

```
[Out] d**2*(Integral(a/x, x) + Integral(2*a*c**2*x, x) + Integral(a*c**4*x**3, x)
 + Integral(b*asinh(c*x)/x, x) + Integral(2*b*c**2*x*asinh(c*x), x) + Integ
ral(b*c**4*x**3*asinh(c*x), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x, x)
```

3.16 $\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=120

$$-\frac{5}{3}bcd^2\sqrt{1+c^2x^2}-\frac{1}{9}bcd^2\left(1+c^2x^2\right)^{3/2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\,\mathrm{s}\right.$$

```
[Out] -1/9*b*c*d^2*(c^2*x^2+1)^(3/2)-d^2*(a+b*arcsinh(c*x))/x+2*c^2*d^2*x*(a+b*ar
csinh(c*x))+1/3*c^4*d^2*x^3*(a+b*arcsinh(c*x))-b*c*d^2*arctanh((c^2*x^2+1)^
(1/2))-5/3*b*c*d^2*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {276, 5803, 12, 1265, 911, 1167, 214}

$$\frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx)\right)+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{9}bcd^2\left(c^2x^2+1\right)^{3/2}-\frac{5}{3}bcd^2\sqrt{c^2x^2+1}-bcd^2\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-5*b*c*d^2*Sqrt[1 + c^2*x^2])/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/9 - (d^2*(
a + b*ArcSinh[c*x]))/x + 2*c^2*d^2*x*(a + b*ArcSinh[c*x]) + (c^4*d^2*x^3*(a
 + b*ArcSinh[c*x]))/3 - b*c*d^2*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2 (a+b\sinh^{-1}(cx))}{x^2} dx = -\frac{d^2(a+b\sinh^{-1}(cx))}{x} + 2c^2d^2x(a+b\sinh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\sinh^{-1}(cx))}{x} + 2c^2d^2x(a+b\sinh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\sinh^{-1}(cx))}{x} + 2c^2d^2x(a+b\sinh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\sinh^{-1}(cx))}{x} + 2c^2d^2x(a+b\sinh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\sinh^{-1}(cx))}{x} + 2c^2d^2x(a+b\sinh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{5}{3}bcd^2\sqrt{1+c^2x^2} - \frac{1}{9}bcd^2(1+c^2x^2)^{3/2} - \frac{d^2(a+b\sinh^{-1}(cx))}{x} +$$

$$= -\frac{5}{3}bcd^2\sqrt{1+c^2x^2} - \frac{1}{9}bcd^2(1+c^2x^2)^{3/2} - \frac{d^2(a+b\sinh^{-1}(cx))}{x} +$$

Mathematica [A]
time = 0.07, size = 124, normalized size = 1.03

$$\frac{d^2\left(-9a+18ac^2x^2+3ac^4x^4-16bcx\sqrt{1+c^2x^2}-bc^3x^3\sqrt{1+c^2x^2}+3b(-3+6c^2x^2+c^4x^4)\sinh^{-1}(cx)+9bcx\log(x)-9bcx\log\left(1+\sqrt{1+c^2x^2}\right)\right)}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-9*a + 18*a*c^2*x^2 + 3*a*c^4*x^4 - 16*b*c*x*Sqrt[1 + c^2*x^2] - b*c^
3*x^3*Sqrt[1 + c^2*x^2] + 3*b*(-3 + 6*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b
*c*x*Log[x] - 9*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(9*x)
```

Maple [A]
time = 0.54, size = 114, normalized size = 0.95

method	result
derivativedivides	$c\left(a\,d^2\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+2\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{c^2x^2\sqrt{c^2}}{9}\right.\right.$
default	$c\left(a\,d^2\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+2\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{c^2x^2\sqrt{c^2}}{9}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a*d^2*(1/3*c^3*x^3+2*c*x-1/c/x)+d^2*b*(1/3*arcsinh(c*x)*c^3*x^3+2*arcsin
h(c*x)*c*x-arcsinh(c*x)/c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-16/9*(c^2*x^2+1)^
(1/2)-arctanh(1/(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.26, size = 143, normalized size = 1.19

$$\frac{1}{3}\,ac^4d^2x^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^4d^2+2\,ac^2d^2x+2\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^2x^2+1}\right)bcd^2-\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}\left(cx\right)}{x}\right)bd^2-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^4*d^2 + 2*a*c^2*d^2*x + 2*(c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*c*d^2 - (c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*
b*d^2 - a*d^2/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(108) = 216$.
time = 0.37, size = 228, normalized size = 1.90

$$\frac{3\,ac^4d^2x^4+18\,ac^2d^2x^2-9\,bcd^2x\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+9\,bcd^2x\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-3\,(bc^4+6\,bc^2-3\,b)d^2x\log\left(-cx+\sqrt{c^2x^2+1}\right)-9\,ad^2+3\,(bc^4d^2x^4+6\,bc^2d^2x^2-(bc^4+6\,bc^2-3\,b)d^2x-3\,bd^2)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bc^3d^2x^3+16\,bcd^2x)\sqrt{c^2x^2+1}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^4*d^2*x^4 + 18*a*c^2*d^2*x^2 - 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x
^2 + 1) + 1) + 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 3*(b*c^4 + 6
*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1)) - 9*a*d^2 + 3*(b*c^4*d^2*
x^4 + 6*b*c^2*d^2*x^2 - (b*c^4 + 6*b*c^2 - 3*b)*d^2*x - 3*b*d^2)*log(c*x +
sqrt(c^2*x^2 + 1)) - (b*c^3*d^2*x^3 + 16*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int 2ac^2\,dx+\int\frac{a}{x^2}\,dx+\int ac^4x^2\,dx+\int 2bc^2\operatorname{asinh}\left(cx\right)dx+\int\frac{b\operatorname{asinh}\left(cx\right)}{x^2}\,dx+\int bc^4x^2\operatorname{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2, x
) + Integral(2*b*c**2*asinh(c*x), x) + Integral(b*asinh(c*x)/x**2, x) + Int
egral(b*c**4*x**2*asinh(c*x), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^2, x)
```

3.17 $\int \frac{(d+c^2dx^2)^2 (a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=187

$$\frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2}-\frac{bcd^2(1+c^2x^2)^{3/2}}{2x}+\frac{1}{4}bc^2d^2\sinh^{-1}(cx)+c^2d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2(1+c^2x^2)^2}{}$$

```
[Out] -1/2*b*c*d^2*(c^2*x^2+1)^(3/2)/x+1/4*b*c^2*d^2*arcsinh(c*x)+c^2*d^2*(c^2*x^
2+1)*(a+b*arcsinh(c*x))-1/2*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/x^2+c^2*d^
2*(a+b*arcsinh(c*x))^2/b+2*c^2*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+
1)^(1/2))^2)-b*c^2*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)+1/4*b*c^3*d^2
*x*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5802, 283, 201, 221, 5801, 5775, 3797, 2221, 2317, 2438}

$$c^2d^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{c^2d^2\left(a+b\sinh^{-1}(cx)\right)^2}{b}+2c^2d^2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-bc^2d^2\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)-\frac{bcd^2(c^2x^2+1)^{3/2}}{2x}+\frac{1}{4}bc^2d^2\sinh^{-1}(cx)+\frac{1}{4}bc^3d^2x\sqrt{c^2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (b*c^3*d^2*x*Sqrt[1 + c^2*x^2])/4 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/(2*x) + (
b*c^2*d^2*ArcSinh[c*x])/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (d
^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(2*x^2) + (c^2*d^2*(a + b*ArcSinh[
c*x])^2)/b + 2*c^2*d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] -
b*c^2*d^2*PolyLog[2, E^(-2*ArcSinh[c*x])]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^2(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)}{2x^2} + (2c^2d)\int \frac{(d+c^2dx^2)\left(a+b\sin\right.}{x}$$

$$= -\frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + c^2d^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right) - \frac{d^2(1+c^2}{}$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + c^2d^2(1+c^2x^2)\left(a+b\sinh\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\sinh^{-1}(cx) + c^2d^2$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\sinh^{-1}(cx) + c^2d^2$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\sinh^{-1}(cx) + c^2d^2$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\sinh^{-1}(cx) + c^2d^2$$

Mathematica [A]
time = 0.13, size = 170, normalized size = 0.91

$$\frac{d^2\left(-2a+2ac^4x^4-2bcx\sqrt{1+c^2x^2}-bc^3x^3\sqrt{1+c^2x^2}+4bc^2x^2\sinh^{-1}(cx)^2+bc^2x^2\tanh^{-1}\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+2b\sinh^{-1}(cx)\left(-1+c^4x^4+4c^2x^2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)+8ac^2x^2\log(x)-4bc^2x^2\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^2*(-2*a + 2*a*c^4*x^4 - 2*b*c*x*Sqrt[1 + c^2*x^2] - b*c^3*x^3*Sqrt[1 + c
^2*x^2] + 4*b*c^2*x^2*ArcSinh[c*x]^2 + b*c^2*x^2*ArcTanh[(c*x)/Sqrt[1 + c^2
*x^2]] + 2*b*ArcSinh[c*x]*(-1 + c^4*x^4 + 4*c^2*x^2*Log[1 - E^(-2*ArcSinh[c
*x])]) + 8*a*c^2*x^2*Log[x] - 4*b*c^2*x^2*PolyLog[2, E^(-2*ArcSinh[c*x])]))
/(4*x^2)
```

Maple [A]
time = 5.10, size = 248, normalized size = 1.33

method	result
derivativedivides	$c^2\left(\frac{a\,d^2c^2x^2}{2}-\frac{a\,d^2}{2c^2x^2}+2a\,d^2\ln\left(cx\right)-d^2b\operatorname{arcsinh}\left(cx\right)^2+\frac{d^2b\operatorname{arcsinh}(cx)c^2x^2}{2}-\frac{bc\,d^2x\sqrt{c^2x^2+}}{4}\right.$
default	$c^2\left(\frac{a\,d^2c^2x^2}{2}-\frac{a\,d^2}{2c^2x^2}+2a\,d^2\ln\left(cx\right)-d^2b\operatorname{arcsinh}\left(cx\right)^2+\frac{d^2b\operatorname{arcsinh}(cx)c^2x^2}{2}-\frac{bc\,d^2x\sqrt{c^2x^2+}}{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a*d^2*c^2*x^2-1/2*a*d^2/c^2/x^2+2*a*d^2*ln(c*x)-d^2*b*arcsinh(c*x)
^2+1/2*d^2*b*arcsinh(c*x)*c^2*x^2-1/4*b*c*d^2*x*(c^2*x^2+1)^(1/2)+1/4*b*d^2
*arcsinh(c*x)+1/2*d^2*b-1/2*d^2*b/c/x*(c^2*x^2+1)^(1/2)-1/2*d^2*b*arcsinh(c
*x)/c^2/x^2+2*d^2*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d^2*b*polylo
g(2,-c*x-(c^2*x^2+1)^(1/2))+2*d^2*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2)
)+2*d^2*b*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^2*x^2 + 2*a*c^2*d^2*log(x) - 1/2*b*d^2*(sqrt(c^2*x^2 + 1)*c/x +
 arcsinh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*b*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\frac{a}{x^3}\,dx+\int\frac{2ac^2}{x}\,dx+\int ac^4x\,dx+\int\frac{b\operatorname{asinh}\left(cx\right)}{x^3}\,dx+\int\frac{2bc^2\operatorname{asinh}\left(cx\right)}{x}\,dx+\int bc^4x\operatorname{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(2*a*c**2/x, x) + Integral(a*c**4*x, x)
 + Integral(b*asinh(c*x)/x**3, x) + Integral(2*b*c**2*asinh(c*x)/x, x) + In
tegral(b*c**4*x*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^3, x)
```

3.18 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=126

$$-bc^3d^2\sqrt{1+c^2x^2}-\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2(a+b\sinh^{-1}(cx))}{3x^3}-\frac{2c^2d^2(a+b\sinh^{-1}(cx))}{x}+c^4d^2x(a+b\sinh^{-1}(c$$

```
[Out] -1/3*d^2*(a+b*arcsinh(c*x))/x^3-2*c^2*d^2*(a+b*arcsinh(c*x))/x+c^4*d^2*x*(a
+b*arcsinh(c*x))-11/6*b*c^3*d^2*arctanh((c^2*x^2+1)^(1/2))-b*c^3*d^2*(c^2*x
^2+1)^(1/2)-1/6*b*c*d^2*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5803, 12, 1265, 911, 1171, 396, 214}

$$c^4d^2x(a+b\sinh^{-1}(cx))-\frac{2c^2d^2(a+b\sinh^{-1}(cx))}{x}-\frac{d^2(a+b\sinh^{-1}(cx))}{3x^3}-\frac{bcd^2\sqrt{c^2x^2+1}}{6x^2}-bc^3d^2\sqrt{c^2x^2+1}-\frac{11}{6}bc^3d^2\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c^3*d^2*Sqrt[1 + c^2*x^2]) - (b*c*d^2*Sqrt[1 + c^2*x^2])/(6*x^2) - (d^2
*(a + b*ArcSinh[c*x]))/(3*x^3) - (2*c^2*d^2*(a + b*ArcSinh[c*x]))/x + c^4*d
^2*x*(a + b*ArcSinh[c*x]) - (11*b*c^3*d^2*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2\,(a+b\sinh^{-1}(cx))}{x^4}\,dx = -\frac{d^2(a+b\sinh^{-1}(cx))}{3x^3} - \frac{2c^2d^2(a+b\sinh^{-1}(cx))}{x} + c^4d^2x(a+b\,\mathrm{si}$$

$$= -\frac{d^2(a+b\sinh^{-1}(cx))}{3x^3} - \frac{2c^2d^2(a+b\sinh^{-1}(cx))}{x} + c^4d^2x(a+b\,\mathrm{si}$$

$$= -\frac{d^2(a+b\sinh^{-1}(cx))}{3x^3} - \frac{2c^2d^2(a+b\sinh^{-1}(cx))}{x} + c^4d^2x(a+b\,\mathrm{si}$$

$$= -\frac{d^2(a+b\sinh^{-1}(cx))}{3x^3} - \frac{2c^2d^2(a+b\sinh^{-1}(cx))}{x} + c^4d^2x(a+b\,\mathrm{si}$$

$$= -\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2} - \frac{d^2(a+b\sinh^{-1}(cx))}{3x^3} - \frac{2c^2d^2(a+b\sinh^{-1}(cx}{x}$$

$$= -bc^3d^2\sqrt{1+c^2x^2} - \frac{bcd^2\sqrt{1+c^2x^2}}{6x^2} - \frac{d^2(a+b\sinh^{-1}(cx))}{3x^3} - \frac{2c}{}$$

$$= -bc^3d^2\sqrt{1+c^2x^2} - \frac{bcd^2\sqrt{1+c^2x^2}}{6x^2} - \frac{d^2(a+b\sinh^{-1}(cx))}{3x^3} - \frac{2c}{}$$

Mathematica [A]
time = 0.07, size = 133, normalized size = 1.06

$$\frac{d^2\left(-2a-12ac^2x^2+6ac^4x^4-bcx\sqrt{1+c^2x^2}-6bc^3x^3\sqrt{1+c^2x^2}+2b(-1-6c^2x^2+3c^4x^4)\sinh^{-1}(cx)+11bc^3x^3\log(x)-11bc^3x^3\log\left(1+\sqrt{1+c^2x^2}\right)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^2*(-2*a - 12*a*c^2*x^2 + 6*a*c^4*x^4 - b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3
*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-1 - 6*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] + 11
*b*c^3*x^3*Log[x] - 11*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(6*x^3)
```

Maple [A]
time = 0.53, size = 114, normalized size = 0.90

method	result
derivativedivides	$c^3\left(a\,d^2\left(cx-\frac{2}{cx}-\frac{1}{3c^3x^3}\right)+d^2b\left(\operatorname{arcsinh}(cx)\,cx-\frac{2\operatorname{arcsinh}(cx)}{cx}-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\sqrt{c^2x^2+1}\right.\right.$

default	$c^{3}\left(a\,d^{2}\left(cx-\frac{2}{cx}-\frac{1}{3c^{3}x^{3}}\right)+d^{2}b\left(\operatorname{arcsinh}\left(cx\right)cx-\frac{2\operatorname{arcsinh}(cx)}{cx}-\frac{\operatorname{arcsinh}(cx)}{3c^{3}x^{3}}-\sqrt{c^{2}x^{2}+1}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a*d^2*(c*x-2/c/x-1/3/c^3/x^3)+d^2*b*(arcsinh(c*x)*c*x-2*arcsinh(c*x)/c
/x-1/3*arcsinh(c*x)/c^3/x^3-(c^2*x^2+1)^(1/2)-11/6*arctanh(1/(c^2*x^2+1)^(1
/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.28, size = 137, normalized size = 1.09

$$ac^{4}d^{2}x+\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^{2}x^{2}+1}\right)bc^{3}d^{2}-2\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}\left(cx\right)}{x}\right)bc^{2}d^{2}+\frac{1}{6}\left(\left(c^{2}\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^{2}x^{2}+1}}{x^{2}}\right)c-\frac{2\operatorname{arsinh}\left(cx\right)}{x^{3}}\right)bd^{2}-\frac{2\,ac^{2}d^{2}}{x}-\frac{ad^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c^3*d^2 - 2*(c*arcsi
nh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*c^2*d^2 + 1/6*((c^2*arcsinh(1/(c*abs(x
))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d^2 - 2*a*c^2*d^2/x
- 1/3*a*d^2/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(114) = 228.
time = 0.37, size = 243, normalized size = 1.93

$$\frac{6\,ac^{4}d^{2}x^{4}-11\,bc^{3}d^{2}x^{3}\log\left(-cx+\sqrt{c^{2}x^{2}+1}+1\right)+11\,bc^{3}d^{2}x^{3}\log\left(-cx+\sqrt{c^{2}x^{2}+1}-1\right)-12\,ac^{2}d^{2}x^{2}-2\,(3\,bc^{4}-6\,bc^{2}-b)d^{2}x^{3}\log\left(-cx+\sqrt{c^{2}x^{2}+1}\right)-2\,ad^{2}+2\,(3\,bc^{4}d^{2}x^{4}-6\,bc^{2}d^{2}x^{2}-(3\,bc^{4}-6\,bc^{2}-b)d^{2}x^{3}-bd^{2})\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)-(6\,bc^{3}d^{2}x^{3}+bcd^{2}x)\sqrt{c^{2}x^{2}+1}}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c^4*d^2*x^4 - 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) +
 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 12*a*c^2*d^2*x^2 - 2*
(3*b*c^4 - 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1)) - 2*a*d^2 + 2
*(3*b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (3*b*c^4 - 6*b*c^2 - b)*d^2*x^3 - b*d
^2)*log(c*x + sqrt(c^2*x^2 + 1)) - (6*b*c^3*d^2*x^3 + b*c*d^2*x)*sqrt(c^2*x
^2 + 1))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int ac^{4}\,dx+\int\frac{a}{x^{4}}\,dx+\int\frac{2ac^{2}}{x^{2}}\,dx+\int bc^{4}\operatorname{asinh}\left(cx\right)dx+\int\frac{b\operatorname{asinh}\left(cx\right)}{x^{4}}\,dx+\int\frac{2bc^{2}\operatorname{asinh}\left(cx\right)}{x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(2*a*c**2/x**2, x
) + Integral(b*c**4*asinh(c*x), x) + Integral(b*asinh(c*x)/x**4, x) + Integ
ral(2*b*c**2*asinh(c*x)/x**2, x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^4, x)
```

3.19 $\int x^4(d+c^2dx^2)^3\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=226

$$-\frac{16bd^3\sqrt{1+c^2x^2}}{1155c^5}-\frac{8bd^3(1+c^2x^2)^{3/2}}{3465c^5}-\frac{2bd^3(1+c^2x^2)^{5/2}}{1925c^5}-\frac{bd^3(1+c^2x^2)^{7/2}}{1617c^5}+\frac{4bd^3(1+c^2x^2)^{9/2}}{297c^5}-\frac{bd^3(1+c^2x^2)^{11/2}}{121c^5}$$

```
[Out] -8/3465*b*d^3*(c^2*x^2+1)^(3/2)/c^5-2/1925*b*d^3*(c^2*x^2+1)^(5/2)/c^5-1/16
17*b*d^3*(c^2*x^2+1)^(7/2)/c^5+4/297*b*d^3*(c^2*x^2+1)^(9/2)/c^5-1/121*b*d^
3*(c^2*x^2+1)^(11/2)/c^5+1/5*d^3*x^5*(a+b*arcsinh(c*x))+3/7*c^2*d^3*x^7*(a+
b*arcsinh(c*x))+1/3*c^4*d^3*x^9*(a+b*arcsinh(c*x))+1/11*c^6*d^3*x^11*(a+b*a
rcsinh(c*x))-16/1155*b*d^3*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A]
time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {276, 5803, 12, 1813, 1634}

$$\frac{1}{11}c^6d^3x^{11}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^3(c^2x^2+1)^{11/2}}{121c^5}+\frac{4bd^3(c^2x^2+1)^{9/2}}{297c^5}-\frac{bd^3(c^2x^2+1)^{7/2}}{1617c^5}-\frac{2bd^3(c^2x^2+1)^{5/2}}{1925c^5}-\frac{8bd^3(c^2x^2+1)^{3/2}}{3465c^5}-\frac{16bd^3\sqrt{c^2x^2+1}}{1155c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(1155*c^5) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(3
465*c^5) - (2*b*d^3*(1 + c^2*x^2)^(5/2))/(1925*c^5) - (b*d^3*(1 + c^2*x^2)^
(7/2))/(1617*c^5) + (4*b*d^3*(1 + c^2*x^2)^(9/2))/(297*c^5) - (b*d^3*(1 + c
^2*x^2)^(11/2))/(121*c^5) + (d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^2*d^3*x
^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcSinh[c*x]))/3 + (c^6*d^
3*x^11*(a + b*ArcSinh[c*x]))/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^4\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x$$

$$=\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x$$

$$=\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x$$

$$=\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x$$

$$=-\frac{16bd^3\sqrt{1+c^2x^2}}{1155c^5}-\frac{8bd^3(1+c^2x^2)^{3/2}}{3465c^5}-\frac{2bd^3(1+c^2x^2)^{5/2}}{1925c^5}-$$

Mathematica [A]
time = 0.08, size = 143, normalized size = 0.63

$$\frac{d^3\left(3465ac^5x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)-b\sqrt{1+c^2x^2}\,(50488-25244c^2x^2+18933c^4x^4+117625c^6x^6+111475c^8x^8+33075c^{10}x^{10})+3465bc^5x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)\sinh^{-1}(cx)\right)}{4002075c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6) - b*Sq
rt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 117625*c^6*x^6 + 1
```

```
11475*c^8*x^8 + 33075*c^10*x^10) + 3465*b*c^5*x^5*(231 + 495*c^2*x^2 + 385*
c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*x]))/(4002075*c^5)
```

Maple [A]
time = 1.01, size = 206, normalized size = 0.91

method	result
derivativedivides	$\frac{d^3a\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\operatorname{arcsinh}(cx)c^9x^9}{3}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-c^{10}\right.}{}$
default	$\frac{d^3a\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\operatorname{arcsinh}(cx)c^9x^9}{3}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-c^{10}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(d^3*a*(1/11*c^11*x^11+1/3*c^9*x^9+3/7*c^7*x^7+1/5*c^5*x^5)+d^3*b*(1/
11*arcsinh(c*x)*c^11*x^11+1/3*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7
+1/5*arcsinh(c*x)*c^5*x^5-1/121*c^10*x^10*(c^2*x^2+1)^(1/2)-91/3267*c^8*x^8
*(c^2*x^2+1)^(1/2)-4705/160083*c^6*x^6*(c^2*x^2+1)^(1/2)-6311/1334025*c^4*x
^4*(c^2*x^2+1)^(1/2)+25244/4002075*c^2*x^2*(c^2*x^2+1)^(1/2)-50488/4002075*
(c^2*x^2+1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(194) = 388.
time = 0.27, size = 465, normalized size = 2.06

$$\frac{1}{11}ac^6d^3x^{11}+\frac{1}{3}ac^4d^3x^9+\frac{3}{7}ac^2d^3x^7+\frac{1}{7623}\left(693x^{11}\operatorname{arsinh}(cx)-\left(\frac{63\sqrt{c^2x^2+1}x^{10}}{c^2}-\frac{70\sqrt{c^2x^2+1}x^8}{c^4}+\frac{80\sqrt{c^2x^2+1}x^6}{c^6}-\frac{96\sqrt{c^2x^2+1}x^4}{c^8}+\frac{128\sqrt{c^2x^2+1}x^2}{c^{10}}-\frac{256\sqrt{c^2x^2+1}}{c^{12}}\right)c\right)bc^6d^3+\frac{1}{945}\left(315x^9\operatorname{arsinh}(cx)-\left(\frac{35\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}x^2}{c^8}+\frac{128\sqrt{c^2x^2+1}}{c^{10}}\right)c\right)bc^4d^3+\frac{1}{5}ad^3x^5+\frac{3}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d^3+\frac{1}{75}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 + 3/7*a*c^2*d^3*x^7 + 1/7623*(693*x
^11*arcsinh(c*x) - (63*sqrt(c^2*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^
8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*s
qrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqrt(c^2*x^2 + 1)/c^12)*c)*b*c^6*d^3 + 1/94
5*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 +
 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 +
 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 + 3/245*(35*x^7*a
rcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 +
8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^3 + 1/75
*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*
x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^3
```

Fricas [A]
time = 0.35, size = 201, normalized size = 0.89

$$\frac{363825\,ac^{11}d^3x^{11}+1334025\,ac^9d^3x^9+1715175\,ac^7d^3x^7+800415\,ac^5d^3x^5+3465\left(105\,bc^{11}d^3x^{11}+385\,bc^9d^3x^9+495\,bc^7d^3x^7+231\,bc^5d^3x^5\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(33075\,bc^{10}d^3x^{10}+111475\,bc^8d^3x^8+117625\,bc^6d^3x^6+18933\,bc^4d^3x^4-25244\,bc^2d^3x^2+50488\,bd^3\right)\sqrt{c^2x^2+1}}{4002075\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4002075*(363825*a*c^11*d^3*x^11 + 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*d
^3*x^7 + 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 + 385*b*c^9*d^3*x
^9 + 495*b*c^7*d^3*x^7 + 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1)) -
(33075*b*c^10*d^3*x^10 + 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 + 1893
3*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 + 50488*b*d^3)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A]
time = 3.25, size = 289, normalized size = 1.28

$$\begin{cases} \frac{ac^6d^3x^{11}}{11} + \frac{ac^4d^3x^9}{3} + \frac{3ac^2d^3x^7}{7} + \frac{ad^3x^5}{5} + \frac{bc^6d^3x^{11}\operatorname{asinh}(cx)}{11} - \frac{bc^5d^3x^{10}\sqrt{c^2x^2+1}}{121} + \frac{bc^4d^3x^9\operatorname{asinh}(cx)}{3} - \frac{91bc^3d^3x^8\sqrt{c^2x^2+1}}{3267} + \frac{3bc^2d^3x^7\operatorname{asinh}(cx)}{7} - \frac{4705bcd^3x^6\sqrt{c^2x^2+1}}{160083} + \frac{bd^3x^5\operatorname{asinh}(cx)}{5} - \frac{6311bd^3x^4\sqrt{c^2x^2+1}}{1334025c} + \frac{25244bd^3x^2\sqrt{c^2x^2+1}}{4002075c^3} - \frac{50488bd^3\sqrt{c^2x^2+1}}{4002075c^5} & \text{for}\ c \neq 0 \\ \frac{ad^3x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 + 3*a*c**2*d**3*x**7/7
 + a*d**3*x**5/5 + b*c**6*d**3*x**11*asinh(c*x)/11 - b*c**5*d**3*x**10*sqrt
(c**2*x**2 + 1)/121 + b*c**4*d**3*x**9*asinh(c*x)/3 - 91*b*c**3*d**3*x**8*s
qrt(c**2*x**2 + 1)/3267 + 3*b*c**2*d**3*x**7*asinh(c*x)/7 - 4705*b*c*d**3*x
**6*sqrt(c**2*x**2 + 1)/160083 + b*d**3*x**5*asinh(c*x)/5 - 6311*b*d**3*x**
4*sqrt(c**2*x**2 + 1)/(1334025*c) + 25244*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(
4002075*c**3) - 50488*b*d**3*sqrt(c**2*x**2 + 1)/(4002075*c**5), Ne(c, 0)),
 (a*d**3*x**5/5, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.20 $\int x^3(d+c^2dx^2)^3\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=199

$$\frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3}+\frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3}+\frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3}+\frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3}-\frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3}+\frac{49b}{}$$

[Out] 49/7680*b*d^3*x*(c^2*x^2+1)^(3/2)/c^3+49/9600*b*d^3*x*(c^2*x^2+1)^(5/2)/c^3
+7/1600*b*d^3*x*(c^2*x^2+1)^(7/2)/c^3-1/100*b*d^3*x*(c^2*x^2+1)^(9/2)/c^3+4
9/5120*b*d^3*arcsinh(c*x)/c^4-1/8*d^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))/c^4+
1/10*d^3*(c^2*x^2+1)^5*(a+b*arcsinh(c*x))/c^4+49/5120*b*d^3*x*(c^2*x^2+1)^(
1/2)/c^3

Rubi [A]
time = 0.12, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {272, 45, 5803, 12, 396, 201, 221}

$$\frac{d^3(c^2x^2+1)^5\left(a+b\sinh^{-1}(cx)\right)}{10c^4}-\frac{d^3(c^2x^2+1)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4}+\frac{49bd^3\sinh^{-1}(cx)}{5120c^4}-\frac{bd^3x(c^2x^2+1)^{9/2}}{100c^3}+\frac{7bd^3x(c^2x^2+1)^{7/2}}{1600c^3}+\frac{49bd^3x(c^2x^2+1)^{5/2}}{9600c^3}+\frac{49bd^3x(c^2x^2+1)^{3/2}}{7680c^3}+\frac{49bd^3x\sqrt{c^2x^2+1}}{5120c^3}$$

Antiderivative was successfully verified.

[In] Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]

[Out] (49*b*d^3*x*Sqrt[1 + c^2*x^2])/(5120*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(3/2)
)/(7680*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(5/2))/(9600*c^3) + (7*b*d^3*x*(1
+ c^2*x^2)^(7/2))/(1600*c^3) - (b*d^3*x*(1 + c^2*x^2)^(9/2))/(100*c^3) + (4
9*b*d^3*ArcSinh[c*x])/(5120*c^4) - (d^3*(1 + c^2*x^2)^4*(a + b*ArcSinh[c*x]
))/(8*c^4) + (d^3*(1 + c^2*x^2)^5*(a + b*ArcSinh[c*x]))/(10*c^4)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)\,dx &= -\frac{d^3(1+c^2x^2)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4}+\frac{d^3(1+c^2x^2)^5\left(a+b\sinh^{-1}(\right.}{10c^4} \\
&= -\frac{d^3(1+c^2x^2)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4}+\frac{d^3(1+c^2x^2)^5\left(a+b\sinh^{-1}(\right.}{10c^4} \\
&= -\frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3}-\frac{d^3(1+c^2x^2)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4}+\frac{d^3(1+}{} \\
&= \frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3}-\frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3}-\frac{d^3(1+c^2x^2)^4\left(a+b\sin\right.}{8c^4} \\
&= \frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3}+\frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3}-\frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3}- \\
&= \frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3}+\frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3}+\frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3} \\
&= \frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3}+\frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3}+\frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3} \\
&= \frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3}+\frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3}+\frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3}
\end{aligned}
$$

Mathematica [A]

time = 0.08, size = 139, normalized size = 0.70

$$
\frac{d^3\left(1920ac^4x^4(10+20c^2x^2+15c^4x^4+4c^6x^6)-bcx\sqrt{1+c^2x^2}\,(-1185+790c^2x^2+3208c^4x^4+2736c^6x^6+768c^8x^8)+15b(-79+1280c^4x^4+2560c^6x^6+1920c^8x^8+512c^{10}x^{10})\sinh^{-1}(cx)\right)}{76800c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(1920*a*c^4*x^4*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - b*c*x*Sqr
t[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8
*x^8) + 15*b*(-79 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x
^10)*ArcSinh[c*x]))/(76800*c^4)
```

Maple [A]

time = 1.79, size = 173, normalized size = 0.87

method	result
derivativedivides	$\frac{d^3a\left(\frac{\left(c^2x^2+1\right)^5}{10}-\frac{\left(c^2x^2+1\right)^4}{8}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{10}x^{10}}{10}+\frac{3\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{79\operatorname{arcsinh}(c}{5120}\right.}{c^4}$

default	$\frac{d^3 a\left(\frac{\left(c^2 x^2+1\right)^5}{10}-\frac{\left(c^2 x^2+1\right)^4}{8}\right)+d^3 b\left(\frac{\operatorname{arcsinh}(cx) c^{10} x^{10}}{10}+\frac{3 \operatorname{arcsinh}(cx) c^8 x^8}{8}+\frac{\operatorname{arcsinh}(cx) c^6 x^6}{2}+\frac{\operatorname{arcsinh}(cx) c^4 x^4}{4}-\frac{79 \operatorname{arcsinh}}{5120}\right.}{c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^3*a*(1/10*(c^2*x^2+1)^5-1/8*(c^2*x^2+1)^4)+d^3*b*(1/10*arcsinh(c*x
)*c^10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c
*x)*c^4*x^4-79/5120*arcsinh(c*x)-1/100*c*x*(c^2*x^2+1)^(9/2)+7/1600*c*x*(c^
2*x^2+1)^(7/2)+49/9600*c*x*(c^2*x^2+1)^(5/2)+49/7680*c*x*(c^2*x^2+1)^(3/2)+
49/5120*(c^2*x^2+1)^(1/2)*c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(173) = 346$.
time = 0.26, size = 429, normalized size = 2.16

$$\frac{1}{10} a c^6 d^3 x^{10} + \frac{3}{8} a c^4 d^3 x^8 + \frac{1}{2} a c^2 d^3 x^6 + \frac{1}{12800}\left(1280 x^{10} \operatorname{arsinh}(cx) - \left(\frac{128 \sqrt{c^2x^2+1}\, x^9}{c^2} - \frac{144 \sqrt{c^2x^2+1}\, x^7}{c^4} + \frac{168 \sqrt{c^2x^2+1}\, x^5}{c^6} - \frac{210 \sqrt{c^2x^2+1}\, x^3}{c^8} + \frac{315 \sqrt{c^2x^2+1}\, x}{c^{10}} - \frac{315 \operatorname{arsinh}(cx)}{c^{11}}\right)c\right) b c^6 d^3 + \frac{1}{1024}\left(384 x^8 \operatorname{arsinh}(cx) - \left(\frac{48 \sqrt{c^2x^2+1}\, x^7}{c^2} - \frac{56 \sqrt{c^2x^2+1}\, x^5}{c^4} + \frac{70 \sqrt{c^2x^2+1}\, x^3}{c^6} - \frac{105 \sqrt{c^2x^2+1}\, x}{c^8} + \frac{105 \operatorname{arsinh}(cx)}{c^9}\right)c\right) b c^4 d^3 + \frac{1}{4} a d^3 x^4 + \frac{1}{96}\left(48 x^6 \operatorname{arsinh}(cx) - \left(\frac{8 \sqrt{c^2x^2+1}\, x^5}{c^2} - \frac{10 \sqrt{c^2x^2+1}\, x^3}{c^4} + \frac{15 \sqrt{c^2x^2+1}\, x}{c^6} - \frac{15 \operatorname{arsinh}(cx)}{c^7}\right)c\right) b c^2 d^3 + \frac{1}{32}\left(8 x^4 \operatorname{arsinh}(cx) - \left(\frac{2 \sqrt{c^2x^2+1}\, x^3}{c^2} - \frac{3 \sqrt{c^2x^2+1}\, x}{c^4} + \frac{3 \operatorname{arsinh}(cx)}{c^5}\right)c\right) b d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 + 1/2*a*c^2*d^3*x^6 + 1/12800*(1280
*x^10*arcsinh(c*x) - (128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)
*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 +
315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c*x)/c^11)*c)*b*c^6*d^3 + 1/1024
*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 +
1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 1
05*arcsinh(c*x)/c^9)*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*
x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(
c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^2*d^3 + 1/32*(8*x^4*arcsin
h(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsi
nh(c*x)/c^5)*c)*b*d^3
```

Fricas [A]
time = 0.38, size = 197, normalized size = 0.99

$$\frac{7680 a c^{10} d^3 x^{10} + 28800 a c^8 d^3 x^8 + 38400 a c^6 d^3 x^6 + 19200 a c^4 d^3 x^4 + 15\left(512 b c^{10} d^3 x^{10} + 1920 b c^8 d^3 x^8 + 2560 b c^6 d^3 x^6 + 1280 b c^4 d^3 x^4 - 79 b d^3\right) \log\left(cx + \sqrt{c^2x^2+1}\right) - \left(768 b c^9 d^3 x^9 + 2736 b c^7 d^3 x^7 + 3208 b c^5 d^3 x^5 + 790 b c^3 d^3 x^3 - 1185 b c d^3 x\right)\sqrt{c^2x^2+1}}{76800 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/76800*(7680*a*c^10*d^3*x^10 + 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6 +
 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 + 1920*b*c^8*d^3*x^8 + 2560*
b*c^6*d^3*x^6 + 1280*b*c^4*d^3*x^4 - 79*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1))
```

```
- (768*b*c^9*d^3*x^9 + 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 + 790*b*c^3
*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A]
time = 2.09, size = 280, normalized size = 1.41

$$\begin{cases} \frac{ac^6d^3x^{10}}{10} + \frac{3ac^4d^3x^8}{8} + \frac{ac^2d^3x^6}{2} + \frac{ad^3x^4}{4} + \frac{bc^6d^3x^{10}\operatorname{asinh}(cx)}{10} - \frac{bc^5d^3x^9\sqrt{c^2x^2+1}}{100} + \frac{3bc^4d^3x^8\operatorname{asinh}(cx)}{8} - \frac{57bc^3d^3x^7\sqrt{c^2x^2+1}}{1600} + \frac{bc^2d^3x^6\operatorname{asinh}(cx)}{2} - \frac{401bcd^3x^5\sqrt{c^2x^2+1}}{9600} + \frac{bd^3x^4\operatorname{asinh}(cx)}{4} - \frac{79bd^3x^3\sqrt{c^2x^2+1}}{7680c} + \frac{79bd^3x\sqrt{c^2x^2+1}}{5120c^3} - \frac{79bd^3\operatorname{asinh}(cx)}{5120c^4} & \text{for}\ c \neq 0 \\ \frac{ad^3x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 + a*c**2*d**3*x**6/2
 + a*d**3*x**4/4 + b*c**6*d**3*x**10*asinh(c*x)/10 - b*c**5*d**3*x**9*sqrt(
c**2*x**2 + 1)/100 + 3*b*c**4*d**3*x**8*asinh(c*x)/8 - 57*b*c**3*d**3*x**7*
sqrt(c**2*x**2 + 1)/1600 + b*c**2*d**3*x**6*asinh(c*x)/2 - 401*b*c*d**3*x**
5*sqrt(c**2*x**2 + 1)/9600 + b*d**3*x**4*asinh(c*x)/4 - 79*b*d**3*x**3*sqrt
(c**2*x**2 + 1)/(7680*c) + 79*b*d**3*x*sqrt(c**2*x**2 + 1)/(5120*c**3) - 79
*b*d**3*asinh(c*x)/(5120*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.21 $\int x^2(d+c^2dx^2)^3\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=202

$$\frac{16bd^3\sqrt{1+c^2x^2}}{315c^3}+\frac{8bd^3(1+c^2x^2)^{3/2}}{945c^3}+\frac{2bd^3(1+c^2x^2)^{5/2}}{525c^3}+\frac{bd^3(1+c^2x^2)^{7/2}}{441c^3}-\frac{bd^3(1+c^2x^2)^{9/2}}{81c^3}+\frac{1}{3}d^3x^3(a+b$$

```
[Out] 8/945*b*d^3*(c^2*x^2+1)^(3/2)/c^3+2/525*b*d^3*(c^2*x^2+1)^(5/2)/c^3+1/441*b
*d^3*(c^2*x^2+1)^(7/2)/c^3-1/81*b*d^3*(c^2*x^2+1)^(9/2)/c^3+1/3*d^3*x^3*(a+
b*arcsinh(c*x))+3/5*c^2*d^3*x^5*(a+b*arcsinh(c*x))+3/7*c^4*d^3*x^7*(a+b*arc
sinh(c*x))+1/9*c^6*d^3*x^9*(a+b*arcsinh(c*x))+16/315*b*d^3*(c^2*x^2+1)^(1/2
)/c^3
```

Rubi [A]

time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {276, 5803, 12, 1813, 1634}

$$\frac{1}{9}c^6d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^3(c^2x^2+1)^{9/2}}{81c^3}+\frac{bd^3(c^2x^2+1)^{7/2}}{441c^3}+\frac{2bd^3(c^2x^2+1)^{5/2}}{525c^3}+\frac{8bd^3(c^2x^2+1)^{3/2}}{945c^3}+\frac{16bd^3\sqrt{c^2x^2+1}}{315c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (16*b*d^3*Sqrt[1 + c^2*x^2])/(315*c^3) + (8*b*d^3*(1 + c^2*x^2)^(3/2))/(945
*c^3) + (2*b*d^3*(1 + c^2*x^2)^(5/2))/(525*c^3) + (b*d^3*(1 + c^2*x^2)^(7/2
))/(441*c^3) - (b*d^3*(1 + c^2*x^2)^(9/2))/(81*c^3) + (d^3*x^3*(a + b*ArcSi
nh[c*x]))/3 + (3*c^2*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^4*d^3*x^7*(a +
b*ArcSinh[c*x]))/7 + (c^6*d^3*x^9*(a + b*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{16bd^3\sqrt{1+c^2x^2}}{315c^3}+\frac{8bd^3\left(1+c^2x^2\right)^{3/2}}{945c^3}+\frac{2bd^3\left(1+c^2x^2\right)^{5/2}}{525c^3}+\frac{bd^3}{}$$

Mathematica [A]
time = 0.07, size = 135, normalized size = 0.67

$$\frac{d^3\left(315ac^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)-b\sqrt{1+c^2x^2}\,(-5258+2629c^2x^2+6297c^4x^4+4675c^6x^6+1225c^8x^8)+315bc^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)\sinh^{-1}(cx)\right)}{99225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(315*a*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - b*Sqrt
[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6 + 1225*c^
8*x^8) + 315*b*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSi
nh[c*x]))/(99225*c^3)
```

Maple [A]
time = 1.02, size = 187, normalized size = 0.93

method	result
derivativedivides	$\frac{d^3a\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^8x^8}{\ldots}\right.}{c^3}$
default	$\frac{d^3a\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^8x^8}{\ldots}\right.}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^3*a*(1/9*c^9*x^9+3/7*c^7*x^7+3/5*c^5*x^5+1/3*c^3*x^3)+d^3*b*(1/9*a
rcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*a
rcsinh(c*x)*c^3*x^3-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^
2+1)^(1/2)-2099/33075*c^4*x^4*(c^2*x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^2
+1)^(1/2)+5258/99225*(c^2*x^2+1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(174) = 348$.
time = 0.26, size = 388, normalized size = 1.92

$$\frac{1}{9}ac^6d^3x^9+\frac{3}{7}ac^4d^3x^7+\frac{1}{2835}\left(315x^9\operatorname{arsinh}(cx)-\left(\frac{35\sqrt{c^2x^2+1}\,x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}\,x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2+1}}{c^{10}}\right)c\right)bc^6d^3+\frac{3}{5}ac^2d^3x^5+\frac{3}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^4d^3+\frac{1}{25}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d^3+\frac{1}{3}ad^3x^3+\frac{1}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
+ 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c
)*b*c^6*d^3 + 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*
x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^
6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^3 + 1/25*(15*x^5*arcsinh(c*x) - (3
*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^3
```

Fricas [A]
time = 0.38, size = 189, normalized size = 0.94

$$\frac{11025\,ac^9d^3x^9+42525\,ac^7d^3x^7+59535\,ac^5d^3x^5+33075\,ac^3d^3x^3+315\left(35\,bc^9d^3x^9+135\,bc^7d^3x^7+189\,bc^5d^3x^5+105\,bc^3d^3x^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(1225\,bc^8d^3x^8+4675\,bc^6d^3x^6+6297\,bc^4d^3x^4+2629\,bc^2d^3x^2-5258\,bd^3\right)\sqrt{c^2x^2+1}}{99225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^3*x^9 + 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5 +
33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 + 135*b*c^7*d^3*x^7 + 189*b*c^5
```

```
*d^3*x^5 + 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8*d^
3*x^8 + 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 + 2629*b*c^2*d^3*x^2 - 5258
*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 1.47, size = 265, normalized size = 1.31

$$\begin{cases} \frac{ac^6d^3x^9}{9} + \frac{3ac^4d^3x^7}{7} + \frac{3ac^2d^3x^5}{5} + \frac{ad^3x^3}{3} + \frac{bc^6d^3x^9\operatorname{asinh}(cx)}{9} - \frac{bc^5d^3x^8\sqrt{c^2x^2+1}}{81} + \frac{3bc^4d^3x^7\operatorname{asinh}(cx)}{7} - \frac{187bc^3d^3x^6\sqrt{c^2x^2+1}}{3969} + \frac{3bc^2d^3x^5\operatorname{asinh}(cx)}{5} - \frac{2099bcd^3x^4\sqrt{c^2x^2+1}}{33075} + \frac{bd^3x^3\operatorname{asinh}(cx)}{3} - \frac{2629bd^3x^2\sqrt{c^2x^2+1}}{99225c} + \frac{5258bd^3\sqrt{c^2x^2+1}}{99225c^3} & \text{for}\: c \neq 0 \\ \frac{ad^3x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 + 3*a*c**2*d**3*x**5/5
 + a*d**3*x**3/3 + b*c**6*d**3*x**9*asinh(c*x)/9 - b*c**5*d**3*x**8*sqrt(c*
*2*x**2 + 1)/81 + 3*b*c**4*d**3*x**7*asinh(c*x)/7 - 187*b*c**3*d**3*x**6*sq
rt(c**2*x**2 + 1)/3969 + 3*b*c**2*d**3*x**5*asinh(c*x)/5 - 2099*b*c*d**3*x*
*4*sqrt(c**2*x**2 + 1)/33075 + b*d**3*x**3*asinh(c*x)/3 - 2629*b*d**3*x**2*
sqrt(c**2*x**2 + 1)/(99225*c) + 5258*b*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3
), Ne(c, 0)), (a*d**3*x**3/3, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.22 $\int x(d+c^2dx^2)^3\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=145

$$-\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c}-\frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c}-\frac{7bd^3x(1+c^2x^2)^{5/2}}{384c}-\frac{bd^3x(1+c^2x^2)^{7/2}}{64c}-\frac{35bd^3\sinh^{-1}(cx)}{1024c^2}+\frac{d^3(\ldots}{}$$

```
[Out] -35/1536*b*d^3*x*(c^2*x^2+1)^(3/2)/c-7/384*b*d^3*x*(c^2*x^2+1)^(5/2)/c-1/64
*b*d^3*x*(c^2*x^2+1)^(7/2)/c-35/1024*b*d^3*arcsinh(c*x)/c^2+1/8*d^3*(c^2*x^
2+1)^4*(a+b*arcsinh(c*x))/c^2-35/1024*b*d^3*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5798, 201, 221}

$$\frac{d^3(c^2x^2+1)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{bd^3x(c^2x^2+1)^{7/2}}{64c}-\frac{7bd^3x(c^2x^2+1)^{5/2}}{384c}-\frac{35bd^3x(c^2x^2+1)^{3/2}}{1536c}-\frac{35bd^3x\sqrt{c^2x^2+1}}{1024c}-\frac{35bd^3\sinh^{-1}(cx)}{1024c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-35*b*d^3*x*Sqrt[1 + c^2*x^2])/(1024*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2))
/(1536*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2))/(384*c) - (b*d^3*x*(1 + c^2*x^2
)^(7/2))/(64*c) - (35*b*d^3*ArcSinh[c*x])/(1024*c^2) + (d^3*(1 + c^2*x^2)^4
*(a + b*ArcSinh[c*x]))/(8*c^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{d^3(1+c^2x^2)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{(bd^3)\int(1+c^2x^2)^{7/2}\,dx}{8c}$$
$$=-\frac{bd^3x(1+c^2x^2)^{7/2}}{64c}+\frac{d^3(1+c^2x^2)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{(7bd^3)\int$$
$$=-\frac{7bd^3x(1+c^2x^2)^{5/2}}{384c}-\frac{bd^3x(1+c^2x^2)^{7/2}}{64c}+\frac{d^3(1+c^2x^2)^4\left(a+b\,\mathrm{si}\right.}{8c^2}$$
$$=-\frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c}-\frac{7bd^3x(1+c^2x^2)^{5/2}}{384c}-\frac{bd^3x(1+c^2x^2)^{7/2}}{64c}+$$
$$=-\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c}-\frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c}-\frac{7bd^3x(1+c^2x^2)^{5/2}}{384c}$$
$$=-\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c}-\frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c}-\frac{7bd^3x(1+c^2x^2)^{5/2}}{384c}$$

Mathematica [A]
time = 0.09, size = 128, normalized size = 0.88

$$\frac{d^3\left(cx\left(384acx(4+6c^2x^2+4c^4x^4+c^6x^6)-b\sqrt{1+c^2x^2}\,(279+326c^2x^2+200c^4x^4+48c^6x^6)\right)+3b(93+512c^2x^2+768c^4x^4+512c^6x^6+128c^8x^8)\sinh^{-1}(cx)\right)}{3072c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(c*x*(384*a*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) - b*Sqrt[1 + c^2
*x^2]*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*b*(93 + 512*c^2*x
^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]))/(3072*c^2)
```

Maple [A]
time = 1.74, size = 143, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{d^3\left(c^2x^2+1\right)^4a}{8}+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{3\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{93\operatorname{arcsinh}(cx)}{1024}-\frac{cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{64}-\right.}{c^2}$
default	$\frac{\frac{d^3\left(c^2x^2+1\right)^4a}{8}+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{3\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{93\operatorname{arcsinh}(cx)}{1024}-\frac{cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{64}-\right.}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/8*d^3*(c^2*x^2+1)^4*a+d^3*b*(1/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(
c*x)*c^6*x^6+3/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+93/1024*arcs
inh(c*x)-1/64*c*x*(c^2*x^2+1)^(7/2)-7/384*c*x*(c^2*x^2+1)^(5/2)-35/1536*c*x
*(c^2*x^2+1)^(3/2)-35/1024*(c^2*x^2+1)^(1/2)*c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(125) = 250$.
time = 0.28, size = 352, normalized size = 2.43

$$\frac{1}{8}ac^6d^3x^8 + \frac{1}{2}ac^4d^3x^6 + \frac{1}{3072}\left(384x^8\operatorname{arsinh}(cx) - \left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2} - \frac{56\sqrt{c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{c^2x^2+1}x^3}{c^6} - \frac{105\sqrt{c^2x^2+1}x}{c^8} + \frac{105\operatorname{arsinh}(cx)}{c^9}\right)c\right)bc^6d^3 + \frac{3}{4}ac^2d^3x^4 + \frac{1}{96}\left(48x^6\operatorname{arsinh}(cx) - \left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{c^2x^2+1}x}{c^6} - \frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)bc^4d^3 + \frac{3}{32}\left(8x^4\operatorname{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)bc^2d^3 + \frac{1}{2}ad^3x^2 + \frac{1}{4}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48*
sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2
+ 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*b*c^6
*d^3 + 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)
*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*a
rcsinh(c*x)/c^7)*c)*b*c^4*d^3 + 3/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2
+ 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*c^2*d^3
 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - a
rcsinh(c*x)/c^3))*b*d^3
```

Fricas [A]
time = 0.36, size = 185, normalized size = 1.28

$$\frac{384\,ac^8d^3x^8 + 1536\,ac^6d^3x^6 + 2304\,ac^4d^3x^4 + 1536\,ac^2d^3x^2 + 3\,(128\,bc^8d^3x^8 + 512\,bc^6d^3x^6 + 768\,bc^4d^3x^4 + 512\,bc^2d^3x^2 + 93\,bd^3)\log\left(cx + \sqrt{c^2x^2+1}\right) - (48\,bc^7d^3x^7 + 200\,bc^5d^3x^5 + 326\,bc^3d^3x^3 + 279\,bcd^3x)\sqrt{c^2x^2+1}}{3072\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3072*(384*a*c^8*d^3*x^8 + 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 + 1536*
a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 + 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*x^
4 + 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (48*b*c^7*
d^3*x^7 + 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 + 279*b*c*d^3*x)*sqrt(c^2*x
^2 + 1))/c^2
```

Sympy [A]
time = 1.06, size = 253, normalized size = 1.74

$$\begin{cases} \frac{ac^6d^3x^8}{8} + \frac{ac^4d^3x^6}{2} + \frac{3ac^2d^3x^4}{4} + \frac{ad^3x^2}{2} + \frac{bc^6d^3x^8\operatorname{asinh}(cx)}{8} - \frac{bc^5d^3x^7\sqrt{c^2x^2+1}}{64} + \frac{bc^4d^3x^6\operatorname{asinh}(cx)}{2} - \frac{25bc^3d^3x^5\sqrt{c^2x^2+1}}{384} + \frac{3bc^2d^3x^4\operatorname{asinh}(cx)}{4} - \frac{163bcd^3x^3\sqrt{c^2x^2+1}}{1536} + \frac{bd^3x^2\operatorname{asinh}(cx)}{2} - \frac{93bd^3x\sqrt{c^2x^2+1}}{1024c} + \frac{93bd^3\operatorname{asinh}(cx)}{1024c^2} & \text{for}\ c \neq 0 \\ \frac{ad^3x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 + 3*a*c**2*d**3*x**4/4 +
 a*d**3*x**2/2 + b*c**6*d**3*x**8*asinh(c*x)/8 - b*c**5*d**3*x**7*sqrt(c**2
*x**2 + 1)/64 + b*c**4*d**3*x**6*asinh(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt(c*
*2*x**2 + 1)/384 + 3*b*c**2*d**3*x**4*asinh(c*x)/4 - 163*b*c*d**3*x**3*sqrt
(c**2*x**2 + 1)/1536 + b*d**3*x**2*asinh(c*x)/2 - 93*b*d**3*x*sqrt(c**2*x**
2 + 1)/(1024*c) + 93*b*d**3*asinh(c*x)/(1024*c**2), Ne(c, 0)), (a*d**3*x**2
/2, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.23 $\int \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=170

$$-\frac{16 b d^3 \sqrt{1+c^2 x^2}}{35 c}-\frac{8 b d^3 \left(1+c^2 x^2\right)^{3/2}}{105 c}-\frac{6 b d^3 \left(1+c^2 x^2\right)^{5/2}}{175 c}-\frac{b d^3 \left(1+c^2 x^2\right)^{7/2}}{49 c}+d^3 x \left(a+b \sinh^{-1}(cx)\right)+c^2 d^3 x$$

```
[Out] -8/105*b*d^3*(c^2*x^2+1)^(3/2)/c-6/175*b*d^3*(c^2*x^2+1)^(5/2)/c-1/49*b*d^3
*(c^2*x^2+1)^(7/2)/c+d^3*x*(a+b*arcsinh(c*x))+c^2*d^3*x^3*(a+b*arcsinh(c*x)
)+3/5*c^4*d^3*x^5*(a+b*arcsinh(c*x))+1/7*c^6*d^3*x^7*(a+b*arcsinh(c*x))-16/
35*b*d^3*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {200, 5784, 12, 1813, 1864}

$$\frac{1}{7} c^6 d^3 x^7 \left(a+b \sinh^{-1}(cx)\right)+\frac{3}{5} c^4 d^3 x^5 \left(a+b \sinh^{-1}(cx)\right)+c^2 d^3 x^3 \left(a+b \sinh^{-1}(cx)\right)+d^3 x \left(a+b \sinh^{-1}(cx)\right)-\frac{b d^3 \left(c^2 x^2+1\right)^{7/2}}{49 c}-\frac{6 b d^3 \left(c^2 x^2+1\right)^{5/2}}{175 c}-\frac{8 b d^3 \left(c^2 x^2+1\right)^{3/2}}{105 c}-\frac{16 b d^3 \sqrt{c^2 x^2+1}}{35 c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(35*c) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(105*c
) - (6*b*d^3*(1 + c^2*x^2)^(5/2))/(175*c) - (b*d^3*(1 + c^2*x^2)^(7/2))/(49
*c) + d^3*x*(a + b*ArcSinh[c*x]) + c^2*d^3*x^3*(a + b*ArcSinh[c*x]) + (3*c^
4*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*d^3*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```

```
, 0] || EqQ[n, 1])
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx = d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\,\mathrm{s}\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\,\mathrm{s}\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\,\mathrm{s}\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\,\mathrm{s}\right.$$

$$= -\frac{16bd^3\sqrt{1+c^2x^2}}{35c}-\frac{8bd^3(1+c^2x^2)^{3/2}}{105c}-\frac{6bd^3(1+c^2x^2)^{5/2}}{175c}-\frac{bd^3(}{}$$

Mathematica [A]
time = 0.07, size = 119, normalized size = 0.70

$$\frac{d^3\left(105acx(35+35c^2x^2+21c^4x^4+5c^6x^6)-b\sqrt{1+c^2x^2}\,(2161+757c^2x^2+351c^4x^4+75c^6x^6)+105bcx(35+35c^2x^2+21c^4x^4+5c^6x^6)\sinh^{-1}(cx)\right)}{3675c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(105*a*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - b*Sqrt[1 + c^2
*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 105*b*c*x*(35 + 35*
c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]))/(3675*c)
```

Maple [A]
time = 0.52, size = 156, normalized size = 0.92

method	result
derivativedivides	$\frac{d^3a\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)c^3x^3+\operatorname{arcsinh}(cx)cx-\frac{c^6x^6\sqrt{c^2x^2}}{49}\right.}{c}$

default	$\frac{d^3a\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)c^3x^3+\operatorname{arcsinh}(cx)cx-\frac{c^6x^6\sqrt{c^2x}}{49}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(d^3*a*(1/7*c^7*x^7+3/5*c^5*x^5+c^3*x^3+c*x)+d^3*b*(1/7*arcsinh(c*x)*c^
7*x^7+3/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/49*c
^6*x^6*(c^2*x^2+1)^(1/2)-117/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-757/3675*c^2*x^
2*(c^2*x^2+1)^(1/2)-2161/3675*(c^2*x^2+1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(150) = 300$.
time = 0.27, size = 301, normalized size = 1.77

$$\frac{1}{7}ac^6d^3x^7+\frac{3}{5}ac^4d^3x^5+\frac{1}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^6d^3+\frac{1}{25}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^3+ac^2d^3x^3+\frac{1}{3}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d^3+ad^3x+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd^3}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*c^4*d^3 + a*c^2*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + a*d^3*x +
 (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

Fricas [A]
time = 0.34, size = 169, normalized size = 0.99

$$\frac{525\,ac^7d^3x^7+2205\,ac^5d^3x^5+3675\,ac^3d^3x^3+3675\,acd^3x+105\,(5\,bc^7d^3x^7+21\,bc^5d^3x^5+35\,bc^3d^3x^3+35\,bcd^3x)\log\left(cx+\sqrt{c^2x^2+1}\right)-(75\,bc^6d^3x^6+351\,bc^4d^3x^4+757\,bc^2d^3x^2+2161\,bd^3)\sqrt{c^2x^2+1}}{3675\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*d^3*x^7 + 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 + 3675*
a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 + 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 + 35
*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d^3*x^6 + 351*b*c^4*d^
3*x^4 + 757*b*c^2*d^3*x^2 + 2161*b*d^3)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A]
time = 0.73, size = 221, normalized size = 1.30

$$\begin{cases}\frac{ac^6d^3x^7}{7}+\frac{3ac^4d^3x^5}{5}+ac^2d^3x^3+ad^3x+\frac{bc^6d^3x^7\operatorname{asinh}(cx)}{7}-\frac{bc^5d^3x^6\sqrt{c^2x^2+1}}{49}+\frac{3bc^4d^3x^5\operatorname{asinh}(cx)}{5}-\frac{117bc^3d^3x^4\sqrt{c^2x^2+1}}{1225}+bc^2d^3x^3\operatorname{asinh}(cx)-\frac{757bcd^3x^2\sqrt{c^2x^2+1}}{3675}+bd^3x\operatorname{asinh}(cx)-\frac{2161bd^3\sqrt{c^2x^2+1}}{3675c} & \text{for } c\neq 0\\ ad^3x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 + a*c**2*d**3*x**3 + a
*d**3*x + b*c**6*d**3*x**7*asinh(c*x)/7 - b*c**5*d**3*x**6*sqrt(c**2*x**2 +
 1)/49 + 3*b*c**4*d**3*x**5*asinh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**2*x
**2 + 1)/1225 + b*c**2*d**3*x**3*asinh(c*x) - 757*b*c*d**3*x**2*sqrt(c**2*x
**2 + 1)/3675 + b*d**3*x*asinh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 + 1)/(3675
*c), Ne(c, 0)), (a*d**3*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.24 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=221

$$-\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}-\frac{19}{48}bd^3\sinh^{-1}(cx)+\frac{1}{2}d^3\left(1+c^2x^2\right)\left(a\right.$$

```
[Out] -7/72*b*c*d^3*x*(c^2*x^2+1)^(3/2)-1/36*b*c*d^3*x*(c^2*x^2+1)^(5/2)-19/48*b*
d^3*arcsinh(c*x)+1/2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/4*d^3*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))+1/6*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))+1/2*d^3*(a+b
*arcsinh(c*x))^2/b+d^3*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)
-1/2*b*d^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-19/48*b*c*d^3*x*(c^2*x^2+
1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5801, 5775, 3797, 2221, 2317, 2438, 201, 221}

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{d^3\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+d^3\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{36}bcd^3x\left(c^2x^2+1\right)^{5/2}-\frac{7}{72}bcd^3x\left(c^2x^2+1\right)^{3/2}-\frac{19}{48}bcd^3x\sqrt{c^2x^2+1}-\frac{1}{2}bd^3\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)-\frac{19}{48}bd^3\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-19*b*c*d^3*x*Sqrt[1 + c^2*x^2])/48 - (7*b*c*d^3*x*(1 + c^2*x^2)^(3/2))/72
 - (b*c*d^3*x*(1 + c^2*x^2)^(5/2))/36 - (19*b*d^3*ArcSinh[c*x])/48 + (d^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[
c*x]))/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/6 + (d^3*(a + b*ArcSi
nh[c*x])^2)/(2*b) + d^3*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] -
 (b*d^3*PolyLog[2, E^(-2*ArcSinh[c*x])])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^3 \left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{6}d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{(d+c^2dx^2)^2 \left(a+b\sinh^{-1}\right.}{x}$$

$$= -\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}+\frac{1}{4}d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}d^3$$

$$= -\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}+\frac{1}{2}d^3\left(1+c^2x^2\right)($$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right.$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right.$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right.$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right.$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right.$$

Mathematica [A]
time = 0.09, size = 189, normalized size = 0.86

$$\frac{1}{144}d^3\left(216ac^2x^2+108ac^4x^4+24ac^6x^6-75bcx\sqrt{1+c^2x^2}-22bc^3x^3\sqrt{1+c^2x^2}-4bc^5x^5\sqrt{1+c^2x^2}-72b\sinh^{-1}(cx)^2+144a\log\left(1-e^{2\sinh^{-1}(cx)}\right)+3\sinh^{-1}(cx)\left(-48a+b(25+72c^2x^2+36c^4x^4+8c^6x^6)+48b\log\left(1-e^{2\sinh^{-1}(cx)}\right)\right)+72b\,\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^3*(216*a*c^2*x^2 + 108*a*c^4*x^4 + 24*a*c^6*x^6 - 75*b*c*x*Sqrt[1 + c^2*
x^2] - 22*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 4*b*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*
b*ArcSinh[c*x]^2 + 144*a*Log[1 - E^(2*ArcSinh[c*x])] + 3*ArcSinh[c*x]*(-48*
a + b*(25 + 72*c^2*x^2 + 36*c^4*x^4 + 8*c^6*x^6) + 48*b*Log[1 - E^(2*ArcSin
h[c*x])]) + 72*b*PolyLog[2, E^(2*ArcSinh[c*x])]))/144
```

Maple [A]
time = 3.60, size = 284, normalized size = 1.29

method	result
derivativedivides	$\frac{a\,d^3c^6x^6}{6}+\frac{3a\,d^3c^4x^4}{4}+\frac{3a\,d^3c^2x^2}{2}+a\,d^3\ln(cx)+d^3b\,\mathrm{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+d^3b\,\mathrm{p}$
default	$\frac{a\,d^3c^6x^6}{6}+\frac{3a\,d^3c^4x^4}{4}+\frac{3a\,d^3c^2x^2}{2}+a\,d^3\ln(cx)+d^3b\,\mathrm{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+d^3b\,\mathrm{p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*d^3*c^6*x^6+3/4*a*d^3*c^4*x^4+3/2*a*d^3*c^2*x^2+a*d^3*ln(c*x)+d^3*b*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^3*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+25/
48*b*d^3*arcsinh(c*x)-1/2*d^3*b*arcsinh(c*x)^2+3/4*d^3*b*arcsinh(c*x)*c^4*x
^4+3/2*d^3*b*arcsinh(c*x)*c^2*x^2+1/6*d^3*b*arcsinh(c*x)*c^6*x^6+d^3*b*arcs
inh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+d^3*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1
)^(1/2))-1/36*d^3*b*c^5*x^5*(c^2*x^2+1)^(1/2)-11/72*d^3*b*c^3*x^3*(c^2*x^2+
1)^(1/2)-25/48*b*c*d^3*x*(c^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 + 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) +
integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x^3*log(
c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^2*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + b*d
^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a}{x}\,dx + \int 3ac^2x\,dx + \int 3ac^4x^3\,dx + \int ac^6x^5\,dx + \int \frac{b\operatorname{asinh}(cx)}{x}\,dx + \int 3bc^2x\operatorname{asinh}(cx)\,dx + \int 3bc^4x^3\operatorname{asinh}(cx)\,dx + \int bc^6x^5\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x,x)
```

```
[Out] d**3*(Integral(a/x, x) + Integral(3*a*c**2*x, x) + Integral(3*a*c**4*x**3,
x) + Integral(a*c**6*x**5, x) + Integral(b*asinh(c*x)/x, x) + Integral(3*b*
c**2*x*asinh(c*x), x) + Integral(3*b*c**4*x**3*asinh(c*x), x) + Integral(b*
c**6*x**5*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x, x)
```

3.25 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=160

$$-\frac{11}{5}bcd^3\sqrt{1+c^2x^2}-\frac{1}{5}bcd^3\left(1+c^2x^2\right)^{3/2}-\frac{1}{25}bcd^3\left(1+c^2x^2\right)^{5/2}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^2d^3x\left(a+b\sinh^{-1}(\right.$$

```
[Out] -1/5*b*c*d^3*(c^2*x^2+1)^(3/2)-1/25*b*c*d^3*(c^2*x^2+1)^(5/2)-d^3*(a+b*arcs
inh(c*x))/x+3*c^2*d^3*x*(a+b*arcsinh(c*x))+c^4*d^3*x^3*(a+b*arcsinh(c*x))+1
/5*c^6*d^3*x^5*(a+b*arcsinh(c*x))-b*c*d^3*arctanh((c^2*x^2+1)^(1/2))-11/5*b
*c*d^3*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {276, 5803, 12, 1813, 1634, 65, 214}

$$\frac{1}{5}c^6d^3x^5\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{25}bcd^3\left(c^2x^2+1\right)^{5/2}-\frac{1}{5}bcd^3\left(c^2x^2+1\right)^{3/2}-\frac{11}{5}bcd^3\sqrt{c^2x^2+1}-bcd^3\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-11*b*c*d^3*Sqrt[1 + c^2*x^2])/5 - (b*c*d^3*(1 + c^2*x^2)^(3/2))/5 - (b*c*
d^3*(1 + c^2*x^2)^(5/2))/25 - (d^3*(a + b*ArcSinh[c*x]))/x + 3*c^2*d^3*x*(a
 + b*ArcSinh[c*x]) + c^4*d^3*x^3*(a + b*ArcSinh[c*x]) + (c^6*d^3*x^5*(a + b
*ArcSinh[c*x]))/5 - b*c*d^3*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^3\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\right.$$

$$= -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\right.$$

$$= -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\right.$$

$$= -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\right.$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2}-\frac{1}{5}bcd^3\left(1+c^2x^2\right)^{3/2}-\frac{1}{25}bcd^3\left(1+c^2x^2\right)^{5/2}-$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2}-\frac{1}{5}bcd^3\left(1+c^2x^2\right)^{3/2}-\frac{1}{25}bcd^3\left(1+c^2x^2\right)^{5/2}-$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2}-\frac{1}{5}bcd^3\left(1+c^2x^2\right)^{3/2}-\frac{1}{25}bcd^3\left(1+c^2x^2\right)^{5/2}-$$

Mathematica [A]
time = 0.08, size = 163, normalized size = 1.02

$$\frac{d^3\left(-25a+75ac^2x^2+25ac^4x^4+5ac^6x^6-61bcx\sqrt{1+c^2x^2}-7bc^3x^3\sqrt{1+c^2x^2}-bc^5x^5\sqrt{1+c^2x^2}+5b(-5+15c^2x^2+5c^4x^4+c^6x^6)\sinh^{-1}(cx)+25bcx\log(x)-25bcx\log\left(1+\sqrt{1+c^2x^2}\right)\right)}{25x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^3*(-25*a + 75*a*c^2*x^2 + 25*a*c^4*x^4 + 5*a*c^6*x^6 - 61*b*c*x*Sqrt[1 +
 c^2*x^2] - 7*b*c^3*x^3*Sqrt[1 + c^2*x^2] - b*c^5*x^5*Sqrt[1 + c^2*x^2] + 5
*b*(-5 + 15*c^2*x^2 + 5*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 25*b*c*x*Log[x] -
 25*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(25*x)
```

Maple [A]
time = 0.64, size = 151, normalized size = 0.94

method	result
derivativedivides	$c\left(a\,d^3\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx)\right.\right.$
default	$c\left(a\,d^3\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx)\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a*d^3*(1/5*c^5*x^5+c^3*x^3+3*c*x-1/c/x)+d^3*b*(1/5*arcsinh(c*x)*c^5*x^5+
arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/25*c^4*x^4*(c^2*
x^2+1)^(1/2)-7/25*c^2*x^2*(c^2*x^2+1)^(1/2)-61/25*(c^2*x^2+1)^(1/2)-arctanh
(1/(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.26, size = 231, normalized size = 1.44

$$\frac{1}{5}ac^6d^3x^5+\frac{1}{75}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^6d^3+ac^4d^3x^3+\frac{1}{3}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^4d^3+3ac^2d^3x+3\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bcd^3-\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}(cx)}{x}\right)bd^3-\frac{ad^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*c^6*d^3*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^
2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^6*d^3 + a
*c^4*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*s
qrt(c^2*x^2 + 1)/c^4))*b*c^4*d^3 + 3*a*c^2*d^3*x + 3*(c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*b*c*d^3 - (c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*d^3
 - a*d^3/x
```

Fricas [A]
time = 0.41, size = 276, normalized size = 1.72

$$\frac{5\,ac^6d^3x^6+25\,ac^4d^3x^4+75\,ac^2d^3x^2-25\,bcd^3x\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+25\,bcd^3x\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-5\,(bc^6+5\,bc^4+15\,bc^2-5\,b)d^3x\log\left(-cx+\sqrt{c^2x^2+1}\right)-25\,ad^3+5\,(bc^6d^3x^6+5\,bc^4d^3x^4+15\,bc^2d^3x^2-(bc^6+5\,bc^4+15\,bc^2-5\,b)d^3x-5\,bd^3)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bc^5d^3x^5+7\,bc^3d^3x^3+61\,bcd^3x)\sqrt{c^2x^2+1}}{25\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/25*(5*a*c^6*d^3*x^6 + 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 25*b*c*d^3*x*
log(-c*x + sqrt(c^2*x^2 + 1) + 1) + 25*b*c*d^3*x*log(-c*x + sqrt(c^2*x^2 +
1) - 1) - 5*(b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x*log(-c*x + sqrt(c^2*x^
2 + 1)) - 25*a*d^3 + 5*(b*c^6*d^3*x^6 + 5*b*c^4*d^3*x^4 + 15*b*c^2*d^3*x^2
- (b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x - 5*b*d^3)*log(c*x + sqrt(c^2*x^
2 + 1)) - (b*c^5*d^3*x^5 + 7*b*c^3*d^3*x^3 + 61*b*c*d^3*x)*sqrt(c^2*x^2 + 1
))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3ac^2\,dx+\int \frac{a}{x^2}\,dx+\int 3ac^4x^2\,dx+\int ac^6x^4\,dx+\int 3bc^2\operatorname{asinh}(cx)\,dx+\int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx+\int 3bc^4x^2\operatorname{asinh}(cx)\,dx+\int bc^6x^4\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c**4*x**2,
 x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*asinh(c*x), x) + Integra
l(b*asinh(c*x)/x**2, x) + Integral(3*b*c**4*x**2*asinh(c*x), x) + Integral(
b*c**6*x**4*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^2, x)
```

3.26 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=249

$$-\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2}-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x}-\frac{3}{32}bc^2d^3\sinh^{-1}(cx)+\frac{3}{2}c^2d^3\left(1+c^2x^2\right)\left(a\right.$$

```
[Out] 7/16*b*c^3*d^3*x*(c^2*x^2+1)^(3/2)-1/2*b*c*d^3*(c^2*x^2+1)^(5/2)/x-3/32*b*c
^2*d^3*arcsinh(c*x)+3/2*c^2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))+3/4*c^2*d^3*
(c^2*x^2+1)^2*(a+b*arcsinh(c*x))-1/2*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/x
^2+3/2*c^2*d^3*(a+b*arcsinh(c*x))^2/b+3*c^2*d^3*(a+b*arcsinh(c*x))*ln(1-1/(
c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b*c^2*d^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))
^2)-3/32*b*c^3*d^3*x*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5802, 283, 201, 221, 5801, 5775, 3797, 2221, 2317, 2438}

$$-\frac{d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{3}{4}c^2d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{2}c^2d^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+3c^2d^3\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{3}{2}bc^2d^3\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)-\frac{bcd^3\left(c^2x^2+1\right)^{5/2}}{2x}-\frac{3}{32}bc^2d^3\sinh^{-1}(cx)+\frac{7}{16}bc^3d^3x\left(c^2x^2+1\right)^{3/2}-\frac{3}{32}bc^3d^3x\sqrt{c^2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (-3*b*c^3*d^3*x*Sqrt[1 + c^2*x^2])/32 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2))
/16 - (b*c*d^3*(1 + c^2*x^2)^(5/2))/(2*x) - (3*b*c^2*d^3*ArcSinh[c*x])/32 +
 (3*c^2*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (3*c^2*d^3*(1 + c^2*x^2
)^2*(a + b*ArcSinh[c*x]))/4 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(2
*x^2) + (3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/(2*b) + 3*c^2*d^3*(a + b*ArcSinh
[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (3*b*c^2*d^3*PolyLog[2, E^(-2*ArcSinh
[c*x])])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
```

```
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\left(3c^2d\right)\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\,\mathrm{si}\cdots\right.}{x}\cdots$$

$$= -\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x}+\frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)-\frac{d^3(1+\cdots}{\cdots}$$

$$= \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2}-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x}+\frac{3}{2}c^2d^3\left(1+c^2x^2\right)\left(a+\cdots\right.$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2}-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x}\cdots$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2}-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x}\cdots$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2}-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x}\cdots$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2}-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x}\cdots$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2}-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x}\cdots$$

Mathematica [A]
time = 0.17, size = 210, normalized size = 0.84

$$\frac{d^3\left(-16a+48ac^4x^4+8ac^6x^6-16bcx\sqrt{1+c^2x^2}-21bc^3x^3\sqrt{1+c^2x^2}-2bc^5x^5\sqrt{1+c^2x^2}+48bc^2x^2\sinh^{-1}(cx)^2+21bc^2x^2\tanh^{-1}\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+8b\sinh^{-1}(cx)\left(-2+6c^4x^4+c^6x^6+12c^2x^2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)+96ac^2x^2\log(x)-48bc^2x^2\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)}{32x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^3*(-16*a + 48*a*c^4*x^4 + 8*a*c^6*x^6 - 16*b*c*x*Sqrt[1 + c^2*x^2] - 21*
b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b*c^5*x^5*Sqrt[1 + c^2*x^2] + 48*b*c^2*x^2*
```

```
ArcSinh[c*x]^2 + 21*b*c^2*x^2*ArcTanh[(c*x)/Sqrt[1 + c^2*x^2]] + 8*b*ArcSin
h[c*x]*(-2 + 6*c^4*x^4 + c^6*x^6 + 12*c^2*x^2*Log[1 - E^(-2*ArcSinh[c*x])])
 + 96*a*c^2*x^2*Log[x] - 48*b*c^2*x^2*PolyLog[2, E^(-2*ArcSinh[c*x])]))/(32
*x^2)
```

Maple [A]
time = 6.02, size = 299, normalized size = 1.20

method	result
derivativedivides	$c^2\left(\frac{a\,d^3c^4x^4}{4}+\frac{3a\,d^3c^2x^2}{2}-\frac{a\,d^3}{2c^2x^2}+3a\,d^3\ln(cx)+3d^3b\,\mathrm{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right.\right.$
default	$c^2\left(\frac{a\,d^3c^4x^4}{4}+\frac{3a\,d^3c^2x^2}{2}-\frac{a\,d^3}{2c^2x^2}+3a\,d^3\ln(cx)+3d^3b\,\mathrm{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/4*a*d^3*c^4*x^4+3/2*a*d^3*c^2*x^2-1/2*a*d^3/c^2/x^2+3*a*d^3*ln(c*x)+
3*d^3*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+3*d^3*b*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))+1/2*d^3*b+3*d^3*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+3
*d^3*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-3/2*d^3*b*arcsinh(c*x)^2+21/32*b*d
^3*arcsinh(c*x)+1/4*d^3*b*arcsinh(c*x)*c^4*x^4+3/2*d^3*b*arcsinh(c*x)*c^2*x
^2-1/2*d^3*b*arcsinh(c*x)/c^2/x^2-1/2*d^3*b/c/x*(c^2*x^2+1)^(1/2)-1/16*d^3*
b*c^3*x^3*(c^2*x^2+1)^(1/2)-21/32*b*c*d^3*x*(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 + 3*a*c^2*d^3*log(x) - 1/2*b*d^3*(sqr
t(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d^3/x^2 + integrate(b*c^6*d^
3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 +
 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x^3, x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a}{x^3}\,dx + \int \frac{3ac^2}{x}\,dx + \int 3ac^4x\,dx + \int ac^6x^3\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x^3}\,dx + \int \frac{3bc^2\,\mathrm{asinh}\,(cx)}{x}\,dx + \int 3bc^4x\,\mathrm{asinh}\,(cx)\,dx + \int bc^6x^3\,\mathrm{asinh}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d**3*(Integral(a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(3*a*c**4*x,
x) + Integral(a*c**6*x**3, x) + Integral(b*asinh(c*x)/x**3, x) + Integral(3
*b*c**2*asinh(c*x)/x, x) + Integral(3*b*c**4*x*asinh(c*x), x) + Integral(b*
c**6*x**3*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^3, x)
```

3.27 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=174

$$-\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2}-\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2}-\frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}$$

```
[Out] -1/9*b*c^3*d^3*(c^2*x^2+1)^(3/2)-1/3*d^3*(a+b*arcsinh(c*x))/x^3-3*c^2*d^3*(
a+b*arcsinh(c*x))/x+3*c^4*d^3*x*(a+b*arcsinh(c*x))+1/3*c^6*d^3*x^3*(a+b*arc
sinh(c*x))-17/6*b*c^3*d^3*arctanh((c^2*x^2+1)^(1/2))-8/3*b*c^3*d^3*(c^2*x^2
+1)^(1/2)-1/6*b*c*d^3*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5803, 12, 1813, 1635, 911, 1167, 214}

$$\frac{1}{3}c^6d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^4d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd^3\sqrt{c^2x^2+1}}{6x^2}-\frac{1}{9}bc^3d^3\left(c^2x^2+1\right)^{3/2}-\frac{8}{3}bc^3d^3\sqrt{c^2x^2+1}-\frac{17}{6}bc^3d^3\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (-8*b*c^3*d^3*Sqrt[1 + c^2*x^2])/3 - (b*c*d^3*Sqrt[1 + c^2*x^2])/(6*x^2) -
(b*c^3*d^3*(1 + c^2*x^2)^(3/2))/9 - (d^3*(a + b*ArcSinh[c*x]))/(3*x^3) - (3
*c^2*d^3*(a + b*ArcSinh[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcSinh[c*x]) + (c^6*
d^3*x^3*(a + b*ArcSinh[c*x]))/3 - (17*b*c^3*d^3*ArcTanh[Sqrt[1 + c^2*x^2]])
/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^3\,(a+b\sinh^{-1}(cx))}{x^4}\,dx = -\frac{d^3(a+b\sinh^{-1}(cx))}{3x^3} - \frac{3c^2d^3(a+b\sinh^{-1}(cx))}{x} + 3c^4d^3x(a+b\,\mathrm{s}$$

$$= -\frac{d^3(a+b\sinh^{-1}(cx))}{3x^3} - \frac{3c^2d^3(a+b\sinh^{-1}(cx))}{x} + 3c^4d^3x(a+b\,\mathrm{s}$$

$$= -\frac{d^3(a+b\sinh^{-1}(cx))}{3x^3} - \frac{3c^2d^3(a+b\sinh^{-1}(cx))}{x} + 3c^4d^3x(a+b\,\mathrm{s}$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3(a+b\sinh^{-1}(cx))}{3x^3} - \frac{3c^2d^3(a+b\sinh^{-1}(cx}{x}$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3(a+b\sinh^{-1}(cx))}{3x^3} - \frac{3c^2d^3(a+b\sinh^{-1}(cx}{x}$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3(a+b\sinh^{-1}(cx))}{3x^3} - \frac{3c^2d^3(a+b\sinh^{-1}(cx}{x}$$

$$= -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2} - \frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{1}{9}bc^3d^3(1+c^2x^2)^{3/2} - \frac{d^3}{}$$

$$= -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2} - \frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{1}{9}bc^3d^3(1+c^2x^2)^{3/2} - \frac{d^3}{}$$

Mathematica [A]
time = 0.09, size = 171, normalized size = 0.98

$$\frac{d^3\left(-6a-54ac^2x^2+54ac^4x^4+6ac^6x^6-3bcx\sqrt{1+c^2x^2}-50bc^3x^3\sqrt{1+c^2x^2}-2bc^5x^5\sqrt{1+c^2x^2}+6b(-1-9c^2x^2+9c^4x^4+c^6x^6)\sinh^{-1}(cx)+51bc^3x^3\log(x)-51bc^3x^3\log\left(1+\sqrt{1+c^2x^2}\right)\right)}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-6*a - 54*a*c^2*x^2 + 54*a*c^4*x^4 + 6*a*c^6*x^6 - 3*b*c*x*Sqrt[1 + c
^2*x^2] - 50*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b*c^5*x^5*Sqrt[1 + c^2*x^2] +
6*b*(-1 - 9*c^2*x^2 + 9*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 51*b*c^3*x^3*Log[
x] - 51*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(18*x^3)
```

Maple [A]
time = 0.53, size = 155, normalized size = 0.89

method	result

derivativedivides	$c^3\left(a\,d^3\left(\frac{c^3x^3}{3}+3cx-\frac{1}{3c^3x^3}-\frac{3}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+3\operatorname{arcsinh}\left(cx\right)cx-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\right.\right.$
default	$c^3\left(a\,d^3\left(\frac{c^3x^3}{3}+3cx-\frac{1}{3c^3x^3}-\frac{3}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+3\operatorname{arcsinh}\left(cx\right)cx-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a*d^3*(1/3*c^3*x^3+3*c*x-1/3/c^3/x^3-3/c/x)+d^3*b*(1/3*arcsinh(c*x)*c^
3*x^3+3*arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-3*arcsinh(c*x)/c/x-1/9*c^
2*x^2*(c^2*x^2+1)^(1/2)-25/9*(c^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2
)-17/6*arctanh(1/(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.26, size = 208, normalized size = 1.20

$$\frac{1}{3}\,ac^6d^3x^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^6d^3+3\,ac^4d^3x+3\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^2x^2+1}\right)bc^3d^3-3\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}\left(cx\right)}{x}\right)bc^2d^3+\frac{1}{6}\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)c-\frac{2\operatorname{arsinh}\left(cx\right)}{x^3}\right)bd^3-\frac{3\,ac^2d^3}{x}-\frac{ad^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^6*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*c^3*d^3 - 3*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)
/x)*b*c^2*d^3 + 1/6*((c^2*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c
- 2*arcsinh(c*x)/x^3)*b*d^3 - 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

Fricas [A]
time = 0.39, size = 289, normalized size = 1.66

$$\frac{6\,ac^6d^3x^6+54\,ac^4d^3x^4-51\,bc^3d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+51\,bc^3d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-54\,ac^2d^3x^2-6\,(bc^6+9\,bc^4-9\,bc^2-b)d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}\right)-6\,ad^3+6\,(bc^6d^3x^6+9\,bc^4d^3x^4-9\,bc^2d^3x^2-(bc^6+9\,bc^4-9\,bc^2-b)d^3x^3-bd^3)\log\left(cx+\sqrt{c^2x^2+1}\right)-(2\,bc^5d^3x^5+50\,bc^3d^3x^3+3\,bcd^3x)\sqrt{c^2x^2+1}}{18\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^6*d^3*x^6 + 54*a*c^4*d^3*x^4 - 51*b*c^3*d^3*x^3*log(-c*x + sqrt
(c^2*x^2 + 1) + 1) + 51*b*c^3*d^3*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 5
4*a*c^2*d^3*x^2 - 6*(b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3*log(-c*x + sqrt
(c^2*x^2 + 1)) - 6*a*d^3 + 6*(b*c^6*d^3*x^6 + 9*b*c^4*d^3*x^4 - 9*b*c^2*d^3
*x^2 - (b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3 - b*d^3)*log(c*x + sqrt(c^2*
```

```
x^2 + 1)) - (2*b*c^5*d^3*x^5 + 50*b*c^3*d^3*x^3 + 3*b*c*d^3*x)*sqrt(c^2*x^2
 + 1))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3ac^4\,dx+\int \frac{a}{x^4}\,dx+\int \frac{3ac^2}{x^2}\,dx+\int ac^6x^2\,dx+\int 3bc^4\operatorname{asinh}(cx)\,dx+\int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx+\int \frac{3bc^2\operatorname{asinh}(cx)}{x^2}\,dx+\int bc^6x^2\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d**3*(Integral(3*a*c**4, x) + Integral(a/x**4, x) + Integral(3*a*c**2/x**2,
 x) + Integral(a*c**6*x**2, x) + Integral(3*b*c**4*asinh(c*x), x) + Integra
l(b*asinh(c*x)/x**4, x) + Integral(3*b*c**2*asinh(c*x)/x**2, x) + Integral(
b*c**6*x**2*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^4, x)
```

3.28 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{d+c^2dx^2}\,dx$

Optimal. Leaf size=156

$$\frac{4b\sqrt{1+c^2x^2}}{3c^5d}-\frac{b(1+c^2x^2)^{3/2}}{9c^5d}-\frac{x(a+b\sinh^{-1}(cx))}{c^4d}+\frac{x^3(a+b\sinh^{-1}(cx))}{3c^2d}+\frac{2(a+b\sinh^{-1}(cx))\,\text{ArcTan}\left(e^{\text{si}}\right.}{c^5d}$$

```
[Out] -1/9*b*(c^2*x^2+1)^(3/2)/c^5/d-x*(a+b*arcsinh(c*x))/c^4/d+1/3*x^3*(a+b*arcs
inh(c*x))/c^2/d+2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d-I*
b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d+I*b*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c^5/d+4/3*b*(c^2*x^2+1)^(1/2)/c^5/d
```

Rubi [A]
time = 0.17, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5812, 5789, 4265, 2317, 2438, 267, 272, 45}

$$\frac{2\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^5d}-\frac{x(a+b\sinh^{-1}(cx))}{c^4d}+\frac{x^3(a+b\sinh^{-1}(cx))}{3c^2d}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c^5d}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c^5d}-\frac{b(c^2x^2+1)^{3/2}}{9c^5d}+\frac{4b\sqrt{c^2x^2+1}}{3c^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (4*b*Sqrt[1 + c^2*x^2])/(3*c^5*d) - (b*(1 + c^2*x^2)^(3/2))/(9*c^5*d) - (x*
(a + b*ArcSinh[c*x]))/(c^4*d) + (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*d) + (2*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d) - (I*b*PolyLog[2, (-I)*
E^ArcSinh[c*x]])/(c^5*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} - \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{3cd}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{c^4} + \frac{b\int \frac{}{\sqrt{1+}}}{c^3}$$

$$= \frac{b\sqrt{1+c^2x^2}}{c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{\text{Subst}\left(\int (a+bx\right.}{}$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b(1+c^2x^2)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b(1+c^2x^2)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b(1+c^2x^2)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}$$

Mathematica [A]
time = 0.18, size = 170, normalized size = 1.09

$$\frac{-9acx+3ac^3x^3+11b\sqrt{1+c^2x^2}-bc^2x^2\sqrt{1+c^2x^2}-9bcx\sinh^{-1}(cx)+3bc^3x^3\sinh^{-1}(cx)+9a\text{ArcTan}(cx)+9ib\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-9ib\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-9ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+9ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{9c^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (-9*a*c*x + 3*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] - b*c^2*x^2*Sqrt[1 + c^2*x
^2] - 9*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x] + (
9*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*b*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] - (9*I)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b*Pol
yLog[2, I*E^ArcSinh[c*x]])/(9*c^5*d)
```

Maple [A]
time = 3.46, size = 245, normalized size = 1.57

method	result
derivativedivides	$\frac{a\,c^3x^3}{3d}-\frac{acx}{d}+\frac{a\arctan(cx)}{d}+\frac{b\,\text{arcsinh}(cx)c^3x^3}{3d}-\frac{b\,\text{arcsinh}(cx)cx}{d}+\frac{b\,\text{arcsinh}(cx)\arctan(cx)}{d}-\frac{b\,c^2x^2\sqrt{c^2x^2+1}}{9d}+\frac{11b\sqrt{c^2x^2}}{9d}$

default	$\frac{a\,c^3x^3}{3d}-\frac{acx}{d}+\frac{a\arctan(cx)}{d}+\frac{b\,\mathrm{arcsinh}(cx)c^3x^3}{3d}-\frac{b\,\mathrm{arcsinh}(cx)cx}{d}+\frac{b\,\mathrm{arcsinh}(cx)\arctan(cx)}{d}-\frac{b\,c^2x^2\sqrt{c^2x^2+1}}{9d}+\frac{11b\sqrt{c^2x}}{9d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/3*a/d*c^3*x^3-a/d*c*x+a/d*arctan(c*x)+1/3*b/d*arcsinh(c*x)*c^3*x^3
-b/d*arcsinh(c*x)*c*x+b/d*arcsinh(c*x)*arctan(c*x)-1/9*b/d*c^2*x^2*(c^2*x^2
+1)^(1/2)+11/9*b/d*(c^2*x^2+1)^(1/2)+b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))-b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*b/d*dil
og(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*b/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(
1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + b*integrate(x^4*l
og(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^2x^2+1}\,dx + \int \frac{bx^4\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**4/(c**2*x**2 + 1), x) + Integral(b*x**4*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,(a+b\,\mathrm{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.29 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{d+c^2dx^2}\,dx$

Optimal. Leaf size=135

$$-\frac{bx\sqrt{1+c^2x^2}}{4c^3d}+\frac{b\sinh^{-1}(cx)}{4c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^2\right.}{c^4d}$$

```
[Out] 1/4*b*arcsinh(c*x)/c^4/d+1/2*x^2*(a+b*arcsinh(c*x))/c^2/d+1/2*(a+b*arcsinh(
c*x))^2/b/c^4/d-(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/
2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/4*b*x*(c^2*x^2+1)^(1/2)/c
^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5812, 5797, 3799, 2221, 2317, 2438, 327, 221}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{b\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{b\sinh^{-1}(cx)}{4c^4d}-\frac{bx\sqrt{c^2x^2+1}}{4c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/4*(b*x*Sqrt[1 + c^2*x^2])/(c^3*d) + (b*ArcSinh[c*x])/(4*c^4*d) + (x^2*(a
 + b*ArcSinh[c*x]))/(2*c^2*d) + (a + b*ArcSinh[c*x])^2/(2*b*c^4*d) - ((a +
b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d) - (b*PolyLog[2, -E^(2*
ArcSinh[c*x])])/(2*c^4*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\int \frac{x(a+b\sinh^{-1}(cx))}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2cd}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\text{Subst}\left(\int(a+bx)\tanh(x)\,dx, x, \text{sin}\right.}{c^4d}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)}{2bc^4d}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)}{2bc^4d}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)}{2bc^4d}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)}{2bc^4d}$$

Mathematica [A]

time = 0.14, size = 181, normalized size = 1.34

$$-\frac{-2ac^2x^2 + bcx\sqrt{1+c^2x^2} - b\sinh^{-1}(cx) - 2bc^2x^2\sinh^{-1}(cx) - 2b\sinh^{-1}(cx)^2 + 4b\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 4b\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) + 2a\log\left(1+c^2x^2\right) + 4b\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 4b\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{4c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/4*(-2*a*c^2*x^2 + b*c*x*Sqrt[1 + c^2*x^2] - b*ArcSinh[c*x] - 2*b*c^2*x^2
*ArcSinh[c*x] - 2*b*ArcSinh[c*x]^2 + 4*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[
c*x])/Sqrt[-c^2]] + 4*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c]
 + 2*a*Log[1 + c^2*x^2] + 4*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4
*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(c^4*d)
```

Maple [A]

time = 3.03, size = 148, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{a\,c^2x^2}{2d} - \frac{a\ln\left(c^2x^2+1\right)}{2d} + \frac{b\,\text{arcsinh}(cx)^2}{2d} + \frac{b\,\text{arcsinh}(cx)c^2x^2}{2d} - \frac{bcx\sqrt{c^2x^2+1}}{4d} + \frac{b\,\text{arcsinh}(cx)}{4d} - \frac{b\,\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c}\right.\right.}{d}}{c^4}$
default	$\frac{\frac{a\,c^2x^2}{2d} - \frac{a\ln\left(c^2x^2+1\right)}{2d} + \frac{b\,\text{arcsinh}(cx)^2}{2d} + \frac{b\,\text{arcsinh}(cx)c^2x^2}{2d} - \frac{bcx\sqrt{c^2x^2+1}}{4d} + \frac{b\,\text{arcsinh}(cx)}{4d} - \frac{b\,\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c}\right.\right.}{d}}{c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*a/d*c^2*x^2-1/2*a/d*ln(c^2*x^2+1)+1/2*b/d*arcsinh(c*x)^2+1/2*b/d
*arcsinh(c*x)*c^2*x^2-1/4*b/d*c*x*(c^2*x^2+1)^(1/2)+1/4*b/d*arcsinh(c*x)-b/
d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))^2)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) - 1/8*b*((2*c^2*x^2 - log(c^
2*x^2 + 1)^2 - 4*(c^2*x^2 - log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 2*log(c^2*x^2 + 1))/(c^4*d) - 8*integrate(-1/2*(c^2*x^2 - log(c^2*x^2 + 1
))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqrt(c^2*x^2 + 1)), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2 + 1), x) + Integral(b*x**3*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.30 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{d+c^2dx^2}\,dx$

Optimal. Leaf size=108

$$-\frac{b\sqrt{1+c^2x^2}}{c^3d}+\frac{x(a+b\sinh^{-1}(cx))}{c^2d}-\frac{2(a+b\sinh^{-1}(cx))\,\mathrm{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d}+\frac{ib\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^3d}$$

```
[Out] x*(a+b*arcsinh(c*x))/c^2/d-2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2
))/c^3/d+I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-I*b*polylog(2,I*(c
*x+(c^2*x^2+1)^(1/2)))/c^3/d-b*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5812, 5789, 4265, 2317, 2438, 267}

$$-\frac{2\mathrm{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^3d}+\frac{x(a+b\sinh^{-1}(cx))}{c^2d}+\frac{ib\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c^3d}-\frac{ib\mathrm{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c^3d}-\frac{b\sqrt{c^2x^2+1}}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -((b*Sqrt[1 + c^2*x^2])/(c^3*d)) + (x*(a + b*ArcSinh[c*x]))/(c^2*d) - (2*(a
 + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d) + (I*b*PolyLog[2, (-I)*E
^ArcSinh[c*x]])/(c^3*d) - (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx &= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{c^2}-\frac{b\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{cd}\\ &= -\frac{b\sqrt{1+c^2x^2}}{c^3d}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\sinh^{-}\right.}{c^3d}\\ &= -\frac{b\sqrt{1+c^2x^2}}{c^3d}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx}\right.}{c^3d}\\ &= -\frac{b\sqrt{1+c^2x^2}}{c^3d}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx}\right.}{c^3d}\\ &= -\frac{b\sqrt{1+c^2x^2}}{c^3d}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx}\right.}{c^3d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 121, normalized size = 1.12

$$\frac{acx-b\sqrt{1+c^2x^2}+bcx\sinh^{-1}(cx)-a\text{ArcTan}(cx)-ib\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+ib\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]

[Out] (a*c*x - b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d)

Maple [A]
time = 2.56, size = 194, normalized size = 1.80

method	result
derivativedivides	$\frac{\frac{acx}{d}-\frac{a\arctan(cx)}{d}+\frac{ib\,\mathrm{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}-\frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}-\frac{ib\,\mathrm{dilog}\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}-b\,\mathrm{ar}}{c^3}$
default	$\frac{\frac{acx}{d}-\frac{a\arctan(cx)}{d}+\frac{ib\,\mathrm{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}-\frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}-\frac{ib\,\mathrm{dilog}\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}-b\,\mathrm{ar}}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)

[Out] 1/c^3*(a/d*c*x-a/d*arctan(c*x)+I*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*b/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b/d*arcsinh(c*x)*arctan(c*x)+b/d*arcsinh(c*x)*c*x+b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b/d*(c^2*x^2+1)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")

[Out] a*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2 + 1), x) + Integral(b*x**2*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.31 $\int \frac{x(a+b\sinh^{-1}(cx))}{d+c^2dx^2}\,dx$

Optimal. Leaf size=73

$$-\frac{(a+b\sinh^{-1}(cx))^2}{2bc^2d}+\frac{(a+b\sinh^{-1}(cx))\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}+\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/b/c^2/d+(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)/c^2/d+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5797, 3799, 2221, 2317, 2438}

$$-\frac{(a+b\sinh^{-1}(cx))^2}{2bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)(a+b\sinh^{-1}(cx))}{c^2d}+\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/2*(a + b*ArcSinh[c*x])^2/(b*c^2*d) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*
ArcSinh[c*x])])/(c^2*d) + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx &= \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^2d}\\ &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\sinh^{-1}(cx)\right)}{c^2d}\\ &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}-\frac{b\text{Subst}(\int \text{l}}{}\\ &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}-\frac{b\text{Subst}\Big(\int}{}\\ &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}+\frac{b\text{Li}_2\Big(-e^{2\text{s}}}{2c^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(73) = 146.
time = 0.05, size = 167, normalized size = 2.29

$$-\frac{b\sinh^{-1}(cx)^2}{2c^2d}+\frac{b\sinh^{-1}(cx)\log\left(1-\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{c^2d}+\frac{b\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{c^2d}+\frac{a\log\left(1+c^2x^2\right)}{2c^2d}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{c^2d}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/2*(b*ArcSinh[c*x]^2)/(c^2*d) + (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^Arc
Sinh[c*x])/c])/(c^2*d) + (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c])/(c^2*d) + (a*Log[1 + c^2*x^2])/(2*c^2*d) + (b*PolyLog[2, -((Sqrt[-c^2
]*E^ArcSinh[c*x])/c)])/(c^2*d) + (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/
c])/(c^2*d)
```

Maple [A]
time = 1.78, size = 90, normalized size = 1.23

method	result	siz
derivativedivides	$\frac{\frac{a\ln\left(c^{2}x^{2}+1\right)}{2d}-\frac{b\operatorname{arcsinh}(cx)^{2}}{2d}+\frac{b\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}\right)}{d}+\frac{b\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}\right)}{2d}}{c^{2}}$	90
default	$\frac{\frac{a\ln\left(c^{2}x^{2}+1\right)}{2d}-\frac{b\operatorname{arcsinh}(cx)^{2}}{2d}+\frac{b\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}\right)}{d}+\frac{b\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}\right)}{2d}}{c^{2}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
[Out] 1/c^2*(1/2*a/d*ln(c^2*x^2+1)-1/2*b/d*arcsinh(c*x)^2+b/d*arcsinh(c*x)*ln(1+(
c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] -1/8*b*((log(c^2*x^2 + 1)^2 - 4*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1
)))/(c^2*d) + 8*integrate(1/2*log(c^2*x^2 + 1)/(c^4*d*x^3 + c^2*d*x + (c^3*
d*x^2 + c*d)*sqrt(c^2*x^2 + 1)), x)) + 1/2*a*log(c^2*d*x^2 + d)/(c^2*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b*x*arcsinh(c*x) + a*x)/(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^{2}x^{2}+1}\,dx+\int \frac{bx\operatorname{asinh}(cx)}{c^{2}x^{2}+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x/(c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(c**2*x**2 + 1)
, x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.32 $\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=70

$$\frac{2\left(a+b\sinh^{-1}(cx)\right)\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{cd}-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{cd}$$

```
[Out] 2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-I*b*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))/c/d+I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Rubi [A]

time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5789, 4265, 2317, 2438}

$$\frac{2\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d) - (I*b*PolyLog[2, (-I
)*E^ArcSinh[c*x]])/(c*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{cd}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(ib)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \sinh^{-1}(c$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\sinh^{-1}(cx)}\right)}{cd} +$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} + \frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right.}{cd}$$

Mathematica [A]
time = 0.07, size = 135, normalized size = 1.93

$$-\frac{c\left(a\sqrt{-c^2}\,\text{ArcTan}(cx)-bc\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+bc\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+bc\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-bc\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)\right)}{\left(-c^2\right)^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

```
[Out] -((c*(a*Sqrt[-c^2]*ArcTan[c*x] - b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b
*c*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*c*PolyLog[2, (Sqrt[-c^2]*E
^ArcSinh[c*x])/c]))/((-c^2)^(3/2)*d))
```

Maple [A]
time = 2.54, size = 157, normalized size = 2.24

method	result
derivativedivides	$\frac{\frac{a\arctan(cx)}{d}+\frac{ib\,\text{dilog}\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}+\frac{b\,\text{arcsinh}(cx)\arctan(cx)}{d}-\frac{ib\,\text{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}-\frac{b\arctan(cx)\ln\left(1-\right.}{d}}{c}$
default	$\frac{\frac{a\arctan(cx)}{d}+\frac{ib\,\text{dilog}\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}+\frac{b\,\text{arcsinh}(cx)\arctan(cx)}{d}-\frac{ib\,\text{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{d}-\frac{b\arctan(cx)\ln\left(1-\right.}{d}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/d*arctan(c*x)+I*b/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+b/d*arcsi
nh(c*x)*arctan(c*x)-I*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b/d*arctan
(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/
(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x) + a*arctan(c*x
)/(c*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^2+1}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**2*x**2 + 1), x)
)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2), x)
```

3.33 $\int \frac{a+b\sinh^{-1}(cx)}{x(d+c^2dx^2)}\,dx$

Optimal. Leaf size=61

$$-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d}-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5799, 5569, 4267, 2317, 2438}

$$-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(2*d) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{x\,(d+c^2dx^2)}\,dx &= \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d} \\
&= \frac{2\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \sinh^{-1}(cx)\right)}{d} \\
&= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\right.}{d} \\
&= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}(cx)}\right)}{2d} \\
&= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d} + \frac{b\text{Li}_2\left(e^{2\sinh^{-1}\right.}{2d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(61) = 122.
time = 0.07, size = 207, normalized size = 3.39

$$-\frac{a\sinh^{-1}(cx)}{d}-\frac{b\sinh^{-1}(cx)\log\left(1-\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{d}-\frac{b\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{d}+\frac{a\log\left(1-e^{2\sinh^{-1}(cx)}\right)}{d}+\frac{b\sinh^{-1}(cx)\log\left(1-e^{2\sinh^{-1}(cx)}\right)}{d}-\frac{a\log\left(1+c^2x^2\right)}{2d}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{d}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{d}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

```
[Out] -((a*ArcSinh[c*x])/d) - (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^ArcSinh[c*x])
/c])/d - (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (a*Log
[1 - E^(2*ArcSinh[c*x])])/d + (b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])])/
d - (a*Log[1 + c^2*x^2])/(2*d) - (b*PolyLog[2, -((Sqrt[-c^2]*E^ArcSinh[c*x]
)/c)])/d - (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (b*PolyLog[2,
E^(2*ArcSinh[c*x])])/(2*d)
```

Maple [A]
time = 2.65, size = 71, normalized size = 1.16

method	result	size
derivativedivides	$-\frac{a\ln(c^2x^2+1)}{2d}+\frac{a\ln(cx)}{d}+\frac{b\left(\operatorname{dilog}\left(\frac{1}{\left(cx+\sqrt{c^2x^2+1}\right)^2}\right)-\frac{\operatorname{dilog}\left(\frac{1}{\left(cx+\sqrt{c^2x^2+1}\right)^4}\right)}{4}\right)}{d}$	71
default	$-\frac{a\ln(c^2x^2+1)}{2d}+\frac{a\ln(cx)}{d}+\frac{b\left(\operatorname{dilog}\left(\frac{1}{\left(cx+\sqrt{c^2x^2+1}\right)^2}\right)-\frac{\operatorname{dilog}\left(\frac{1}{\left(cx+\sqrt{c^2x^2+1}\right)^4}\right)}{4}\right)}{d}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*ln(c^2*x^2+1)+a/d*ln(c*x)+b/d*(dilog(1/(c*x+(c^2*x^2+1)^(1/2))^2)-
1/4*dilog(1/(c*x+(c^2*x^2+1)^(1/2))^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c^2*x
^2 + 1))/(c^2*d*x^3 + d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^3+x}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^2x^3+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**2*x**3 + x), x)
)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)), x)
```

3.34 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)}\,dx$

Optimal. Leaf size=101

$$-\frac{a+b\sinh^{-1}(cx)}{dx}-\frac{2c\left(a+b\sinh^{-1}(cx)\right)\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{d}-\frac{bc\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)}{d}+\frac{ibc\text{PolyLog}\left(2,-ie\right.}{d}$$

[Out] (-a-b*arcsinh(c*x))/d/x-2*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)/d-b*c*arctanh((c^2*x^2+1)^(1/2))/d+I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)^(1
/2)))/d-I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d

Rubi [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {5809, 5789, 4265, 2317, 2438, 272, 65, 214}

$$-\frac{2c\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{a+b\sinh^{-1}(cx)}{dx}-\frac{bc\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{d}+\frac{ibc\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{ibc\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]

[Out] -((a + b*ArcSinh[c*x])/(d*x)) - (2*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[
c*x]])/d - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d + (I*b*c*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/d - (I*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{dx} - c^2\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx + \frac{(bc)\int \frac{1}{x\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{c\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d} + \frac{(bc)\text{Subst}\Big(\int \frac{}{x}}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} + \frac{b\text{Subst}\Big(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}}{c}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{bc\tanh^{-1}\Big(\sqrt{1+c}}{d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{bc\tanh^{-1}\Big(\sqrt{1+c}}{d}$$

<u>Mathematica</u> [A]

time = 0.10, size = 182, normalized size = 1.80

$$-\frac{a+b\sinh^{-1}(cx)+acx\text{ArcTan}(cx)+bcx\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)+b\sqrt{-c^2}\,x\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-b\sqrt{-c^2}\,x\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)-b\sqrt{-c^2}\,x\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+b\sqrt{-c^2}\,x\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x] + a*c*x*ArcTan[c*x] + b*c*x*ArcTanh[Sqrt[1 + c^2*x^2]
] + b*Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*
Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*Sqrt[-
c^2]*x*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + b*Sqrt[-c^2]*x*PolyLog[2
, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(d*x))
```

<u>Maple</u> [A]

time = 0.70, size = 203, normalized size = 2.01

method	result
derivativedivides	$c\left(-\frac{a\arctan(cx)}{d}-\frac{a}{dcx}-\frac{b\,\text{arcsinh}(cx)\arctan(cx)}{d}-\frac{b\,\text{arcsinh}(cx)}{dcx}-\frac{b\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{d}-\frac{b\arctan(}{}\right.$
default	$c\left(-\frac{a\arctan(cx)}{d}-\frac{a}{dcx}-\frac{b\,\text{arcsinh}(cx)\arctan(cx)}{d}-\frac{b\,\text{arcsinh}(cx)}{dcx}-\frac{b\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{d}-\frac{b\arctan(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)

[Out] c*(-a/d*arctan(c*x)-a/d/c/x-b/d*arcsinh(c*x)*arctan(c*x)-b/d*arcsinh(c*x)/c/x-b/d*arctanh(1/(c^2*x^2+1)^(1/2))-b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*b/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="maxima")

[Out] -a*(c*arctan(c*x)/d + 1/(d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^4 + d*x^2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="fricas")

[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^4 + d*x^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^4+x^2}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^4+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d),x)

[Out] (Integral(a/(c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c**2*x**4 + x**2), x))/d

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^2\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)), x)
```

3.35 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(d+c^2dx^2)}\,dx$

Optimal. Leaf size=113

$$-\frac{bc\sqrt{1+c^2x^2}}{2dx}-\frac{a+b\sinh^{-1}(cx)}{2dx^2}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d}+\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(c$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/d/x^2+2*c^2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^
2+1)^(1/2))^2)/d+1/2*b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*c^
2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*c*(c^2*x^2+1)^(1/2)/d/x
```

Rubi [A]

time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5809, 5799, 5569, 4267, 2317, 2438, 270}

$$\frac{2c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{a+b\sinh^{-1}(cx)}{2dx^2}+\frac{bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -1/2*(b*c*Sqrt[1 + c^2*x^2])/(d*x) - (a + b*ArcSinh[c*x])/(2*d*x^2) + (2*c^
2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d + (b*c^2*PolyLog[2, -
E^(2*ArcSinh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2} - c^2\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx + \frac{(bc)\int \frac{1}{x^2\sqrt{1+c^2x^2}}\,dx}{2d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} - \frac{c^2\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \sinh$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} - \frac{(2c^2)\,\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \sinh^{-1}(\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} +$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} +$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} +$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(113) = 226.
time = 0.18, size = 240, normalized size = 2.12

$$\frac{-\frac{bc\sqrt{1+c^2x^2}}{x}-bc^2\sinh^{-1}(cx)^2-\frac{a+b\sinh^{-1}(cx)}{x^2}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{b}+2bc^2\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+2bc^2\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+ac^2\log\left(1+c^2x^2\right)+2bc^2\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+2bc^2\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)-c^2\left(2\left(a+b\sinh^{-1}(cx)\right)\log\left(1-e^{2\sinh^{-1}(cx)}\right)+b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] (-((b*c*Sqrt[1 + c^2*x^2])/x) - b*c^2*ArcSinh[c*x]^2 - (a + b*ArcSinh[c*x])
/x^2 + (c^2*(a + b*ArcSinh[c*x])^2)/b + 2*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] + 2*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSi
nh[c*x])/c] + a*c^2*Log[1 + c^2*x^2] + 2*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + 2*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - c^2*(2*
(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSin
h[c*x])]))/(2*d)
```

Maple [A]
time = 2.96, size = 251, normalized size = 2.22

method	result
derivativedivides	$c^2\left(\frac{a\ln\left(c^2x^2+1\right)}{2d}-\frac{a}{2d\,c^2x^2}-\frac{a\ln(cx)}{d}-\frac{b\sqrt{c^2x^2+1}}{2dcx}+\frac{b}{2d}-\frac{b\,\mathrm{arcsinh}(cx)}{2d\,c^2x^2}-\frac{b\,\mathrm{arcsinh}(cx)\ln\left(1+cx+\right.}{d}\right.$
default	$c^2\left(\frac{a\ln\left(c^2x^2+1\right)}{2d}-\frac{a}{2d\,c^2x^2}-\frac{a\ln(cx)}{d}-\frac{b\sqrt{c^2x^2+1}}{2dcx}+\frac{b}{2d}-\frac{b\,\mathrm{arcsinh}(cx)}{2d\,c^2x^2}-\frac{b\,\mathrm{arcsinh}(cx)\ln\left(1+cx+\right.}{d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a/d*ln(c^2*x^2+1)-1/2*a/d/c^2/x^2-a/d*ln(c*x)-1/2*b/d/c/x*(c^2*x^2
+1)^(1/2)+1/2*b/d-1/2*b/d*arcsinh(c*x)/c^2/x^2-b/d*arcsinh(c*x)*ln(1+c*x+(c
^2*x^2+1)^(1/2))-b/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b/d*arcsinh(c*x)*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-
b/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-b/d*polylog(2,c*x+(c^2*x^2+1)^
(1/2)))
```

Maple [F]

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a + b*integrate(l
og(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^5+x^3}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^5+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c**2*x**5 + x**
3), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{x^3\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)), x)
```

3.36 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)}\,dx$

Optimal. Leaf size=156

$$-\frac{bc\sqrt{1+c^2x^2}}{6dx^2}-\frac{a+b\sinh^{-1}(cx)}{3dx^3}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx}+\frac{2c^3\left(a+b\sinh^{-1}(cx)\right)\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{d}+\frac{7bc^3}{}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3+c^2*(a+b*arcsinh(c*x))/d/x+2*c^3*(a+b*arcsinh
(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d+7/6*b*c^3*arctanh((c^2*x^2+1)^(1/2))
/d-I*b*c^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d+I*b*c^3*polylog(2,I*(c*x
+(c^2*x^2+1)^(1/2)))/d-1/6*b*c*(c^2*x^2+1)^(1/2)/d/x^2
```

Rubi [A]

time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5809, 5789, 4265, 2317, 2438, 272, 65, 214, 44}

$$\frac{2c^3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx}-\frac{a+b\sinh^{-1}(cx)}{3dx^3}-\frac{ibc^3\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{ibc^3\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{bc\sqrt{c^2x^2+1}}{6dx^2}+\frac{7bc^3\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[1 + c^2*x^2])/(d*x^2) - (a + b*ArcSinh[c*x])/(3*d*x^3) + (c^
2*(a + b*ArcSinh[c*x]))/(d*x) + (2*c^3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSin
h[c*x]])/d + (7*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d) - (I*b*c^3*PolyLog[
2, (-I)*E^ArcSinh[c*x]])/d + (I*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3} - c^2\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)}\,dx + \frac{(bc)\int \frac{1}{x^3\sqrt{1+c^2x^2}}\,dx}{3d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + c^4\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx + \frac{(bc)\text{Subs}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{c^3\text{Subst}\left(\int (a+bx)\text{s}\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx\right.}{}$$

Mathematica [A]
time = 0.14, size = 247, normalized size = 1.58

$$\frac{-2a+6ac^2x^2-bcx\sqrt{1+c^2x^2}-2b\sinh^{-1}(cx)+6bc^2x^2\sinh^{-1}(cx)+6ac^3x^3\text{ArcTan}(cx)+7bc^3x^3\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)-6b(-c^2)^{3/2}x^3\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+6b(-c^2)^{3/2}x^3\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+6b(-c^2)^{3/2}x^3\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-6b(-c^2)^{3/2}x^3\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{6dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + 6*b*c^2*
x^2*ArcSinh[c*x] + 6*a*c^3*x^3*ArcTan[c*x] + 7*b*c^3*x^3*ArcTanh[Sqrt[1 + c
^2*x^2]] - 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqr
t[-c^2]] + 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[
c*x])/c] + 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] -
 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*d*x^3)
```

Maple [A]
time = 0.68, size = 252, normalized size = 1.62

method	result
derivativedivides	$c^3\left(\frac{a\arctan(cx)}{d} - \frac{a}{3d\,c^3x^3} + \frac{a}{dcx} + \frac{b\,\text{arcsinh}(cx)\arctan(cx)}{d} - \frac{b\,\text{arcsinh}(cx)}{3d\,c^3x^3} + \frac{b\,\text{arcsinh}(cx)}{dcx} - \frac{b\sqrt{c^2x^2+}}{6d\,c^2x^2}\right.$

default	$c^3\left(\frac{a\arctan(cx)}{d}-\frac{a}{3d\,c^3x^3}+\frac{a}{dcx}+\frac{b\,\mathrm{arcsinh}(cx)\arctan(cx)}{d}-\frac{b\,\mathrm{arcsinh}(cx)}{3d\,c^3x^3}+\frac{b\,\mathrm{arcsinh}(cx)}{dcx}-\frac{b\sqrt{c^2x^2+}}{6d\,c^2x^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/d*arctan(c*x)-1/3*a/d/c^3/x^3+a/d/c/x+b/d*arcsinh(c*x)*arctan(c*x)-1
/3*b/d*arcsinh(c*x)/c^3/x^3+b/d*arcsinh(c*x)/c/x-1/6*b/d/c^2/x^2*(c^2*x^2+1
)^(1/2)+7/6*b/d*arctanh(1/(c^2*x^2+1)^(1/2))+b/d*arctan(c*x)*ln(1+I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))-b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I
*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*b/d*dilog(1-I*(1+I*c*x)/(c^2*
x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a + b*integrate(log(c*x
 + sqrt(c^2*x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^6 + d*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^6+x^4}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{c^2x^6+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c**2*x**6 + x**
4), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)),x)
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)), x)
```

3.37 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=171

$$\frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{3\left(a+b\sinh^{-1}(cx)\right)\mathrm{ArcTan}(}{c^5d^2}$$

```
[Out] 3/2*x*(a+b*arcsinh(c*x))/c^4/d^2-1/2*x^3*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^
2+1)-3*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d^2+3/2*I*b*pol
ylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3/2*I*b*polylog(2,I*(c*x+(c^2*x^
2+1)^(1/2)))/c^5/d^2+1/2*b/c^5/d^2/(c^2*x^2+1)^(1/2)-b*(c^2*x^2+1)^(1/2)/c^
5/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5810, 5812, 5789, 4265, 2317, 2438, 267, 272, 45}

$$-\frac{3\mathrm{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{3ib\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{3ib\mathrm{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{b\sqrt{c^2x^2+1}}{c^5d^2}+\frac{b}{2c^5d^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] b/(2*c^5*d^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[1 + c^2*x^2])/(c^5*d^2) + (3*x*(a
 + b*ArcSinh[c*x]))/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1
+ c^2*x^2)) - (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) + (
((3*I)/2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^2) - (((3*I)/2)*b*PolyL
og[2, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
```

```
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{x^3}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{(3b)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{2c^3d^2}+\frac{b\text{Subs}\ldots}{\ldots}$$

$$= -\frac{3b\sqrt{1+c^2x^2}}{2c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{3\text{Subst}\left(\int(\ldots\right.}{\ldots}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(c\ldots\right.}{2c^2d^2\left(1+c^2x^2\right)}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(c\ldots\right.}{2c^2d^2\left(1+c^2x^2\right)}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(c\ldots\right.}{2c^2d^2\left(1+c^2x^2\right)}$$

Mathematica [A]
time = 0.17, size = 268, normalized size = 1.57

$$\frac{3acx+2ac^3x^3-b\sqrt{1+c^2x^2}-2bc^2x^2\sqrt{1+c^2x^2}+3bcx\sinh^{-1}(cx)+2bc^3x^3\sinh^{-1}(cx)-3a\text{ArcTan}(cx)-3ac^2x^2\text{ArcTan}(cx)-3ib\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-3ibc^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+3ib\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+3ibc^2x^2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+3ib(1+c^2x^2)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-3ib(1+c^2x^2)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2\left(1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (3*a*c*x + 2*a*c^3*x^3 - b*Sqrt[1 + c^2*x^2] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^2
] + 3*b*c*x*ArcSinh[c*x] + 2*b*c^3*x^3*ArcSinh[c*x] - 3*a*ArcTan[c*x] - 3*a
*c^2*x^2*ArcTan[c*x] - (3*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*
I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)*b*ArcSinh[c*x]*
Log[1 + I*E^ArcSinh[c*x]] + (3*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] + (3*I)*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (3*I)*b*(
1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^5*d^2*(1 + c^2*x^2))
```

Maple [A]
time = 2.52, size = 260, normalized size = 1.52

method	result
derivativedivides	$\frac{\frac{acx}{d^2}+\frac{acx}{2d^2(c^2x^2+1)}-\frac{3a\arctan(cx)}{2d^2}+\frac{b\,\mathrm{arcsinh}(cx)cx}{d^2}+\frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)}-\frac{3b\,\mathrm{arcsinh}(cx)\arctan(cx)}{2d^2}-\frac{b\,c^2x^2}{d^2\sqrt{c^2x^2+1}}-\frac{}{2d^2\sqrt{c}}}{}$
default	$\frac{\frac{acx}{d^2}+\frac{acx}{2d^2(c^2x^2+1)}-\frac{3a\arctan(cx)}{2d^2}+\frac{b\,\mathrm{arcsinh}(cx)cx}{d^2}+\frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)}-\frac{3b\,\mathrm{arcsinh}(cx)\arctan(cx)}{2d^2}-\frac{b\,c^2x^2}{d^2\sqrt{c^2x^2+1}}-\frac{}{2d^2\sqrt{c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/d^2*c*x+1/2*a/d^2*c*x/(c^2*x^2+1)-3/2*a/d^2*arctan(c*x)+b/d^2*arcs
inh(c*x)*c*x+1/2*b/d^2*arcsinh(c*x)*c*x/(c^2*x^2+1)-3/2*b/d^2*arcsinh(c*x)*
arctan(c*x)-b/d^2*c^2*x^2/(c^2*x^2+1)^(1/2)-1/2*b/d^2/(c^2*x^2+1)^(1/2)-3/2
*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*b/d^2*arctan(c*x
)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b/d^2*dilog(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))-3/2*I*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2))
 + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^
2 + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^4\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**4*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.38 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2(a+b\sinh^{-1}(cx))}{2c^2d^2(1+c^2x^2)}-\frac{(a+b\sinh^{-1}(cx))^2}{2bc^4d^2}+\frac{(a+b\sinh^{-1}(cx))\log\left(1+\right.}{c^4d^2}$$

```
[Out] 1/2*b*arcsinh(c*x)/c^4/d^2-1/2*x^2*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)-1
/2*(a+b*arcsinh(c*x))^2/b/c^4/d^2+(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)/c^4/d^2+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*b
*x/c^3/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5810, 5797, 3799, 2221, 2317, 2438, 294, 221}

$$-\frac{(a+b\sinh^{-1}(cx))^2}{2bc^4d^2}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)(a+b\sinh^{-1}(cx))}{c^4d^2}-\frac{x^2(a+b\sinh^{-1}(cx))}{2c^2d^2(c^2x^2+1)}+\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{bx}{2c^3d^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*(b*x)/(c^3*d^2*Sqrt[1 + c^2*x^2]) + (b*ArcSinh[c*x])/(2*c^4*d^2) - (x^
2*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/
(2*b*c^4*d^2) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d^2
) + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2} + \frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{c^2d}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x,\right.}{c^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}} + \frac{b\sinh^{-1}(cx)}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(c\right.}{2bc^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}} + \frac{b\sinh^{-1}(cx)}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(c\right.}{2bc^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}} + \frac{b\sinh^{-1}(cx)}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(c\right.}{2bc^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}} + \frac{b\sinh^{-1}(cx)}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(c\right.}{2bc^4d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 241, normalized size = 1.66

$$\frac{a-bcx\sqrt{1+c^2x^2}+b\sinh^{-1}(cx)-b\sinh^{-1}(cx)^2-bc^2x^2\sinh^{-1}(cx)^2+2b\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+2bc^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+2b\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+2bc^2x^2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+a\log\left(1+c^2x^2\right)+ac^2x^2\log\left(1+c^2x^2\right)+2b\left(1+c^2x^2\right)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+2b\left(1+c^2x^2\right)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2c^4d^2\left(1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a - b*c*x*Sqrt[1 + c^2*x^2] + b*ArcSinh[c*x] - b*ArcSinh[c*x]^2 - b*c^2*x^
2*ArcSinh[c*x]^2 + 2*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*c^2*x^2
*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*ArcSinh[c*x]*Log[1 + I*E^ArcS
inh[c*x]] + 2*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + a*Log[1 +
c^2*x^2] + a*c^2*x^2*Log[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E
^ArcSinh[c*x]] + 2*b*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^4*d^2
*(1 + c^2*x^2))
```

Maple [A]
time = 4.88, size = 187, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{a\ln\left(c^2x^2+1\right)}{2d^2}+\frac{a}{2d^2\left(c^2x^2+1\right)}-\frac{b\,\text{arcsinh}(cx)^2}{2d^2}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}+\frac{b\,c^2x^2}{2d^2\left(c^2x^2+1\right)}+\frac{b\,\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2d^2\left(c^2x^2+1\right)}+\frac{b\,\text{arcsinh}}{}}{c^4}$

default	$\frac{\frac{a\ln\left(c^{2}x^{2}+1\right)}{2d^{2}}+\frac{a}{2d^{2}\left(c^{2}x^{2}+1\right)}-\frac{b\,\mathrm{arcsinh}(cx)^{2}}{2d^{2}}-\frac{bcx}{2d^{2}\sqrt{c^{2}x^{2}+1}}+\frac{b\,c^{2}x^{2}}{2d^{2}\left(c^{2}x^{2}+1\right)}+\frac{b\,\mathrm{arcsinh}(cx)}{2d^{2}\left(c^{2}x^{2}+1\right)}+\frac{b}{2d^{2}\left(c^{2}x^{2}+1\right)}+\frac{b\,\mathrm{arcsinh}(c}{}}{c^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*a/d^2*ln(c^2*x^2+1)+1/2*a/d^2/(c^2*x^2+1)-1/2*b/d^2*arcsinh(c*x)
^2-1/2*b*c*x/d^2/(c^2*x^2+1)^(1/2)+1/2*b/d^2/(c^2*x^2+1)*c^2*x^2+1/2*b/d^2*
arcsinh(c*x)/(c^2*x^2+1)+1/2*b/d^2/(c^2*x^2+1)+b/d^2*arcsinh(c*x)*ln(1+(c*x
+(c^2*x^2+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(((c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - 4*((c^2*x^2 + 1)*log(c^2*x^2 +
1) + 1)*log(c*x + sqrt(c^2*x^2 + 1)) - 2)/(c^6*d^2*x^2 + c^4*d^2) + 8*integ
rate(1/2*((c^2*x^2 + 1)*log(c^2*x^2 + 1) + 1)/(c^8*d^2*x^5 + 2*c^6*d^2*x^3
+ c^4*d^2*x + (c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x
)) + 1/2*a*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^{3}}{c^{4}x^{4}+2c^{2}x^{2}+1}\,dx+\int \frac{bx^{3}\,\mathrm{asinh}\,(cx)}{c^{4}x^{4}+2c^{2}x^{2}+1}\,dx}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**3*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.39 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=127

$$-\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x(a+b\sinh^{-1}(cx))}{2c^2d^2(1+c^2x^2)}+\frac{(a+b\sinh^{-1}(cx))\,\mathrm{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}-\frac{ib\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}$$

```
[Out] -1/2*x*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))*arctan(c*x
+(c^2*x^2+1)^(1/2))/c^3/d^2-1/2*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c
^3/d^2+1/2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2-1/2*b/c^3/d^2/(
c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5810, 5789, 4265, 2317, 2438, 267}

$$\frac{\mathrm{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^3d^2}-\frac{x(a+b\sinh^{-1}(cx))}{2c^2d^2(c^2x^2+1)}-\frac{ib\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}+\frac{ib\mathrm{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}-\frac{b}{2c^3d^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*b/(c^3*d^2*Sqrt[1 + c^2*x^2]) - (x*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1
 + c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d^2) - ((
I/2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + ((I/2)*b*PolyLog[2, I*E
^ArcSinh[c*x]])/(c^3*d^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx &= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}+\frac{\int\frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{2c^2d}\\
&= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\mathrm{Subst}\left(\int(a+bx)\mathrm{sech}(x)\,dx,x,\mathrm{s}\right.}{2c^3d^2}\\
&= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-}}\right.}{c^3d^2}\\
&= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-}}\right.}{c^3d^2}\\
&= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-}}\right.}{c^3d^2}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 221, normalized size = 1.74

$$-\frac{acx + b\sqrt{1+c^2x^2} + bcx\sinh^{-1}(cx) - a\text{ArcTan}(cx) - ac^2x^2\text{ArcTan}(cx) - ib\sinh^{-1}(cx)\log\left(1 - ie^{\sinh^{-1}(cx)}\right) - ibc^2x^2\sinh^{-1}(cx)\log\left(1 - ie^{\sinh^{-1}(cx)}\right) + ib\sinh^{-1}(cx)\log\left(1 + ie^{\sinh^{-1}(cx)}\right) + ibc^2x^2\sinh^{-1}(cx)\log\left(1 + ie^{\sinh^{-1}(cx)}\right) + ib(1+c^2x^2)\text{PolyLog}\left(2, -ie^{\sinh^{-1}(cx)}\right) - ib(1+c^2x^2)\text{PolyLog}\left(2, ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*(a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - a*
c^2*x^2*ArcTan[c*x] - I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*c^2*
x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] + I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + I*b*(1
+ c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*(1 + c^2*x^2)*PolyLog[2, I
*E^ArcSinh[c*x]])/(c^3*d^2*(1 + c^2*x^2))
```

Maple [A]
time = 3.12, size = 219, normalized size = 1.72

method	result
derivativedivides	$\frac{-\frac{acx}{2d^2(c^2x^2+1)} + \frac{a\arctan(cx)}{2d^2} - \frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)} + \frac{b\,\mathrm{arcsinh}(cx)\arctan(cx)}{2d^2} + \frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2d^2} - \frac{b\arctan(cx)\ln\left(\cdots\right.}{\cdots}}{c^3}$
default	$\frac{-\frac{acx}{2d^2(c^2x^2+1)} + \frac{a\arctan(cx)}{2d^2} - \frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)} + \frac{b\,\mathrm{arcsinh}(cx)\arctan(cx)}{2d^2} + \frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2d^2} - \frac{b\arctan(cx)\ln\left(\cdots\right.}{\cdots}}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/2*a/d^2*c*x/(c^2*x^2+1)+1/2*a/d^2*arctan(c*x)-1/2*b/d^2*arcsinh(c
*x)*c*x/(c^2*x^2+1)+1/2*b/d^2*arcsinh(c*x)*arctan(c*x)+1/2*b/d^2*arctan(c*x
)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b/d^2*arctan(c*x)*ln(1-I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))-1/2*I*b/d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2
*I*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b/d^2/(c^2*x^2+1)^(1/2)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + b*integrate(x^
2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**2*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.40 $\int \frac{x(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{bx}{2cd^2\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(1+c^2x^2\right)}$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)+1/2*b*x/c/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5798, 197}

$$\frac{bx}{2cd^2\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*d^2*(1 + c^
2*x^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int \frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}\\ &= \frac{bx}{2cd^2\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(1+c^2x^2\right)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 1.35

$$-\frac{a}{2c^2d^2\left(1+c^2x^2\right)}+\frac{bx}{2cd^2\sqrt{1+c^2x^2}}-\frac{b\sinh^{-1}(cx)}{2c^2d^2\left(1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
[Out] -1/2*a/(c^2*d^2*(1 + c^2*x^2)) + (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (b*Arc
Sinh[c*x])/(2*c^2*d^2*(1 + c^2*x^2))
```

Maple [A]
time = 0.55, size = 61, normalized size = 1.11

method	result	size
derivativedivides	$\frac{-\frac{a}{2d^2\left(c^2x^2+1\right)}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2\left(c^2x^2+1\right)}+\frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2}}{c^2}$	61
default	$\frac{-\frac{a}{2d^2\left(c^2x^2+1\right)}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2\left(c^2x^2+1\right)}+\frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2}}{c^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
[Out] 1/c^2*(-1/2*a/d^2/(c^2*x^2+1)+b/d^2*(-1/2/(c^2*x^2+1)*arcsinh(c*x)+1/2*c*x/
(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] -1/4*b*((2*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^4*d^2*x^2 + c^2*d^2) - 4*in
tegrate(1/2/(c^6*d^2*x^5 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3
*d^2*x^2 + c*d^2)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a/(c^4*d^2*x^2 + c^2*d^2)
```

Fricas [A]
time = 0.35, size = 65, normalized size = 1.18

$$\frac{ac^2x^2+\sqrt{c^2x^2+1}\,bcx-b\log\left(cx+\sqrt{c^2x^2+1}\right)}{2\left(c^4d^2x^2+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x - b*log(c*x + sqrt(c^2*x^2 + 1)))/
(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx\,\mathrm{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(
c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.41 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=124

$$\frac{b}{2cd^2\sqrt{1+c^2x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2}-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2cd^2}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))*arctan(c*x+(c^2
*x^2+1)^(1/2))/c/d^2-1/2*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+1/
2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+1/2*b/c/d^2/(c^2*x^2+1)^(1
/2)
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5788, 5789, 4265, 2317, 2438, 267}

$$\frac{\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{2cd^2}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

```
[Out] b/(2*c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^
2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d^2) - ((I/2)*b*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/(c*d^2) + ((I/2)*b*PolyLog[2, I*E^ArcSinh[c*x]
])/(c*d^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^2}\,dx = \frac{x(a+b\sinh^{-1}(cx))}{2d^2(1+c^2x^2)} - \frac{(bc)\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{2d}$$

$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\sinh^{-1}(cx))}{2d^2(1+c^2x^2)} + \frac{\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \sinh^{-1}(cx\right.}{2cd^2}$$

$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\sinh^{-1}(cx))}{2d^2(1+c^2x^2)} + \frac{(a+b\sinh^{-1}(cx))\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} -$$

$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\sinh^{-1}(cx))}{2d^2(1+c^2x^2)} + \frac{(a+b\sinh^{-1}(cx))\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} -$$

$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\sinh^{-1}(cx))}{2d^2(1+c^2x^2)} + \frac{(a+b\sinh^{-1}(cx))\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} -$$

Mathematica [A]
time = 0.07, size = 216, normalized size = 1.74

$$\frac{acx+b\sqrt{1+c^2x^2}+bcx\sinh^{-1}(cx)+a\text{ArcTan}(cx)+ac^2x^2\text{ArcTan}(cx)+ib\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+ibc^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-ib\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-ibc^2x^2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-ib(1+c^2x^2)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+ib(1+c^2x^2)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2d^2(c+c^3x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] + a*ArcTan[c*x] + a*c^2*x
^2*ArcTan[c*x] + I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*c^2*x^2*A
rcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] - I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*b*(1 + c^2
*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*(1 + c^2*x^2)*PolyLog[2, I*E^Ar
cSinh[c*x]])/(2*d^2*(c + c^3*x^2))
```

Maple [A]
time = 0.64, size = 219, normalized size = 1.77

method	result
derivativedivides	$\frac{\frac{acx}{2d^2(c^2x^2+1)}+\frac{a\arctan(cx)}{2d^2}+\frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)}+\frac{b\,\mathrm{arcsinh}(cx)\arctan(cx)}{2d^2}+\frac{b}{2d^2\sqrt{c^2x^2+1}}+\frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+}}\right.}{2d^2}}{c}$ …
default	$\frac{\frac{acx}{2d^2(c^2x^2+1)}+\frac{a\arctan(cx)}{2d^2}+\frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)}+\frac{b\,\mathrm{arcsinh}(cx)\arctan(cx)}{2d^2}+\frac{b}{2d^2\sqrt{c^2x^2+1}}+\frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+}}\right.}{2d^2}}{c}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*a/d^2*c*x/(c^2*x^2+1)+1/2*a/d^2*arctan(c*x)+1/2*b/d^2*arcsinh(c*x)
*c*x/(c^2*x^2+1)+1/2*b/d^2*arcsinh(c*x)*arctan(c*x)+1/2*b/d^2/(c^2*x^2+1)^(
1/2)+1/2*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b/d^2*ar
ctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*b/d^2*dilog(1+I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))+1/2*I*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + b*integrate(log(c*x +
 sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**4
*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2, x)
```

3.42 $\int \frac{a+b\sinh^{-1}(cx)}{x(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{bcx}{2d^2\sqrt{1+c^2x^2}}+\frac{a+b\sinh^{-1}(cx)}{2d^2(1+c^2x^2)}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^2}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))*arctanh((c*x+(c
^2*x^2+1)^(1/2))^2)/d^2-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2
*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-1/2*b*c*x/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5811, 5799, 5569, 4267, 2317, 2438, 197}

$$\frac{a+b\sinh^{-1}(cx)}{2d^2(c^2x^2+1)}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d^2}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(b*c*x)/(d^2*Sqrt[1 + c^2*x^2]) + (a + b*ArcSinh[c*x])/(2*d^2*(1 + c^2
*x^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 - (b*Poly
Log[2, -E^(2*ArcSinh[c*x])])/(2*d^2) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(
2*d^2)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x(d+c^2dx^2)}\,dx}{d}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \text{sinh}\right.}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} + \frac{2\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \sinh^{-1}(c\right.}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 234 vs. $2(110) = 220$.
time = 0.30, size = 234, normalized size = 2.13

$$-\frac{\frac{a^2}{b} - \frac{a}{1+c^2x^2} + \frac{bcx}{\sqrt{1+c^2x^2}} + 2a\sinh^{-1}(cx) - \frac{b\sinh^{-1}(cx)}{1+c^2x^2} + 2b\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 2b\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) - 2a\log\left(1-e^{2\sinh^{-1}(cx)}\right) - 2b\sinh^{-1}(cx)\log\left(1-e^{2\sinh^{-1}(cx)}\right) + a\log\left(1+c^2x^2\right) + 2b\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 2b\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) - b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(a^2/b - a/(1 + c^2*x^2) + (b*c*x)/Sqrt[1 + c^2*x^2] + 2*a*ArcSinh[c*x
] - (b*ArcSinh[c*x])/(1 + c^2*x^2) + 2*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[
c*x])/Sqrt[-c^2]] + 2*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c]
 - 2*a*Log[1 - E^(2*ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[
c*x])] + a*Log[1 + c^2*x^2] + 2*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
 + 2*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*PolyLog[2, E^(2*ArcSin
h[c*x])]])/d^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(127) = 254$.
time = 2.87, size = 283, normalized size = 2.57

method	result
derivativedivides	$-\frac{a\ln(c^2x^2+1)}{2d^2} + \frac{a}{2d^2(c^2x^2+1)} + \frac{a\ln(cx)}{d^2} - \frac{bcx}{2d^2\sqrt{c^2x^2+1}} + \frac{b\,c^2x^2}{2d^2(c^2x^2+1)} + \frac{b\,\text{arcsinh}(cx)}{2d^2(c^2x^2+1)} + \frac{b}{2d^2(c^2x^2}$

default	$-\frac{a\ln(c^2x^2+1)}{2d^2}+\frac{a}{2d^2(c^2x^2+1)}+\frac{a\ln(cx)}{d^2}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}+\frac{b\,c^2x^2}{2d^2(c^2x^2+1)}+\frac{b\operatorname{arcsinh}(cx)}{2d^2(c^2x^2+1)}+\frac{b}{2d^2(c^2x^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d^2*ln(c^2*x^2+1)+1/2*a/d^2/(c^2*x^2+1)+a/d^2*ln(c*x)-1/2*b*c*x/d^2/
(c^2*x^2+1)^(1/2)+1/2*b/d^2/(c^2*x^2+1)*c^2*x^2+1/2*b/d^2*arcsinh(c*x)/(c^2
*x^2+1)+1/2*b/d^2/(c^2*x^2+1)+b/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2)
)+b/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2
*x^2+1)^(1/2))^2)-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b/d^2*arc
sinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + b*int
egrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^5+2c^2x^3+x}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**4
*x**5 + 2*c**2*x**3 + x), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^2), x)
```

3.43 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{bc}{2d^2\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{d^2x\,(1+c^2x^2)}-\frac{3c^2x\big(a+b\sinh^{-1}(cx)\big)}{2d^2\,(1+c^2x^2)}-\frac{3c\big(a+b\sinh^{-1}(cx)\big)\,\mathrm{ArcTan}\Big(e^{\sinh^{-1}(cx)}\Big)}{d^2}-\frac{bc}{}$$

```
[Out] (-a-b*arcsinh(c*x))/d^2/x/(c^2*x^2+1)-3/2*c^2*x*(a+b*arcsinh(c*x))/d^2/(c^2
*x^2+1)-3*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2-b*c*arctan
h((c^2*x^2+1)^(1/2))/d^2+3/2*I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^
2-3/2*I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-1/2*b*c/d^2/(c^2*x^2+1
)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5809, 5788, 5789, 4265, 2317, 2438, 267, 272, 53, 65, 214}

$$-\frac{3c\mathrm{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\,(c^2x^2+1)}-\frac{a+b\sinh^{-1}(cx)}{d^2x\,(c^2x^2+1)}-\frac{bc}{2d^2\sqrt{c^2x^2+1}}-\frac{bc\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{d^2}+\frac{3ibc\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{2d^2}-\frac{3ibc\mathrm{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(d^2*x*(1 + c^2*x
^2)) - (3*c^2*x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (3*c*(a + b*A
rcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/d^2 - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])
/d^2 + (((3*I)/2)*b*c*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - (((3*I)/2)*b*c
*PolyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
```

```
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x(1+c^2x^2)^{3/2}}\,dx}{d^2}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{(bc)\text{Subst}\left(\int \frac{1}{x(1+c^2x)^{3/2}}\,dx, x, x^2\right)}{2d^2} +$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{(3c)\text{Subst}\left(\int(a+\right.}{}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(c\right.}{}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(c\right.}{}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(c\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.39, size = 253, normalized size = 1.51

$$-\frac{\frac{3a}{x}-\frac{a}{x+c^2x^3}+\frac{3b\sinh^{-1}(cx)}{x}-\frac{b\sinh^{-1}(cx)}{x+c^2x^3}+3ac\text{ArcTan}(cx)+3bc\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)+\frac{bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+c^2x^2\right)}{\sqrt{1+c^2x^2}}+3b\sqrt{-c^2}\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-3b\sqrt{-c^2}\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)-3b\sqrt{-c^2}\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+3b\sqrt{-c^2}\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*((3*a)/x - a/(x + c^2*x^3) + (3*b*ArcSinh[c*x])/x - (b*ArcSinh[c*x])/(
x + c^2*x^3) + 3*a*c*ArcTan[c*x] + 3*b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (b*c*
Hypergeometric2F1[-1/2, 1, 1/2, 1 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 3*b*Sqrt[
-c^2]*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 3*b*Sqrt[-c^2]*
ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 3*b*Sqrt[-c^2]*PolyLo
g[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 3*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]
*E^ArcSinh[c*x])/c])/d^2
```

Maple [A]
time = 0.67, size = 263, normalized size = 1.57

method	result
derivativedivides	$c\left(-\frac{acx}{2d^2(c^2x^2+1)}-\frac{3a\arctan(cx)}{2d^2}-\frac{a}{d^2cx}-\frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)}-\frac{3b\,\mathrm{arcsinh}(cx)\arctan(cx)}{2d^2}-\frac{b\,\mathrm{arcsinh}(cx)}{d^2cx}-\right.$
default	$c\left(-\frac{acx}{2d^2(c^2x^2+1)}-\frac{3a\arctan(cx)}{2d^2}-\frac{a}{d^2cx}-\frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)}-\frac{3b\,\mathrm{arcsinh}(cx)\arctan(cx)}{2d^2}-\frac{b\,\mathrm{arcsinh}(cx)}{d^2cx}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*a/d^2*c*x/(c^2*x^2+1)-3/2*a/d^2*arctan(c*x)-a/d^2/c/x-1/2*b/d^2*arc
sinh(c*x)*c*x/(c^2*x^2+1)-3/2*b/d^2*arcsinh(c*x)*arctan(c*x)-b/d^2*arcsinh(
c*x)/c/x-3/2*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*b/d^
2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b/d^2*dilog(1+I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-1/2*b/d^2/(c^2*x^2+1)^(1/2)-b/d^2*arctanh(1/(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + b*in
tegrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c
**4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^2\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^2), x)
```

3.44 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{bc}{2d^2x\sqrt{1+c^2x^2}}-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2}+$$

[Out] -c^2*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+1/2*(-a-b*arcsinh(c*x))/d^2/x^2/(c^2*x^2+1)+4*c^2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c^2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-1/2*b*c/d^2/x/(c^2*x^2+1)^(1/2)

Rubi [A]
time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5809, 5811, 5799, 5569, 4267, 2317, 2438, 197, 277}

$$-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{4c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{bc}{2d^2x\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]

[Out] -1/2*(b*c)/(d^2*x*Sqrt[1 + c^2*x^2]) - (c^2*(a + b*ArcSinh[c*x]))/(d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(2*d^2*x^2*(1 + c^2*x^2)) + (4*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 + (b*c^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\int \frac{a+b\sinh^{-1}(}{x(d+c^2dx^2}}{d}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\text{Subst}\left(\int(a\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(4c^2\right)\text{Subst}\left(\int(a\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 326 vs. 2(146) = 292.
time = 0.32, size = 326, normalized size = 2.23

$$\frac{\frac{2a^2c^2}{b} - \frac{2a}{x^2} + \frac{bc}{x\sqrt{1+c^2x^2}} + \frac{2bc^3x}{\sqrt{1+c^2x^2}} - \frac{2bc\sqrt{1+c^2x^2}}{x} + \frac{a}{x^2+c^2x^4} + 4ac^2\sinh^{-1}(cx) - \frac{2b\sinh^{-1}(cx)}{x^2} + \frac{b\sinh^{-1}(cx)}{x^2+c^2x^4} + 4bc^2\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 4bc^2\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) - 4ac^2\log\left(1-e^{2\sinh^{-1}(cx)}\right) - 4bc^2\sinh^{-1}(cx)\log\left(1-e^{2\sinh^{-1}(cx)}\right) + 2ac^2\log\left(1+c^2x^2\right) + 4bc^2\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 4bc^2\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) - 2bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((2*a^2*c^2)/b - (2*a)/x^2 + (b*c)/(x*Sqrt[1 + c^2*x^2]) + (2*b*c^3*x)/Sqrt
[1 + c^2*x^2] - (2*b*c*Sqrt[1 + c^2*x^2])/x + a/(x^2 + c^2*x^4) + 4*a*c^2*A
rcSinh[c*x] - (2*b*ArcSinh[c*x])/x^2 + (b*ArcSinh[c*x])/(x^2 + c^2*x^4) + 4
*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*ArcSin
h[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 4*a*c^2*Log[1 - E^(2*ArcSin
h[c*x])] - 4*b*c^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a*c^2*Log[1
 + c^2*x^2] + 4*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*P
olyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b*c^2*PolyLog[2, E^(2*ArcSinh[
c*x])])/(2*d^2)
```

Maple [**A**]
time = 3.70, size = 293, normalized size = 2.01

method	result
derivativedivides	$c^2\left(-\frac{a}{2d^2c^2x^2}-\frac{2a\ln(cx)}{d^2}+\frac{a\ln(c^2x^2+1)}{d^2}-\frac{a}{2d^2(c^2x^2+1)}-\frac{b\,\mathrm{arcsinh}(cx)}{d^2(c^2x^2+1)}-\frac{b}{2d^2cx\sqrt{c^2x^2+1}}-\frac{b\,\mathrm{arc}}{2d^2c^2x}\right.$
default	$c^2\left(-\frac{a}{2d^2c^2x^2}-\frac{2a\ln(cx)}{d^2}+\frac{a\ln(c^2x^2+1)}{d^2}-\frac{a}{2d^2(c^2x^2+1)}-\frac{b\,\mathrm{arcsinh}(cx)}{d^2(c^2x^2+1)}-\frac{b}{2d^2cx\sqrt{c^2x^2+1}}-\frac{b\,\mathrm{arc}}{2d^2c^2x}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a/d^2/c^2/x^2-2*a/d^2*ln(c*x)+a/d^2*ln(c^2*x^2+1)-1/2*a/d^2/(c^2*
x^2+1)-b/d^2*arcsinh(c*x)/(c^2*x^2+1)-1/2*b/d^2/c/x/(c^2*x^2+1)^(1/2)-1/2*b
/d^2/c^2/x^2/(c^2*x^2+1)*arcsinh(c*x)-2*b/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^
2+1)^(1/2))-2*b/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b/d^2*arcsinh(c*x)*
ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2
-2*b/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*b/d^2*polylog(2,c*x+(c^
2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c^2
*d^2*x^4 + d^2*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^
7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^7+2c^2x^5+x^3}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c
**4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^2), x)
```

3.45 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=239

$$\frac{bc^3}{3d^2\sqrt{1+c^2x^2}}-\frac{bc}{6d^2x^2\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\,(1+c^2x^2)}+\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\,(1+c^2x^2)}+\frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\,(1+c^2x^2)}+\frac{5c^3\left(\right.}{}$$

[Out] 1/3*(-a-b*arcsinh(c*x))/d^2/x^3/(c^2*x^2+1)+5/3*c^2*(a+b*arcsinh(c*x))/d^2/x/(c^2*x^2+1)+5/2*c^4*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+5*c^3*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2+13/6*b*c^3*arctanh((c^2*x^2+1)^(1/2))/d^2-5/2*I*b*c^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+5/2*I*b*c^3*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+1/3*b*c^3/d^2/(c^2*x^2+1)^(1/2)-1/6*b*c/d^2/x^2/(c^2*x^2+1)^(1/2)

Rubi [A]
time = 0.21, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5809, 5788, 5789, 4265, 2317, 2438, 267, 272, 53, 65, 214, 44}

$$\frac{5c^3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\,(c^2x^2+1)}-\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\,(c^2x^2+1)}+\frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\,(c^2x^2+1)}-\frac{5ibc^3\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{2d^2}+\frac{5ibc^3\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{2d^2}-\frac{bc}{6d^2x^2\sqrt{c^2x^2+1}}+\frac{bc^3}{3d^2\sqrt{c^2x^2+1}}+\frac{13bc^3\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{6d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]

[Out] (b*c^3)/(3*d^2*Sqrt[1 + c^2*x^2]) - (b*c)/(6*d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*d^2*x^3*(1 + c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x]))/(3*d^2*x*(1 + c^2*x^2)) + (5*c^4*x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) + (5*c^3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/d^2 + (13*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d^2) - (((5*I)/2)*b*c^3*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 + (((5*I)/2)*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d^2

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} - \frac{1}{3}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)^{3/2}}\,dx}{3d^2}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)} + \left(5c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)}{}$$

$$= \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)} + \frac{5c^4x\left(a+b\sinh^{-}\right.}{2d^2\left(1+c^2x\right.}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a\right.}{3d^2}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a\right.}{3d^2}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a\right.}{3d^2}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a\right.}{3d^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.42, size = 311, normalized size = 1.30

$$\frac{-\frac{5a}{3x^3} + \frac{5ac^2}{x} - \frac{5bc\sqrt{1+c^2x^2}}{6x^2} + \frac{a}{x^3+c^2x^5} - \frac{5b\sinh^{-1}(cx)}{3x^3} + \frac{5bc^2\sinh^{-1}(cx)}{x} + \frac{b\sinh^{-1}(cx)}{x^3+c^2x^5} + 5ac^3\text{ArcTan}(cx) + \frac{35}{6}bc^3\tanh^{-1}\left(\sqrt{1+c^2x^2}\right) + \frac{bc^3\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1+c^2x^2\right)}{\sqrt{1+c^2x^2}} - 5b(-c^2)^{3/2}\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 5b(-c^2)^{3/2}\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) + 5b(-c^2)^{3/2}\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - 5b(-c^2)^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((-5*a)/(3*x^3) + (5*a*c^2)/x - (5*b*c*Sqrt[1 + c^2*x^2])/(6*x^2) + a/(x^3
+ c^2*x^5) - (5*b*ArcSinh[c*x])/(3*x^3) + (5*b*c^2*ArcSinh[c*x])/x + (b*Arc
Sinh[c*x])/(x^3 + c^2*x^5) + 5*a*c^3*ArcTan[c*x] + (35*b*c^3*ArcTanh[Sqrt[1
 + c^2*x^2]])/6 + (b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x^2])/Sqrt
[1 + c^2*x^2] - 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sq
rt[-c^2]] + 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x
])/c] + 5*b*(-c^2)^(3/2)*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 5*b*(-
c^2)^(3/2)*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(2*d^2)
```

Maple [A]
time = 2.53, size = 316, normalized size = 1.32

method	result
derivativedivides	$c^3\left(-\frac{a}{3d^2c^3x^3}+\frac{2a}{d^2cx}+\frac{acx}{2d^2(c^2x^2+1)}+\frac{5a\arctan(cx)}{2d^2}-\frac{b\,\mathrm{arcsinh}(cx)}{3d^2c^3x^3}+\frac{2b\,\mathrm{arcsinh}(cx)}{d^2cx}+\frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)}+\right.$
default	$c^3\left(-\frac{a}{3d^2c^3x^3}+\frac{2a}{d^2cx}+\frac{acx}{2d^2(c^2x^2+1)}+\frac{5a\arctan(cx)}{2d^2}-\frac{b\,\mathrm{arcsinh}(cx)}{3d^2c^3x^3}+\frac{2b\,\mathrm{arcsinh}(cx)}{d^2cx}+\frac{b\,\mathrm{arcsinh}(cx)cx}{2d^2(c^2x^2+1)}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a/d^2/c^3/x^3+2*a/d^2/c/x+1/2*a/d^2*c*x/(c^2*x^2+1)+5/2*a/d^2*arc
tan(c*x)-1/3*b/d^2*arcsinh(c*x)/c^3/x^3+2*b/d^2*arcsinh(c*x)/c/x+1/2*b/d^2*
arcsinh(c*x)*c*x/(c^2*x^2+1)+5/2*b/d^2*arcsinh(c*x)*arctan(c*x)+1/3*b/d^2/(
c^2*x^2+1)^(1/2)+13/6*b/d^2*arctanh(1/(c^2*x^2+1)^(1/2))-1/6*b/d^2/c^2/x^2/
(c^2*x^2+1)^(1/2)+5/2*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
-5/2*b/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-5/2*I*b/d^2*dilo
g(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+5/2*I*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2
+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
d^2*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8 + 2*c^2
*d^2*x^6 + d^2*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c
**4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^2), x)
```

3.46 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=186

$$\frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)}+\frac{3\left(a+b\sinh^{-1}(cx)\right)\,A}{4c^5d}$$

```
[Out] 1/12*b/c^5/d^3/(c^2*x^2+1)^(3/2)-1/4*x^3*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^
2+1)^2-3/8*x*(a+b*arcsinh(c*x))/c^4/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))*
arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d^3-3/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c^5/d^3+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3-5/8*b
/c^5/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5810, 5789, 4265, 2317, 2438, 267, 272, 45}

$$\frac{3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(c^2x^2+1\right)}-\frac{3ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}+\frac{3ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}-\frac{5b}{8c^5d^3\sqrt{c^2x^2+1}}+\frac{b}{12c^5d^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c^5*d^3*(1 + c^2*x^2)^(3/2)) - (5*b)/(8*c^5*d^3*Sqrt[1 + c^2*x^2]) -
(x^3*(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a + b*ArcSin
h[c*x]))/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(4*c^5*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*
d^3) + (((3*I)/8)*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{b\int \frac{x^3}{(1+c^2x^2)^{5/2}}\,dx}{4cd^3} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{(d+c^2dx^2)^2}\,dx}{4c^2d}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} - \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)} + \frac{(3b)\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{8c^3d^3} + \frac{b\text{Subs}\ldots}{\ldots}$$

$$= -\frac{3b}{8c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} - \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)} + \frac{3\text{Subst}(\ldots}{\ldots}$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}} - \frac{5b}{8c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} - \frac{3x\left(a+b\,\text{si}\ldots}{8c^4d^3\left(1\ldots\right.}$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}} - \frac{5b}{8c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} - \frac{3x\left(a+b\,\text{si}\ldots}{8c^4d^3\left(1\ldots\right.}$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}} - \frac{5b}{8c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} - \frac{3x\left(a+b\,\text{si}\ldots}{8c^4d^3\left(1\ldots\right.}$$

Mathematica [A]
time = 0.38, size = 341, normalized size = 1.83

$$-\frac{9acx+15ac^3x^3+13b\sqrt{1+c^2x^2}+15bc^2x^2\sqrt{1+c^2x^2}+9bcx\sinh^{-1}(cx)+15bc^3x^3\sinh^{-1}(cx)-9a\text{ArcTan}(cx)-18ac^2x^2\text{ArcTan}(cx)-9ac^4x^4\text{ArcTan}(cx)-9ib\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-18ibc^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-9ibc^4x^4\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+9ib\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+18ibc^2x^2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+9ibc^4x^4\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+9ib(1+c^2x^2)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-9ib(1+c^2x^2)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{24c^5d^3\left(1+c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/24*(9*a*c*x + 15*a*c^3*x^3 + 13*b*Sqrt[1 + c^2*x^2] + 15*b*c^2*x^2*Sqrt[
1 + c^2*x^2] + 9*b*c*x*ArcSinh[c*x] + 15*b*c^3*x^3*ArcSinh[c*x] - 9*a*ArcTa
n[c*x] - 18*a*c^2*x^2*ArcTan[c*x] - 9*a*c^4*x^4*ArcTan[c*x] - (9*I)*b*ArcSi
nh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I
*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] +
 (9*I)*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[
c*x]*Log[1 + I*E^ArcSinh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] + (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9
*I)*b*(1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3*(1 + c^2*x^2)^
2)
```

Maple [A]
time = 0.66, size = 292, normalized size = 1.57

method	result

derivativedivides	$\frac{-\frac{5a\,c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{3acx}{8d^3(c^2x^2+1)^2}+\frac{3a\arctan(cx)}{8d^3}-\frac{5b\,\text{arcsinh}(cx)c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{3b\,\text{arcsinh}(cx)cx}{8d^3(c^2x^2+1)^2}+\frac{3b\,\text{arcsinh}(cx)\arctan(cx)}{8d^3}+\frac{3b\arctan(c}{}}{}$
default	$\frac{-\frac{5a\,c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{3acx}{8d^3(c^2x^2+1)^2}+\frac{3a\arctan(cx)}{8d^3}-\frac{5b\,\text{arcsinh}(cx)c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{3b\,\text{arcsinh}(cx)cx}{8d^3(c^2x^2+1)^2}+\frac{3b\,\text{arcsinh}(cx)\arctan(cx)}{8d^3}+\frac{3b\arctan(c}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-5/8*a/d^3/(c^2*x^2+1)^2*c^3*x^3-3/8*a/d^3/(c^2*x^2+1)^2*c*x+3/8*a/d
^3*arctan(c*x)-5/8*b/d^3*arcsinh(c*x)/(c^2*x^2+1)^2*c^3*x^3-3/8*b/d^3*arcsi
nh(c*x)/(c^2*x^2+1)^2*c*x+3/8*b/d^3*arcsinh(c*x)*arctan(c*x)+3/8*b/d^3*arct
an(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*b/d^3*arctan(c*x)*ln(1-I*(1
+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))+3/8*I*b/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-5/8*b/d^3*c^2*x^2/(c^
2*x^2+1)^(3/2)-13/24*b/d^3/(c^2*x^2+1)^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arcta
n(c*x)/(c^5*d^3)) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x
^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{bx^4\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.47 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=97

$$\frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{b\sinh^{-1}(cx)}{4c^4d^3}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}$$

```
[Out] 1/12*b*x^3/c/d^3/(c^2*x^2+1)^(3/2)-1/4*b*arcsinh(c*x)/c^4/d^3+1/4*x^4*(a+b*
arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/4*b*x/c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5800, 294, 221}

$$\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{b\sinh^{-1}(cx)}{4c^4d^3}+\frac{bx^3}{12cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b*x^3)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(4*c^3*d^3*Sqrt[1 + c^2*x^2]
) - (b*ArcSinh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c
^2*x^2)^2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
```

```
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx &= \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{(bc)\int\frac{x^4}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3}\\ &= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{b\int\frac{x^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{4cd^3}\\ &= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{1+c^2x^2}}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{b\int\frac{1}{\sqrt{1+c^2}}}{4c^3d^3}\\ &= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{b\sinh^{-1}(cx)}{4c^4d^3}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.81

$$\frac{-3a(1+2c^2x^2)+bcx\sqrt{1+c^2x^2}\,(3+4c^2x^2)-3(b+2bc^2x^2)\sinh^{-1}(cx)}{12c^4d^3\left(1+c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a*(1 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2) - 3*(b + 2*
b*c^2*x^2)*ArcSinh[c*x])/(12*c^4*d^3*(1 + c^2*x^2)^2)
```

Maple [A]
time = 0.53, size = 108, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\frac{a\left(-\frac{1}{2\left(c^2x^2+1\right)}+\frac{1}{4\left(c^2x^2+1\right)^2}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2\left(c^2x^2+1\right)}+\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}-\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^2x^2+1}}\right)}{d^3}}{c^4}$	108
default	$\frac{\frac{a\left(-\frac{1}{2\left(c^2x^2+1\right)}+\frac{1}{4\left(c^2x^2+1\right)^2}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2\left(c^2x^2+1\right)}+\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}-\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^2x^2+1}}\right)}{d^3}}{c^4}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/d^3*(-1/2/(c^2*x^2+1)+1/4/(c^2*x^2+1)^2)+b/d^3*(-1/2/(c^2*x^2+1)*a
rcsinh(c*x)+1/4*arcsinh(c*x)/(c^2*x^2+1)^2-1/12/(c^2*x^2+1)^(3/2)*c*x+1/3*c
*x/(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1)) + 3)/(
c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 16*integrate(1/4*(2*c^2*x^2 + 1)/(
c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^3*x + (c^9*d^3*x^6 + 3
*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2 + 1)), x)) - 1/4*(2*c^
2*x^2 + 1)*a/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

Fricas [A]
time = 0.35, size = 99, normalized size = 1.02

$$\frac{3\,ac^4x^4 - 3\,(2\,bc^2x^2 + b)\log\left(cx + \sqrt{c^2x^2 + 1}\right) + (4\,bc^3x^3 + 3\,bcx)\sqrt{c^2x^2 + 1}}{12\,(c^8d^3x^4 + 2\,c^6d^3x^2 + c^4d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 - 3*(2*b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + (4*b
*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d
^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.48 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=184

$$-\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)A}{4c^3}$$

```
[Out] -1/12*b/c^3/d^3/(c^2*x^2+1)^(3/2)-1/4*x*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^2
+1)^2+1/8*x*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^2+1)+1/4*(a+b*arcsinh(c*x))*a
rctan(c*x+(c^2*x^2+1)^(1/2))/c^3/d^3-1/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))/c^3/d^3+1/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/8*b/
c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$,
Rules used = {5810, 5788, 5789, 4265, 2317, 2438, 267}

$$\frac{\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(c^2x^2+1\right)}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{b}{8c^3d^3\sqrt{c^2x^2+1}}-\frac{b}{12c^3d^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*b/(c^3*d^3*(1 + c^2*x^2)^(3/2)) + b/(8*c^3*d^3*Sqrt[1 + c^2*x^2]) - (
x*(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSinh[c*x
]))/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]
])/(4*c^3*d^3) - ((I/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/
8)*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{b\int\frac{x}{(1+c^2x^2)^{5/2}}\,dx}{4cd^3} + \frac{\int\frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^2}\,dx}{4c^2d}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(1+c^2x^2\right)} - \frac{b\int\frac{}{(1\cdots}}{}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b}{8c^3d^3\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{x\left(a+b\,\mathrm{s}\cdots\right.}{8c^2d^3\left(\cdots\right.}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b}{8c^3d^3\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{x\left(a+b\,\mathrm{s}\cdots\right.}{8c^2d^3\left(\cdots\right.}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b}{8c^3d^3\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{x\left(a+b\,\mathrm{s}\cdots\right.}{8c^2d^3\left(\cdots\right.}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b}{8c^3d^3\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{x\left(a+b\,\mathrm{s}\cdots\right.}{8c^2d^3\left(\cdots\right.}$$

Mathematica [A]
time = 0.14, size = 340, normalized size = 1.85

$$\frac{-3acx+3ac^3x^3+b\sqrt{1+c^2x^2}+3bc^2x^2\sqrt{1+c^2x^2}-3bcx\sinh^{-1}(cx)+3bc^3x^3\sinh^{-1}(cx)+3a\mathrm{ArcTan}(cx)+6ac^2x^2\mathrm{ArcTan}(cx)+3ac^4x^4\mathrm{ArcTan}(cx)+3ib\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+6ibc^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+3ibc^4x^4\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-3ib\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-6ibc^2x^2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-3ibc^4x^4\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-3ib(1+c^2x^2)^2\,\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+3ib(1+c^2x^2)^2\,\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{24c^3d^3\left(1+c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a*c*x + 3*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 3*b*c^2*x^2*Sqrt[1 + c^2*x^
2] - 3*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 3*a*ArcTan[c*x] + 6*
a*c^2*x^2*ArcTan[c*x] + 3*a*c^4*x^4*ArcTan[c*x] + (3*I)*b*ArcSinh[c*x]*Log[
1 - I*E^ArcSinh[c*x]] + (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*
x]] + (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*b*ArcS
inh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I
*E^ArcSinh[c*x]] - (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] -
 (3*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*b*(1 + c^2
*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c^3*d^3*(1 + c^2*x^2)^2)
```

Maple [A]
time = 0.66, size = 292, normalized size = 1.59

method	result

derivativedivides	$\frac{\frac{a\,c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{acx}{8d^3(c^2x^2+1)^2}+\frac{a\arctan(cx)}{8d^3}+\frac{b\,\mathrm{arcsinh}(cx)c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{b\,\mathrm{arcsinh}(cx)cx}{8d^3(c^2x^2+1)^2}+\frac{b\,\mathrm{arcsinh}(cx)\arctan(cx)}{8d^3}+\frac{b\arctan(cx)\ln\left(1+\right.}{8}}{}$
default	$\frac{\frac{a\,c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{acx}{8d^3(c^2x^2+1)^2}+\frac{a\arctan(cx)}{8d^3}+\frac{b\,\mathrm{arcsinh}(cx)c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{b\,\mathrm{arcsinh}(cx)cx}{8d^3(c^2x^2+1)^2}+\frac{b\,\mathrm{arcsinh}(cx)\arctan(cx)}{8d^3}+\frac{b\arctan(cx)\ln\left(1+\right.}{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/8*a/d^3/(c^2*x^2+1)^2*c^3*x^3-1/8*a/d^3/(c^2*x^2+1)^2*c*x+1/8*a/d^
3*arctan(c*x)+1/8*b/d^3*arcsinh(c*x)/(c^2*x^2+1)^2*c^3*x^3-1/8*b/d^3*arcsin
h(c*x)/(c^2*x^2+1)^2*c*x+1/8*b/d^3*arcsinh(c*x)*arctan(c*x)+1/8*b/d^3*arcta
n(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*b/d^3*arctan(c*x)*ln(1-I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))-1/8*I*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+1/8*I*b/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/8*b/d^3*c^2*x^2/(c^2
*x^2+1)^(3/2)+1/24*b/d^3/(c^2*x^2+1)^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x)/
(c^3*d^3)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{bx^2\,\mathrm{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.49 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=80

$$\frac{bx}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx}{6cd^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(1+c^2x^2\right)^2}$$

```
[Out] 1/12*b*x/c/d^3/(c^2*x^2+1)^(3/2)+1/4*(-a-b*arcsinh(c*x))/c^2/d^3/(c^2*x^2+1
)^2+1/6*b*x/c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 198, 197}

$$-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{bx}{6cd^3\sqrt{c^2x^2+1}}+\frac{bx}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b*x)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(6*c*d^3*Sqrt[1 + c^2*x^2]) -
(a + b*ArcSinh[c*x])/(4*c^2*d^3*(1 + c^2*x^2)^2)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{4c^2d^3(1+c^2x^2)^2} + \frac{b\int \frac{1}{(1+c^2x^2)^{5/2}}\,dx}{4cd^3}$$
$$= \frac{bx}{12cd^3(1+c^2x^2)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{4c^2d^3(1+c^2x^2)^2} + \frac{b\int \frac{1}{(1+c^2x^2)^{3/2}}\,dx}{6cd^3}$$
$$= \frac{bx}{12cd^3(1+c^2x^2)^{3/2}} + \frac{bx}{6cd^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{4c^2d^3(1+c^2x^2)^2}$$

Mathematica [**A**]
time = 0.04, size = 56, normalized size = 0.70

$$\frac{-3a+bcx\sqrt{1+c^2x^2}\,(3+2c^2x^2)-3b\sinh^{-1}(cx)}{12d^3(c+c^3x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
[Out] (-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2) - 3*b*ArcSinh[c*x])/(12*d^3
*(c + c^3*x^2)^2)
```

Maple [**A**]
time = 0.53, size = 76, normalized size = 0.95

method	result	size
derivativedivides	$\frac{-\frac{a}{4d^3(c^2x^2+1)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{4(c^2x^2+1)^2}+\frac{cx}{12(c^2x^2+1)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3}}{c^2}$	76
default	$\frac{-\frac{a}{4d^3(c^2x^2+1)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{4(c^2x^2+1)^2}+\frac{cx}{12(c^2x^2+1)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3}}{c^2}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
[Out] 1/c^2*(-1/4*a/d^3/(c^2*x^2+1)^2+b/d^3*(-1/4*arcsinh(c*x)/(c^2*x^2+1)^2+1/12
/(c^2*x^2+1)^(3/2)*c*x+1/6*c*x/(c^2*x^2+1)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*b*((4*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2
+ c^2*d^3) - 16*integrate(1/4/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3
+ c^2*d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^
2*x^2 + 1)), x)) - 1/4*a/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Fricas [A]
time = 0.38, size = 98, normalized size = 1.22

$$\frac{3\,ac^4x^4+6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(2\,bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}}{12\,\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 + 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 + 1)) + (2*b*c
^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{bx\operatorname{asinh}\left(cx\right)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*
x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.50 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=178

$$\frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{3b}{8cd^3\sqrt{1+c^2x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}+\frac{3\left(a+b\sinh^{-1}(cx)\right)\text{Arc}}{4cd^3}$$

```
[Out] 1/12*b/c/d^3/(c^2*x^2+1)^(3/2)+1/4*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+3
/8*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))*arctan(c*x+(
c^2*x^2+1)^(1/2))/c/d^3-3/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3
+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/8*b/c/d^3/(c^2*x^2+1)
^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5788, 5789, 4265, 2317, 2438, 267}

$$\frac{3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}+\frac{3b}{8cd^3\sqrt{c^2x^2+1}}+\frac{b}{12cd^3\left(c^2x^2+1\right)^{3/2}}-\frac{3ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{8cd^3}+\frac{3ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{8cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (3*b)/(8*c*d^3*Sqrt[1 + c^2*x^2]) + (x*(
a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*x]))/(
8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*c
*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/8)*
b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^3}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x}{(1+c^2x^2)^{5/2}}\,dx}{4d^3} + \frac{3\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^2}\,dx}{4d}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)} - \frac{(3bc)\int \frac{\cdots}{(1+c^{\cdots}}}{8d^3}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(\cdots\right.}{8d^3\left(1+c^2x^2\right)\cdots}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(\cdots\right.}{8d^3\left(1+c^2x^2\right)\cdots}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(\cdots\right.}{8d^3\left(1+c^2x^2\right)\cdots}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(\cdots\right.}{8d^3\left(1+c^2x^2\right)\cdots}$$

Mathematica [A]

time = 0.10, size = 341, normalized size = 1.92

$$\frac{15acx+9ac^3x^3+11b\sqrt{1+c^2x^2}+9bc^2x^2\sqrt{1+c^2x^2}+15bcx\sinh^{-1}(cx)+9bc^3x^3\sinh^{-1}(cx)+9a\text{ArcTan}(cx)+18ac^2x^2\text{ArcTan}(cx)+9ac^4x^4\text{ArcTan}(cx)+9ib\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+18ibc^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+9ibc^4x^4\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-9ib\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-18ibc^2x^2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-9ibc^4x^4\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-9ib(1+c^2x^2)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+9ib(1+c^2x^2)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{24cd^3(1+c^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

```
[Out] (15*a*c*x + 9*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] + 9*b*c^2*x^2*Sqrt[1 + c^2
*x^2] + 15*b*c*x*ArcSinh[c*x] + 9*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x]
+ 18*a*c^2*x^2*ArcTan[c*x] + 9*a*c^4*x^4*ArcTan[c*x] + (9*I)*b*ArcSinh[c*x]
*Log[1 - I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcS
inh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*
b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Lo
g[1 + I*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[
c*x]] - (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b*(
1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c*d^3*(1 + c^2*x^2)^2)
```

Maple [A]

time = 0.67, size = 284, normalized size = 1.60

method	result
derivativedivides	$\frac{\frac{acx}{4d^3(c^2x^2+1)^2}+\frac{3acx}{8d^3(c^2x^2+1)}+\frac{3a\arctan(cx)}{8d^3}+\frac{b\operatorname{arcsinh}(cx)cx}{4d^3(c^2x^2+1)^2}+\frac{3b\operatorname{arcsinh}(cx)cx}{8d^3(c^2x^2+1)}+\frac{3b\operatorname{arcsinh}(cx)\arctan(cx)}{8d^3}+\frac{3b\,c^2x^2}{8d^3(c^2x^2+1)^{\frac{3}{2}}}+\cdots}{}$
default	$\frac{\frac{acx}{4d^3(c^2x^2+1)^2}+\frac{3acx}{8d^3(c^2x^2+1)}+\frac{3a\arctan(cx)}{8d^3}+\frac{b\operatorname{arcsinh}(cx)cx}{4d^3(c^2x^2+1)^2}+\frac{3b\operatorname{arcsinh}(cx)cx}{8d^3(c^2x^2+1)}+\frac{3b\operatorname{arcsinh}(cx)\arctan(cx)}{8d^3}+\frac{3b\,c^2x^2}{8d^3(c^2x^2+1)^{\frac{3}{2}}}+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*a/d^3/(c^2*x^2+1)^2*c*x+3/8*a/d^3*c*x/(c^2*x^2+1)+3/8*a/d^3*arctan
(c*x)+1/4*b/d^3*arcsinh(c*x)/(c^2*x^2+1)^2*c*x+3/8*b/d^3*arcsinh(c*x)*c*x/(
c^2*x^2+1)+3/8*b/d^3*arcsinh(c*x)*arctan(c*x)+3/8*b/d^3*c^2*x^2/(c^2*x^2+1)
^(3/2)+11/24*b/d^3/(c^2*x^2+1)^(3/2)+3/8*b/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-3/8*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))-3/8*I*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/8*I*b/d^3*dilog(1-I
*(1+I*c*x)/(c^2*x^2+1)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c*x
)/(c*d^3)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*
d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
+ d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*as
inh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3, x)
```

3.51 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=159

$$-\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{2bcx}{3d^3\sqrt{1+c^2x^2}}+\frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2}+\frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^3}$$

```
[Out] -1/12*b*c*x/d^3/(c^2*x^2+1)^(3/2)+1/4*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+
1/2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))*arctanh((c*x+(c
^2*x^2+1)^(1/2))^2)/d^3-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+1/2
*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-2/3*b*c*x/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5811, 5799, 5569, 4267, 2317, 2438, 197, 198}

$$\frac{a+b\sinh^{-1}(cx)}{2d^3\left(c^2x^2+1\right)}+\frac{a+b\sinh^{-1}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{2bcx}{3d^3\sqrt{c^2x^2+1}}-\frac{bcx}{12d^3\left(c^2x^2+1\right)^{3/2}}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/12*(b*c*x)/(d^3*(1 + c^2*x^2)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[1 + c^2*x^2
]) + (a + b*ArcSinh[c*x])/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])/(2
*d^3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/
d^3 - (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*d^3) + (b*PolyLog[2, E^(2*ArcS
inh[c*x])])/(2*d^3)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx &= \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{1}{(1+c^2x^2)^{5/2}}\,dx}{4d^3} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x(d+c^2dx^2)^2}\,dx}{d} \\
&= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{1}{(1+c^2x^2)^{3/2}}\,dx}{6d^3} \\
&= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} + \\
&= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} + \\
&= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \\
&= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \\
&= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} -
\end{aligned}
$$

Mathematica [A]
time = 0.43, size = 289, normalized size = 1.82

$$\frac{-\frac{2a^2}{b}+\frac{a}{(1+c^2x^2)^2}-\frac{bcx}{3(1+c^2x^2)^{3/2}}+\frac{2a}{1+c^2x^2}-\frac{8bcx}{3\sqrt{1+c^2x^2}}-4a\sinh^{-1}(cx)+\frac{b\sinh^{-1}(cx)}{(1+c^2x^2)^2}+\frac{2b\sinh^{-1}(cx)}{1+c^2x^2}-4b\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-4b\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)+4a\log\left(1-e^{2\sinh^{-1}(cx)}\right)+4b\sinh^{-1}(cx)\log\left(1-e^{2\sinh^{-1}(cx)}\right)-2a\log\left(1+c^2x^2\right)-4b\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-4b\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)+2b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-2*a^2)/b + a/(1 + c^2*x^2)^2 - (b*c*x)/(3*(1 + c^2*x^2)^(3/2)) + (2*a)/(
1 + c^2*x^2) - (8*b*c*x)/(3*Sqrt[1 + c^2*x^2]) - 4*a*ArcSinh[c*x] + (b*ArcS
inh[c*x])/(1 + c^2*x^2)^2 + (2*b*ArcSinh[c*x])/(1 + c^2*x^2) - 4*b*ArcSinh[
c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*ArcSinh[c*x]*Log[1 + (Sqr
t[-c^2]*E^ArcSinh[c*x])/c] + 4*a*Log[1 - E^(2*ArcSinh[c*x])] + 4*b*ArcSinh[
c*x]*Log[1 - E^(2*ArcSinh[c*x])] - 2*a*Log[1 + c^2*x^2] - 4*b*PolyLog[2, (c
*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c
] + 2*b*PolyLog[2, E^(2*ArcSinh[c*x])])/(4*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(170) = 340$.
time = 4.46, size = 451, normalized size = 2.84

method	result
derivativedivides	$-\frac{a\ln(c^2x^2+1)}{2d^3}+\frac{a}{4d^3(c^2x^2+1)^2}+\frac{a}{2d^3(c^2x^2+1)}+\frac{a\ln(cx)}{d^3}-\frac{2b\sqrt{c^2x^2+1}\,c^3x^3}{3d^3(c^4x^4+2c^2x^2+1)}+\frac{2b\,c^4x^4}{3d^3(c^4x^4+2c^2x^2+1)}$
default	$-\frac{a\ln(c^2x^2+1)}{2d^3}+\frac{a}{4d^3(c^2x^2+1)^2}+\frac{a}{2d^3(c^2x^2+1)}+\frac{a\ln(cx)}{d^3}-\frac{2b\sqrt{c^2x^2+1}\,c^3x^3}{3d^3(c^4x^4+2c^2x^2+1)}+\frac{2b\,c^4x^4}{3d^3(c^4x^4+2c^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d^3*ln(c^2*x^2+1)+1/4*a/d^3/(c^2*x^2+1)^2+1/2*a/d^3/(c^2*x^2+1)+a/d^
3*ln(c*x)-2/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)*(c^2*x^2+1)^(1/2)*c^3*x^3+2/3*b/d
^3/(c^4*x^4+2*c^2*x^2+1)*c^4*x^4+1/2*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*
x)*c^2*x^2-3/4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c*x*(c^2*x^2+1)^(1/2)+4/3*b/d^3/
(c^4*x^4+2*c^2*x^2+1)*c^2*x^2+3/4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+
2/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)+b/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))+b/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-b/d^3*arcsinh(c*x)*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+b/d^3*a
rcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^2
+ 1)/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^
3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
+ d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b*as
inh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{x\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^3), x)
```

3.52 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=222

$$-\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{7bc}{8d^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{15c^2x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}$$

```
[Out] -1/12*b*c/d^3/(c^2*x^2+1)^(3/2)+(-a-b*arcsinh(c*x))/d^3/x/(c^2*x^2+1)^2-5/4
*c^2*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2-15/8*c^2*x*(a+b*arcsinh(c*x))/d
^3/(c^2*x^2+1)-15/4*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3-
b*c*arctanh((c^2*x^2+1)^(1/2))/d^3+15/8*I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/d^3-15/8*I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-7/8*b*c/d^
3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5809, 5788, 5789, 4265, 2317, 2438, 267, 272, 53, 65, 214}

$$-\frac{15c\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}-\frac{15c^2x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{a+b\sinh^{-1}(cx)}{d^3x\left(c^2x^2+1\right)^2}-\frac{7bc}{8d^3\sqrt{c^2x^2+1}}-\frac{bc}{12d^3\left(c^2x^2+1\right)^{3/2}}-\frac{bc\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{d^3}+\frac{15ibc\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{8d^3}-\frac{15ibc\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/12*(b*c)/(d^3*(1 + c^2*x^2)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[1 + c^2*x^2]) -
 (a + b*ArcSinh[c*x])/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c*x
]))/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c
^2*x^2)) - (15*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (b*
c*ArcTanh[Sqrt[1 + c^2*x^2]])/d^3 + (((15*I)/8)*b*c*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/d^3 - (((15*I)/8)*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
```

```
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx+\frac{(bc)\int \frac{1}{x(1+c^2x^2)^{5/2}}\,dx}{d^3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}+\frac{(bc)\mathrm{Subst}\left(\int \frac{1}{x(1+c^2x)^{5/2}}\,dx,x,x^2\right)}{2d^3}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{15c^2x\left(a+b\right.}{8d^3\left(1+\right.}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{7bc}{8d^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x\left(a+b\sinh^{-1}(cx\right.}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{7bc}{8d^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x\left(a+b\sinh^{-1}(cx\right.}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{7bc}{8d^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x\left(a+b\sinh^{-1}(cx\right.}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{7bc}{8d^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x\left(a+b\sinh^{-1}(cx\right.}{4d^3\left(1+c^2x^2\right)^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.74, size = 298, normalized size = 1.34

$$-\frac{\frac{45(a+b\sinh^{-1}(cx))}{x}-\frac{6(a+b\sinh^{-1}(cx))}{x(1+c^2x^2)^2}-\frac{15(a+b\sinh^{-1}(cx))}{x+c^2x^3}+45ac\text{ArcTan}(cx)+45bc\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)+\frac{2bc\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+c^2x^2\right)}{(1+c^2x^2)^{3/2}}+\frac{15bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+c^2x^2\right)}{\sqrt{1+c^2x^2}}+45b\sqrt{-c^2}\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-45b\sqrt{-c^2}\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)-45b\sqrt{-c^2}\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+45b\sqrt{-c^2}\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/24*((45*(a + b*ArcSinh[c*x]))/x - (6*(a + b*ArcSinh[c*x]))/(x*(1 + c^2*x
^2)^2) - (15*(a + b*ArcSinh[c*x]))/(x + c^2*x^3) + 45*a*c*ArcTan[c*x] + 45*
b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (2*b*c*Hypergeometric2F1[-3/2, 1, -1/2, 1
+ c^2*x^2])/(1 + c^2*x^2)^(3/2) + (15*b*c*Hypergeometric2F1[-1/2, 1, 1/2, 1
 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (c*E^
ArcSinh[c*x])/Sqrt[-c^2]] - 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2
]*E^ArcSinh[c*x])/c] - 45*b*Sqrt[-c^2]*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-
c^2]] + 45*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d^3
```

Maple [A]
time = 3.42, size = 352, normalized size = 1.59

method	result
derivativedivides	$c\left(-\frac{a}{d^3cx}-\frac{7a\,c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{9acx}{8d^3(c^2x^2+1)^2}-\frac{15a\arctan(cx)}{8d^3}-\frac{b\,\mathrm{arcsinh}(cx)}{d^3cx}-\frac{7b\,\mathrm{arcsinh}(cx)c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{9b\,\mathrm{arcs}}{8d^3(c}\right.$
default	$c\left(-\frac{a}{d^3cx}-\frac{7a\,c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{9acx}{8d^3(c^2x^2+1)^2}-\frac{15a\arctan(cx)}{8d^3}-\frac{b\,\mathrm{arcsinh}(cx)}{d^3cx}-\frac{7b\,\mathrm{arcsinh}(cx)c^3x^3}{8d^3(c^2x^2+1)^2}-\frac{9b\,\mathrm{arcs}}{8d^3(c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a/d^3/c/x-7/8*a/d^3/(c^2*x^2+1)^2*c^3*x^3-9/8*a/d^3/(c^2*x^2+1)^2*c*x-1
5/8*a/d^3*arctan(c*x)-b/d^3*arcsinh(c*x)/c/x-7/8*b/d^3*arcsinh(c*x)/(c^2*x^
2+1)^2*c^3*x^3-9/8*b/d^3*arcsinh(c*x)/(c^2*x^2+1)^2*c*x-15/8*b/d^3*arcsinh(
c*x)*arctan(c*x)-15/8*b/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
+15/8*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-15/8*I*b/d^3*di
log(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+15/8*I*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))-15/8*b/d^3*c^2*x^2/(c^2*x^2+1)^(3/2)-47/24*b/d^3/(c^2*x^2+1)^
(3/2)+b/d^3/(c^2*x^2+1)^(1/2)-b/d^3*arctanh(1/(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*x)
 + 15*c*arctan(c*x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^
3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
+ d^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integral(b
*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{x^2\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^3), x)
```

3.53 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=232

$$-\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}}-\frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}}+\frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2}-\frac{3c^2(a}{2d}$$

```
[Out] -1/2*b*c/d^3/x/(c^2*x^2+1)^(3/2)-5/12*b*c^3*x/d^3/(c^2*x^2+1)^(3/2)-3/4*c^2
*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/2*(-a-b*arcsinh(c*x))/d^3/x^2/(c^2*
x^2+1)^2-3/2*c^2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+6*c^2*(a+b*arcsinh(c*x)
)*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*b*c^2*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)/d^3-3/2*b*c^2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+2/3*b*
c^3*x/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5809, 5811, 5799, 5569, 4267, 2317, 2438, 197, 198, 277}

$$-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^3\left(c^2x^2+1\right)}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{6c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}+\frac{3bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{3bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{bc}{2d^3x\left(c^2x^2+1\right)^{3/2}}+\frac{2bc^3x}{3d^3\sqrt{c^2x^2+1}}-\frac{5bc^3x}{12d^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/2*(b*c)/(d^3*x*(1 + c^2*x^2)^(3/2)) - (5*b*c^3*x)/(12*d^3*(1 + c^2*x^2)^
(3/2)) + (2*b*c^3*x)/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a + b*ArcSinh[c*x]
))/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])/(2*d^3*x^2*(1 + c^2*x^2)^
2) - (3*c^2*(a + b*ArcSinh[c*x]))/(2*d^3*(1 + c^2*x^2)) + (6*c^2*(a + b*Arc
Sinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (3*b*c^2*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} - \left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} + \frac{\left(3bc^3\right)\int \frac{1}{\left(1+c^2x\right.}}{4d^3}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sinh^{-1}(}{2d^3x^2\left(1+c^2\right.}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right.}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right.}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right.}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right.}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right.}{4d^3\left(1+c^2x^2\right)^2}$$

Mathematica [A]
time = 0.57, size = 353, normalized size = 1.52

$$\frac{-\frac{18bc\sqrt{1+c^2x^2}}{x}+\frac{9bc\left(1+2c^2x^2\right)}{x\sqrt{1+c^2x^2}}+\frac{bc\left(3+12c^2x^2+8c^4x^4\right)}{x\left(1+c^2x^2\right)^{3/2}}-18bc^2\sinh^{-1}(cx)^2-\frac{18\left(a+b\sinh^{-1}(cx)\right)}{x^2}+\frac{3\left(a+b\sinh^{-1}(cx)\right)}{\left(x+c^2x^3\right)^2}+\frac{9\left(a+b\sinh^{-1}(cx)\right)}{x^2+c^2x^4}+\frac{18c^2\left(a+b\sinh^{-1}(cx)\right)^2}{b}+36bc^2\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+36bc^2\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+18ac^2\log\left(1+c^2x^2\right)+36bc^2\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+36bc^2\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)-18c^2\left(2\left(a+b\sinh^{-1}(cx)\right)\log\left(1-e^{2\sinh^{-1}(cx)}\right)+b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\right)}{12d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-18*b*c*Sqrt[1 + c^2*x^2])/x + (9*b*c*(1 + 2*c^2*x^2))/(x*Sqrt[1 + c^2*x^
2]) + (b*c*(3 + 12*c^2*x^2 + 8*c^4*x^4))/(x*(1 + c^2*x^2)^(3/2)) - 18*b*c^2
*ArcSinh[c*x]^2 - (18*(a + b*ArcSinh[c*x]))/x^2 + (3*(a + b*ArcSinh[c*x]))/
(x + c^2*x^3)^2 + (9*(a + b*ArcSinh[c*x]))/(x^2 + c^2*x^4) + (18*c^2*(a + b
*ArcSinh[c*x])^2)/b + 36*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt
[-c^2]] + 36*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 18
*a*c^2*Log[1 + c^2*x^2] + 36*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]
] + 36*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 18*c^2*(2*(a + b*A
rcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSinh[c*x])]
))/(12*d^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(240) = 480$.
time = 4.84, size = 549, normalized size = 2.37

method	result
derivativedivides	$c^2\left(-\frac{a}{d^3(c^2x^2+1)}+\frac{3a\ln(c^2x^2+1)}{2d^3}-\frac{a}{4d^3(c^2x^2+1)^2}-\frac{a}{2d^3c^2x^2}-\frac{3a\ln(cx)}{d^3}+\frac{2b\sqrt{c^2x^2+1}\,c^3x^3}{3d^3(c^4x^4+2c^2x^2+1)}-\frac{}{3c}\right.$
default	$c^2\left(-\frac{a}{d^3(c^2x^2+1)}+\frac{3a\ln(c^2x^2+1)}{2d^3}-\frac{a}{4d^3(c^2x^2+1)^2}-\frac{a}{2d^3c^2x^2}-\frac{3a\ln(cx)}{d^3}+\frac{2b\sqrt{c^2x^2+1}\,c^3x^3}{3d^3(c^4x^4+2c^2x^2+1)}-\frac{}{3c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-a/d^3/(c^2*x^2+1)+3/2*a/d^3*ln(c^2*x^2+1)-1/4*a/d^3/(c^2*x^2+1)^2-1/2
*a/d^3/c^2/x^2-3*a/d^3*ln(c*x)+2/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)*(c^2*x^2+1)^
(1/2)*c^3*x^3-2/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c^4*x^4-3/2*b/d^3/(c^4*x^4+2*
c^2*x^2+1)*arcsinh(c*x)*c^2*x^2+1/4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c*x*(c^2*x^
2+1)^(1/2)-4/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c^2*x^2-9/4*b/d^3/(c^4*x^4+2*c^2
*x^2+1)*arcsinh(c*x)-1/2*b/d^3/(c^4*x^4+2*c^2*x^2+1)/c/x*(c^2*x^2+1)^(1/2)-
2/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)-1/2*b/d^3/(c^4*x^4+2*c^2*x^2+1)/c^2/x^2*arc
sinh(c*x)-3*b/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-3*b/d^3*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))+3*b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)+3/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-3*b/d^3*arcsinh(c*x)*ln
(1-c*x-(c^2*x^2+1)^(1/2))-3*b/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^2)
 - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + b*integrate(log(c*x +
sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5
+ d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + Integral(b
*asinh(c*x)/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^3), x)
```

3.54 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=295

$$-\frac{bc^3}{12d^3(1+c^2x^2)^{3/2}}-\frac{bc}{6d^3x^2(1+c^2x^2)^{3/2}}+\frac{29bc^3}{24d^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{3d^3x^3(1+c^2x^2)^2}+\frac{7c^2(a+b\sinh^{-1}(cx))}{3d^3x(1+c^2x^2)^2}+\frac{35}{}$$

```
[Out] -1/12*b*c^3/d^3/(c^2*x^2+1)^(3/2)-1/6*b*c/d^3/x^2/(c^2*x^2+1)^(3/2)+1/3*(-a
-b*arcsinh(c*x))/d^3/x^3/(c^2*x^2+1)^2+7/3*c^2*(a+b*arcsinh(c*x))/d^3/x/(c^
2*x^2+1)^2+35/12*c^4*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+35/8*c^4*x*(a+b
*arcsinh(c*x))/d^3/(c^2*x^2+1)+35/4*c^3*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*
x^2+1)^(1/2))/d^3+19/6*b*c^3*arctanh((c^2*x^2+1)^(1/2))/d^3-35/8*I*b*c^3*po
lylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+35/8*I*b*c^3*polylog(2,I*(c*x+(c^2*
x^2+1)^(1/2)))/d^3+29/24*b*c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5809, 5788, 5789, 4265, 2317, 2438, 267, 272, 53, 65, 214, 44}

$$\frac{35c^3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{4d^3}+\frac{7c^2(a+b\sinh^{-1}(cx))}{3d^3x(c^2x^2+1)^2}-\frac{a+b\sinh^{-1}(cx)}{3d^3x^3(c^2x^2+1)^2}+\frac{35c^4x(a+b\sinh^{-1}(cx))}{8d^3(c^2x^2+1)}+\frac{35c^4x(a+b\sinh^{-1}(cx))}{12d^3(c^2x^2+1)^2}-\frac{35ibc^3\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{8d^3}+\frac{35ibc^3\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{8d^3}-\frac{bc}{6d^3x^2(c^2x^2+1)^{3/2}}+\frac{29bc^3}{24d^3\sqrt{c^2x^2+1}}-\frac{bc^3}{12d^3(c^2x^2+1)^{3/2}}+\frac{19bc^3\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/12*(b*c^3)/(d^3*(1 + c^2*x^2)^(3/2)) - (b*c)/(6*d^3*x^2*(1 + c^2*x^2)^(3
/2)) + (29*b*c^3)/(24*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*d^3*
x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x]))/(3*d^3*x*(1 + c^2*x^2)^
2) + (35*c^4*x*(a + b*ArcSinh[c*x]))/(12*d^3*(1 + c^2*x^2)^2) + (35*c^4*x*(
a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*ArcSinh[c*x])*A
rcTan[E^ArcSinh[c*x]])/(4*d^3) + (19*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d
^3) - (((35*I)/8)*b*c^3*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)/8)*
b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} - \frac{1}{3}\left(7c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)^{5/2}}\,dx}{3d^3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^3x\left(1+c^2x^2\right)^2} + \frac{1}{3}\left(35c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx + \cdots$$

$$= \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^3x\left(1+c^2x^2\right)^2} + \cdots$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \cdots$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.64, size = 380, normalized size = 1.29

$$\frac{-\frac{70a}{x^3}+\frac{210ac^2}{x}+\frac{12a}{x^3(1+c^2x^2)^2}-\frac{35bc\sqrt{1+c^2x^2}}{x^2}+\frac{42a}{x^3+c^2x^5}-\frac{70b\sinh^{-1}(cx)}{x^3}+\frac{210bc^2\sinh^{-1}(cx)}{x}+\frac{12b\sinh^{-1}(cx)}{x^3(1+c^2x^2)^2}+\frac{42b\sinh^{-1}(cx)}{x^3+c^2x^5}+210ac^3\mathrm{ArcTan}(cx)+245bc^3\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)+\frac{4bc^3\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};1+c^2x^2\right)}{(1+c^2x^2)^{3/2}}+\frac{42bc^3\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1+c^2x^2\right)}{\sqrt{1+c^2x^2}}-210b(-c^2)^{3/2}\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+210b(-c^2)^{3/2}\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+210b(-c^2)^{3/2}\mathrm{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-210b(-c^2)^{3/2}\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{48d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-70*a)/x^3 + (210*a*c^2)/x + (12*a)/(x^3*(1 + c^2*x^2)^2) - (35*b*c*Sqrt[
1 + c^2*x^2])/x^2 + (42*a)/(x^3 + c^2*x^5) - (70*b*ArcSinh[c*x])/x^3 + (210
*b*c^2*ArcSinh[c*x])/x + (12*b*ArcSinh[c*x])/(x^3*(1 + c^2*x^2)^2) + (42*b*
ArcSinh[c*x])/(x^3 + c^2*x^5) + 210*a*c^3*ArcTan[c*x] + 245*b*c^3*ArcTanh[S
qrt[1 + c^2*x^2]] + (4*b*c^3*Hypergeometric2F1[-3/2, 2, -1/2, 1 + c^2*x^2])
/(1 + c^2*x^2)^(3/2) + (42*b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x^
2])/Sqrt[1 + c^2*x^2] - 210*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
```

```
h[c*x])/Sqrt[-c^2]] + 210*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E
^ArcSinh[c*x])/c] + 210*b*(-c^2)^(3/2)*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-
c^2]] - 210*b*(-c^2)^(3/2)*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(48*d
^3)
```

Maple [A]
time = 2.58, size = 406, normalized size = 1.38

method	result
derivativedivides	$c^3\left(\frac{11a\,c^3x^3}{8d^3(c^2x^2+1)^2}+\frac{13acx}{8d^3(c^2x^2+1)^2}+\frac{35a\arctan(cx)}{8d^3}-\frac{a}{3d^3c^3x^3}+\frac{3a}{d^3cx}+\frac{11b\,\mathrm{arcsinh}(cx)c^3x^3}{8d^3(c^2x^2+1)^2}+\frac{13b\,\mathrm{arcsin}}{8d^3(c^2x}\right.$
default	$c^3\left(\frac{11a\,c^3x^3}{8d^3(c^2x^2+1)^2}+\frac{13acx}{8d^3(c^2x^2+1)^2}+\frac{35a\arctan(cx)}{8d^3}-\frac{a}{3d^3c^3x^3}+\frac{3a}{d^3cx}+\frac{11b\,\mathrm{arcsinh}(cx)c^3x^3}{8d^3(c^2x^2+1)^2}+\frac{13b\,\mathrm{arcsin}}{8d^3(c^2x}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(11/8*a/d^3/(c^2*x^2+1)^2*c^3*x^3+13/8*a/d^3/(c^2*x^2+1)^2*c*x+35/8*a/d
^3*arctan(c*x)-1/3*a/d^3/c^3/x^3+3*a/d^3/c/x+11/8*b/d^3*arcsinh(c*x)/(c^2*x
^2+1)^2*c^3*x^3+13/8*b/d^3*arcsinh(c*x)/(c^2*x^2+1)^2*c*x+35/8*b/d^3*arcsin
h(c*x)*arctan(c*x)-1/3*b/d^3*arcsinh(c*x)/c^3/x^3+3*b/d^3*arcsinh(c*x)/c/x+
35/8*b/d^3*c^2*x^2/(c^2*x^2+1)^(3/2)+103/24*b/d^3/(c^2*x^2+1)^(3/2)-1/6*b/d
^3/c^2/x^2/(c^2*x^2+1)^(3/2)-19/6*b/d^3/(c^2*x^2+1)^(1/2)+19/6*b/d^3*arctan
h(1/(c^2*x^2+1)^(1/2))+35/8*b/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))-35/8*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+35/8*I*b/
d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-35/8*I*b/d^3*dilog(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2 -
 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6
 + d^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integral(
b*asinh(c*x)/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^3), x)
```

3.55 $\int x^3\sqrt{\pi + c^2\pi x^2}\,\left(a + b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=109

$$\frac{2b\sqrt{\pi}\,x}{15c^3}-\frac{b\sqrt{\pi}\,x^3}{45c}-\frac{1}{25}bc\sqrt{\pi}\,x^5-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi}+\frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4\pi^2}$$

```
[Out] -1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/c^4/Pi+1/5*(Pi*c^2*x^2+Pi)^(5
/2)*(a+b*arcsinh(c*x))/c^4/Pi^2+2/15*b*x*Pi^(1/2)/c^3-1/45*b*x^3*Pi^(1/2)/c
-1/25*b*c*x^5*Pi^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45, 5804, 12}

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^4}+\frac{2\sqrt{\pi}\,bx}{15c^3}-\frac{1}{25}\sqrt{\pi}\,bcx^5-\frac{\sqrt{\pi}\,bx^3}{45c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Sqrt[Pi]*x)/(15*c^3) - (b*Sqrt[Pi]*x^3)/(45*c) - (b*c*Sqrt[Pi]*x^5)/25
 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*Pi) + ((Pi + c^2*P
i*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*Pi^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
```

```
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)dx &= -\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{5/2}\left(a+b\right.}{5c^4}\\ &= -\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{5/2}\left(a+b\right.}{5c^4}\\ &= \frac{2b\sqrt{\pi}\,x}{15c^3}-\frac{b\sqrt{\pi}\,x^3}{45c}-\frac{1}{25}bc\sqrt{\pi}\,x^5-\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh\right.}{3c^4}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 106, normalized size = 0.97

$$\frac{\sqrt{\pi}\left(15a\sqrt{1+c^2x^2}\left(-2+c^2x^2+3c^4x^4\right)+b\left(30cx-5c^3x^3-9c^5x^5\right)+15b\sqrt{1+c^2x^2}\left(-2+c^2x^2+3c^4x^4\right)\sinh^{-1}(cx)\right)}{225c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(15*a*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4) + b*(30*c*x -
5*c^3*x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*
ArcSinh[c*x]))/(225*c^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3(a+b\,\mathrm{arcsinh}\,(cx))\sqrt{\pi\,c^2x^2+\pi}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

Maxima [A]
time = 0.27, size = 134, normalized size = 1.23

$$\frac{1}{15}b\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)\mathrm{arsinh}\,(cx)+\frac{1}{15}a\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)-\frac{\left(9\sqrt{\pi}\,c^4x^5+5\sqrt{\pi}\,c^2x^3-30\sqrt{\pi}\,x\right)b}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*b*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(3/2)/
(pi*c^4))*arcsinh(c*x) + 1/15*a*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2
*(pi + pi*c^2*x^2)^(3/2)/(pi*c^4)) - 1/225*(9*sqrt(pi)*c^4*x^5 + 5*sqrt(pi)
*c^2*x^3 - 30*sqrt(pi)*x)*b/c^3
```

Fricas [A]
time = 0.47, size = 158, normalized size = 1.45

$$\frac{15\sqrt{\pi+\pi c^2x^2}\,(3\,bc^6x^6+4\,bc^4x^4-bc^2x^2-2\,b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(45\,ac^6x^6+60\,ac^4x^4-15\,ac^2x^2-(9\,bc^5x^5+5\,bc^3x^3-30\,bcx)\sqrt{c^2x^2+1}-30\,a\right)}{225\,(c^6x^2+c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*
b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 + 60*
a*c^4*x^4 - 15*a*c^2*x^2 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*
x^2 + 1) - 30*a))/(c^6*x^2 + c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(100) = 200$.
time = 1.06, size = 221, normalized size = 2.03

$$\begin{cases} \frac{\sqrt{\pi}\,ax^4\sqrt{c^2x^2+1}}{5}+\frac{\sqrt{\pi}\,ax^2\sqrt{c^2x^2+1}}{15c^2}-\frac{2\sqrt{\pi}\,a\sqrt{c^2x^2+1}}{15c^4}-\frac{\sqrt{\pi}\,bcx^5}{25}+\frac{\sqrt{\pi}\,bx^4\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{5}-\frac{\sqrt{\pi}\,bx^3}{45c}+\frac{\sqrt{\pi}\,bx^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{15c^2}+\frac{2\sqrt{\pi}\,bx}{15c^3}-\frac{2\sqrt{\pi}\,b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{15c^4} & \text{for}\ c\neq 0\\ \frac{\sqrt{\pi}\,ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((sqrt(pi)*a*x**4*sqrt(c**2*x**2 + 1)/5 + sqrt(pi)*a*x**2*sqrt(c**
2*x**2 + 1)/(15*c**2) - 2*sqrt(pi)*a*sqrt(c**2*x**2 + 1)/(15*c**4) - sqrt(p
i)*b*c*x**5/25 + sqrt(pi)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5 - sqrt(pi
)*b*x**3/(45*c) + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**2)
+ 2*sqrt(pi)*b*x/(15*c**3) - 2*sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
5*c**4), Ne(c, 0)), (sqrt(pi)*a*x**4/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\sqrt{\Pi\,c^2\,x^2+\Pi}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.56 $\int x^2 \sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=119

$$-\frac{b\sqrt{\pi}\, x^2}{16c}-\frac{1}{16}bc\sqrt{\pi}\, x^4+\frac{\sqrt{\pi}\, x\sqrt{1+c^2x^2}\, \left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}\, \left(a+b\sinh^{-1}(cx)\right)-\frac{\sqrt{\pi}\, (a$$

```
[Out] -1/16*b*x^2*Pi^(1/2)/c-1/16*b*c*x^4*Pi^(1/2)-1/16*(a+b*arcsinh(c*x))^2*Pi^(
1/2)/b/c^3+1/8*x*(a+b*arcsinh(c*x))*Pi^(1/2)*(c^2*x^2+1)^(1/2)/c^2+1/4*x^3*
(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5806, 5812, 5783, 30}

$$-\frac{\sqrt{\pi}\, \left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3}+\frac{\sqrt{\pi}\, x\sqrt{c^2x^2+1}\, \left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}\, \left(a+b\sinh^{-1}(cx)\right)-\frac{1}{16}\sqrt{\pi}\, bcx^4-\frac{\sqrt{\pi}\, bx^2}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/16*(b*Sqrt[Pi]*x^2)/c - (b*c*Sqrt[Pi]*x^4)/16 + (Sqrt[Pi]*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(8*c^2) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*Arc
Sinh[c*x]))/4 - (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi+c^2\pi x^2}\int\frac{x^2(a+b\sinh^{-1}(}{\sqrt{1+c^2x^2}}}{4\sqrt{1+c^2x^2}}$$

$$=-\frac{bcx^4\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{\pi}$$

$$=-\frac{bx^2\sqrt{\pi+c^2\pi x^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sin\right.}{8c^2}$$

Mathematica [A]
time = 0.11, size = 79, normalized size = 0.66

$$\frac{\sqrt{\pi}\left(16acx\sqrt{1+c^2x^2}\left(1+2c^2x^2\right)-8b\sinh^{-1}(cx)^2-b\cosh\left(4\sinh^{-1}(cx)\right)+\sinh^{-1}(cx)\left(-16a+4b\sinh\left(4\sinh^{-1}(cx)\right)\right)\right)}{128c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
[Out] (Sqrt[Pi]*(16*a*c*x*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2) - 8*b*ArcSinh[c*x]^2
- b*Cosh[4*ArcSinh[c*x]] + ArcSinh[c*x]*(-16*a + 4*b*Sinh[4*ArcSinh[c*x]]))
)/(128*c^3)
```

Maple [A]
time = 0.80, size = 156, normalized size = 1.31

method	result
default	$\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4\pi c^2}-\frac{ax\sqrt{\pi c^2x^2+\pi}}{8c^2}-\frac{a\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8c^2\sqrt{\pi c^2}}-\frac{b\sqrt{\pi}\left(-4\sqrt{c^2x^2+1}\ \operatorname{arcsinh}(cx)x^3c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-1/8*a/c^2*x*(Pi*c^2*x^2+Pi)^(1/2)-1/8*
a/c^2*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1
/16*b*Pi^(1/2)*(-4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3*c^3+c^4*x^4-2*arcsinh
(c*x)*(c^2*x^2+1)^(1/2)*x*c+c^2*x^2+arcsinh(c*x)^2)/c^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int ax^2\sqrt{c^2x^2+1}\,dx+\int bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] sqrt(pi)*(Integral(a*x**2*sqrt(c**2*x**2 + 1), x) + Integral(b*x**2*sqrt(c*
*2*x**2 + 1)*asinh(c*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\sqrt{\Pi\,c^2\,x^2+\Pi}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.57 $\int x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=61

$$-\frac{b\sqrt{\pi}\,x}{3c} - \frac{1}{9}bc\sqrt{\pi}\,x^3 + \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{3c^2\pi}$$

```
[Out] 1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/c^2/Pi-1/3*b*x*Pi^(1/2)/c-1/9*
b*c*x^3*Pi^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{5798\}$

$$\frac{\left(\pi c^2x^2 + \pi\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{3\pi c^2} - \frac{1}{9}\sqrt{\pi}\,bcx^3 - \frac{\sqrt{\pi}\,bx}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/3*(b*Sqrt[Pi]*x)/c - (b*c*Sqrt[Pi]*x^3)/9 + ((Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/(3*c^2*Pi)
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{3c^2\pi} - \frac{\left(b\sqrt{\pi + c^2\pi x^2}\right)\int \left(1 + c^2x^2\right.}{3c\sqrt{1 + c^2x^2}}$$

$$= -\frac{bx\sqrt{\pi + c^2\pi x^2}}{3c\sqrt{1 + c^2x^2}} - \frac{bcx^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\,\mathrm{si}\right.}{3c^2\pi}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 1.03

$$\frac{\sqrt{\pi}\left(3a\left(1 + c^2x^2\right)^{3/2} - bcx\left(3 + c^2x^2\right) + 3b\left(1 + c^2x^2\right)^{3/2}\sinh^{-1}(cx)\right)}{9c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(3*a*(1 + c^2*x^2)^(3/2) - b*c*x*(3 + c^2*x^2) + 3*b*(1 + c^2*x^2
)^(3/2)*ArcSinh[c*x]))/(9*c^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{arcsinh}(cx))\sqrt{\pi c^2x^2+\pi}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

Maxima [A]
time = 0.27, size = 73, normalized size = 1.20

$$\frac{(\pi+\pi c^2x^2)^{\frac{3}{2}}b\operatorname{arsinh}(cx)}{3\pi c^2}-\frac{\left(\pi^{\frac{3}{2}}c^2x^3+3\pi^{\frac{3}{2}}x\right)b}{9\pi c}+\frac{(\pi+\pi c^2x^2)^{\frac{3}{2}}a}{3\pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*c^2) - 1/9*(pi^(3/2)*c^2*x^3
 + 3*pi^(3/2)*x)*b/(pi*c) + 1/3*(pi + pi*c^2*x^2)^(3/2)*a/(pi*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(49) = 98$.
time = 0.38, size = 127, normalized size = 2.08

$$\frac{3\sqrt{\pi+\pi c^2x^2}\,(bc^4x^4+2\,bc^2x^2+b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(3\,ac^4x^4+6\,ac^2x^2-(bc^3x^3+3\,bcx)\sqrt{c^2x^2+1}+3\,a\right)}{9\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(c
^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^
3 + 3*b*c*x)*sqrt(c^2*x^2 + 1) + 3*a))/(c^4*x^2 + c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(53) = 106$.
time = 0.30, size = 141, normalized size = 2.31

$$\begin{cases}\frac{\sqrt{\pi}\,ax^2\sqrt{c^2x^2+1}}{3}+\frac{\sqrt{\pi}\,a\sqrt{c^2x^2+1}}{3c^2}-\frac{\sqrt{\pi}\,bcx^3}{9}+\frac{\sqrt{\pi}\,bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3}-\frac{\sqrt{\pi}\,bx}{3c}+\frac{\sqrt{\pi}\,b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^2} & \text{for } c\neq 0\\ \frac{\sqrt{\pi}\,ax^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((sqrt(pi)*a*x**2*sqrt(c**2*x**2 + 1)/3 + sqrt(pi)*a*sqrt(c**2*x**
2 + 1)/(3*c**2) - sqrt(pi)*b*c*x**3/9 + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)
*asinh(c*x)/3 - sqrt(pi)*b*x/(3*c) + sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(3*c**2), Ne(c, 0)), (sqrt(pi)*a*x**2/2, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2+\Pi}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.58 $\int \sqrt{\pi + c^2\pi x^2}\,\left(a + b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=67

$$-\frac{1}{4}bc\sqrt{\pi}\,x^2 + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}\,\left(a + b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi}\,\left(a + b\sinh^{-1}(cx)\right)^2}{4bc}$$

```
[Out] -1/4*b*c*x^2*Pi^(1/2)+1/4*(a+b*arcsinh(c*x))^2*Pi^(1/2)/b/c+1/2*x*(a+b*arcs
inh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5785, 5783, 30}

$$\frac{1}{2}x\sqrt{\pi c^2x^2 + \pi}\,\left(a + b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi}\,\left(a + b\sinh^{-1}(cx)\right)^2}{4bc} - \frac{1}{4}\sqrt{\pi}\,bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/4*(b*c*Sqrt[Pi]*x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2
+ (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(4*b*c)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))\,dx = \frac{1}{2}x\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx)) + \frac{\sqrt{\pi+c^2\pi x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,d}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcx^2\sqrt{\pi+c^2\pi x^2}}{4\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx)) + \frac{\sqrt{\pi+}}{}$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 1.03

$$\frac{\sqrt{\pi}\left(4acx\sqrt{1+c^2x^2}+2b\sinh^{-1}(cx)^2-b\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(2a+b\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(4*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSin
h[c*x]] + 2*ArcSinh[c*x]*(2*a + b*Sinh[2*ArcSinh[c*x]])))/(8*c)
```

Maple [A]
time = 1.00, size = 100, normalized size = 1.49

method	result
default	$\frac{ax\sqrt{\pi c^2x^2+\pi}}{2}+\frac{a\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{2\sqrt{\pi c^2}}+\frac{b\sqrt{\pi}\left(2\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,xc-c^2x^2+\mathrm{arcsinh}(cx)^2-\right.}{4c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x
^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b*Pi^(1/2)*(2*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*x*c-c^2*x^2+arcsinh(c*x)^2-1)/c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int a\sqrt{c^2x^2+1}\,dx+\int b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 +
1)*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2+\Pi}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.59 $\int \frac{\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=89

$$-bc\sqrt{\pi}\,x+\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)-2\sqrt{\pi}\,\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)-b\sqrt{\pi}\,\mathrm{PolyLog}\Big(2$$

```
[Out] -b*c*x*Pi^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2
)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^
(1/2))*Pi^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5806, 5816, 4267, 2317, 2438, 8}

$$\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)-2\sqrt{\pi}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\sqrt{\pi}\,b\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\sqrt{\pi}\,b\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)+\sqrt{\pi}\,(-b)cx$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -(b*c*Sqrt[Pi]*x) + Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) - 2*Sqrt[Pi]
*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Sqrt[Pi]*PolyLog[2, -E^Ar
cSinh[c*x]] + b*Sqrt[Pi]*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi+c^2\pi x^2}\,\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} - \cdots$$

$$= -\frac{bcx\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}} + \sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi+c^2\pi x^2}\cdots}{\cdots}$$

$$= -\frac{bcx\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}} + \sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi+c^2\pi x^2}\cdots}{\cdots}$$

$$= -\frac{bcx\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}} + \sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi+c^2\pi x^2}\cdots}{\cdots}$$

$$= -\frac{bcx\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}} + \sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi+c^2\pi x^2}\cdots}{\cdots}$$

Mathematica [A]
time = 0.13, size = 131, normalized size = 1.47

$$\sqrt{\pi}\left(a\sqrt{1+c^2x^2}+a\log(x)-a\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right)+b\left(-cx+\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)+\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]

[Out] Sqrt[Pi]*(a*Sqrt[1 + c^2*x^2] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))

Maple [A]
time = 4.37, size = 171, normalized size = 1.92

method	result
default	$a\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\ \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)+\ln\left(1-cx-\sqrt{c^2x^2+1}\right)\sqrt{\pi}\ \operatorname{arcsinh}(c$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x,x,method=_RETURNVERBOSE)

[Out] a*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2)))+ln(1-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)*arcsinh(c*x)*b+(c^2*x^2+1)^(1/2)*Pi^(1/2)*arcsinh(c*x)*b-Pi^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*b-b*c*x*Pi^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="maxima")

[Out] -(sqrt(pi)*arcsinh(1/(c*abs(x))) - sqrt(pi + pi*c^2*x^2))*a + b*integrate(sqrt(pi + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int\frac{a\sqrt{c^2x^2+1}}{x}\,dx+\int\frac{b\sqrt{c^2x^2+1}\ \operatorname{asinh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2
+ 1)*asinh(c*x)/x, x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2+\Pi}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x, x)
```

3.60 $\int \frac{\sqrt{\pi + c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{\pi + c^2\pi x^2}\,(a + b\sinh^{-1}(cx))}{x} + \frac{c\sqrt{\pi}\,(a + b\sinh^{-1}(cx))^2}{2b} + bc\sqrt{\pi}\,\log(x)$$

```
[Out] 1/2*c*(a+b*arcsinh(c*x))^2*Pi^(1/2)/b+b*c*ln(x)*Pi^(1/2)-(a+b*arcsinh(c*x))
*(Pi*c^2*x^2+Pi)^(1/2)/x
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5805, 29, 5783}

$$-\frac{\sqrt{\pi c^2x^2 + \pi}\,(a + b\sinh^{-1}(cx))}{x} + \frac{\sqrt{\pi}\,c(a + b\sinh^{-1}(cx))^2}{2b} + \sqrt{\pi}\,bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[Pi]*(a + b*ArcS
inh[c*x])^2)/(2*b) + b*c*Sqrt[Pi]*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\pi + c^2\pi x^2}\,(a + b\sinh^{-1}(cx))}{x^2}\,dx = -\frac{\sqrt{\pi + c^2\pi x^2}\,(a + b\sinh^{-1}(cx))}{x} + \frac{\left(bc\sqrt{\pi + c^2\pi x^2}\right)\int \frac{1}{x}\,dx}{\sqrt{1 + c^2x^2}} + \frac{\left(c^2\right.}{}$$
$$= -\frac{\sqrt{\pi + c^2\pi x^2}\,(a + b\sinh^{-1}(cx))}{x} + \frac{c\sqrt{\pi + c^2\pi x^2}\,(a + b\sinh^{-1}(cx)}{2b\sqrt{1 + c^2x^2}}$$

Mathematica [**A**]
time = 0.11, size = 75, normalized size = 1.23

$$\frac{\sqrt{\pi}\left(-2a\sqrt{1 + c^2x^2} + 2\left(acx - b\sqrt{1 + c^2x^2}\right)\sinh^{-1}(cx) + bcx\sinh^{-1}(cx)^2 + 2bcx\log(cx)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Sqrt[Pi]*(-2*a*Sqrt[1 + c^2*x^2] + 2*(a*c*x - b*Sqrt[1 + c^2*x^2])*ArcSinh
[c*x] + b*c*x*ArcSinh[c*x]^2 + 2*b*c*x*Log[c*x]))/(2*x)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(53) = 106.
time = 3.63, size = 155, normalized size = 2.54

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{\pi x} + a c^2 x\sqrt{\pi c^2x^2 + \pi} + \frac{a c^2\pi \ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}} + \sqrt{\pi c^2x^2 + \pi}\right)}{\sqrt{\pi c^2}} + \frac{bc\sqrt{\pi}\ \operatorname{arcsinh}(cx)^2}{2} - bc\sqrt{\pi}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^2*Pi*ln(Pi*
c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c*Pi^(1/2)
*arcsinh(c*x)^2-b*c*Pi^(1/2)*arcsinh(c*x)-b*Pi^(1/2)*arcsinh(c*x)/x*(c^2*x^
2+1)^(1/2)+b*c*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] (sqrt(pi)*c*arcsinh(c*x) - sqrt(pi + pi*c^2*x^2)/x)*a + b*integrate(sqrt(pi
 + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^2, x)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(54) = 108$.
time = 1.84, size = 110, normalized size = 1.80

$$-\frac{\sqrt{\pi}\,ac^2x}{\sqrt{c^2x^2+1}}+\sqrt{\pi}\,ac\operatorname{asinh}(cx)-\frac{\sqrt{\pi}\,a}{x\sqrt{c^2x^2+1}}+\sqrt{\pi}\,bc\log(x)+\frac{\sqrt{\pi}\,bc\operatorname{asinh}^2(cx)}{2}-\frac{\sqrt{\pi}\,b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**2,x)
```

```
[Out] -sqrt(pi)*a*c**2*x/sqrt(c**2*x**2 + 1) + sqrt(pi)*a*c*asinh(c*x) - sqrt(pi)
*a/(x*sqrt(c**2*x**2 + 1)) + sqrt(pi)*b*c*log(x) + sqrt(pi)*b*c*asinh(c*x)*
*2/2 - sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2+\Pi}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^2, x)
```

3.61 $\int \frac{\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=113

$$-\frac{bc\sqrt{\pi}}{2x}-\frac{\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{2x^2}-c^2\sqrt{\pi}\,(a+b\sinh^{-1}(cx))\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)-\frac{1}{2}bc^2\sqrt{\pi}\,\text{PolyLog}$$

```
[Out] -1/2*b*c*Pi^(1/2)/x-c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*P
i^(1/2)-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+1/2*b*c^2*poly
log(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)-1/2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)/x^2
```

Rubi [A]
time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5805, 30, 5816, 4267, 2317, 2438}

$$-\frac{\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{2x^2}-\sqrt{\pi}\,c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))-\frac{1}{2}\sqrt{\pi}\,bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\frac{1}{2}\sqrt{\pi}\,bc^2\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)-\frac{\sqrt{\pi}\,bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Sqrt[Pi])/x - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*x^2
) - c^2*Sqrt[Pi]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - (b*c^2*Sqrt
[Pi]*PolyLog[2, -E^ArcSinh[c*x]])/2 + (b*c^2*Sqrt[Pi]*PolyLog[2, E^ArcSinh[
c*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx &= -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{\left(bc\sqrt{\pi+c^2\pi x^2}\right)\int\frac{1}{x^2}\,dx}{2\sqrt{1+c^2x^2}}+ \\
&= -\frac{bc\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{\left(c^2\sqrt{\pi+c\right.}{} \\
&= -\frac{bc\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{\pi+c^2}}{} \\
&= -\frac{bc\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{\pi+c^2}}{} \\
&= -\frac{bc\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{\pi+c^2}}{}
\end{aligned}$$

Mathematica [A]
time = 2.18, size = 185, normalized size = 1.64

$$\frac{1}{8}\sqrt{\pi}\left(-\frac{4a\sqrt{1+c^2x^2}}{x^2}+4ac^2\log(x)-4ac^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right)+bc^2\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\operatorname{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Sqrt[Pi]*((-4*a*Sqrt[1 + c^2*x^2])/x^2 + 4*a*c^2*Log[x] - 4*a*c^2*Log[Pi*(
1 + Sqrt[1 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csc
h[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh
[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*Pol
yLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[A
rcSinh[c*x]/2])))/8
```

Maple [A]
time = 6.16, size = 241, normalized size = 2.13

method	result
default	$a\left(-\frac{\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{2\pi x^2}+\frac{c^2\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)}{2}\right)-\frac{b\sqrt{\pi}\,\operatorname{arcsinh}(cx)c^2}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{\pi}}{2x}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(3/2)+1/2*c^2*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2
)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))-1/2*b*Pi^(1/2)/(c^2*x^2+1)^(1/2
)*arcsinh(c*x)*c^2-1/2*b*c*Pi^(1/2)/x-1/2*b*Pi^(1/2)/(c^2*x^2+1)^(1/2)/x^2*
arcsinh(c*x)-1/2*b*c^2*Pi^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/
2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+1/2*b*c^2*Pi^(1/2)*arcsi
nh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/
2))*Pi^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(sqrt(pi)*c^2*arcsinh(1/(c*abs(x))) - sqrt(pi + pi*c^2*x^2)*c^2 + (pi
+ pi*c^2*x^2)^(3/2)/(pi*x^2))*a + b*integrate(sqrt(pi + pi*c^2*x^2)*log(c*x
 + sqrt(c^2*x^2 + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int\frac{a\sqrt{c^2x^2+1}}{x^3}\,dx+\int\frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}\left(cx\right)}{x^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**3,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(b*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/x**3, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\sqrt{\Pi\,c^2\,x^2+\Pi}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^3, x)
```

3.62 $\int \frac{\sqrt{\pi + c^2 \pi x^2}\,(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=62

$$-\frac{bc\sqrt{\pi}}{6x^2}-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{1}{3}bc^3\sqrt{\pi}\,\log(x)$$

```
[Out] -1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/Pi/x^3-1/6*b*c*Pi^(1/2)/x^2+1
/3*b*c^3*ln(x)*Pi^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5800, 14}

$$-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{1}{3}\sqrt{\pi}\,bc^3\log(x)-\frac{\sqrt{\pi}\,bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[Pi])/x^2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3
*Pi*x^3) + (b*c^3*Sqrt[Pi]*Log[x])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\pi + c^2\pi x^2}\,(a + b\sinh^{-1}(cx))}{x^4}\,dx = -\frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\sinh^{-1}(cx))}{3\pi x^3} + \frac{\left(bc\sqrt{\pi + c^2\pi x^2}\right)\int \frac{1+c^2x^2}{x^3}\,d}{3\sqrt{1 + c^2x^2}}$$

$$= -\frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\sinh^{-1}(cx))}{3\pi x^3} + \frac{\left(bc\sqrt{\pi + c^2\pi x^2}\right)\int \left(\frac{1}{x^3} + \right.}{3\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{6x^2\sqrt{1 + c^2x^2}} - \frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\sinh^{-1}(cx))}{3\pi x^3} + \frac{bc^3\sqrt{\pi +}}{3\sqrt{1}}$$

Mathematica [**A**]
time = 0.08, size = 78, normalized size = 1.26

$$-\frac{\sqrt{\pi}\left(bcx + 3bc^3x^3 + 2a(1 + c^2x^2)^{3/2} + 2b(1 + c^2x^2)^{3/2}\sinh^{-1}(cx)\right)}{6x^3} + \frac{1}{3}bc^3\sqrt{\pi}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(Sqrt[Pi]*(b*c*x + 3*b*c^3*x^3 + 2*a*(1 + c^2*x^2)^(3/2) + 2*b*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]))/x^3 + (b*c^3*Sqrt[Pi]*Log[x])/3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(50) = 100$.
time = 6.54, size = 501, normalized size = 8.08

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi x^3} - \frac{2b c^3\sqrt{\pi}\ \operatorname{arcsinh}(cx)}{3} + \frac{b\sqrt{\pi}\ x^4\operatorname{arcsinh}(cx)c^7}{3c^4x^4+3c^2x^2+1} - \frac{b\sqrt{\pi}\ x^3\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\ c^6}{3c^4x^4+3c^2x^2+1} + \frac{b\sqrt{\pi}}{18c^4x^4+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(3/2)-2/3*b*c^3*Pi^(1/2)*arcsinh(c*x)+b*Pi^(1
/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-b*Pi^(1/2)/(3*c^4*x^4+3*c^
2*x^2+1)*x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6+1/6*b*Pi^(1/2)/(3*c^4*x^4+3
*c^2*x^2+1)*x^4*c^7-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*(c^2*x^2+1)*
c^5+b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-2*b*Pi^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4-1/3*b*Pi^(1/2)/(
3*c^4*x^4+3*c^2*x^2+1)*(c^2*x^2+1)*c^3+1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+
1)*arcsinh(c*x)*c^3-4/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)*c^2-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2*(c^2*x^2+1)
```

```
*c-1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)+1/3*b*c^3*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(50) = 100$.
time = 0.28, size = 133, normalized size = 2.15

$$-\frac{\left(\pi^{\frac{3}{2}}(-1)^{2\pi+2\pi c^2x^2}c^2\log\left(2\pi c^2+\frac{2\pi}{x^2}\right)-\pi^{\frac{3}{2}}c^2\log\left(x^2+\frac{1}{c^2}\right)+\frac{\pi\sqrt{\pi+\pi c^4x^4+2\pi c^2x^2}}{x^2}\right)bc}{6\pi}-\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}b\operatorname{arsinh}\left(cx\right)}{3\pi x^3}-\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}a}{3\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*(pi^(3/2)*(-1)^(2*pi + 2*pi*c^2*x^2)*c^2*log(2*pi*c^2 + 2*pi/x^2) - pi
^(3/2)*c^2*log(x^2 + 1/c^2) + pi*sqrt(pi + pi*c^4*x^4 + 2*pi*c^2*x^2)/x^2)*
b*c/pi - 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*x^3) - 1/3*(pi + pi
*c^2*x^2)^(3/2)*a/(pi*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(50) = 100$.
time = 0.38, size = 217, normalized size = 3.50

$$-\frac{2\sqrt{\pi+\pi c^2x^2}\left(bc^4x^4+2bc^2x^2+b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\sqrt{\pi}\left(bc^5x^5+bc^3x^3\right)\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}\left(x^4-1\right)}{c^2x^4+x^2}\right)+\sqrt{\pi+\pi c^2x^2}\left(2ac^4x^4+4ac^2x^2-\left(bcx^3-bcx\right)\sqrt{c^2x^2+1}+2a\right)}{6\left(c^2x^5+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(
c^2*x^2 + 1)) - sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + pi*
c^2*x^2 + pi*x^4 + sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 -
1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(2*a*c^4*x^4 + 4*a*c^2*x^2 - (
b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) + 2*a))/(c^2*x^5 + x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int\frac{a\sqrt{c^2x^2+1}}{x^4}\,dx+\int\frac{b\sqrt{c^2x^2+1}\operatorname{asinh}\left(cx\right)}{x^4}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**4,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(b*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/x**4, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^4, x)
```

3.63 $\int x^3(\pi + c^2\pi x^2)^{3/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=125

$$\frac{2b\pi^{3/2}x}{35c^3} - \frac{b\pi^{3/2}x^3}{105c} - \frac{8}{175}bc\pi^{3/2}x^5 - \frac{1}{49}bc^3\pi^{3/2}x^7 - \frac{(\pi + c^2\pi x^2)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{5c^4\pi} + \frac{(\pi + c^2\pi x^2)^{7/2}\left(a + b\sinh^{-1}(cx)\right)}{7c^4\pi^2}$$

```
[Out] 2/35*b*Pi^(3/2)*x/c^3-1/105*b*Pi^(3/2)*x^3/c-8/175*b*c*Pi^(3/2)*x^5-1/49*b*
c^3*Pi^(3/2)*x^7-1/5*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/c^4/Pi+1/7*(P
i*c^2*x^2+Pi)^(7/2)*(a+b*arcsinh(c*x))/c^4/Pi^2
```

Rubi [A]
time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {272, 45, 5804, 12, 380}

$$\frac{(\pi c^2x^2 + \pi)^{7/2}\left(a + b\sinh^{-1}(cx)\right)}{7\pi^2c^4} - \frac{(\pi c^2x^2 + \pi)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{5\pi c^4} - \frac{1}{49}\pi^{3/2}bc^3x^7 + \frac{2\pi^{3/2}bx}{35c^3} - \frac{8}{175}\pi^{3/2}bcx^5 - \frac{\pi^{3/2}bx^3}{105c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Pi^(3/2)*x)/(35*c^3) - (b*Pi^(3/2)*x^3)/(105*c) - (8*b*c*Pi^(3/2)*x^5)
/175 - (b*c^3*Pi^(3/2)*x^7)/49 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*
x]))/(5*c^4*Pi) + ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*Pi^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx &= -\frac{\pi^{3/2}(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}+\frac{\pi^{3/2}(1+c^2x^2)^{7/2}\left(a\right.}{7c^4} \\
&= -\frac{\pi^{3/2}(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}+\frac{\pi^{3/2}(1+c^2x^2)^{7/2}\left(a\right.}{7c^4} \\
&= -\frac{\pi^{3/2}(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}+\frac{\pi^{3/2}(1+c^2x^2)^{7/2}\left(a\right.}{7c^4} \\
&= \frac{2b\pi^{3/2}x}{35c^3}-\frac{b\pi^{3/2}x^3}{105c}-\frac{8}{175}bc\pi^{3/2}x^5-\frac{1}{49}bc^3\pi^{3/2}x^7-\frac{\pi^{3/2}(1+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 100, normalized size = 0.80

$$\frac{\pi^{3/2}\left(105a(1+c^2x^2)^{5/2}\left(-2+5c^2x^2\right)-bcx\left(-210+35c^2x^2+168c^4x^4+75c^6x^6\right)+105b(1+c^2x^2)^{5/2}\left(-2+5c^2x^2\right)\sinh^{-1}(cx)\right)}{3675c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(105*a*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x^2) - b*c*x*(-210 + 35*c^
2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x
^2)*ArcSinh[c*x]))/(3675*c^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.27, size = 145, normalized size = 1.16

$$\frac{1}{35}\left(\frac{5\,(\pi+\pi c^2x^2)^{\frac{5}{2}}x^2}{\pi c^2}-\frac{2\,(\pi+\pi c^2x^2)^{\frac{5}{2}}}{\pi c^4}\right)b\,\mathrm{arsinh}\,(cx)+\frac{1}{35}\left(\frac{5\,(\pi+\pi c^2x^2)^{\frac{5}{2}}x^2}{\pi c^2}-\frac{2\,(\pi+\pi c^2x^2)^{\frac{5}{2}}}{\pi c^4}\right)a-\frac{\left(75\,\pi^{\frac{3}{2}}c^6x^7+168\,\pi^{\frac{3}{2}}c^4x^5+35\,\pi^{\frac{3}{2}}c^2x^3-210\,\pi^{\frac{3}{2}}x\right)b}{3675\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(5/2)/(p
i*c^4))*b*arcsinh(c*x) + 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(
pi + pi*c^2*x^2)^(5/2)/(pi*c^4))*a - 1/3675*(75*pi^(3/2)*c^6*x^7 + 168*pi^(
3/2)*c^4*x^5 + 35*pi^(3/2)*c^2*x^3 - 210*pi^(3/2)*x)*b/c^3
```

Fricas [A]
time = 0.39, size = 199, normalized size = 1.59

$$\frac{105\,\sqrt{\pi+\pi c^2x^2}\,(5\,\pi bc^8x^8+13\,\pi bc^6x^6+9\,\pi bc^4x^4-\pi bc^2x^2-2\,\pi b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(525\,\pi ac^8x^8+1365\,\pi ac^6x^6+945\,\pi ac^4x^4-105\,\pi ac^2x^2-210\,\pi a-(75\,\pi bc^7x^7+168\,\pi bc^5x^5+35\,\pi bc^3x^3-210\,\pi bcx)\sqrt{c^2x^2+1}\right)}{3675\,(c^6x^2+c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] 1/3675*(105*sqrt(pi + pi*c^2*x^2)*(5*pi*b*c^8*x^8 + 13*pi*b*c^6*x^6 + 9*pi*
b*c^4*x^4 - pi*b*c^2*x^2 - 2*pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi +
 pi*c^2*x^2)*(525*pi*a*c^8*x^8 + 1365*pi*a*c^6*x^6 + 945*pi*a*c^4*x^4 - 105
*pi*a*c^2*x^2 - 210*pi*a - (75*pi*b*c^7*x^7 + 168*pi*b*c^5*x^5 + 35*pi*b*c^
3*x^3 - 210*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(117) = 234$.
time = 11.90, size = 301, normalized size = 2.41

$$\begin{cases}\frac{\pi^{\frac{3}{2}}ac^2x^6\sqrt{c^2x^2+1}}{7}+\frac{8\pi^{\frac{3}{2}}ax^4\sqrt{c^2x^2+1}}{35}+\frac{\pi^{\frac{3}{2}}ax^2\sqrt{c^2x^2+1}}{35c^2}-\frac{2\pi^{\frac{3}{2}}a\sqrt{c^2x^2+1}}{35c^4}-\frac{\pi^{\frac{3}{2}}bc^3x^7}{49}+\frac{\pi^{\frac{3}{2}}bc^2x^6\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{7}-\frac{8\pi^{\frac{3}{2}}bcx^5}{175}+\frac{8\pi^{\frac{3}{2}}bx^4\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{35}-\frac{\pi^{\frac{3}{2}}bx^3}{105c}+\frac{\pi^{\frac{3}{2}}bx^2\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{35c^2}+\frac{2\pi^{\frac{3}{2}}bx}{35c^3}-\frac{2\pi^{\frac{3}{2}}b\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{35c^4} & \text{for}\ c\neq 0\\ \frac{\pi^{\frac{3}{2}}ax^4}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**6*sqrt(c**2*x**2 + 1)/7 + 8*pi**(3/2)*a*x**4
*sqrt(c**2*x**2 + 1)/35 + pi**(3/2)*a*x**2*sqrt(c**2*x**2 + 1)/(35*c**2) -
```

```
2*pi**(3/2)*a*sqrt(c**2*x**2 + 1)/(35*c**4) - pi**(3/2)*b*c**3*x**7/49 + pi
**(3/2)*b*c**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - 8*pi**(3/2)*b*c*x**5
/175 + 8*pi**(3/2)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/35 - pi**(3/2)*b*x
**3/(105*c) + pi**(3/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(35*c**2) + 2
*pi**(3/2)*b*x/(35*c**3) - 2*pi**(3/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(35
*c**4), Ne(c, 0)), (pi**(3/2)*a*x**4/4, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,(a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.64 $\int x^2(\pi + c^2\pi x^2)^{3/2} \left(a + b\sinh^{-1}(cx)\right)\, dx$

Optimal. Leaf size=165

$$-\frac{b\pi^{3/2}x^2}{32c} - \frac{7}{96}bc\pi^{3/2}x^4 - \frac{1}{36}bc^3\pi^{3/2}x^6 + \frac{\pi^{3/2}x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{16c^2} + \frac{1}{8}\pi x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(\right.$$

```
[Out] -1/32*b*Pi^(3/2)*x^2/c-7/96*b*c*Pi^(3/2)*x^4-1/36*b*c^3*Pi^(3/2)*x^6+1/6*x^
3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))-1/32*Pi^(3/2)*(a+b*arcsinh(c*x))
^2/b/c^3+1/16*Pi^(3/2)*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^2+1/8*Pi*x^
3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5808, 5806, 5812, 5783, 30, 14}

$$-\frac{\pi^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc^3} + \frac{\pi^{3/2}x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{16c^2} + \frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{8}\pi x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{36}\pi^{3/2}bc^3x^6 - \frac{7}{96}\pi^{3/2}bcx^4 - \frac{\pi^{3/2}bx^2}{32c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(b*Pi^(3/2)*x^2)/c - (7*b*c*Pi^(3/2)*x^4)/96 - (b*c^3*Pi^(3/2)*x^6)/3
6 + (Pi^(3/2)*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (Pi*x^3*
Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (x^3*(Pi + c^2*Pi*x^2)^(3/2
)*(a + b*ArcSinh[c*x]))/6 - (Pi^(3/2)*(a + b*ArcSinh[c*x])^2)/(32*b*c^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}\pi\int x^2\sqrt{\pi+c^2\pi x^2}$$

$$=\frac{1}{8}\pi x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a\right.$$

$$=-\frac{7bc\pi x^4\sqrt{\pi+c^2\pi x^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^6\sqrt{\pi+c^2\pi x^2}}{36\sqrt{1+c^2x^2}}+\frac{\pi x\sqrt{\pi+c^2\pi x}}{}$$

$$=-\frac{b\pi x^2\sqrt{\pi+c^2\pi x^2}}{32c\sqrt{1+c^2x^2}}-\frac{7bc\pi x^4\sqrt{\pi+c^2\pi x^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^6\sqrt{\pi+c^2}}{36\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.24, size = 154, normalized size = 0.93

$$\frac{\pi^{3/2}\left(144acx\sqrt{1+c^2x^2}+672ac^3x^3\sqrt{1+c^2x^2}+384ac^5x^5\sqrt{1+c^2x^2}-72b\sinh^{-1}(cx)^2+18b\cosh\left(2\sinh^{-1}(cx)\right)-9b\cosh\left(4\sinh^{-1}(cx)\right)-2b\cosh\left(6\sinh^{-1}(cx)\right)-12\sinh^{-1}(cx)\left(12a+3b\sinh\left(2\sinh^{-1}(cx)\right)-3b\sinh\left(4\sinh^{-1}(cx)\right)-b\sinh\left(6\sinh^{-1}(cx)\right)\right)\right)}{2304c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(144*a*c*x*Sqrt[1 + c^2*x^2] + 672*a*c^3*x^3*Sqrt[1 + c^2*x^2] +
384*a*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*b*ArcSinh[c*x]^2 + 18*b*Cosh[2*ArcSinh
[c*x]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 12*ArcSinh[c
*x]*(12*a + 3*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4*ArcSinh[c*x]] - b*Sinh[6*
ArcSinh[c*x]])))/(2304*c^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(\pi \,c^{2} x^{2}+\pi \right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^4 + pi*a*x^2 + (pi*b*c^2*x^4 + p
i*b*x^2)*arcsinh(c*x)), x)
```

Sympy [A]

time = 7.01, size = 262, normalized size = 1.59

$$\begin{cases} \frac{\pi^{\frac{3}{2}}ac^2x^5\sqrt{c^2x^2+1}}{6} + \frac{7\pi^{\frac{3}{2}}ax^3\sqrt{c^2x^2+1}}{24} + \frac{\pi^{\frac{3}{2}}ax\sqrt{c^2x^2+1}}{16c^2} - \frac{\pi^{\frac{3}{2}}a\operatorname{asinh}(cx)}{16c^3} - \frac{\pi^{\frac{3}{2}}bc^3x^6}{36} + \frac{\pi^{\frac{3}{2}}bc^2x^5\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{6} - \frac{7\pi^{\frac{3}{2}}bcx^4}{96} + \frac{7\pi^{\frac{3}{2}}bx^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{24} - \frac{\pi^{\frac{3}{2}}bx^2}{32c} + \frac{\pi^{\frac{3}{2}}bx\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{16c^2} - \frac{\pi^{\frac{3}{2}}b\operatorname{asinh}^2(cx)}{32c^3} & \text{for } c \neq 0 \\ \frac{\pi^{\frac{3}{2}}ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**5*sqrt(c**2*x**2 + 1)/6 + 7*pi**(3/2)*a*x**3
*sqrt(c**2*x**2 + 1)/24 + pi**(3/2)*a*x*sqrt(c**2*x**2 + 1)/(16*c**2) - pi*
*(3/2)*a*asinh(c*x)/(16*c**3) - pi**(3/2)*b*c**3*x**6/36 + pi**(3/2)*b*c**2
*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/6 - 7*pi**(3/2)*b*c*x**4/96 + 7*pi**(3
/2)*b*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/24 - pi**(3/2)*b*x**2/(32*c) + pi
**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c**2) - pi**(3/2)*b*asinh(c*
x)**2/(32*c**3), Ne(c, 0)), (pi**(3/2)*a*x**3/3, True))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.65 $\int x(\pi+c^2\pi x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=77

$$-\frac{b\pi^{3/2}x}{5c}-\frac{2}{15}bc\pi^{3/2}x^3-\frac{1}{25}bc^3\pi^{3/2}x^5+\frac{(\pi+c^2\pi x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2\pi}$$

```
[Out] -1/5*b*Pi^(3/2)*x/c-2/15*b*c*Pi^(3/2)*x^3-1/25*b*c^3*Pi^(3/2)*x^5+1/5*(Pi*c
^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/c^2/Pi
```

Rubi [A]

time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {5798, 200}

$$\frac{(\pi c^2x^2+\pi)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^2}-\frac{1}{25}\pi^{3/2}bc^3x^5-\frac{2}{15}\pi^{3/2}bcx^3-\frac{\pi^{3/2}bx}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/5*(b*Pi^(3/2)*x)/c - (2*b*c*Pi^(3/2)*x^3)/15 - (b*c^3*Pi^(3/2)*x^5)/25 +
 ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x(\pi+c^2\pi x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx &= \frac{(\pi+c^2\pi x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2\pi}-\frac{\left(b\pi\sqrt{\pi+c^2\pi x^2}\right)\int (1+c^2\\ &= \frac{(\pi+c^2\pi x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2\pi}-\frac{\left(b\pi\sqrt{\pi+c^2\pi x^2}\right)\int (1+2c\\ &= -\frac{b\pi x\sqrt{\pi+c^2\pi x^2}}{5c\sqrt{1+c^2x^2}}-\frac{2bc\pi x^3\sqrt{\pi+c^2\pi x^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^5\sqrt{\pi+c^2\pi x^2}}{25\sqrt{1+c^2x^2}}\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 72, normalized size = 0.94

$$\frac{\pi^{3/2}\left(15a(1+c^2x^2)^{5/2}-bcx(15+10c^2x^2+3c^4x^4)+15b(1+c^2x^2)^{5/2}\sinh^{-1}(cx)\right)}{75c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(15*a*(1 + c^2*x^2)^(5/2) - b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) +
 15*b*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]))/(75*c^2)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(\pi\, c^2x^2+\pi\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [**A**]
time = 0.30, size = 85, normalized size = 1.10

$$\frac{(\pi+\pi c^2x^2)^{\frac{5}{2}}b\,\mathrm{arsinh}\left(cx\right)}{5\,\pi c^2}+\frac{(\pi+\pi c^2x^2)^{\frac{5}{2}}a}{5\,\pi c^2}-\frac{\left(3\,\pi^{\frac{5}{2}}c^4x^5+10\,\pi^{\frac{5}{2}}c^2x^3+15\,\pi^{\frac{5}{2}}x\right)b}{75\,\pi c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(pi + pi*c^2*x^2)^(5/2)*b*arcsinh(c*x)/(pi*c^2) + 1/5*(pi + pi*c^2*x^2)
^(5/2)*a/(pi*c^2) - 1/75*(3*pi^(5/2)*c^4*x^5 + 10*pi^(5/2)*c^2*x^3 + 15*pi^
(5/2)*x)*b/(pi*c)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(61) = 122.
time = 0.38, size = 167, normalized size = 2.17

$$\frac{15\,\sqrt{\pi+\pi c^2x^2}\,(\pi bc^6x^6+3\,\pi bc^4x^4+3\,\pi bc^2x^2+\pi b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(15\,\pi ac^6x^6+45\,\pi ac^4x^4+45\,\pi ac^2x^2+15\,\pi a-(3\,\pi bc^5x^5+10\,\pi bc^3x^3+15\,\pi bcx)\sqrt{c^2x^2+1}\right)}{75\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*sqrt(pi + pi*c^2*x^2)*(pi*b*c^6*x^6 + 3*pi*b*c^4*x^4 + 3*pi*b*c^2*
x^2 + pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(15*pi*a*c
^6*x^6 + 45*pi*a*c^4*x^4 + 45*pi*a*c^2*x^2 + 15*pi*a - (3*pi*b*c^5*x^5 + 10
*pi*b*c^3*x^3 + 15*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(70) = 140$.
time = 3.96, size = 221, normalized size = 2.87

$$\begin{cases} \frac{\pi^{\frac{3}{2}}ac^2x^4\sqrt{c^2x^2+1}}{5} + \frac{2\pi^{\frac{3}{2}}ax^2\sqrt{c^2x^2+1}}{5} + \frac{\pi^{\frac{3}{2}}a\sqrt{c^2x^2+1}}{5c^2} - \frac{\pi^{\frac{3}{2}}bc^3x^5}{25} + \frac{\pi^{\frac{3}{2}}bc^2x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5} - \frac{2\pi^{\frac{3}{2}}bcx^3}{15} + \frac{2\pi^{\frac{3}{2}}bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5} - \frac{\pi^{\frac{3}{2}}bx}{5c} + \frac{\pi^{\frac{3}{2}}b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5c^2} & \text{for } c \neq 0 \\ \frac{\pi^{\frac{3}{2}}ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**4*sqrt(c**2*x**2 + 1)/5 + 2*pi**(3/2)*a*x**2
*sqrt(c**2*x**2 + 1)/5 + pi**(3/2)*a*sqrt(c**2*x**2 + 1)/(5*c**2) - pi**(3/
2)*b*c**3*x**5/25 + pi**(3/2)*b*c**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5
- 2*pi**(3/2)*b*c*x**3/15 + 2*pi**(3/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*
x)/5 - pi**(3/2)*b*x/(5*c) + pi**(3/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(5*
c**2), Ne(c, 0)), (pi**(3/2)*a*x**2/2, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.66 $\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=111

$$-\frac{5}{16}bc\pi^{3/2}x^2-\frac{1}{16}bc^3\pi^{3/2}x^4+\frac{3}{8}\pi x\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3\pi}{}$$

```
[Out] -5/16*b*c*Pi^(3/2)*x^2-1/16*b*c^3*Pi^(3/2)*x^4+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*
(a+b*arcsinh(c*x))+3/16*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b/c+3/8*Pi*x*(a+b*arc
sinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5786, 5785, 5783, 30, 14}

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{3\pi^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc}-\frac{1}{16}\pi^{3/2}bc^3x^4-\frac{5}{16}\pi^{3/2}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*Pi^(3/2)*x^2)/16 - (b*c^3*Pi^(3/2)*x^4)/16 + (3*Pi*x*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh
[c*x]))/4 + (3*Pi^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{4}x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}(3\pi)\int\sqrt{\pi+c^2\pi x^2}\,(a+$$

$$=\frac{3}{8}\pi x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}x\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\sin$$

$$=-\frac{5bc\pi x^2\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^4\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}}+\frac{3}{8}\pi x\sqrt{\pi+c^2\pi x^2}$$

Mathematica [A]
time = 0.16, size = 111, normalized size = 1.00

$$\frac{\pi^{3/2}\left(80acx\sqrt{1+c^2x^2}+32ac^3x^3\sqrt{1+c^2x^2}+24b\sinh^{-1}(cx)^2-16b\cosh\left(2\sinh^{-1}(cx)\right)-b\cosh\left(4\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\left(12a+8b\sinh\left(2\sinh^{-1}(cx)\right)+b\sinh\left(4\sinh^{-1}(cx)\right)\right)\right)}{128c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(80*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24
*b*ArcSinh[c*x]^2 - 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*
ArcSinh[c*x]*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]])))/(
128*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x)), x)
```

Sympy [A]
time = 1.93, size = 185, normalized size = 1.67

$$\begin{cases} \frac{\pi^{\frac{3}{2}} a c^{2} x^{3} \sqrt{c^{2} x^{2} + 1}}{4} + \frac{5 \pi^{\frac{3}{2}} a x \sqrt{c^{2} x^{2} + 1}}{8} + \frac{3 \pi^{\frac{3}{2}} a \operatorname{asinh}{\left(c x \right)}}{8 c} - \frac{\pi^{\frac{3}{2}} b c^{3} x^{4}}{16} + \frac{\pi^{\frac{3}{2}} b c^{2} x^{3} \sqrt{c^{2} x^{2} + 1} \operatorname{asinh}{\left(c x \right)}}{4} - \frac{5 \pi^{\frac{3}{2}} b c x^{2}}{16} + \frac{5 \pi^{\frac{3}{2}} b x \sqrt{c^{2} x^{2} + 1} \operatorname{asinh}{\left(c x \right)}}{8} + \frac{3 \pi^{\frac{3}{2}} b \operatorname{asinh}^{2}{\left(c x \right)}}{16 c} & \text{for}\: c \neq 0 \\ \pi^{\frac{3}{2}} a x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*x*sq
rt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a*asinh(c*x)/(8*c) - pi**(3/2)*b*c**3*x**
4/16 + pi**(3/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/4 - 5*pi**(3/2)
*b*c*x**2/16 + 5*pi**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/8 + 3*pi**(3/
2)*b*asinh(c*x)**2/(16*c), Ne(c, 0)), (pi**(3/2)*a*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.67 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=134

$$-\frac{4}{3}bc\pi^{3/2}x-\frac{1}{9}bc^3\pi^{3/2}x^3+\pi\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-2\pi^{3/2}(a+$$

```
[Out] -4/3*b*c*Pi^(3/2)*x-1/9*b*c^3*Pi^(3/2)*x^3+1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*a
rcsinh(c*x))-2*Pi^(3/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-b
*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(3/2)*polylog(2,c*x+(c^2*x
^2+1)^(1/2))+Pi*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5808, 5806, 5816, 4267, 2317, 2438, 8}

$$\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\pi\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-2\pi^{3/2}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{9}\pi^{3/2}bc^3x^3-\pi^{3/2}b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\pi^{3/2}b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)-\frac{4}{3}\pi^{3/2}bcx$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-4*b*c*Pi^(3/2)*x)/3 - (b*c^3*Pi^(3/2)*x^3)/9 + Pi*Sqrt[Pi + c^2*Pi*x^2]*(
a + b*ArcSinh[c*x]) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - 2*
Pi^(3/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Pi^(3/2)*PolyLog[
2, -E^ArcSinh[c*x]] + b*Pi^(3/2)*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\sinh^{-1}(cx))}{x}\,dx = \frac{1}{3}(\pi + c^2\pi x^2)^{3/2}\,(a + b\sinh^{-1}(cx)) + \pi \int \frac{\sqrt{\pi + c^2\pi x^2}\,(a + b\,\mathrm{si}}{x}$$

$$= -\frac{bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\,(a +$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\,(a$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\,(a$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\,(a$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\,(a$$

Mathematica [A]
time = 0.26, size = 180, normalized size = 1.34

$$\frac{1}{9}\pi^{3/2}\left(3a\sqrt{1+c^2x^2}\,(4+c^2x^2) - b\left(3cx + c^3x^3 - 3(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\right) + 9a\log(x) - 9a\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) + 9b\left(-cx + \sqrt{1+c^2x^2}\sinh^{-1}(cx) + \sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) - \sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) + \mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - \mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (Pi^(3/2)*(3*a*Sqrt[1 + c^2*x^2]*(4 + c^2*x^2) - b*(3*c*x + c^3*x^3 - 3*(1
+ c^2*x^2)^(3/2)*ArcSinh[c*x]) + 9*a*Log[x] - 9*a*Log[Pi*(1 + Sqrt[1 + c^2*
x^2])] + 9*b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1
- E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2,
 -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])])))/9
```

Maple [A]
time = 5.18, size = 228, normalized size = 1.70

method	result
default	$a\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3} + \pi\left(\sqrt{\pi c^2x^2+\pi} - \sqrt{\pi}\ \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right) + \frac{b\sqrt{c^2x^2+1}\ \operatorname{arcsinh}(cx)\pi^{\frac{3}{}}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^
(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+1/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(3/2
)*x^2*c^2-b*Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*Pi^(3/2)*ar
csinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/9*b*c^3*Pi^(3/2)*x^3+4/3*b*(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*Pi^(3/2)-4/3*b*c*Pi^(3/2)*x+b*Pi^(3/2)*polylog(2,c*
x+(c^2*x^2+1)^(1/2))-b*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*pi^(3/2)*arcsinh(1/(c*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2) - (pi +
 pi*c^2*x^2)^(3/2))*a + b*integrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqrt(
c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{3}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x}\,dx+\int ac^2x\sqrt{c^2x^2+1}\,dx+\int \frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x}\,dx+\int bc^2x\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(a*c**2*x*sqrt(c*
*2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Integr
al(b*c**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x, x)
```

3.68 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{1}{4}bc^3\pi^{3/2}x^2+\frac{3}{2}c^2\pi x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{3c\pi^{3/2}\left(a+b\sinh^{-1}\right.}{4b}$$

```
[Out] -1/4*b*c^3*Pi^(3/2)*x^2-(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x+3/4*c*Pi
^(3/2)*(a+b*arcsinh(c*x))^2/b+b*c*Pi^(3/2)*ln(x)+3/2*c^2*Pi*x*(a+b*arcsinh(
c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5807, 5785, 5783, 30, 14}

$$\frac{3}{2}\pi c^2 x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{3\pi^{3/2}c\left(a+b\sinh^{-1}(cx)\right)^2}{4b}-\frac{1}{4}\pi^{3/2}bc^3x^2+\pi^{3/2}bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -1/4*(b*c^3*Pi^(3/2)*x^2) + (3*c^2*Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSin
h[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x + (3*c*Pi^(3/
2)*(a + b*ArcSinh[c*x])^2)/(4*b) + b*c*Pi^(3/2)*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{(\pi + c^2\pi x^2)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x} + \left(3c^2\pi\right)\int \sqrt{\pi + c^2\pi x^2}\,(a$$

$$= \frac{3}{2}c^2\pi x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{(\pi + c^2\pi x^2)^{3/2}\left(a + b\sin\right.}{x}$$

$$= -\frac{bc^3\pi x^2\sqrt{\pi + c^2\pi x^2}}{4\sqrt{1 + c^2x^2}} + \frac{3}{2}c^2\pi x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) -$$

Mathematica [A]
time = 0.20, size = 122, normalized size = 1.13

$$\frac{\pi^{3/2}\left(-8a\sqrt{1+c^2x^2} + 4ac^2x^2\sqrt{1+c^2x^2} + 6bcx\sinh^{-1}(cx)^2 - bcx\cosh\left(2\sinh^{-1}(cx)\right) + 8bcx\log(cx) + 2\sinh^{-1}(cx)\left(6acx - 4b\sqrt{1+c^2x^2} + bcx\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Pi^(3/2)*(-8*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 6*b*c*x
*ArcSinh[c*x]^2 - b*c*x*Cosh[2*ArcSinh[c*x]] + 8*b*c*x*Log[c*x] + 2*ArcSinh
[c*x]*(6*a*c*x - 4*b*Sqrt[1 + c^2*x^2] + b*c*x*Sinh[2*ArcSinh[c*x]])))/(8*x
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(92) = 184.
time = 4.48, size = 222, normalized size = 2.06

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{\pi x}+a\,c^2x(\pi\,c^2x^2+\pi)^{\frac{3}{2}}+\frac{3a\,c^2\pi x\sqrt{\pi\,c^2x^2+\pi}}{2}+\frac{3a\,c^2\pi^2\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2\sqrt{\pi\,c^2}}+\frac{3bc}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/2*a*c^2*Pi*x*
(Pi*c^2*x^2+Pi)^(1/2)+3/2*a*c^2*Pi^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2
+Pi)^(1/2))/(Pi*c^2)^(1/2)+3/4*b*c*Pi^(3/2)*arcsinh(c*x)^2+1/2*b*arcsinh(c*
x)*Pi^(3/2)*(c^2*x^2+1)^(1/2)*x*c^2-1/4*b*c^3*Pi^(3/2)*x^2-b*c*Pi^(3/2)*arc
sinh(c*x)-1/8*b*Pi^(3/2)*c-b*Pi^(3/2)*arcsinh(c*x)/x*(c^2*x^2+1)^(1/2)+b*c*
Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{3}{2}}\left(\int ac^2\sqrt{c^2x^2+1}\,dx+\int\frac{a\sqrt{c^2x^2+1}}{x^2}\,dx+\int bc^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,dx+\int\frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] pi**(3/2)*(Integral(a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*x
**2 + 1)/x**2, x) + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + In
tegral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^2, x)
```

3.69 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=155

$$-\frac{bc\pi^{3/2}}{2x}-bc^3\pi^{3/2}x+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))-\frac{(\pi+c^2\pi x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{2x^2}-3c^2\pi^{3/2}(a+b$$

```
[Out] -1/2*b*c*Pi^(3/2)/x-b*c^3*Pi^(3/2)*x-1/2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh
(c*x))/x^2-3*c^2*Pi^(3/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))
-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^2*Pi^(3/2)*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))+3/2*c^2*Pi*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5807, 5806, 5816, 4267, 2317, 2438, 8, 14}

$$\frac{3}{2}\pi c^2\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))-\frac{(\pi c^2x^2+\pi)^{3/2}\,(a+b\sinh^{-1}(cx))}{2x^2}-3\pi^{3/2}c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))+\pi^{3/2}(-b)c^3x-\frac{3}{2}\pi^{3/2}bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\frac{3}{2}\pi^{3/2}bc^2\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)-\frac{\pi^{3/2}bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Pi^(3/2))/x - b*c^3*Pi^(3/2)*x + (3*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*
(a + b*ArcSinh[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2
*x^2) - 3*c^2*Pi^(3/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - (3*b*
c^2*Pi^(3/2)*PolyLog[2, -E^ArcSinh[c*x]])/2 + (3*b*c^2*Pi^(3/2)*PolyLog[2,
E^ArcSinh[c*x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{(\pi+c^2\pi x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}(3c^2\pi)\int \frac{\sqrt{\pi+c^2\pi x^2}\,(a$$

$$=\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{(\pi+c^2\pi x^2)^{3/2}\left(a+b\sinh\right.}{2x^2}$$

$$=-\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3\pi x\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,(a+$$

$$=-\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3\pi x\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,(a+$$

$$=-\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3\pi x\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,(a+$$

$$=-\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3\pi x\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,(a+$$

Mathematica [A]
time = 1.08, size = 292, normalized size = 1.88

$$\frac{\pi^{3/2}\left(-8bc^3x^3-4a\sqrt{1+c^2x^2}+8ac^2x^2\sqrt{1+c^2x^2}+8bc^2x^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)-bc^3x^3\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)-bc^2x^2\sinh^{-1}(cx)\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+12bc^2x^2\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-12bc^2x^2\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+12ac^2x^2\log(x)-12ac^2x^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right)+12bc^2x^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-12bc^2x^2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+4bcx\sinh^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)-4b\sinh^{-1}(cx)\sinh^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Pi^(3/2)*(-8*b*c^3*x^3 - 4*a*Sqrt[1 + c^2*x^2] + 8*a*c^2*x^2*Sqrt[1 + c^2*
x^2] + 8*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*c^3*x^3*Csch[ArcSinh[
c*x]/2]^2 - b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 12*b*c^2*x^2*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*ArcSinh[c*x]*Log[1 + E
^(-ArcSinh[c*x])] + 12*a*c^2*x^2*Log[x] - 12*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 +
 c^2*x^2])] + 12*b*c^2*x^2*PolyLog[2, -E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*Po
lyLog[2, E^(-ArcSinh[c*x])] + 4*b*c*x*Sinh[ArcSinh[c*x]/2]^2 - 4*b*ArcSinh[
c*x]*Sinh[ArcSinh[c*x]/2]^2))/(8*x^2)
```

Maple [A]
time = 10.32, size = 291, normalized size = 1.88

method	result

default	$a\left(-\frac{(\pi\,c^{2}x^{2}+\pi)^{\frac{5}{2}}}{2\pi\,x^{2}}+\frac{3c^{2}\left(\frac{(\pi\,c^{2}x^{2}+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi\,c^{2}x^{2}+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)\right)\right)}{2}\right)+b\sqrt{c^{2}x^{2}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(5/2)+3/2*c^2*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*
((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2)))))+
b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(3/2)*c^2-b*c^3*Pi^(3/2)*x-1/2*b*Pi^(3/
2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c*Pi^(3/2)/x-1/2*b*Pi^(3/2)/(c^
2*x^2+1)^(1/2)/x^2*arcsinh(c*x)-3/2*b*c^2*Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c
^2*x^2+1)^(1/2))-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3/2*b
*c^2*Pi^(3/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^2*Pi^(3/2)*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(3*pi^(3/2)*c^2*arcsinh(1/(c*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2)*c^2
 - (pi + pi*c^2*x^2)^(3/2)*c^2 + (pi + pi*c^2*x^2)^(5/2)/(pi*x^2))*a + b*in
tegrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{3}{2}}\left(\int\frac{a\sqrt{c^{2}x^{2}+1}}{x^{3}}\,dx+\int\frac{ac^{2}\sqrt{c^{2}x^{2}+1}}{x}\,dx+\int\frac{b\sqrt{c^{2}x^{2}+1}\,\operatorname{asinh}(cx)}{x^{3}}\,dx+\int\frac{bc^{2}\sqrt{c^{2}x^{2}+1}\,\operatorname{asinh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(a*c**2*sqrt(c
**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x) +
Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^3, x)
```

3.70 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=115

$$-\frac{bc\pi^{3/2}}{6x^2}-\frac{c^2\pi\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{x}-\frac{(\pi+c^2\pi x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{3x^3}+\frac{c^3\pi^{3/2}(a+b\sinh^{-1}(cx))}{2b}$$

```
[Out] -1/6*b*c*Pi^(3/2)/x^2-1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3+1/2*
c^3*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b+4/3*b*c^3*Pi^(3/2)*ln(x)-c^2*Pi*(a+b*ar
csinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x
```

Rubi [A]
time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5807, 5805, 29, 5783, 14}

$$\frac{\pi^{3/2}c^3(a+b\sinh^{-1}(cx))^2}{2b}-\frac{\pi c^2\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{x}-\frac{(\pi c^2x^2+\pi)^{3/2}\,(a+b\sinh^{-1}(cx))}{3x^3}+\frac{4}{3}\pi^{3/2}bc^3\log(x)-\frac{\pi^{3/2}bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Pi^(3/2))/x^2 - (c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
))/x - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*x^3) + (c^3*Pi^(3/
2)*(a + b*ArcSinh[c*x])^2)/(2*b) + (4*b*c^3*Pi^(3/2)*Log[x])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{(\pi + c^2\pi x^2)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{3x^3} + \left(c^2\pi\right)\int \frac{\sqrt{\pi + c^2\pi x^2}\,(a +}{x^2}$$

$$= -\frac{c^2\pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{x} - \frac{(\pi + c^2\pi x^2)^{3/2}\left(a + b\sinh\right.}{3x^3}$$

$$= -\frac{bc\pi\sqrt{\pi + c^2\pi x^2}}{6x^2\sqrt{1 + c^2x^2}} - \frac{c^2\pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{x} - \frac{(\pi + c}{}$$

Mathematica [A]
time = 0.14, size = 125, normalized size = 1.09

$$\frac{\pi^{3/2}\left(-bcx - 2a\sqrt{1 + c^2x^2} - 8ac^2x^2\sqrt{1 + c^2x^2} + \left(6ac^3x^3 - 2b\sqrt{1 + c^2x^2}\,(1 + 4c^2x^2)\right)\sinh^{-1}(cx) + 3bc^3x^3\sinh^{-1}(cx)^2 + 8bc^3x^3\log(cx)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (Pi^(3/2)*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2] - 8*a*c^2*x^2*Sqrt[1 + c^2*x^2]
 + (6*a*c^3*x^3 - 2*b*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2))*ArcSinh[c*x] + 3*b
*c^3*x^3*ArcSinh[c*x]^2 + 8*b*c^3*x^3*Log[c*x]))/(6*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(97) = 194$.
time = 6.62, size = 622, normalized size = 5.41

method	result
default	$-\frac{a\left(\pi\,c^{2} x^{2}+\pi\right)^{\frac{5}{2}}}{3 \pi\,x^{3}}-\frac{2 a\,c^{2} \left(\pi\,c^{2} x^{2}+\pi\right)^{\frac{5}{2}}}{3 \pi x}+\frac{2 a\,c^{4} x \left(\pi\,c^{2} x^{2}+\pi\right)^{\frac{3}{2}}}{3}+a\,c^{4} \pi x \sqrt{\pi\,c^{2} x^{2}+\pi}+\frac{a\,c^{4} \pi^{2} \ln\left(\frac{\pi\,c^{2} x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}}\right.}{\sqrt{\pi\,c^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(5/2)-2/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+2/
3*a*c^4*x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^4*Pi^2
*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c^3
*Pi^(3/2)*arcsinh(c*x)^2-8/3*b*c^3*Pi^(3/2)*arcsinh(c*x)+32*b*Pi^(3/2)/(24*
c^4*x^4+9*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-32*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x
^2+1)*x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6+8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c
^2*x^2+1)*x^4*c^7-8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*(c^2*x^2+1)*c
^5+12*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-20*b*Pi^(3/2
)/(24*c^4*x^4+9*c^2*x^2+1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4-4/3*b*Pi^(3
/2)/(24*c^4*x^4+9*c^2*x^2+1)*(c^2*x^2+1)*c^3+4/3*b*Pi^(3/2)/(24*c^4*x^4+9*c
^2*x^2+1)*arcsinh(c*x)*c^3-13/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)*c^2-1/6*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^2*(
c^2*x^2+1)*c-1/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^3*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)+4/3*b*c^3*Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{3}{2}}\left(\int\frac{a\sqrt{c^2x^2+1}}{x^4}\,dx+\int\frac{ac^2\sqrt{c^2x^2+1}}{x^2}\,dx+\int\frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x^4}\,dx+\int\frac{bc^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(a*c**2*sqrt(c
**2*x**2 + 1)/x**2, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**4, x)
 + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^4, x)
```

3.71 $\int x^3(\pi + c^2\pi x^2)^{5/2}\left(a + b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=141

$$\frac{2b\pi^{5/2}x}{63c^3}-\frac{b\pi^{5/2}x^3}{189c}-\frac{1}{21}bc\pi^{5/2}x^5-\frac{19}{441}bc^3\pi^{5/2}x^7-\frac{1}{81}bc^5\pi^{5/2}x^9-\frac{(\pi+c^2\pi x^2)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4\pi}+(\pi+c^2\pi x$$

```
[Out] 2/63*b*Pi^(5/2)*x/c^3-1/189*b*Pi^(5/2)*x^3/c-1/21*b*c*Pi^(5/2)*x^5-19/441*b
*c^3*Pi^(5/2)*x^7-1/81*b*c^5*Pi^(5/2)*x^9-1/7*(Pi*c^2*x^2+Pi)^(7/2)*(a+b*ar
csinh(c*x))/c^4/Pi+1/9*(Pi*c^2*x^2+Pi)^(9/2)*(a+b*arcsinh(c*x))/c^4/Pi^2
```

Rubi [A]
time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {272, 45, 5804, 12, 380}

$$\frac{(\pi c^2x^2+\pi)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9\pi^2c^4}-\frac{(\pi c^2x^2+\pi)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi c^4}-\frac{1}{81}\pi^{5/2}bc^5x^9-\frac{19}{441}\pi^{5/2}bc^3x^7+\frac{2\pi^{5/2}bx}{63c^3}-\frac{1}{21}\pi^{5/2}bcx^5-\frac{\pi^{5/2}bx^3}{189c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Pi^(5/2)*x)/(63*c^3) - (b*Pi^(5/2)*x^3)/(189*c) - (b*c*Pi^(5/2)*x^5)/2
1 - (19*b*c^3*Pi^(5/2)*x^7)/441 - (b*c^5*Pi^(5/2)*x^9)/81 - ((Pi + c^2*Pi*x
^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*Pi) + ((Pi + c^2*Pi*x^2)^(9/2)*(a +
b*ArcSinh[c*x]))/(9*c^4*Pi^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx &= -\frac{\pi^{5/2}(1+c^2x^2)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}(1+c^2x^2)^{9/2}\left(a+\right.}{9c^4}\\ &= -\frac{\pi^{5/2}(1+c^2x^2)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}(1+c^2x^2)^{9/2}\left(a+\right.}{9c^4}\\ &= -\frac{\pi^{5/2}(1+c^2x^2)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}(1+c^2x^2)^{9/2}\left(a+\right.}{9c^4}\\ &= \frac{2b\pi^{5/2}x}{63c^3}-\frac{b\pi^{5/2}x^3}{189c}-\frac{1}{21}bc\pi^{5/2}x^5-\frac{19}{441}bc^3\pi^{5/2}x^7-\frac{1}{81}bc^5\pi^{5/2}x\end{aligned}$$

Mathematica [A]
time = 0.12, size = 108, normalized size = 0.77

$$\frac{\pi^{5/2}\left(63a(1+c^2x^2)^{7/2}(-2+7c^2x^2)-bcx(-126+21c^2x^2+189c^4x^4+171c^6x^6+49c^8x^8)+63b(1+c^2x^2)^{7/2}(-2+7c^2x^2)\sinh^{-1}(cx)\right)}{3969c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
[Out] (Pi^(5/2)*(63*a*(1 + c^2*x^2)^(7/2)*(-2 + 7*c^2*x^2) - b*c*x*(-126 + 21*c^2
*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8) + 63*b*(1 + c^2*x^2)^(7/2)*(
-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3\left(\pi\,c^2x^2+\pi\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.29, size = 156, normalized size = 1.11

$$\frac{1}{63}\left(\frac{7\,(\pi+\pi c^2x^2)^{\frac{7}{2}}x^2}{\pi c^2}-\frac{2\,(\pi+\pi c^2x^2)^{\frac{7}{2}}}{\pi c^4}\right)b\,\mathrm{arsinh}\,(cx)+\frac{1}{63}\left(\frac{7\,(\pi+\pi c^2x^2)^{\frac{7}{2}}x^2}{\pi c^2}-\frac{2\,(\pi+\pi c^2x^2)^{\frac{7}{2}}}{\pi c^4}\right)a-\frac{\left(49\,\pi^{\frac{5}{2}}c^8x^9+171\,\pi^{\frac{5}{2}}c^6x^7+189\,\pi^{\frac{5}{2}}c^4x^5+21\,\pi^{\frac{5}{2}}c^2x^3-126\,\pi^{\frac{5}{2}}x\right)b}{3969\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(7/2)/(p
i*c^4))*b*arcsinh(c*x) + 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(
pi + pi*c^2*x^2)^(7/2)/(pi*c^4))*a - 1/3969*(49*pi^(5/2)*c^8*x^9 + 171*pi^(
5/2)*c^6*x^7 + 189*pi^(5/2)*c^4*x^5 + 21*pi^(5/2)*c^2*x^3 - 126*pi^(5/2)*x)
*b/c^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(113) = 226$.
time = 0.38, size = 263, normalized size = 1.87

$$\frac{63\,\sqrt{\pi+\pi c^2x^2}\,(7\,\pi^2bc^{10}x^{10}+26\,\pi^2bc^8x^8+34\,\pi^2bc^6x^6+16\,\pi^2bc^4x^4-\pi^2bc^2x^2-2\,\pi^2b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(441\,\pi^2ac^{10}x^{10}+1638\,\pi^2ac^8x^8+2142\,\pi^2ac^6x^6+1008\,\pi^2ac^4x^4-63\,\pi^2ac^2x^2-126\,\pi^2a-(49\,\pi^2bc^9x^9+171\,\pi^2bc^7x^7+189\,\pi^2bc^5x^5+21\,\pi^2bc^3x^3-126\,\pi^2bcx)\sqrt{c^2x^2+1}\right)}{3969\,(c^6x^2+c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] 1/3969*(63*sqrt(pi + pi*c^2*x^2)*(7*pi^2*b*c^10*x^10 + 26*pi^2*b*c^8*x^8 +
34*pi^2*b*c^6*x^6 + 16*pi^2*b*c^4*x^4 - pi^2*b*c^2*x^2 - 2*pi^2*b)*log(c*x
+ sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(441*pi^2*a*c^10*x^10 + 1638*p
i^2*a*c^8*x^8 + 2142*pi^2*a*c^6*x^6 + 1008*pi^2*a*c^4*x^4 - 63*pi^2*a*c^2*x
^2 - 126*pi^2*a - (49*pi^2*b*c^9*x^9 + 171*pi^2*b*c^7*x^7 + 189*pi^2*b*c^5*
x^5 + 21*pi^2*b*c^3*x^3 - 126*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^
4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,(a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.72 $\int x^2(\pi + c^2\pi x^2)^{5/2} \left(a + b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=213

$$-\frac{5b\pi^{5/2}x^2}{256c}-\frac{59}{768}bc\pi^{5/2}x^4-\frac{17}{288}bc^3\pi^{5/2}x^6-\frac{1}{64}bc^5\pi^{5/2}x^8+\frac{5\pi^{5/2}x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{128c^2}+\frac{5}{64}\pi^2x^3\sqrt{\pi -$$

```
[Out] -5/256*b*Pi^(5/2)*x^2/c-59/768*b*c*Pi^(5/2)*x^4-17/288*b*c^3*Pi^(5/2)*x^6-1
/64*b*c^5*Pi^(5/2)*x^8+5/48*Pi*x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))
+1/8*x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))-5/256*Pi^(5/2)*(a+b*arcsi
nh(c*x))^2/b/c^3+5/128*Pi^(5/2)*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^2+
5/64*Pi^2*x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5808, 5806, 5812, 5783, 30, 14, 272, 45}

$$-\frac{5\pi^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{256bc^3}+\frac{5\pi^{5/2}x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{128c^2}+\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{64}\pi^2x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{64}\pi^{5/2}bc^5x^8-\frac{17}{288}\pi^{5/2}bc^3x^6-\frac{59}{768}\pi^{5/2}bcx^4-\frac{5\pi^{5/2}bx^2}{256c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*Pi^(5/2)*x^2)/(256*c) - (59*b*c*Pi^(5/2)*x^4)/768 - (17*b*c^3*Pi^(5/2
)*x^6)/288 - (b*c^5*Pi^(5/2)*x^8)/64 + (5*Pi^(5/2)*x*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(128*c^2) + (5*Pi^2*x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/64 + (5*Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48
+ (x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*Pi^(5/2)*(a + b
*ArcSinh[c*x])^2)/(256*b*c^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}(5\pi)\int x^2\left(\pi+c^2\pi x\right.$$

$$=\frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/2}$$

$$=\frac{5}{64}\pi^2x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/}$$

$$=-\frac{59bc\pi^2x^4\sqrt{\pi+c^2\pi x^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3\pi^2x^6\sqrt{\pi+c^2\pi x^2}}{288\sqrt{1+c^2x^2}}-\frac{bc^5\pi^2x^8}{64\sqrt{}}$$

$$=-\frac{5b\pi^2x^2\sqrt{\pi+c^2\pi x^2}}{256c\sqrt{1+c^2x^2}}-\frac{59bc\pi^2x^4\sqrt{\pi+c^2\pi x^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3\pi^2x^6\sqrt{}}{288\sqrt{1}}$$

Mathematica [A]
time = 0.41, size = 196, normalized size = 0.92

$$\frac{\pi^{5/2}\left(2880acx\sqrt{1+c^2x^2}+22656ac^3x^3\sqrt{1+c^2x^2}+26112ac^5x^5\sqrt{1+c^2x^2}+9216ac^7x^7\sqrt{1+c^2x^2}-1440b\sinh^{-1}(cx)^2+576b\cosh\left(2\sinh^{-1}(cx)\right)-144b\cosh\left(4\sinh^{-1}(cx)\right)-64b\cosh\left(6\sinh^{-1}(cx)\right)-9b\cosh\left(8\sinh^{-1}(cx)\right)-24\sinh^{-1}(cx)\left(120a+48b\sinh\left(2\sinh^{-1}(cx)\right)-24b\sinh\left(4\sinh^{-1}(cx)\right)-16b\sinh\left(6\sinh^{-1}(cx)\right)-3b\sinh\left(8\sinh^{-1}(cx)\right)\right)\right)}{73728c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(2880*a*c*x*Sqrt[1 + c^2*x^2] + 22656*a*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2] - 1
440*b*ArcSinh[c*x]^2 + 576*b*Cosh[2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*
x]] - 64*b*Cosh[6*ArcSinh[c*x]] - 9*b*Cosh[8*ArcSinh[c*x]] - 24*ArcSinh[c*x
]*(120*a + 48*b*Sinh[2*ArcSinh[c*x]] - 24*b*Sinh[4*ArcSinh[c*x]] - 16*b*Sin
h[6*ArcSinh[c*x]] - 3*b*Sinh[8*ArcSinh[c*x]])))/(73728*c^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(\pi\,c^2x^2+\pi\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^6 + 2*pi^2*a*c^2*x^4 + pi^2*a*
x^2 + (pi^2*b*c^4*x^6 + 2*pi^2*b*c^2*x^4 + pi^2*b*x^2)*arcsinh(c*x)), x)
```

Sympy [A]
time = 63.80, size = 350, normalized size = 1.64

$$\begin{cases} \frac{\pi^{\frac{5}{2}} a c^4 x^7 \sqrt{c^2x^2+1}}{8} + \frac{17\pi^{\frac{5}{2}} a c^2 x^5\sqrt{c^2x^2+1}}{48} + \frac{59\pi^{\frac{5}{2}} a x^3 \sqrt{c^2x^2+1}}{192} + \frac{5\pi^{\frac{5}{2}} a x\sqrt{c^2x^2+1}}{128c^2} - \frac{5\pi^{\frac{5}{2}} a \operatorname{asinh}(cx)}{128c^3} - \frac{\pi^{\frac{5}{2}} b c^5 x^8}{64} + \frac{\pi^{\frac{5}{2}} b c^4 x^7\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{8} - \frac{17\pi^{\frac{5}{2}} b c^3 x^6}{288} + \frac{17\pi^{\frac{5}{2}} b c^2 x^5 \sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{48} - \frac{59\pi^{\frac{5}{2}} b c x^4}{768} + \frac{59\pi^{\frac{5}{2}} b x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{192} - \frac{5\pi^{\frac{5}{2}} b x^2}{256c} + \frac{5\pi^{\frac{5}{2}} b x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{128c^2} - \frac{5\pi^{\frac{5}{2}} b \operatorname{asinh}^2(cx)}{256c^3} & \text{for}\ c\neq 0 \\ \frac{\pi^{\frac{5}{2}} a x^3}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(5/2)*a*c**4*x**7*sqrt(c**2*x**2 + 1)/8 + 17*pi**(5/2)*a*c**
2*x**5*sqrt(c**2*x**2 + 1)/48 + 59*pi**(5/2)*a*x**3*sqrt(c**2*x**2 + 1)/192
 + 5*pi**(5/2)*a*x*sqrt(c**2*x**2 + 1)/(128*c**2) - 5*pi**(5/2)*a*asinh(c*x
)/(128*c**3) - pi**(5/2)*b*c**5*x**8/64 + pi**(5/2)*b*c**4*x**7*sqrt(c**2*x
**2 + 1)*asinh(c*x)/8 - 17*pi**(5/2)*b*c**3*x**6/288 + 17*pi**(5/2)*b*c**2*
x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/48 - 59*pi**(5/2)*b*c*x**4/768 + 59*pi*
*(5/2)*b*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/192 - 5*pi**(5/2)*b*x**2/(256*
c) + 5*pi**(5/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(128*c**2) - 5*pi**(5/2
)*b*asinh(c*x)**2/(256*c**3), Ne(c, 0)), (pi**(5/2)*a*x**3/3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.73 $\int x(\pi + c^2\pi x^2)^{5/2} \left(a + b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=93

$$-\frac{b\pi^{5/2}x}{7c} - \frac{1}{7}bc\pi^{5/2}x^3 - \frac{3}{35}bc^3\pi^{5/2}x^5 - \frac{1}{49}bc^5\pi^{5/2}x^7 + \frac{(\pi + c^2\pi x^2)^{7/2}\left(a + b\sinh^{-1}(cx)\right)}{7c^2\pi}$$

```
[Out] -1/7*b*Pi^(5/2)*x/c-1/7*b*c*Pi^(5/2)*x^3-3/35*b*c^3*Pi^(5/2)*x^5-1/49*b*c^5
*Pi^(5/2)*x^7+1/7*(Pi*c^2*x^2+Pi)^(7/2)*(a+b*arcsinh(c*x))/c^2/Pi
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5798, 200}

$$\frac{(\pi c^2x^2 + \pi)^{7/2}\left(a + b\sinh^{-1}(cx)\right)}{7\pi c^2} - \frac{1}{49}\pi^{5/2}bc^5x^7 - \frac{3}{35}\pi^{5/2}bc^3x^5 - \frac{1}{7}\pi^{5/2}bcx^3 - \frac{\pi^{5/2}bx}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/7*(b*Pi^(5/2)*x)/c - (b*c*Pi^(5/2)*x^3)/7 - (3*b*c^3*Pi^(5/2)*x^5)/35 -
(b*c^5*Pi^(5/2)*x^7)/49 + ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7
*c^2*Pi)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\pi + c^2\pi x^2\right)^{7/2}\left(a + b\sinh^{-1}(cx)\right)}{7c^2\pi} - \frac{\left(b\pi^2\sqrt{\pi + c^2\pi x^2}\right)\int(1 +}{7c\sqrt{1 + c^2x^2}}$$

$$= \frac{\left(\pi + c^2\pi x^2\right)^{7/2}\left(a + b\sinh^{-1}(cx)\right)}{7c^2\pi} - \frac{\left(b\pi^2\sqrt{\pi + c^2\pi x^2}\right)\int(1 +}{7c\sqrt{1 -}}$$

$$= -\frac{b\pi^2 x\sqrt{\pi + c^2\pi x^2}}{7c\sqrt{1 + c^2x^2}} - \frac{bc\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{7\sqrt{1 + c^2x^2}} - \frac{3bc^3\pi^2x^5\sqrt{\pi + c}}{35\sqrt{1 + c^2x^2}}$$

Mathematica [A]
time = 0.10, size = 80, normalized size = 0.86

$$\frac{\pi^{5/2}\left(35a(1 + c^2x^2)^{7/2} - bcx(35 + 35c^2x^2 + 21c^4x^4 + 5c^6x^6) + 35b(1 + c^2x^2)^{7/2}\sinh^{-1}(cx)\right)}{245c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
[Out] (Pi^(5/2)*(35*a*(1 + c^2*x^2)^(7/2) - b*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 +
 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^(7/2)*ArcSinh[c*x]))/(245*c^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(\pi\,c^2x^2 + \pi\right)^{\frac{5}{2}}\left(a + b\,\mathrm{arcsinh}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
[Out] int(x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.27, size = 96, normalized size = 1.03

$$\frac{(\pi + \pi c^2x^2)^{\frac{7}{2}}b\,\mathrm{arsinh}\,(cx)}{7\,\pi c^2} + \frac{(\pi + \pi c^2x^2)^{\frac{7}{2}}a}{7\,\pi c^2} - \frac{\left(5\,\pi^{\frac{7}{2}}c^6x^7 + 21\,\pi^{\frac{7}{2}}c^4x^5 + 35\,\pi^{\frac{7}{2}}c^2x^3 + 35\,\pi^{\frac{7}{2}}x\right)b}{245\,\pi c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] 1/7*(pi + pi*c^2*x^2)^(7/2)*b*arcsinh(c*x)/(pi*c^2) + 1/7*(pi + pi*c^2*x^2)
^(7/2)*a/(pi*c^2) - 1/245*(5*pi^(7/2)*c^6*x^7 + 21*pi^(7/2)*c^4*x^5 + 35*pi
^(7/2)*c^2*x^3 + 35*pi^(7/2)*x)*b/(pi*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(73) = 146$.
time = 0.37, size = 225, normalized size = 2.42

$$\frac{35\sqrt{\pi+\pi c^2x^2}\left(\pi^2bc^8x^8+4\,\pi^2bc^6x^6+6\,\pi^2bc^4x^4+4\,\pi^2bc^2x^2+\pi^2b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(35\,\pi^2ac^8x^8+140\,\pi^2ac^6x^6+210\,\pi^2ac^4x^4+140\,\pi^2ac^2x^2+35\,\pi^2a-\left(5\,\pi^2bc^7x^7+21\,\pi^2bc^5x^5+35\,\pi^2bc^3x^3+35\,\pi^2bcx\right)\sqrt{c^2x^2+1}\right)}{245\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*sqrt(pi + pi*c^2*x^2)*(pi^2*b*c^8*x^8 + 4*pi^2*b*c^6*x^6 + 6*pi^2
*b*c^4*x^4 + 4*pi^2*b*c^2*x^2 + pi^2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt
(pi + pi*c^2*x^2)*(35*pi^2*a*c^8*x^8 + 140*pi^2*a*c^6*x^6 + 210*pi^2*a*c^4*
x^4 + 140*pi^2*a*c^2*x^2 + 35*pi^2*a - (5*pi^2*b*c^7*x^7 + 21*pi^2*b*c^5*x^
5 + 35*pi^2*b*c^3*x^3 + 35*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(85) = 170$.
time = 37.57, size = 299, normalized size = 3.22

$$\begin{cases} \frac{\pi^{\frac{5}{2}}ac^4x^6\sqrt{c^2x^2+1}}{7}+\frac{3\pi^{\frac{5}{2}}ac^2x^4\sqrt{c^2x^2+1}}{7}+\frac{3\pi^{\frac{5}{2}}ax^2\sqrt{c^2x^2+1}}{7}+\frac{\pi^{\frac{5}{2}}a\sqrt{c^2x^2+1}}{7c^2}-\frac{\pi^{\frac{5}{2}}bc^5x^7}{49}+\frac{\pi^{\frac{5}{2}}bc^4x^6\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{7}-\frac{3\pi^{\frac{5}{2}}bc^3x^5}{35}+\frac{3\pi^{\frac{5}{2}}bc^2x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{7}-\frac{\pi^{\frac{5}{2}}bcx^3}{7}+\frac{3\pi^{\frac{5}{2}}bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{7}-\frac{\pi^{\frac{5}{2}}bx}{7c}+\frac{\pi^{\frac{5}{2}}b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{7c^2} & \text{for}\: c\neq 0 \\ \frac{\pi^{\frac{5}{2}}ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(5/2)*a*c**4*x**6*sqrt(c**2*x**2 + 1)/7 + 3*pi**(5/2)*a*c**2
*x**4*sqrt(c**2*x**2 + 1)/7 + 3*pi**(5/2)*a*x**2*sqrt(c**2*x**2 + 1)/7 + pi
**(5/2)*a*sqrt(c**2*x**2 + 1)/(7*c**2) - pi**(5/2)*b*c**5*x**7/49 + pi**(5/
2)*b*c**4*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - 3*pi**(5/2)*b*c**3*x**5/3
5 + 3*pi**(5/2)*b*c**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - pi**(5/2)*b*
c*x**3/7 + 3*pi**(5/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - pi**(5/2)*
b*x/(7*c) + pi**(5/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(7*c**2), Ne(c, 0)),
 (pi**(5/2)*a*x**2/2, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.74 $\int (\pi + c^2\pi x^2)^{5/2} \left(a + b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=165

$$-\frac{25}{96}bc\pi^{5/2}x^2-\frac{5}{96}bc^3\pi^{5/2}x^4-\frac{b\pi^{5/2}(1+c^2x^2)^3}{36c}+\frac{5}{16}\pi^2x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}\pi x(\pi+c^2\pi x^2)^{3/2}\left(\right.$$

```
[Out] -25/96*b*c*Pi^(5/2)*x^2-5/96*b*c^3*Pi^(5/2)*x^4-1/36*b*Pi^(5/2)*(c^2*x^2+1)
^3/c+5/24*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(Pi*c^2*x^2+P
i)^(5/2)*(a+b*arcsinh(c*x))+5/32*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b/c+5/16*Pi^
2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5786, 5785, 5783, 30, 14, 267}

$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}\pi^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{5\pi^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc}-\frac{5}{96}\pi^{5/2}bc^3x^4-\frac{\pi^{5/2}b(c^2x^2+1)^3}{36c}-\frac{25}{96}\pi^{5/2}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*Pi^(5/2)*x^2)/96 - (5*b*c^3*Pi^(5/2)*x^4)/96 - (b*Pi^(5/2)*(1 + c^
2*x^2)^3)/(36*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/16
 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (x*(Pi + c^2*
Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)
/(32*b*c)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right)\, dx = \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) + \frac{1}{6}(5\pi)\int \left(\pi + c^2\pi x^2\right)^{3/2}$$

$$= -\frac{b\pi^2(1 + c^2x^2)^{5/2}\sqrt{\pi + c^2\pi x^2}}{36c} + \frac{5}{24}\pi x\left(\pi + c^2\pi x^2\right)^{3/2} (a + b\,\mathrm{si}$$

$$= -\frac{b\pi^2(1 + c^2x^2)^{5/2}\sqrt{\pi + c^2\pi x^2}}{36c} + \frac{5}{16}\pi^2 x\sqrt{\pi + c^2\pi x^2}\ (a + b\,\mathrm{sin}$$

$$= -\frac{25bc\pi^2x^2\sqrt{\pi + c^2\pi x^2}}{96\sqrt{1 + c^2x^2}} - \frac{5bc^3\pi^2x^4\sqrt{\pi + c^2\pi x^2}}{96\sqrt{1 + c^2x^2}} - \frac{b\pi^2(1 + c^2x}{}$$

Mathematica [A]
time = 0.26, size = 153, normalized size = 0.93

$$\frac{\pi^{5/2}\left(1584acx\sqrt{1+c^2x^2} + 1248ac^3x^3\sqrt{1+c^2x^2} + 384ac^5x^5\sqrt{1+c^2x^2} + 360b\sinh^{-1}(cx)^2 - 270b\cosh\left(2\sinh^{-1}(cx)\right) - 27b\cosh\left(4\sinh^{-1}(cx)\right) - 2b\cosh\left(6\sinh^{-1}(cx)\right) + 12\sinh^{-1}(cx)\left(60a + 45b\sinh\left(2\sinh^{-1}(cx)\right) + 9b\sinh\left(4\sinh^{-1}(cx)\right) + b\sinh\left(6\sinh^{-1}(cx)\right)\right)\right)}{2304c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(1584*a*c*x*Sqrt[1 + c^2*x^2] + 1248*a*c^3*x^3*Sqrt[1 + c^2*x^2]
+ 384*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 360*b*ArcSinh[c*x]^2 - 270*b*Cosh[2*Arc
Sinh[c*x]] - 27*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] + 12*ArcS
inh[c*x]*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*S
inh[6*ArcSinh[c*x]])))/(2304*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\pi \,c^{2} x^{2}+\pi \right)^{\frac{5}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 21.67, size = 265, normalized size = 1.61

$$\begin{cases} \frac{\pi^{\frac{5}{2}} a c^{4} x^{5} \sqrt{c^{2} x^{2} + 1}}{6} + \frac{13 \pi^{\frac{5}{2}} a c^{2} x^{3} \sqrt{c^{2} x^{2} + 1}}{24} + \frac{11 \pi^{\frac{5}{2}} a x \sqrt{c^{2} x^{2} + 1}}{16} + \frac{5 \pi^{\frac{5}{2}} a \operatorname{asinh}{\left(c x \right)}}{16 c} - \frac{\pi^{\frac{5}{2}} b c^{5} x^{6}}{36} + \frac{\pi^{\frac{5}{2}} b c^{4} x^{5} \sqrt{c^{2} x^{2} + 1} \operatorname{asinh}{\left(c x \right)}}{6} - \frac{13 \pi^{\frac{5}{2}} b c^{3} x^{4}}{96} + \frac{13 \pi^{\frac{5}{2}} b c^{2} x^{3} \sqrt{c^{2} x^{2} + 1} \operatorname{asinh}{\left(c x \right)}}{24} - \frac{11 \pi^{\frac{5}{2}} b c x^{2}}{32} + \frac{11 \pi^{\frac{5}{2}} b x \sqrt{c^{2} x^{2} + 1} \operatorname{asinh}{\left(c x \right)}}{16} + \frac{5 \pi^{\frac{5}{2}} b \operatorname{asinh}^{2}{\left(c x \right)}}{32 c} & \text{for}\: c \neq 0 \\\pi^{\frac{5}{2}} a x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(5/2)*a*c**4*x**5*sqrt(c**2*x**2 + 1)/6 + 13*pi**(5/2)*a*c**
2*x**3*sqrt(c**2*x**2 + 1)/24 + 11*pi**(5/2)*a*x*sqrt(c**2*x**2 + 1)/16 + 5
*pi**(5/2)*a*asinh(c*x)/(16*c) - pi**(5/2)*b*c**5*x**6/36 + pi**(5/2)*b*c**
4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/6 - 13*pi**(5/2)*b*c**3*x**4/96 + 13*
pi**(5/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/24 - 11*pi**(5/2)*b*c*
x**2/32 + 11*pi**(5/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/16 + 5*pi**(5/2)*
b*asinh(c*x)**2/(32*c), Ne(c, 0)), (pi**(5/2)*a*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.75 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=179

$$-\frac{23}{15}bc\pi^{5/2}x-\frac{11}{45}bc^3\pi^{5/2}x^3-\frac{1}{25}bc^5\pi^{5/2}x^5+\pi^2\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))+\frac{1}{3}\pi(\pi+c^2\pi x^2)^{3/2}\,(a+b\sinh^{-1}$$

```
[Out] -23/15*b*c*Pi^(5/2)*x-11/45*b*c^3*Pi^(5/2)*x^3-1/25*b*c^5*Pi^(5/2)*x^5+1/3*
Pi*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/5*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*
arcsinh(c*x))-2*Pi^(5/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-
b*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*polylog(2,c*x+(c^2*
x^2+1)^(1/2))+Pi^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5808, 5806, 5816, 4267, 2317, 2438, 8, 200}

$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\sinh^{-1}(cx))+\frac{1}{3}\pi\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\sinh^{-1}(cx))+\pi^2\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))-2\pi^{5/2}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))-\frac{1}{25}\pi^{5/2}bc^5x^5-\frac{11}{45}\pi^{5/2}bc^3x^3-\pi^{5/2}b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\pi^{5/2}b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)-\frac{23}{15}\pi^{5/2}bcx$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-23*b*c*Pi^(5/2)*x)/15 - (11*b*c^3*Pi^(5/2)*x^3)/45 - (b*c^5*Pi^(5/2)*x^5)
/25 + Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) + (Pi*(Pi + c^2*Pi*x^
2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/5 - 2*Pi^(5/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Pi^(
5/2)*PolyLog[2, -E^ArcSinh[c*x]] + b*Pi^(5/2)*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} \left(a + b\sinh^{-1}(cx)\right)}{x} dx = \frac{1}{5}\left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) + \pi \int \frac{(\pi + c^2\pi x^2)^{3/2} \left(a + b\,\mathrm{si}\right.}{x}$$

$$= \frac{1}{3}\pi\left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) + \frac{1}{5}\left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\,\mathrm{sin}\right.$$

$$= -\frac{8bc\pi^2 x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2 x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2 x^5\sqrt{\pi +}}{25\sqrt{1 + c^2}}$$

$$= -\frac{23bc\pi^2 x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2 x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2 x^5\sqrt{\pi +}}{25\sqrt{1 + c}}$$

$$= -\frac{23bc\pi^2 x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2 x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2 x^5\sqrt{\pi +}}{25\sqrt{1 + c}}$$

$$= -\frac{23bc\pi^2 x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2 x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2 x^5\sqrt{\pi +}}{25\sqrt{1 + c}}$$

$$= -\frac{23bc\pi^2 x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2 x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2 x^5\sqrt{\pi +}}{25\sqrt{1 + c}}$$

Mathematica [A]
time = 0.24, size = 257, normalized size = 1.44

$$\frac{1}{225}\pi^{5/2}\left(-345bcx - 55bc^3x^3 - 9bc^5x^5 + 345a\sqrt{1+c^2x^2} + 165ac^2x^2\sqrt{1+c^2x^2} + 45ac^4x^4\sqrt{1+c^2x^2} + 345b\sqrt{1+c^2x^2}\sinh^{-1}(cx) + 165bc^2x^2\sqrt{1+c^2x^2}\sinh^{-1}(cx) + 45bc^4x^4\sqrt{1+c^2x^2}\sinh^{-1}(cx) + 225b\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) - 225b\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) + 225a\log(x) - 225a\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) + 225b\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 225b\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (Pi^(5/2)*(-345*b*c*x - 55*b*c^3*x^3 - 9*b*c^5*x^5 + 345*a*Sqrt[1 + c^2*x^2
] + 165*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 45*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 345*
b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 165*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + 45*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 225*b*ArcSinh[c*x]*Log
[1 - E^(-ArcSinh[c*x])] - 225*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 2
25*a*Log[x] - 225*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 225*b*PolyLog[2, -E^(
-ArcSinh[c*x])] - 225*b*PolyLog[2, E^(-ArcSinh[c*x])]))/225
```

Maple [A]
time = 5.79, size = 284, normalized size = 1.59

method	result

default	$a\left(\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5}+\pi\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\ \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)\right)-\frac{b\,c^5\pi^{\frac{5}{2}}}{25}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*(Pi*c^2*x^2+Pi)^(5/2)+Pi*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2+
Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2)))))-1/25*b*c^5*Pi
^(5/2)*x^5-11/45*b*c^3*Pi^(5/2)*x^3+23/15*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)-b*Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*arc
sinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+11/15*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)*x^2*c^2-23/15*b*c*Pi^(5/2)*x+1/5*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)*x^4*c^4+b*Pi^(5/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-b*Pi^(5/2)*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*pi^(5/2)*arcsinh(1/(c*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2*x^2) -
5*pi*(pi + pi*c^2*x^2)^(3/2) - 3*(pi + pi*c^2*x^2)^(5/2))*a + b*integrate((
pi + pi*c^2*x^2)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{5}{2}}\left(\int\frac{a\sqrt{c^2x^2+1}}{x}\,dx+\int 2ac^2x\sqrt{c^2x^2+1}\,dx+\int ac^4x^3\sqrt{c^2x^2+1}\,dx+\int\frac{b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x}\,dx+\int 2bc^2x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\,dx+\int bc^4x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] pi**(5/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(2*a*c**2*x*sqrt(
c**2*x**2 + 1), x) + Integral(a*c**4*x**3*sqrt(c**2*x**2 + 1), x) + Integra
l(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Integral(2*b*c**2*x*sqrt(c**2*x*
*2 + 1)*asinh(c*x), x) + Integral(b*c**4*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x
), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x, x)
```

3.76 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\sinh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=157

$$-\frac{9}{16}bc^3\pi^{5/2}x^2-\frac{1}{16}bc^5\pi^{5/2}x^4+\frac{15}{8}c^2\pi^2x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{4}c^2\pi x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.$$

```
[Out] -9/16*b*c^3*Pi^(5/2)*x^2-1/16*b*c^5*Pi^(5/2)*x^4+5/4*c^2*Pi*x*(Pi*c^2*x^2+P
i)^(3/2)*(a+b*arcsinh(c*x))-(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x+15/1
6*c*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b+b*c*Pi^(5/2)*ln(x)+15/8*c^2*Pi^2*x*(a+b
*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5807, 5786, 5785, 5783, 30, 14, 272, 45}

$$\frac{5}{4}\pi c^2x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{15}{8}\pi^2c^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{15\pi^{5/2}c\left(a+b\sinh^{-1}(cx)\right)^2}{16b}-\frac{1}{16}\pi^{5/2}bc^5x^4-\frac{9}{16}\pi^{5/2}bc^3x^2+\pi^{5/2}bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-9*b*c^3*Pi^(5/2)*x^2)/16 - (b*c^5*Pi^(5/2)*x^4)/16 + (15*c^2*Pi^2*x*Sqrt[
Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*Pi*x*(Pi + c^2*Pi*x^2)^(3
/2)*(a + b*ArcSinh[c*x]))/4 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])
)/x + (15*c*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)/(16*b) + b*c*Pi^(5/2)*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{(\pi + c^2\pi x^2)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x} + \left(5c^2\pi\right)\int \left(\pi + c^2\pi x^2\right)^{3/2} (\dots$$

$$= \frac{5}{4}c^2\pi x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) - \frac{(\pi + c^2\pi x^2)^{5/2}\left(a + b \dots\right.}{x}$$

$$= \frac{15}{8}c^2\pi^2 x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{5}{4}c^2\pi x\left(\pi + c^2\pi x^2\right)^{3/2} \dots$$

$$= -\frac{9bc^3\pi^2x^2\sqrt{\pi + c^2\pi x^2}}{16\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^4\sqrt{\pi + c^2\pi x^2}}{16\sqrt{1 + c^2x^2}} + \frac{15}{8}c^2\pi^2 x\sqrt{\pi \dots}$$

Mathematica [A]
time = 0.27, size = 168, normalized size = 1.07

$$\frac{\pi^{5/2}\left(-128a\sqrt{1+c^2x^2} + 144ac^2x^2\sqrt{1+c^2x^2} + 32ac^4x^4\sqrt{1+c^2x^2} + 120bcx\sinh^{-1}(cx)^2 - 32bcx\cosh\left(2\sinh^{-1}(cx)\right) - bcx\cosh\left(4\sinh^{-1}(cx)\right) + 128bcx\log(cx) + 4\sinh^{-1}(cx)\left(60acx - 32b\sqrt{1+c^2x^2} + 16bcx\sinh\left(2\sinh^{-1}(cx)\right) + bcx\sinh\left(4\sinh^{-1}(cx)\right)\right)\right)}{128x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Pi^(5/2)*(-128*a*Sqrt[1 + c^2*x^2] + 144*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 32*
a*c^4*x^4*Sqrt[1 + c^2*x^2] + 120*b*c*x*ArcSinh[c*x]^2 - 32*b*c*x*Cosh[2*Ar
cSinh[c*x]] - b*c*x*Cosh[4*ArcSinh[c*x]] + 128*b*c*x*Log[c*x] + 4*ArcSinh[c
*x]*(60*a*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]] + b*
c*x*Sinh[4*ArcSinh[c*x]])))/(128*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(133) = 266$.
time = 6.40, size = 283, normalized size = 1.80

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{\pi x} + a\,c^2x(\pi\,c^2x^2+\pi)^{\frac{5}{2}} + \frac{5a\,c^2\pi x(\pi\,c^2x^2+\pi)^{\frac{3}{2}}}{4} + \frac{15a\,c^2\pi^2x\sqrt{\pi\,c^2x^2+\pi}}{8} + \frac{15a\,c^2\pi^3\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2 \dots}}\dots\right.}{8\sqrt{\dots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/4*a*c^2*Pi*x*
(Pi*c^2*x^2+Pi)^(3/2)+15/8*a*c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+15/8*a*c^2*Pi
^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-33/128*
```

```
b*Pi^(5/2)*c+15/16*b*c*Pi^(5/2)*arcsinh(c*x)^2-b*c*Pi^(5/2)*arcsinh(c*x)+b*
c*Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-b*Pi^(5/2)*arcsinh(c*x)/x*(c^2*x
^2+1)^(1/2)+1/4*b*arcsinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)*x^3*c^4+9/8*b*arc
sinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)*x*c^2-1/16*b*c^5*Pi^(5/2)*x^4-9/16*b*c
^3*Pi^(5/2)*x^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{5}{2}} \left(\int 2ac^2\sqrt{c^2x^2+1}\, dx + \int \frac{a\sqrt{c^2x^2+1}}{x^2}\, dx + \int ac^4x^2\sqrt{c^2x^2+1}\, dx + \int 2bc^2\sqrt{c^2x^2+1} \operatorname{asinh}{(cx)}\, dx + \int \frac{b\sqrt{c^2x^2+1} \operatorname{asinh}{(cx)}}{x^2}\, dx + \int bc^4x^2\sqrt{c^2x^2+1} \operatorname{asinh}{(cx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] pi**(5/2)*(Integral(2*a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2
*x**2 + 1)/x**2, x) + Integral(a*c**4*x**2*sqrt(c**2*x**2 + 1), x) + Integr
al(2*b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*sqrt(c**2*x**2
+ 1)*asinh(c*x)/x**2, x) + Integral(b*c**4*x**2*sqrt(c**2*x**2 + 1)*asinh(c
*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))\ \left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^2, x)
```

3.77 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=205

$$-\frac{bc\pi^{5/2}}{2x}-\frac{7}{3}bc^3\pi^{5/2}x-\frac{1}{9}bc^5\pi^{5/2}x^3+\frac{5}{2}c^2\pi^2\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))+\frac{5}{6}c^2\pi(\pi+c^2\pi x^2)^{3/2}\,(a+b\sinh^{-1}($$

```
[Out] -1/2*b*c*Pi^(5/2)/x-7/3*b*c^3*Pi^(5/2)*x-1/9*b*c^5*Pi^(5/2)*x^3+5/6*c^2*Pi*
(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))-1/2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arc
sinh(c*x))/x^2-5*c^2*Pi^(5/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1
/2))-5/2*b*c^2*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/2*b*c^2*Pi^(5/2
)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+5/2*c^2*Pi^2*(a+b*arcsinh(c*x))*(Pi*c^2*
x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5807, 5808, 5806, 5816, 4267, 2317, 2438, 8, 276}

$$\frac{5}{6}\pi c^2(\pi c^2x^2+\pi)^{3/2}(a+b\sinh^{-1}(cx))+\frac{5}{2}\pi^2c^2\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))-\frac{(\pi c^2x^2+\pi)^{5/2}(a+b\sinh^{-1}(cx))}{2x^2}-5\pi^{5/2}c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))-\frac{1}{9}\pi^{5/2}bc^5x^3-\frac{7}{3}\pi^{5/2}bc^3x-\frac{5}{2}\pi^{5/2}bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\frac{5}{2}\pi^{5/2}bc^2\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)-\frac{\pi^{5/2}bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Pi^(5/2))/x - (7*b*c^3*Pi^(5/2)*x)/3 - (b*c^5*Pi^(5/2)*x^3)/9 + (
5*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 + (5*c^2*Pi*(Pi +
c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b
*ArcSinh[c*x]))/(2*x^2) - 5*c^2*Pi^(5/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^Arc
Sinh[c*x]] - (5*b*c^2*Pi^(5/2)*PolyLog[2, -E^ArcSinh[c*x]])/2 + (5*b*c^2*Pi
^(5/2)*PolyLog[2, E^ArcSinh[c*x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{(\pi + c^2\pi x^2)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} + \frac{1}{2}(5c^2\pi)\int \frac{(\pi + c^2\pi x^2)^{3/2}\,(}{}$$

$$= \frac{5}{6}c^2\pi(\pi + c^2\pi x^2)^{3/2}\left(a + b\sinh^{-1}(cx)\right) - \frac{(\pi + c^2\pi x^2)^{5/2}\left(a + b\sin\right.}{2x^2}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} + \frac{bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{6\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x}}{9\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x}}{9\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x}}{9\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x}}{9\sqrt{1 + c^2x^2}}$$

Mathematica [A]
time = 1.26, size = 349, normalized size = 1.70

$$\frac{\pi^{5/2}\left(-168bc^3x^3 - 8bc^5x^5 - 36a\sqrt{1+c^2x^2} + 168ac^2x^2\sqrt{1+c^2x^2} + 24ac^4x^4\sqrt{1+c^2x^2} + 168bc^2x^2\sqrt{1+c^2x^2}\sinh^{-1}(cx) + 24bc^4x^4\sqrt{1+c^2x^2}\sinh^{-1}(cx) - 9bc^3x^3\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) - 9bc^2x^2\sinh^{-1}(cx)\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) + 180bc^2x^2\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) - 180bc^2x^2\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) + 180ac^2x^2\log(x) - 180ac^2x^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) + 180bc^2x^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 180bc^2x^2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) + 36bcx\sinh^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) - 36b\sinh^{-1}(cx)\sinh^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{72x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Pi^(5/2)*(-168*b*c^3*x^3 - 8*b*c^5*x^5 - 36*a*Sqrt[1 + c^2*x^2] + 168*a*c^
2*x^2*Sqrt[1 + c^2*x^2] + 24*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 168*b*c^2*x^2*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] + 24*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
- 9*b*c^3*x^3*Csch[ArcSinh[c*x]/2]^2 - 9*b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSin
h[c*x]/2]^2 + 180*b*c^2*x^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 180*b
*c^2*x^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 180*a*c^2*x^2*Log[x] - 1
```

```
80*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 180*b*c^2*x^2*PolyLog[2, -E^
(-ArcSinh[c*x])] - 180*b*c^2*x^2*PolyLog[2, E^(-ArcSinh[c*x])] + 36*b*c*x*S
inh[ArcSinh[c*x]/2]^2 - 36*b*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2]^2))/(72*x^2)
```

Maple [A]
time = 11.02, size = 348, normalized size = 1.70

method	result
default	$a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{7}{2}}}{2\pi x^2}+\frac{5c^2\left(\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5}+\pi\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)\right)}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(7/2)+5/2*c^2*(1/5*(Pi*c^2*x^2+Pi)^(5/2)+Pi*
(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1
/2)/(Pi*c^2*x^2+Pi)^(1/2))))))-1/2*b*Pi^(5/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x
)*c^2+1/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)*x^2*c^4-5/2*b*c^2*Pi^(5
/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/9*b*c^5*Pi^(5/2)*x^3-7/3*b*c
^3*Pi^(5/2)*x-5/2*b*c^2*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/2*b*c^
2*Pi^(5/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2*Pi^(5/2)*arcsinh(c*x)
*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*b*c*Pi^(5/2)/x-1/2*b*Pi^(5/2)/(c^2*x^2+1)^
(1/2)/x^2*arcsinh(c*x)+7/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(15*pi^(5/2)*c^2*arcsinh(1/(c*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2*x^2)
*c^2 - 5*pi*(pi + pi*c^2*x^2)^(3/2)*c^2 - 3*(pi + pi*c^2*x^2)^(5/2)*c^2 + 3
*(pi + pi*c^2*x^2)^(7/2)/(pi*x^2))*a + b*integrate((pi + pi*c^2*x^2)^(5/2)*
log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{5}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^3}\,dx + \int \frac{2ac^2\sqrt{c^2x^2+1}}{x}\,dx + \int ac^4x\sqrt{c^2x^2+1}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{2bc^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x}\,dx + \int bc^4x\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] pi**(5/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(2*a*c**2*sqrt
(c**2*x**2 + 1)/x, x) + Integral(a*c**4*x*sqrt(c**2*x**2 + 1), x) + Integra
l(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x) + Integral(2*b*c**2*sqrt(c**2*x
**2 + 1)*asinh(c*x)/x, x) + Integral(b*c**4*x*sqrt(c**2*x**2 + 1)*asinh(c*x
), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^3, x)
```

3.78 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=166

$$-\frac{bc\pi^{5/2}}{6x^2}-\frac{1}{4}bc^5\pi^{5/2}x^2+\frac{5}{2}c^4\pi^2x\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))-\frac{5c^2\pi(\pi+c^2\pi x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{3x}-\frac{(\pi$$

```
[Out] -1/6*b*c*Pi^(5/2)/x^2-1/4*b*c^5*Pi^(5/2)*x^2-5/3*c^2*Pi*(Pi*c^2*x^2+Pi)^(3/
2)*(a+b*arcsinh(c*x))/x-1/3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3+5/
4*c^3*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b+7/3*b*c^3*Pi^(5/2)*ln(x)+5/2*c^4*Pi^2
*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5807, 5785, 5783, 30, 14, 272, 45}

$$\frac{5\pi^{5/2}c^3(a+b\sinh^{-1}(cx))^2}{4b}-\frac{5\pi c^2(\pi c^2x^2+\pi)^{3/2}\,(a+b\sinh^{-1}(cx))}{3x}-\frac{(\pi c^2x^2+\pi)^{5/2}\,(a+b\sinh^{-1}(cx))}{3x^3}+\frac{5}{2}\pi^2c^4x\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))-\frac{1}{4}\pi^{5/2}bc^5x^2+\frac{7}{3}\pi^{5/2}bc^3\log(x)-\frac{\pi^{5/2}bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Pi^(5/2))/x^2 - (b*c^5*Pi^(5/2)*x^2)/4 + (5*c^4*Pi^2*x*Sqrt[Pi +
c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/(3*x) - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(
3*x^3) + (5*c^3*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)/(4*b) + (7*b*c^3*Pi^(5/2)*
Log[x])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{(\pi + c^2\pi x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3} + \frac{1}{3}(5c^2\pi)\int \frac{(\pi + c^2\pi x^2)^{3/2}\,(}{}$$

$$= -\frac{5c^2\pi(\pi + c^2\pi x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x} - \frac{(\pi + c^2\pi x^2)^{5/2}\left(a+b\,\mathrm{s}\right.}{3x^3}$$

$$= \frac{5}{2}c^4\pi^2x\sqrt{\pi + c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{5c^2\pi(\pi + c^2\pi x^2)^{3/2}\left(a+\right.}{3x}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{bc^5\pi^2x^2\sqrt{\pi + c^2\pi x^2}}{4\sqrt{1+c^2x^2}} + \frac{5}{2}c^4\pi^2x\sqrt{\pi + c^2\pi x^2}$$

Mathematica [A]
time = 0.25, size = 179, normalized size = 1.08

$$\frac{\pi^{5/2}\left(-4bcx-8a\sqrt{1+c^2x^2}-56ac^2x^2\sqrt{1+c^2x^2}+12ac^4x^4\sqrt{1+c^2x^2}+30bc^3x^3\sinh^{-1}(cx)^2-3bc^3x^3\cosh\left(2\sinh^{-1}(cx)\right)+56bc^3x^3\log(cx)+\sinh^{-1}(cx)\left(60ac^3x^3-8b\sqrt{1+c^2x^2}\,(1+7c^2x^2)+6bc^3x^3\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (Pi^(5/2)*(-4*b*c*x - 8*a*Sqrt[1 + c^2*x^2] - 56*a*c^2*x^2*Sqrt[1 + c^2*x^2
] + 12*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 30*b*c^3*x^3*ArcSinh[c*x]^2 - 3*b*c^3*
x^3*Cosh[2*ArcSinh[c*x]] + 56*b*c^3*x^3*Log[c*x] + ArcSinh[c*x]*(60*a*c^3*x
^3 - 8*b*Sqrt[1 + c^2*x^2]*(1 + 7*c^2*x^2) + 6*b*c^3*x^3*Sinh[2*ArcSinh[c*x
]])))/(24*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(138) = 276$.
time = 7.28, size = 692, normalized size = 4.17

method	result
default	$-\frac{a\left(\pi\,c^2x^2+\pi\right)^{\frac{7}{2}}}{3\pi\,x^3}-\frac{4a\,c^2\left(\pi\,c^2x^2+\pi\right)^{\frac{7}{2}}}{3\pi x}+\frac{4a\,c^4x\left(\pi\,c^2x^2+\pi\right)^{\frac{5}{2}}}{3}+\frac{5a\,c^4\pi x\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}{3}+\frac{5a\,c^4\pi^2x\sqrt{\pi\,c^2x^2+\pi}}{2}+\frac{5a\,c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(7/2)-4/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+4/
3*a*c^4*x*(Pi*c^2*x^2+Pi)^(5/2)+5/3*a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/2*a*
c^4*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/2*a*c^4*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+
(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+5/4*b*c^3*Pi^(5/2)*arcsinh(c*x)^2-147
*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^
6-56*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
c^4-22/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x*(c^2*x^2+1)^(1/2)*arcsinh(c
*x)*c^2+7/3*b*c^3*Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-14/3*b*c^3*Pi^(5
/2)*arcsinh(c*x)+147*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^4*arcsinh(c*x)*
c^7-49/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^2*(c^2*x^2+1)*c^5+35*b*Pi^(
5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-7/3*b*Pi^(5/2)/(63*c^4*
x^4+15*c^2*x^2+1)*(c^2*x^2+1)*c^3+7/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*
arcsinh(c*x)*c^3-1/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x^2*(c^2*x^2+1)*c
-1/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x
)+1/2*b*Pi^(5/2)*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c^4-1/4*b*c^5*Pi^(5/2)*x^
2+49/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^4*c^7-1/8*b*Pi^(5/2)*c^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{5}{2}}\left(\int ac^4\sqrt{c^2x^2+1}\,dx+\int\frac{a\sqrt{c^2x^2+1}}{x^4}\,dx+\int\frac{2ac^2\sqrt{c^2x^2+1}}{x^2}\,dx+\int bc^4\sqrt{c^2x^2+1}\,\operatorname{asinh}{(cx)}\,dx+\int\frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}{(cx)}}{x^4}\,dx+\int\frac{2bc^2\sqrt{c^2x^2+1}\,\operatorname{asinh}{(cx)}}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] pi**(5/2)*(Integral(a*c**4*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*x
**2 + 1)/x**4, x) + Integral(2*a*c**2*sqrt(c**2*x**2 + 1)/x**2, x) + Integr
al(b*c**4*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*sqrt(c**2*x**2 +
1)*asinh(c*x)/x**4, x) + Integral(2*b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x
**2, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^4,x)
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^4, x)
```

3.79 $\int \sqrt{1+x^2} \sinh^{-1}(x)\, dx$

Optimal. Leaf size=32

$$-\frac{x^2}{4}+\frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

```
[Out] -1/4*x^2+1/4*arcsinh(x)^2+1/2*x*arcsinh(x)*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5785, 5783, 30}

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}\,x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{1+x^2}\,\sinh^{-1}(x)\,dx = \frac{1}{2}x\sqrt{1+x^2}\,\sinh^{-1}(x) - \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx$$
$$= -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1+x^2}\,\sinh^{-1}(x) + \frac{1}{4}\sinh^{-1}(x)^2$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-x^2+2x\sqrt{1+x^2}\,\sinh^{-1}(x)+\sinh^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
[Out] (-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

Maple [A]
time = 1.54, size = 26, normalized size = 0.81

method	result	size
default	$\frac{x\,\mathrm{arcsinh}(x)\sqrt{x^2+1}}{2}+\frac{\mathrm{arcsinh}(x)^2}{4}-\frac{x^2}{4}-\frac{1}{4}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x)*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

Maxima [A]
time = 0.52, size = 28, normalized size = 0.88

$$-\frac{1}{4}x^2+\frac{1}{2}\left(\sqrt{x^2+1}\,x+\operatorname{arsinh}(x)\right)\operatorname{arsinh}(x)-\frac{1}{4}\operatorname{arsinh}(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^2
```

Fricas [A]
time = 0.36, size = 40, normalized size = 1.25

$$\frac{1}{2}\sqrt{x^2+1}\,x\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}x^2+\frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^2
 + 1))^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\operatorname{asinh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x)*(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*asinh(x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{asinh}(x)\,\sqrt{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x)*(x^2 + 1)^(1/2),x)
```

```
[Out] int(asinh(x)*(x^2 + 1)^(1/2), x)
```

3.80 $\int \frac{x^5(a+b\sinh^{-1}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=149

$$-\frac{8bx}{15c^5\sqrt{\pi}}+\frac{4bx^3}{45c^3\sqrt{\pi}}-\frac{bx^5}{25c\sqrt{\pi}}+\frac{8\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{15c^6\pi}-\frac{4x^2\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{15c^4\pi}+$$

```
[Out] -8/15*b*x/c^5/Pi^(1/2)+4/45*b*x^3/c^3/Pi^(1/2)-1/25*b*x^5/c/Pi^(1/2)+8/15*(
a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi-4/15*x^2*(a+b*arcsinh(c*x))*
(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi+1/5*x^4*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/
2)/c^2/Pi
```

Rubi [A]
time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{5812, 5798, 8, 30\}$

$$\frac{x^4\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{5\pi c^2}+\frac{8\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{15\pi c^6}-\frac{4x^2\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{15\pi c^4}-\frac{8bx}{15\sqrt{\pi}\,c^5}+\frac{4bx^3}{45\sqrt{\pi}\,c^3}-\frac{bx^5}{25\sqrt{\pi}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-8*b*x)/(15*c^5*Sqrt[Pi]) + (4*b*x^3)/(45*c^3*Sqrt[Pi]) - (b*x^5)/(25*c*Sq
rt[Pi]) + (8*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(15*c^6*Pi) - (4*x
^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*Pi) + (x^4*Sqrt[Pi +
 c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^4\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2\pi} - \frac{4\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{5c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\ldots}{5c\sqrt{\pi+c^2\pi\ldots}}$$

$$= -\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} - \frac{4x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^4\pi} + \frac{x^4\sqrt{\pi+c^2\pi x^2}\,(a\ldots}{5c^2\ldots}$$

$$= \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} + \frac{8\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^6\pi} - \ldots$$

$$= -\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{\pi+c^2\pi x^2}} + \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} + \frac{8\sqrt{\pi+c^2\pi x^2}\,(\ldots}{15\ldots}$$

Mathematica [**A**]
time = 0.10, size = 108, normalized size = 0.72

$$\frac{15a\sqrt{1+c^2x^2}\,(8-4c^2x^2+3c^4x^4)+b(-120cx+20c^3x^3-9c^5x^5)+15b\sqrt{1+c^2x^2}\,(8-4c^2x^2+3c^4x^4)\sinh^{-1}(cx)}{225c^6\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (15*a*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4) + b*(-120*c*x + 20*c^3*
x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4)*ArcSi
nh[c*x])/(225*c^6*Sqrt[Pi])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}{\sqrt{\pi\,c^2x^2+\pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)

[Out] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)

Maxima [A]
time = 0.32, size = 174, normalized size = 1.17

$$\frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}\,x^4}{\pi c^2}-\frac{4\sqrt{\pi+\pi c^2x^2}\,x^2}{\pi c^4}+\frac{8\sqrt{\pi+\pi c^2x^2}}{\pi c^6}\right)b\,\mathrm{arsinh}\,(cx)+\frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}\,x^4}{\pi c^2}-\frac{4\sqrt{\pi+\pi c^2x^2}\,x^2}{\pi c^4}+\frac{8\sqrt{\pi+\pi c^2x^2}}{\pi c^6}\right)a-\frac{(9\,c^4x^5-20\,c^2x^3+120\,x)b}{225\,\sqrt{\pi}\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")

[Out] 1/15*(3*sqrt(pi + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(p
i*c^4) + 8*sqrt(pi + pi*c^2*x^2)/(pi*c^6))*b*arcsinh(c*x) + 1/15*(3*sqrt(pi
 + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^4) + 8*sqrt
(pi + pi*c^2*x^2)/(pi*c^6))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*b/(s
qrt(pi)*c^5)

Fricas [A]
time = 0.37, size = 161, normalized size = 1.08

$$\frac{15\sqrt{\pi+\pi c^2x^2}\,(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(45\,ac^6x^6-15\,ac^4x^4+60\,ac^2x^2-(9\,bc^5x^5-20\,bc^3x^3+120\,bcx)\sqrt{c^2x^2+1}+120\,a\right)}{225\,(\pi c^8x^2+\pi c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")

[Out] 1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*
b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 - 15*
a*c^4*x^4 + 60*a*c^2*x^2 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^
2*x^2 + 1) + 120*a))/(pi*c^8*x^2 + pi*c^6)

Sympy [A]
time = 5.00, size = 182, normalized size = 1.22

$$\frac{a\left(\begin{cases}\frac{x^4\sqrt{c^2x^2+1}}{5c^2}-\frac{4x^2\sqrt{c^2x^2+1}}{15c^4}+\frac{8\sqrt{c^2x^2+1}}{15c^6} & \text{for}\ c\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x^5}{25c}+\frac{x^4\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{5c^2}+\frac{4x^3}{45c^3}-\frac{4x^2\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{15c^4}-\frac{8x}{15c^5}+\frac{8\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{15c^6} & \text{for}\ c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)

[Out] a*Piecewise((x**4*sqrt(c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(c**2*x**2 + 1)
/(15*c**4) + 8*sqrt(c**2*x**2 + 1)/(15*c**6), Ne(c, 0)), (x**6/6, True))/sq
rt(pi) + b*Piecewise((-x**5/(25*c) + x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(5
*c**2) + 4*x**3/(45*c**3) - 4*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**4)

```
 - 8*x/(15*c**5) + 8*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**6), Ne(c, 0)), (
0, True))/sqrt(pi)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.81 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{3bx^2}{16c^3\sqrt{\pi}}-\frac{bx^4}{16c\sqrt{\pi}}-\frac{3x\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{8c^4\pi}+\frac{x^3\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{4c^2\pi}+\frac{3(a+b\sinh^{-}}{16bc^5\sqrt{\pi}}$$

```
[Out] 3/16*b*x^2/c^3/Pi^(1/2)-1/16*b*x^4/c/Pi^(1/2)+3/16*(a+b*arcsinh(c*x))^2/b/c
^5/Pi^(1/2)-3/8*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi+1/4*x^3*(
a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A]
time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5812, 5783, 30}

$$\frac{3(a+b\sinh^{-1}(cx))^2}{16\sqrt{\pi}\,bc^5}+\frac{x^3\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{4\pi c^2}-\frac{3x\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{8\pi c^4}+\frac{3bx^2}{16\sqrt{\pi}\,c^3}-\frac{bx^4}{16\sqrt{\pi}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (3*b*x^2)/(16*c^3*Sqrt[Pi]) - (b*x^4)/(16*c*Sqrt[Pi]) - (3*x*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x]))/(8*c^4*Pi) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/(4*c^2*Pi) + (3*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[Pi]
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
```

```
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{4c^2\pi} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)}{4c\sqrt{\pi+c^2\pi}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{\pi+c^2\pi x^2}} - \frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4\pi} + \frac{x^3\sqrt{\pi+c^2\pi x^2}\,(a+}{4c^2\pi}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{\pi+c^2\pi x^2}} - \frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4\pi} +$$

Mathematica [A]
time = 0.15, size = 111, normalized size = 0.88

$$\frac{-48acx\sqrt{1+c^2x^2}+32ac^3x^3\sqrt{1+c^2x^2}+24b\sinh^{-1}(cx)^2+16b\cosh\left(2\sinh^{-1}(cx)\right)-b\cosh\left(4\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\left(12a-8b\sinh\left(2\sinh^{-1}(cx)\right)+b\sinh\left(4\sinh^{-1}(cx)\right)\right)}{128c^5\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-48*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24*b*ArcSin
h[c*x]^2 + 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c
*x]*(12*a - 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))/(128*c^5*Sq
rt[Pi])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\,\mathrm{arcsinh}\,(cx))}{\sqrt{\pi\,c^2x^2+\pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [A]
time = 5.31, size = 185, normalized size = 1.47

$$\frac{ax^5}{4\sqrt{\pi}\,\sqrt{c^2x^2+1}} - \frac{ax^3}{8\sqrt{\pi}\,c^2\sqrt{c^2x^2+1}} - \frac{3ax}{8\sqrt{\pi}\,c^4\sqrt{c^2x^2+1}} + \frac{3a\operatorname{asinh}(cx)}{8\sqrt{\pi}\,c^5} + \frac{b\left(\begin{cases} -\frac{x^4}{16c} + \frac{x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{4c^2} + \frac{3x^2}{16c^3} - \frac{3x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{8c^4} + \frac{3\operatorname{asinh}^2(cx)}{16c^5} & \text{for}\ c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*x**5/(4*sqrt(pi)*sqrt(c**2*x**2 + 1)) - a*x**3/(8*sqrt(pi)*c**2*sqrt(c**2
*x**2 + 1)) - 3*a*x/(8*sqrt(pi)*c**4*sqrt(c**2*x**2 + 1)) + 3*a*asinh(c*x)/
(8*sqrt(pi)*c**5) + b*Piecewise((-x**4/(16*c) + x**3*sqrt(c**2*x**2 + 1)*as
inh(c*x)/(4*c**2) + 3*x**2/(16*c**3) - 3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(
8*c**4) + 3*asinh(c*x)**2/(16*c**5), Ne(c, 0)), (0, True))/sqrt(pi)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.82 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=98

$$\frac{2bx}{3c^3\sqrt{\pi}}-\frac{bx^3}{9c\sqrt{\pi}}-\frac{2\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi}+\frac{x^2\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi}$$

```
[Out] 2/3*b*x/c^3/Pi^(1/2)-1/9*b*x^3/c/Pi^(1/2)-2/3*(a+b*arcsinh(c*x))*(Pi*c^2*x^
2+Pi)^(1/2)/c^4/Pi+1/3*x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5812, 5798, 8, 30}

$$\frac{x^2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^4}+\frac{2bx}{3\sqrt{\pi}\,c^3}-\frac{bx^3}{9\sqrt{\pi}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (2*b*x)/(3*c^3*Sqrt[Pi]) - (b*x^3)/(9*c*Sqrt[Pi]) - (2*Sqrt[Pi + c^2*Pi*x^2
]*(a + b*ArcSinh[c*x]))/(3*c^4*Pi) + (x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/(3*c^2*Pi)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi} - \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{3c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int}{3c\sqrt{\pi+c^2\pi x}}$$

$$= -\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\right.}{3c^2\pi}$$

$$= \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi} + \frac{x^2\sqrt{}}{}$$

Mathematica [**A**]
time = 0.08, size = 82, normalized size = 0.84

$$\frac{3a(-2+c^2x^2)\sqrt{1+c^2x^2}+b(6cx-c^3x^3)+3b(-2+c^2x^2)\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{9c^4\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
[Out] (3*a*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2] + b*(6*c*x - c^3*x^3) + 3*b*(-2 + c^2
*x^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(9*c^4*Sqrt[Pi])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{arcsinh}\,(cx))}{\sqrt{\pi\,c^2x^2+\pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
[Out] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

Maxima [**A**]
time = 0.28, size = 117, normalized size = 1.19

$$\frac{1}{3}\,b\left(\frac{\sqrt{\pi+\pi c^2x^2}\,x^2}{\pi c^2}-\frac{2\,\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)\mathrm{arsinh}\,(cx)+\frac{1}{3}\,a\left(\frac{\sqrt{\pi+\pi c^2x^2}\,x^2}{\pi c^2}-\frac{2\,\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)-\frac{(c^2x^3-6\,x)b}{9\,\sqrt{\pi}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + pi*c^2*x^2)/(pi*c^4
))*arcsinh(c*x) + 1/3*a*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + p
i*c^2*x^2)/(pi*c^4)) - 1/9*(c^2*x^3 - 6*x)*b/(sqrt(pi)*c^3)
```

Fricas [A]
time = 0.39, size = 132, normalized size = 1.35

$$\frac{3\,\sqrt{\pi+\pi c^2x^2}\,(bc^4x^4-bc^2x^2-2\,b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(3\,ac^4x^4-3\,ac^2x^2-(bc^3x^3-6\,bcx)\sqrt{c^2x^2+1}-6\,a\right)}{9\,(\pi c^6x^2+\pi c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*log(c*x + sqrt(c
^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^
3 - 6*b*c*x)*sqrt(c^2*x^2 + 1) - 6*a))/(pi*c^6*x^2 + pi*c^4)
```

Sympy [A]
time = 2.01, size = 122, normalized size = 1.24

$$\frac{a\left(\begin{cases}\frac{x^2\sqrt{c^2x^2+1}}{3c^2}-\frac{2\sqrt{c^2x^2+1}}{3c^4} & \text{for}\ c\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x^3}{9c}+\frac{x^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{3c^2}+\frac{2x}{3c^3}-\frac{2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{3c^4} & \text{for}\ c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**2*sqrt(c**2*x**2 + 1)/(3*c**2) - 2*sqrt(c**2*x**2 + 1)/(3*c
**4), Ne(c, 0)), (x**4/4, True))/sqrt(pi) + b*Piecewise((-x**3/(9*c) + x**2
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**2) + 2*x/(3*c**3) - 2*sqrt(c**2*x**2
+ 1)*asinh(c*x)/(3*c**4), Ne(c, 0)), (0, True))/sqrt(pi)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.83 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=75

$$-\frac{bx^2}{4c\sqrt{\pi}}+\frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2\pi}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{\pi}}$$

```
[Out] -1/4*b*x^2/c/Pi^(1/2)-1/4*(a+b*arcsinh(c*x))^2/b/c^3/Pi^(1/2)+1/2*x*(a+b*ar
csinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5812, 5783, 30}

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4\sqrt{\pi}\,bc^3}+\frac{x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] -1/4*(b*x^2)/(c*Sqrt[Pi]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/
(2*c^2*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*Sqrt[Pi])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2\pi} - \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{2c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x\,d}{2c\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{\pi+c^2\pi x^2}} + \frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2\pi} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{\pi}}$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 0.92

$$\frac{4acx\sqrt{1+c^2x^2} - 2b\sinh^{-1}(cx)^2 - b\cosh\left(2\sinh^{-1}(cx)\right) + \sinh^{-1}(cx)\left(-4a+2b\sinh\left(2\sinh^{-1}(cx)\right)\right)}{8c^3\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
[Out] (4*a*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSinh[c*x]] +
ArcSinh[c*x]*(-4*a + 2*b*Sinh[2*ArcSinh[c*x]]))/(8*c^3*Sqrt[Pi])
```

Maple [A]
time = 1.21, size = 107, normalized size = 1.43

method	result	s
default	$\frac{ax\sqrt{\pi\,c^2x^2+\pi}}{2\pi\,c^2} - \frac{a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2c^2\sqrt{\pi\,c^2}} - \frac{b\left(-2\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\;xc+c^2x^2+\mathrm{arcsinh}(cx)^2+1\right)}{4\sqrt{\pi}\;c^3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-1/2*a/c^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(
Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1/4*b/Pi^(1/2)*(-2*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*x*c+c^2*x^2+arcsinh(c*x)^2+1)/c^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [A]
time = 2.66, size = 92, normalized size = 1.23

$$\frac{ax\sqrt{c^2x^2+1}}{2\sqrt{\pi}\,c^2} - \frac{a\operatorname{asinh}(cx)}{2\sqrt{\pi}\,c^3} + \frac{b\left(\begin{cases} -\frac{x^2}{4c} + \frac{x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{2c^2} - \frac{\operatorname{asinh}^2(cx)}{4c^3} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*x*sqrt(c**2*x**2 + 1)/(2*sqrt(pi)*c**2) - a*asinh(c*x)/(2*sqrt(pi)*c**3)
+ b*Piecewise((-x**2/(4*c) + x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c**2) - as
inh(c*x)**2/(4*c**3), Ne(c, 0)), (0, True))/sqrt(pi)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.84 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{bx}{c\sqrt{\pi}}+\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi}$$

```
[Out] -b*x/c/Pi^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 8}

$$\frac{\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2}-\frac{bx}{\sqrt{\pi}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] -((b*x)/(c*Sqrt[Pi])) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^2*P
i)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx &= \frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{\pi+c^2\pi x^2}}\\ &= -\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.17

$$\frac{-bcx+a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{c^2\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c^2*Sq
rt[Pi])
```

Maple [A]
time = 1.15, size = 72, normalized size = 1.71

method	result	size
default	$\frac{a\sqrt{\pi\,c^2x^2+\pi}}{\pi\,c^2}+\frac{b\left(\operatorname{arcsinh}(cx)x^2c^2+\operatorname{arcsinh}(cx)-\sqrt{c^2x^2+1}\,cx\right)}{c^2\sqrt{\pi}\,\sqrt{c^2x^2+1}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)+b/c^2/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*
x)*x^2*c^2+arcsinh(c*x)-(c^2*x^2+1)^(1/2)*c*x)
```

Maxima [A]
time = 0.33, size = 55, normalized size = 1.31

$$-\frac{bx}{\sqrt{\pi}\,c}+\frac{\sqrt{\pi+\pi c^2x^2}\,b\operatorname{arsinh}(cx)}{\pi c^2}+\frac{\sqrt{\pi+\pi c^2x^2}\,a}{\pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*x/(sqrt(pi)*c) + sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*c^2) + sqrt(pi
 + pi*c^2*x^2)*a/(pi*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(38) = 76$.
time = 0.37, size = 96, normalized size = 2.29

$$\frac{\sqrt{\pi+\pi c^2x^2}\,(bc^2x^2+b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(ac^2x^2-\sqrt{c^2x^2+1}\,bcx+a\right)}{\pi c^4x^2+\pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(
pi + pi*c^2*x^2)*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a))/(pi*c^4*x^2 + p
i*c^2)
```

Sympy [**A**]
time = 1.39, size = 60, normalized size = 1.43

$$\frac{a\left(\begin{cases}\frac{x^2}{2} & \text{for } c^2=0\\ \frac{\sqrt{c^2x^2+1}}{c^2} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x}{c}+\frac{\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c^2} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**2/2, Eq(c**2, 0)), (sqrt(c**2*x**2 + 1)/c**2, True))/sqrt(p
i) + b*Piecewise((-x/c + sqrt(c**2*x**2 + 1)*asinh(c*x)/c**2, Ne(c, 0)), (0
, True))/sqrt(pi)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.85 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{\pi}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/b/c/Pi^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{\pi}\,bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{\pi}}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(21) = 42$.
time = 0.67, size = 53, normalized size = 2.12

method	result	size
default	$\frac{a \ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b \operatorname{arcsinh}(c x)^2}{2 c \sqrt{\pi}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c/
Pi^(1/2)*arcsinh(c*x)^2
```

Maxima [A]
time = 0.30, size = 28, normalized size = 1.12

$$\frac{b \operatorname{arsinh}(cx)^2}{2 \sqrt{\pi} c}+\frac{a \operatorname{arsinh}(cx)}{\sqrt{\pi} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a*arcsinh(c*x)/(sqrt(pi)*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(19) = 38$.

time = 1.50, size = 85, normalized size = 3.40

$$\begin{cases} a\left(\begin{cases} \frac{\sqrt{-\frac{1}{c^2}}\operatorname{asin}\left(x\sqrt{-c^2}\right)}{\sqrt{\pi}} & \text{for } \pi c^2 < 0 \\ \frac{\sqrt{\frac{1}{c^2}}\operatorname{asinh}\left(x\sqrt{c^2}\right)}{\sqrt{\pi}} & \text{for } \pi c^2 > 0 \end{cases}\right) & \text{for } b = 0 \\ \frac{ax}{\sqrt{\pi}} & \text{for } c = 0 \\ \frac{(a+b\operatorname{asinh}(cx))^2}{2\sqrt{\pi}\,bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((a*Piecewise((sqrt(-1/c**2)*asin(x*sqrt(-c**2))/sqrt(pi), pi*c**2
 < 0), (sqrt(c**(-2))*asinh(x*sqrt(c**2))/sqrt(pi), pi*c**2 > 0)), Eq(b, 0)
), (a*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**2/(2*sqrt(pi)*b*c), True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.86 $\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)-b*polylog(2,-
c*x-(c^2*x^2+1)^(1/2))/Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5816, 4267, 2317, 2438}

$$-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}}-\frac{b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}+\frac{b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[Pi] - (b*PolyLog[2,
-E^ArcSinh[c*x]])/Sqrt[Pi] + (b*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[Pi]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{\pi}}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(c\right.}{\sqrt{\pi}}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} +$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} + \frac{b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right.}{\sqrt{\pi}}$$

Mathematica [A]
time = 0.10, size = 96, normalized size = 1.71

$$\frac{b\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) - b\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) + a\log(x) - a\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) + b\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - b\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c*x]*Log[1 + E^(-Arc
Sinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + b*PolyLog[2, -
E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Sqrt[Pi]
```

Maple [A]
time = 1.10, size = 72, normalized size = 1.29

method	result	size
default	$-\frac{a\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\sqrt{\pi}} + \frac{b\left(4\,\text{dilog}\left(\frac{1}{cx+\sqrt{c^2x^2+1}}\right) - \text{dilog}\left(\frac{1}{\left(cx+\sqrt{c^2x^2+1}\right)^2}\right)\right)}{2\sqrt{\pi}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+1/2*b*(4*dilog(1/(c*x+(
c^2*x^2+1)^(1/2)))-dilog(1/(c*x+(c^2*x^2+1)^(1/2))^2))/Pi^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x), x) - a*
arcsinh(1/(c*abs(x)))/sqrt(pi)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^3 + pi*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{x\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{x\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x*sqrt(c**
2*x**2 + 1)), x))/sqrt(pi)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

3.87 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{\pi x}+\frac{bc\log(x)}{\sqrt{\pi}}$$

```
[Out] b*c*ln(x)/Pi^(1/2)-(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/Pi/x
```

Rubi [A]
time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5800, 29}

$$\frac{bc\log(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{\pi x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(Pi*x)) + (b*c*Log[x])/Sqrt[
Pi]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx &= -\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{\pi x}+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x}\,dx}{\sqrt{\pi+c^2\pi x^2}}\\ &= -\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{\pi x}+\frac{bc\sqrt{1+c^2x^2}\,\log(x)}{\sqrt{\pi+c^2\pi x^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 42, normalized size = 1.02

$$-\frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}\,x}+\frac{bc\log(x)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[Pi]*x)) + (b*c*Log[x])/Sqr
t[Pi]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.
time = 2.05, size = 84, normalized size = 2.05

method	result	size
default	$-\frac{a\sqrt{\pi\,c^2x^2+\pi}}{\pi x}-\frac{bc\,\mathrm{arcsinh}(cx)}{\sqrt{\pi}}-\frac{b\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}}{\sqrt{\pi}\,x}+\frac{bc\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)}{\sqrt{\pi}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(1/2)-b*c/Pi^(1/2)*arcsinh(c*x)-b/Pi^(1/2)*arcsinh(
c*x)/x*(c^2*x^2+1)^(1/2)+b*c/Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(37) = 74.
time = 0.31, size = 101, normalized size = 2.46

$$-\frac{\left(\sqrt{\pi}\,(-1)^{2\pi+2\pi c^2x^2}\log\left(2\pi c^2+\frac{2\pi}{x^2}\right)-\sqrt{\pi}\,\log\left(x^2+\frac{1}{c^2}\right)\right)bc}{2\pi}-\frac{\sqrt{\pi+\pi c^2x^2}\,b\,\mathrm{arsinh}\,(cx)}{\pi x}-\frac{\sqrt{\pi+\pi c^2x^2}\,a}{\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/2*(sqrt(pi)*(-1)^(2*pi + 2*pi*c^2*x^2)*log(2*pi*c^2 + 2*pi/x^2) - sqrt(p
i)*log(x^2 + 1/c^2))*b*c/pi - sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*x) -
 sqrt(pi + pi*c^2*x^2)*a/(pi*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(37) = 74.
time = 0.43, size = 132, normalized size = 3.22

$$\frac{\sqrt{\pi}\,bcx\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{\pi}\,\sqrt{\pi+\pi c^2x^2}\,\sqrt{c^2x^2+1}\,(x^4-1)}{c^2x^4+x^2}\right)-2\sqrt{\pi+\pi c^2x^2}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{\pi+\pi c^2x^2}\,a}{2\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(pi)*b*c*x*log((pi + pi*c^2*x^6 + pi*c^2*x^2 + pi*x^4 + sqrt(pi)*s
qrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 - 1))/(c^2*x^4 + x^2)) - 2*sqrt
(pi + pi*c^2*x^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*
a)/(pi*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{x^2\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x^2\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**2*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^2\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

3.88 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{bc}{2\sqrt{\pi}\,x}-\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}+\frac{bc^2\text{PolyLog}\left(2,-e^{\text{si}}\right.}{2\sqrt{\pi}}$$

```
[Out] -1/2*b*c/x/Pi^(1/2)+c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/P
i^(1/2)+1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*b*c^2*poly
log(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)/Pi/x^2
```

Rubi [A]
time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5809, 5816, 4267, 2317, 2438, 30}

$$-\frac{\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2}+\frac{c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}}+\frac{bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc^2\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc}{2\sqrt{\pi}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -1/2*(b*c)/(Sqrt[Pi]*x) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*P
i*x^2) + (c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[Pi] + (b*c
^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[Pi]) - (b*c^2*PolyLog[2, E^ArcSinh[
c*x]])/(2*Sqrt[Pi])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} - \frac{1}{2}c^2\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)}{2\sqrt{\pi+c^2\pi x}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} - \frac{c^2\text{Subst}\left(\int(a+bx)\text{csch}(x)\,c\right.}{2\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-}}{\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-}}{\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-}}{\sqrt{\pi}}$$

Mathematica [A]
time = 1.80, size = 185, normalized size = 1.61

$$\frac{-\frac{4a\sqrt{1+c^2x^2}}{x^2} - 4ac^2\log(x) + 4ac^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) + bc^2\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right) - \sinh^{-1}(cx)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) - 4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) + 4\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) - 4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) + 4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) - \sinh^{-1}(cx)\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) + 2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{8\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] ((-4*a*Sqrt[1 + c^2*x^2])/x^2 - 4*a*c^2*Log[x] + 4*a*c^2*Log[Pi*(1 + Sqrt[1
 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[
c*x]/2]^2 - 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[
1 + E^(-ArcSinh[c*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^
(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x
]/2]))/(8*Sqrt[Pi])
```

Maple [A]
time = 4.29, size = 226, normalized size = 1.97

method	result
default	$a\left(-\frac{\sqrt{\pi\,c^{2}x^{2}+\pi}}{2\pi\,x^{2}}+\frac{c^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)}{2\sqrt{\pi}}\right)-\frac{b\,\mathrm{arcsinh}(cx)c^{2}}{2\sqrt{\pi}\,\sqrt{c^{2}x^{2}+1}}-\frac{bc}{2x\sqrt{\pi}}-\frac{b\,\mathrm{arcsinh}(cx)}{2\sqrt{\pi}\,\sqrt{c^{2}x^{2}+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(1/2)+1/2/Pi^(1/2)*c^2*arctanh(Pi^(1/2)/(Pi*
c^2*x^2+Pi)^(1/2)))-1/2*b/Pi^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b
*c/x/Pi^(1/2)-1/2*b/Pi^(1/2)/(c^2*x^2+1)^(1/2)/x^2*arcsinh(c*x)+1/2*b*c^2/P
i^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+1/2*b*c^2*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*b*c^2/Pi^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^
2+1)^(1/2))-1/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(pi) - sqrt(pi + pi*c^2*x^2)/(pi*x^2))*a
 + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^5 + pi*x^3),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{x^3\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}{(cx)}}{x^3\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**3*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

3.89 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=97

$$-\frac{bc}{6\sqrt{\pi}\,x^2}-\frac{\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{3\pi x^3}+\frac{2c^2\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{3\pi x}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}$$

```
[Out] -1/6*b*c/x^2/Pi^(1/2)-2/3*b*c^3*ln(x)/Pi^(1/2)-1/3*(a+b*arcsinh(c*x))*(Pi*c
^2*x^2+Pi)^(1/2)/Pi/x^3+2/3*c^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/Pi
/x
```

Rubi [A]
time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5809, 5800, 29, 30}

$$\frac{2c^2\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{3\pi x}-\frac{\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{3\pi x^3}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}-\frac{bc}{6\sqrt{\pi}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -1/6*(b*c)/(Sqrt[Pi]*x^2) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(3
*Pi*x^3) + (2*c^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(3*Pi*x) - (2
*b*c^3*Log[x])/(3*Sqrt[Pi])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx &= -\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}-\frac{1}{3}\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right.}{3\sqrt{\pi+c}}\\ &= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{\pi+c^2\pi x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{2c^2\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh\right.}{3\pi x}\\ &= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{\pi+c^2\pi x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{2c^2\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh\right.}{3\pi x}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 99, normalized size = 1.02

$$\frac{2a\sqrt{1+c^2x^2}\,(-1+2c^2x^2)+bcx(-1+6c^2x^2)+2b\sqrt{1+c^2x^2}\,(-1+2c^2x^2)\sinh^{-1}(cx)}{6\sqrt{\pi}\,x^3}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (2*a*Sqrt[1 + c^2*x^2]*(-1 + 2*c^2*x^2) + b*c*x*(-1 + 6*c^2*x^2) + 2*b*Sqrt
[1 + c^2*x^2]*(-1 + 2*c^2*x^2)*ArcSinh[c*x])/(6*Sqrt[Pi]*x^3) - (2*b*c^3*Lo
g[x])/(3*Sqrt[Pi])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(81) = 162.
time = 3.88, size = 373, normalized size = 3.85

method	result
default	$a\left(-\frac{\sqrt{\pi c^2x^2+\pi}}{3\pi x^3}+\frac{2c^2\sqrt{\pi c^2x^2+\pi}}{3\pi x}\right)+\frac{4b\,c^3\,\mathrm{arcsinh}(cx)}{3\sqrt{\pi}}-\frac{2b\,x^4c^7}{3\sqrt{\pi}\,(3c^2x^2-1)}+\frac{2b\,x^2(c^2x^2+1)c^5}{3\sqrt{\pi}\,(3c^2x^2-1)}-\frac{2b\,x^2\,\mathrm{arc}}{\sqrt{\pi}\,(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/Pi/x^3*(Pi*c^2*x^2+Pi)^(1/2)+2/3/Pi*c^2/x*(Pi*c^2*x^2+Pi)^(1/2))+4/
3*b*c^3/Pi^(1/2)*arcsinh(c*x)-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*x^4*c^7+2/3*b/Pi
^(1/2)/(3*c^2*x^2-1)*x^2*(c^2*x^2+1)*c^5-2*b/Pi^(1/2)/(3*c^2*x^2-1)*x^2*arc
sinh(c*x)*c^5+2*b/Pi^(1/2)/(3*c^2*x^2-1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c
^4-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*(c^2*x^2+1)*c^3+2/3*b/Pi^(1/2)/(3*c^2*x^2-1
)*arcsinh(c*x)*c^3-5/3*b/Pi^(1/2)/(3*c^2*x^2-1)/x*(c^2*x^2+1)^(1/2)*arcsinh
(c*x)*c^2+1/6*b/Pi^(1/2)/(3*c^2*x^2-1)/x^2*(c^2*x^2+1)*c+1/3*b/Pi^(1/2)/(3*
c^2*x^2-1)/x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2/3*b*c^3/Pi^(1/2)*ln((c*x+(c
^2*x^2+1)^(1/2))^2-1)
```

Maxima [A]
time = 0.30, size = 121, normalized size = 1.25

$$-\frac{1}{6}\left(\frac{4\,c^2\log\left(x\right)}{\sqrt{\pi}}+\frac{1}{\sqrt{\pi}\,x^2}\right)bc+\frac{1}{3}\,b\left(\frac{2\,\sqrt{\pi+\pi c^2x^2}\,c^2}{\pi x}-\frac{\sqrt{\pi+\pi c^2x^2}}{\pi x^3}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{2\,\sqrt{\pi+\pi c^2x^2}\,c^2}{\pi x}-\frac{\sqrt{\pi+\pi c^2x^2}}{\pi x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*(4*c^2*log(x)/sqrt(pi) + 1/(sqrt(pi)*x^2))*b*c + 1/3*b*(2*sqrt(pi + pi
*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))*arcsinh(c*x) + 1/3*a
*(2*sqrt(pi + pi*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(81) = 162.
time = 0.40, size = 222, normalized size = 2.29

$$\frac{2\,\sqrt{\pi+\pi c^2x^2}\,\left(2\,bc^4x^4+bc^2x^2-b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+2\,\sqrt{\pi}\,\left(bc^5x^5+bc^3x^3\right)\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4-\sqrt{\pi}\,\sqrt{\pi+\pi c^2x^2}\,\sqrt{c^2x^2+1}\,\left(x^4-1\right)}{c^2x^4+x^2}\right)+\sqrt{\pi+\pi c^2x^2}\,\left(4\,ac^4x^4+2\,ac^2x^2+\left(bcx^3-bcx\right)\sqrt{c^2x^2+1}-2\,a\right)}{6\left(\pi c^2x^5+\pi x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*sqrt(pi + pi*c^2*x^2)*(2*b*c^4*x^4 + b*c^2*x^2 - b)*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + pi
*c^2*x^2 + pi*x^4 - sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 -
 1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(4*a*c^4*x^4 + 2*a*c^2*x^2 +
(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) - 2*a))/(pi*c^2*x^5 + pi*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{x^4\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}\left(cx\right)}{x^4\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**4*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

3.90 $\int \frac{x^5(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{5bx}{3c^5\pi^{3/2}}-\frac{bx^3}{9c^3\pi^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^6\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{c^6\pi^2}+\frac{(\pi+c^2\pi x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{3c^6\pi^3}$$

```
[Out] 5/3*b*x/c^5/Pi^(3/2)-1/9*b*x^3/c^3/Pi^(3/2)+1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*
arcsinh(c*x))/c^6/Pi^3+b*arctan(c*x)/c^6/Pi^(3/2)+(-a-b*arcsinh(c*x))/c^6/P
i/(Pi*c^2*x^2+Pi)^(1/2)-2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi^2
```

Rubi [A]
time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 45, 5804, 12, 1167, 209}

$$\frac{(\pi c^2x^2+\pi)^{3/2}\,(a+b\sinh^{-1}(cx))}{3\pi^3c^6}-\frac{2\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{\pi^2c^6}-\frac{a+b\sinh^{-1}(cx)}{\pi c^6\sqrt{\pi c^2x^2+\pi}}+\frac{b\text{ArcTan}(cx)}{\pi^{3/2}c^6}+\frac{5bx}{3\pi^{3/2}c^5}-\frac{bx^3}{9\pi^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (5*b*x)/(3*c^5*Pi^(3/2)) - (b*x^3)/(9*c^3*Pi^(3/2)) - (a + b*ArcSinh[c*x])/
(c^6*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*
x]))/(c^6*Pi^2) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6*Pi^
3) + (b*ArcTan[c*x])/(c^6*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}}+\frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.}{3c^6\pi^{3/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}}+\frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.}{3c^6\pi^{3/2}}$$

$$= \frac{5bx}{3c^5\pi^{3/2}}-\frac{bx^3}{9c^3\pi^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}}+\frac{(1}{}$$

$$= \frac{5bx}{3c^5\pi^{3/2}}-\frac{bx^3}{9c^3\pi^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}}+\frac{(1}{}$$

Mathematica [A]
time = 0.12, size = 131, normalized size = 0.96

$$\frac{-24a-12ac^2x^2+3ac^4x^4+15bcx\sqrt{1+c^2x^2}-bc^3x^3\sqrt{1+c^2x^2}+3b(-8-4c^2x^2+c^4x^4)\sinh^{-1}(cx)+9b\sqrt{1+c^2x^2}\,\mathrm{ArcTan}(cx)}{9c^6\pi^{3/2}\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-24*a - 12*a*c^2*x^2 + 3*a*c^4*x^4 + 15*b*c*x*Sqrt[1 + c^2*x^2] - b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 3*b*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b*Sqr
t[1 + c^2*x^2]*ArcTan[c*x])/(9*c^6*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 5.19, size = 229, normalized size = 1.67

method	result
default	$a\left(\frac{x^{4}}{3\pi\,c^{2}\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{4\left(\frac{x^{2}}{\pi\,c^{2}\sqrt{\pi\,c^{2}x^{2}+\pi}}+\frac{2}{\pi\,c^{4}\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)}{3c^{2}}\right)-\frac{5b\,\mathrm{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}}{3\pi^{\frac{3}{2}}c^{6}}-\frac{ib\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)-4/3/c^2*(x^2/Pi/c^2/(Pi*c^2*x^2+Pi)
^(1/2)+2/Pi/c^4/(Pi*c^2*x^2+Pi)^(1/2)))-5/3*b/Pi^(3/2)/c^6*arcsinh(c*x)*(c^
2*x^2+1)^(1/2)-I*b/c^6/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)+I*b/c^6/Pi^(3/2
)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-1/9*b*x^3/c^3/Pi^(3/2)+5/3*b*x/c^5/Pi^(3/2)+1
/3*b/Pi^(3/2)/c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2-b/Pi^(3/2)/(c^2*x^2+1)
^(1/2)/c^6*arcsinh(c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a*(x^4/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - 4*x^2/(pi*sqrt(pi + pi*c^2*x^2)
*c^4) - 8/(pi*sqrt(pi + pi*c^2*x^2)*c^6)) + 1/3*b*((sqrt(pi)*c^4*x^4 - 4*sq
rt(pi)*c^2*x^2 - 8*sqrt(pi))*log(c*x + sqrt(c^2*x^2 + 1))/(pi^2*sqrt(c^2*x^
2 + 1)*c^6) - integrate((sqrt(pi)*c^4*x^4 - 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi)
)/(sqrt(c^2*x^2 + 1)*x), x)/(pi^2*c^6) + 3*integrate(1/3*(sqrt(pi)*c^4*x^4
- 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi))/(pi^2*c^9*x^4 + pi^2*c^7*x^2 + (pi^2*c^8
*x^3 + pi^2*c^6*x)*sqrt(c^2*x^2 + 1)), x))
```

Fricas [A]
time = 0.43, size = 196, normalized size = 1.43

$$-\frac{9\sqrt{\pi}\,(bc^{2}x^{2}+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^{2}x^{2}}\sqrt{c^{2}x^{2}+1}\,cx}{\pi-\pi c^{4}x^{4}}\right)-6\sqrt{\pi+\pi c^{2}x^{2}}\,(bc^{4}x^{4}-4\,bc^{2}x^{2}-8\,b)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)-2\sqrt{\pi+\pi c^{2}x^{2}}\left(3\,ac^{4}x^{4}-12\,ac^{2}x^{2}-(bc^{3}x^{3}-15\,bcx)\sqrt{c^{2}x^{2}+1}-24\,a\right)}{18\,(\pi^{2}c^{8}x^{2}+\pi^{2}c^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/18*(9*sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*
sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) - 6*sqrt(pi + pi*c^2*x^2)*(b*c^4*x
^4 - 4*b*c^2*x^2 - 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x
^2)*(3*a*c^4*x^4 - 12*a*c^2*x^2 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1)
- 24*a))/(pi^2*c^8*x^2 + pi^2*c^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^5}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^5\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**5/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**5*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.91 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{bx^2}{4c^3\pi^{3/2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2}-\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^5\pi^{3/2}}-\frac{b\log\left(1+c^2x^2\right)}{2c^5\pi^{3/2}}$$

```
[Out] -1/4*b*x^2/c^3/Pi^(3/2)-3/4*(a+b*arcsinh(c*x))^2/b/c^5/Pi^(3/2)-1/2*b*ln(c^
2*x^2+1)/c^5/Pi^(3/2)-x^3*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)+3
/2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi^2
```

Rubi [A]

time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5810, 5812, 5783, 30, 272, 45}

$$-\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{3/2}bc^5}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{3x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2c^4}-\frac{bx^2}{4\pi^{3/2}c^3}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -1/4*(b*x^2)/(c^3*Pi^(3/2)) - (x^3*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi +
c^2*Pi*x^2]) + (3*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*c^4*Pi^2
) - (3*(a + b*ArcSinh[c*x])^2)/(4*b*c^5*Pi^(3/2)) - (b*Log[1 + c^2*x^2])/(2
*c^5*Pi^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^2\pi}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^3}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2}-\frac{3\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}}{2c^4\pi}$$

$$= -\frac{3bx^2\sqrt{1+c^2x^2}}{4c^3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2}$$

Mathematica [A]
time = 0.26, size = 147, normalized size = 1.12

$$\frac{12acx+4ac^3x^3-6b\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2-b\sqrt{1+c^2x^2}\cosh\left(2\sinh^{-1}(cx)\right)-4b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)+\sinh^{-1}(cx)\left(9bcx-12a\sqrt{1+c^2x^2}+b\sinh\left(3\sinh^{-1}(cx)\right)\right)}{8c^5\pi^{3/2}\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (12*a*c*x + 4*a*c^3*x^3 - 6*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[1 +
 c^2*x^2]*Cosh[2*ArcSinh[c*x]] - 4*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + A
rcSinh[c*x]*(9*b*c*x - 12*a*Sqrt[1 + c^2*x^2] + b*Sinh[3*ArcSinh[c*x]]))/(8
*c^5*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(113) = 226$.
time = 5.09, size = 269, normalized size = 2.05

method	result
default	$\frac{a\,x^{3}}{2\pi\,c^{2}\sqrt{\pi\,c^{2}x^{2}+\pi}}+\frac{3ax}{2c^{4}\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{3a\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{2c^{4}\pi\sqrt{\pi\,c^{2}}}-\frac{3b\,\mathrm{arcsinh}(cx)^{2}}{4c^{5}\pi^{\frac{3}{2}}}+\frac{b\sqrt{c^{2}x^{2}+}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+3/2*a/c^4*x/Pi/(Pi*c^2*x^2+Pi)^(1/2)
-3/2*a/c^4/Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1
/2)-3/4*b/c^5/Pi^(3/2)*arcsinh(c*x)^2+1/2*b/Pi^(3/2)/c^4*(c^2*x^2+1)^(1/2)*
arcsinh(c*x)*x-1/4*b*x^2/c^3/Pi^(3/2)+2*b/c^5/Pi^(3/2)*arcsinh(c*x)-1/8*b/P
i^(3/2)/c^5-b/Pi^(3/2)*arcsinh(c*x)/c^3/(c^2*x^2+1)*x^2+b/Pi^(3/2)*arcsinh(
c*x)/c^4/(c^2*x^2+1)^(1/2)*x-b/Pi^(3/2)*arcsinh(c*x)/c^5/(c^2*x^2+1)-b/c^5/
Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*a*(x^3/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 3*x/(pi*sqrt(pi + pi*c^2*x^2)*c
^4) - 3*arcsinh(c*x)/(pi^(3/2)*c^5)) + b*integrate(x^4*log(c*x + sqrt(c^2*x
^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^2*c^4*x^4 +
 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^4\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**4/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**4*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.92 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{bx}{c^3\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{c^4\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}\,(a+b\sinh^{-1}(cx))}{c^4\pi^2}-\frac{b\text{ArcTan}(cx)}{c^4\pi^{3/2}}$$

```
[Out] -b*x/c^3/Pi^(3/2)-b*arctan(c*x)/c^4/Pi^(3/2)+(a+b*arcsinh(c*x))/c^4/Pi/(Pi*
c^2*x^2+Pi)^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi^2
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 45, 5804, 12, 396, 209}

$$\frac{\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{\pi^2c^4}+\frac{a+b\sinh^{-1}(cx)}{\pi c^4\sqrt{\pi c^2x^2+\pi}}-\frac{b\text{ArcTan}(cx)}{\pi^{3/2}c^4}-\frac{bx}{\pi^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((b*x)/(c^3*Pi^(3/2))) + (a + b*ArcSinh[c*x])/(c^4*Pi*Sqrt[Pi + c^2*Pi*x^2
]) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^4*Pi^2) - (b*ArcTan[c*
x])/(c^4*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{(bc)\int\frac{2+c^2x^2}{c^4+c^6x^2}\,dx}{\pi^{3/2}}\\ &= -\frac{bx}{c^3\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{(bc)\int\frac{1}{c^4+c^6x^2}}{\pi^{3/2}}\\ &= -\frac{bx}{c^3\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{b\tan^{-1}(cx)}{c^4\pi^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 87, normalized size = 1.01

$$\frac{2a+ac^2x^2-bcx\sqrt{1+c^2x^2}+b(2+c^2x^2)\sinh^{-1}(cx)-b\sqrt{1+c^2x^2}\,\mathrm{ArcTan}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
[Out] (2*a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] + b*(2 + c^2*x^2)*ArcSinh[c*x] -
 b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^4*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 5.15, size = 159, normalized size = 1.85

method	result
default	$a\left(\frac{x^{2}}{\pi c^{2}\sqrt{\pi c^{2}x^{2}+\pi}}+\frac{2}{\pi c^{4}\sqrt{\pi c^{2}x^{2}+\pi}}\right)+\frac{b\,\mathrm{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}}{\pi^{\frac{3}{2}}c^{4}}-\frac{bx}{c^{3}\pi^{\frac{3}{2}}}+\frac{b\,\mathrm{arcsinh}(cx)}{\pi^{\frac{3}{2}}\sqrt{c^{2}x^{2}+1}\,c^{4}}+\frac{ib}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+2/Pi/c^4/(Pi*c^2*x^2+Pi)^(1/2))+b/Pi^(3
/2)/c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-b*x/c^3/Pi^(3/2)+b/Pi^(3/2)/(c^2*x^2
+1)^(1/2)/c^4*arcsinh(c*x)+I*b/c^4/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-I*b
/c^4/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

Maxima [A]
time = 0.53, size = 119, normalized size = 1.38

$$-bc\left(\frac{x}{\pi^{\frac{3}{2}}c^{4}}+\frac{\arctan\left(cx\right)}{\pi^{\frac{3}{2}}c^{5}}\right)+b\left(\frac{x^{2}}{\pi\sqrt{\pi+\pi c^{2}x^{2}}\,c^{2}}+\frac{2}{\pi\sqrt{\pi+\pi c^{2}x^{2}}\,c^{4}}\right)\operatorname{arsinh}\left(cx\right)+a\left(\frac{x^{2}}{\pi\sqrt{\pi+\pi c^{2}x^{2}}\,c^{2}}+\frac{2}{\pi\sqrt{\pi+\pi c^{2}x^{2}}\,c^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] -b*c*(x/(pi^(3/2)*c^4) + arctan(c*x)/(pi^(3/2)*c^5)) + b*(x^2/(pi*sqrt(pi +
 pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))*arcsinh(c*x) + a*(x^2
/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(78) = 156.
time = 0.42, size = 165, normalized size = 1.92

$$\frac{\sqrt{\pi}\,(bc^{2}x^{2}+b)\arctan\left(-\frac{2\sqrt{\pi}\,\sqrt{\pi+\pi c^{2}x^{2}}\,\sqrt{c^{2}x^{2}+1}\,cx}{\pi-\pi c^{4}x^{4}}\right)+2\sqrt{\pi+\pi c^{2}x^{2}}\,(bc^{2}x^{2}+2\,b)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+2\sqrt{\pi+\pi c^{2}x^{2}}\,\left(ac^{2}x^{2}-\sqrt{c^{2}x^{2}+1}\,bcx+2\,a\right)}{2\,(\pi^{2}c^{6}x^{2}+\pi^{2}c^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt
(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 +
 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*(a*c^2*x^2 - s
qrt(c^2*x^2 + 1)*b*c*x + 2*a))/(pi^2*c^6*x^2 + pi^2*c^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**3*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.93 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{x(a+b\sinh^{-1}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{(a+b\sinh^{-1}(cx))^2}{2bc^3\pi^{3/2}}+\frac{b\log(1+c^2x^2)}{2c^3\pi^{3/2}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/b/c^3/Pi^(3/2)+1/2*b*ln(c^2*x^2+1)/c^3/Pi^(3/2)-x*
(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5810, 5783, 266}

$$\frac{(a+b\sinh^{-1}(cx))^2}{2\pi^{3/2}bc^3}-\frac{x(a+b\sinh^{-1}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\log(c^2x^2+1)}{2\pi^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (a + b*ArcSinh
[c*x])^2/(2*b*c^3*Pi^(3/2)) + (b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
```

[m, 1]

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^2\pi}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}$$
$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3\pi^{3/2}}+\frac{b\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{2c^3\pi\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [**A**]
time = 0.21, size = 78, normalized size = 0.98

$$\frac{-\frac{2acx}{\sqrt{1+c^2x^2}}+\left(2a-\frac{2bcx}{\sqrt{1+c^2x^2}}\right)\sinh^{-1}(cx)+b\sinh^{-1}(cx)^2+b\log\left(1+c^2x^2\right)}{2c^3\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] ((-2*a*c*x)/Sqrt[1 + c^2*x^2] + (2*a - (2*b*c*x)/Sqrt[1 + c^2*x^2])*ArcSinh
[c*x] + b*ArcSinh[c*x]^2 + b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(70) = 140.
time = 3.33, size = 196, normalized size = 2.45

method	result
default	$-\frac{ax}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\pi c^2\sqrt{\pi c^2}}+\frac{b\,\mathrm{arcsinh}(cx)^2}{2c^3\pi^{\frac{3}{2}}}-\frac{2b\,\mathrm{arcsinh}(cx)}{c^3\pi^{\frac{3}{2}}}+\frac{b\,\mathrm{arcsinh}(cx)x^2}{\pi^{\frac{3}{2}}c(c^2x^2+1)}-\frac{b}{\pi^{\frac{3}{2}}c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*x/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+a/Pi/c^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c
^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c^3/Pi^(3/2)*arcsinh(c*x)^2-2*b/c^3/
Pi^(3/2)*arcsinh(c*x)+b/Pi^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)*x^2-b/Pi^(3/2)*
arcsinh(c*x)/c^2/(c^2*x^2+1)^(1/2)*x+b/Pi^(3/2)*arcsinh(c*x)/c^3/(c^2*x^2+1
)+b/c^3/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] -a*(x/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - arcsinh(c*x)/(pi^(3/2)*c^3)) + b*int
egrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^2*c^4*x^4 +
 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**2*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.94 $\int \frac{x(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{a+b\sinh^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\text{ArcTan}(cx)}{c^2\pi^{3/2}}$$

```
[Out] b*arctan(c*x)/c^2/Pi^(3/2)+(-a-b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5798, 209}

$$\frac{b\text{ArcTan}(cx)}{\pi^{3/2}c^2}-\frac{a+b\sinh^{-1}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (b*ArcTan[c*x])/(c
^2*Pi^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 52, normalized size = 1.16

$$\frac{-a-b\sinh^{-1}(cx)+b\sqrt{1+c^2x^2}\,\mathrm{ArcTan}(cx)}{c^2\pi^{3/2}\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-a - b*ArcSinh[c*x] + b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^2*Pi^(3/2)*Sqrt[
1 + c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 2.38, size = 103, normalized size = 2.29

method	result	size
default	$-\frac{a}{\pi c^2\sqrt{\pi c^2x^2+\pi}}-\frac{b\,\mathrm{arcsinh}(cx)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,c^2}+\frac{ib\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{c^2\pi^{\frac{3}{2}}}-\frac{ib\ln\left(cx+\sqrt{c^2x^2+1}-i\right)}{c^2\pi^{\frac{3}{2}}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)-b/Pi^(3/2)/(c^2*x^2+1)^(1/2)/c^2*arcsinh(c*
x)+I*b/c^2/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-I*b/c^2/Pi^(3/2)*ln(c*x+(c^
2*x^2+1)^(1/2)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(pi^(3/2)*c^2) - log(c*x + sqrt(c^
2*x^2 + 1))/(pi^(3/2)*sqrt(c^2*x^2 + 1)*c^2) - integrate(1/(pi^(3/2)*c^5*x^
4 + pi^(3/2)*c^3*x^2 + (pi^(3/2)*c^4*x^3 + pi^(3/2)*c^2*x)*sqrt(c^2*x^2 + 1
)), x)) - a/(pi*sqrt(pi + pi*c^2*x^2)*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(41) = 82$.
time = 0.46, size = 127, normalized size = 2.82

$$-\frac{\sqrt{\pi}\,(bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\,\sqrt{\pi+\pi c^2x^2}\,\sqrt{c^2x^2+1}\,cx}{\pi-\pi c^4x^4}\right)+2\sqrt{\pi+\pi c^2x^2}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{\pi+\pi c^2x^2}\,a}{2\,(\pi^2c^4x^2+\pi^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqr
t(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*b*log(c*x +
 sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*a)/(pi^2*c^4*x^2 + pi^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + I
ntegral(b*x*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)
), x))/pi**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.95 $\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\log\left(1+c^2x^2\right)}{2c\pi^{3/2}}$$

```
[Out] -1/2*b*ln(c^2*x^2+1)/c/Pi^(3/2)+x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5787, 266}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Log[1 + c^2*x^2])/
(2*c*Pi^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}\\ &= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{2c\pi\sqrt{\pi+c^2\pi x^2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 66, normalized size = 1.29

$$\frac{2acx + 2bcx\sinh^{-1}(cx) - b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c\pi^{3/2}\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*
c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(45) = 90$.
time = 1.60, size = 132, normalized size = 2.59

method	result
default	$\frac{ax}{\pi\sqrt{\pi\, c^2x^2+\pi}} + \frac{2b\,\mathrm{arcsinh}(cx)}{c\,\pi^{\frac{3}{2}}} - \frac{b\,\mathrm{arcsinh}(cx)c\,x^2}{\pi^{\frac{3}{2}}(c^2x^2+1)} + \frac{b\,\mathrm{arcsinh}(cx)x}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}} - \frac{b\,\mathrm{arcsinh}(cx)}{\pi^{\frac{3}{2}}c(c^2x^2+1)} - \frac{b\ln\left(1+\left(cx+\sqrt{c^2x^2+\right.\right.}{c\,\pi^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+2*b/c/Pi^(3/2)*arcsinh(c*x)-b/Pi^(3/2)*arcsinh
(c*x)*c/(c^2*x^2+1)*x^2+b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*x-b/Pi^(3
/2)*arcsinh(c*x)/c/(c^2*x^2+1)-b/c/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [A]
time = 0.29, size = 58, normalized size = 1.14

$$\frac{bx\,\mathrm{arsinh}\,(cx)}{\pi\sqrt{\pi+\pi c^2x^2}} + \frac{ax}{\pi\sqrt{\pi+\pi c^2x^2}} - \frac{b\log\left(x^2+\frac{1}{c^2}\right)}{2\,\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a*x/(pi*sqrt(pi + pi*c^2*x^2)
) - 1/2*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^4 + 2*pi^2*
c^2*x^2 + pi^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + Int
egral(b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x
))/pi**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.96 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\text{ArcTan}(cx)}{\pi^{3/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{b\text{Po}}{}$$

```
[Out] -b*arctan(c*x)/Pi^(3/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))
/Pi^(3/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(3/2)+b*polylog(2,c*x+(c^2
*x^2+1)^(1/2))/Pi^(3/2)+(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5811, 5816, 4267, 2317, 2438, 209}

$$\frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}}-\frac{b\text{ArcTan}(cx)}{\pi^{3/2}}-\frac{b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*ArcTan[c*x])/Pi^(3/2)
- (2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(3/2) - (b*PolyLog[2,
 -E^ArcSinh[c*x]])/Pi^(3/2) + (b*PolyLog[2, E^ArcSinh[c*x]])/Pi^(3/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{\pi}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}\\
&= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\mathrm{Subst}\left(\int (a+bx)\mathrm{csch}(x)\,dx,x,\sinh^{-1}\right.}{\pi^{3/2}}\\
&= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx}\right.}{\pi^{3/2}}\\
&= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx}\right.}{\pi^{3/2}}\\
&= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx}\right.}{\pi^{3/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.23, size = 143, normalized size = 1.52

$$\frac{\frac{a}{\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}-2b\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+b\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-b\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+a\log(x)-a\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right)+b\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-b\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)}{\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (a/Sqrt[1 + c^2*x^2] + (b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 2*b*ArcTan[Tanh
[ArcSinh[c*x]/2]] + b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c
*x]*Log[1 + E^(-ArcSinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2]
)] + b*PolyLog[2, -E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Pi
^(3/2)
```

Maple [A]
time = 3.54, size = 157, normalized size = 1.67

method	result
default	$a\left(\frac{1}{\pi\sqrt{\pi\,c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}\right)+\frac{b\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}-\frac{2b\arctan\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}}-\frac{b\operatorname{dil}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(
1/2)))+b/Pi^(3/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2*b/Pi^(3/2)*arctan(c*x+(c
^2*x^2+1)^(1/2))-b/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2))-b/Pi^(3/2)*dilog(1
+c*x+(c^2*x^2+1)^(1/2))-b/Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(1/(c*abs(x)))/pi^(3/2) - 1/(pi*sqrt(pi + pi*c^2*x^2))) + b*inte
grate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^5 + 2*pi^2*
c^2*x^3 + pi^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x) + I
ntegral(b*asinh(c*x)/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)
), x))/pi**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{x\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

3.97 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{a+b\sinh^{-1}(cx)}{\pi x\sqrt{\pi+c^2\pi x^2}}-\frac{2c^2x(a+b\sinh^{-1}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{bc\log(x)}{\pi^{3/2}}+\frac{bc\log(1+c^2x^2)}{2\pi^{3/2}}$$

```
[Out] b*c*ln(x)/Pi^(3/2)+1/2*b*c*ln(c^2*x^2+1)/Pi^(3/2)+(-a-b*arcsinh(c*x))/Pi/x/
(Pi*c^2*x^2+Pi)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {277, 197, 5804, 12, 457, 78}

$$-\frac{2c^2x(a+b\sinh^{-1}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}+\frac{bc\log(c^2x^2+1)}{2\pi^{3/2}}+\frac{bc\log(x)}{\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(Pi*x*Sqrt[Pi + c^2*Pi*x^2])) - (2*c^2*x*(a + b*ArcS
inh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*Log[x])/Pi^(3/2) + (b*c*Log[1
+ c^2*x^2])/(2*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\int\frac{-1-2c^2x^2}{x(1+c^2x^2)}\,dx}{\pi^{3/2}}\\
&= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\text{Subst}\left(\int\frac{-1-2c^2x}{x(1+c^2x)}\,dx,x,x^2\right)}{2\pi^{3/2}}\\
&= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\text{Subst}\left(\int\left(-\frac{1}{x}-\frac{c^2}{1+c^2x}\right)\,dx,x,\right.}{2\pi^{3/2}}\\
&= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{bc\log(x)}{\pi^{3/2}}+\frac{bc\log\left(1+c^2x^2\right)}{2\pi^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 69, normalized size = 0.74

$$-\frac{\left(1+2c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}x\sqrt{1+c^2x^2}}+\frac{b\left(c\log(x)+\frac{1}{2}c\log\left(1+c^2x^2\right)\right)}{\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -(((1 + 2*c^2*x^2)*(a + b*ArcSinh[c*x]))/(Pi^(3/2)*x*Sqrt[1 + c^2*x^2])) +
(b*(c*Log[x] + (c*Log[1 + c^2*x^2])/2))/Pi^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(85) = 170$.
time = 2.49, size = 181, normalized size = 1.95

method	result
default	$a\left(-\frac{1}{\pi x\sqrt{\pi\, c^2x^2+\pi}}-\frac{2c^2x}{\pi\sqrt{\pi\, c^2x^2+\pi}}\right)-\frac{4bc\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}}+\frac{2b\operatorname{arcsinh}(cx)x^2c^3}{\pi^{\frac{3}{2}}(c^2x^2+1)}-\frac{2b\operatorname{arcsinh}(cx)x\,c^2}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}+\frac{2b\operatorname{arc}}{\pi^{\frac{3}{2}}(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)-2/Pi*c^2*x/(Pi*c^2*x^2+Pi)^(1/2))-4*b*c/Pi
^(3/2)*arcsinh(c*x)+2*b/Pi^(3/2)*arcsinh(c*x)*x^2/(c^2*x^2+1)*c^3-2*b/Pi^(3
/2)*arcsinh(c*x)*x/(c^2*x^2+1)^(1/2)*c^2+2*b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2
+1)*c-b/Pi^(3/2)*arcsinh(c*x)/x/(c^2*x^2+1)^(1/2)+b*c/Pi^(3/2)*ln((c*x+(c^2
*x^2+1)^(1/2))^4-1)
```

Maxima [A]
time = 0.30, size = 119, normalized size = 1.28

$$\frac{1}{2}bc\left(\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{3}{2}}}+\frac{2\log\left(x\right)}{\pi^{\frac{3}{2}}}\right)-\left(\frac{2c^2x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}\,x}\right)b\operatorname{arsinh}\left(cx\right)-\left(\frac{2c^2x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}\,x}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*b*c*(log(c^2*x^2 + 1)/pi^(3/2) + 2*log(x)/pi^(3/2)) - (2*c^2*x/(pi*sqrt
(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*b*arcsinh(c*x) - (2*c^
2*x/(pi*sqrt(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^6 + 2*pi^2*
c^2*x^4 + pi^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{x^2\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

3.98 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{bc}{2\pi^{3/2}x}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}+\frac{bc^2\text{ArcTan}(cx)}{\pi^{3/2}}+\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh}\right.}{\pi^{3/2}}$$

```
[Out] -1/2*b*c/Pi^(3/2)/x+b*c^2*arctan(c*x)/Pi^(3/2)+3*c^2*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)+3/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))/Pi^(3/2)-3/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)-3/2*c^2*
(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)+1/2*(-a-b*arcsinh(c*x))/Pi/x^2/
(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5809, 5811, 5816, 4267, 2317, 2438, 209, 331}

$$-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{3c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}}+\frac{bc^2\text{ArcTan}(cx)}{\pi^{3/2}}+\frac{3bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}-\frac{3bc^2\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}-\frac{bc}{2\pi^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -1/2*(b*c)/(Pi^(3/2)*x) - (3*c^2*(a + b*ArcSinh[c*x]))/(2*Pi*Sqrt[Pi + c^2*
Pi*x^2]) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c^2*A
rcTan[c*x])/Pi^(3/2) + (3*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])
/Pi^(3/2) + (3*b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Pi^(3/2)) - (3*b*c^2*P
olyLog[2, E^ArcSinh[c*x]])/(2*Pi^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2(1+c}}{2\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} - \frac{\left(3c^2\right)\int \frac{a+b}{x\sqrt{\pi}}}{2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x}}{\pi\sqrt{\pi+}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x}}{\pi\sqrt{\pi+}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x}}{\pi\sqrt{\pi+}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x}}{\pi\sqrt{\pi+}}$$

Mathematica [A]
time = 2.76, size = 269, normalized size = 1.66

$$\frac{-\frac{8ac^2}{\sqrt{1+c^2x^2}} - \frac{4a\sqrt{1+c^2x^2}}{x^2} - \frac{8bc^2\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + 16bc^2\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right) - 2bc^2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right) - bc^2\sinh^{-1}(cx)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) - 12bc^2\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) + 12bc^2\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) - 12ac^2\log(x) + 12ac^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) - 12bc^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) + 12bc^2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) - bc^2\sinh^{-1}(cx)\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) + 2bc^2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)}{8\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] ((-8*a*c^2)/Sqrt[1 + c^2*x^2] - (4*a*Sqrt[1 + c^2*x^2])/x^2 - (8*b*c^2*ArcS
inh[c*x])/Sqrt[1 + c^2*x^2] + 16*b*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 2*b*c
^2*Coth[ArcSinh[c*x]/2] - b*c^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*b*
c^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*b*c^2*ArcSinh[c*x]*Log[1 +
 E^(-ArcSinh[c*x])] - 12*a*c^2*Log[x] + 12*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x
^2])] - 12*b*c^2*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*b*c^2*PolyLog[2, E^(-A
rcSinh[c*x])] - b*c^2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*b*c^2*Tanh[Ar
cSinh[c*x]/2])/(8*Pi^(3/2))
```

Maple [A]
time = 4.22, size = 233, normalized size = 1.44

method	result

default	$a\left(-\frac{1}{2\pi x^{2}\sqrt{\pi c^{2}x^{2}+\pi}}-\frac{3c^{2}\left(\frac{1}{\pi\sqrt{\pi c^{2}x^{2}+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^{2}x^{2}+\pi}}\right)}{\pi^{\frac{3}{2}}}\right)}{2}\right)-\frac{3b\operatorname{arcsinh}(cx)c^{2}}{2\pi^{\frac{3}{2}}\sqrt{c^{2}x^{2}+1}}-\frac{bc}{2\pi^{\frac{3}{2}}x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2/(Pi*c^2*x^2+Pi)^(1/2)-3/2*c^2*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/
Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))-3/2*b/Pi^(3/2)/(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c/Pi^(3/2)/x-1/2*b/Pi^(3/2)/(c^2*x^2+1)^(1/
2)/x^2*arcsinh(c*x)+2*b*c^2/Pi^(3/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+3/2*b*c^
2/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2))+3/2*b*c^2/Pi^(3/2)*dilog(1+c*x+(c^2
*x^2+1)^(1/2))+3/2*b*c^2/Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/pi^(3/2) - 3*c^2/(pi*sqrt(pi + pi*c^2*x^2)
) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^2))*a + b*integrate(log(c*x + sqrt(c^2*x^
2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^7 + 2*pi^2*
c^2*x^5 + pi^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{x^3\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

3.99 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{bc}{6\pi^{3/2}x^2}-\frac{a+b\sinh^{-1}(cx)}{3\pi x^3\sqrt{\pi+c^2\pi x^2}}+\frac{4c^2(a+b\sinh^{-1}(cx))}{3\pi x\sqrt{\pi+c^2\pi x^2}}+\frac{8c^4x(a+b\sinh^{-1}(cx))}{3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{5bc^3\log(x)}{3\pi^{3/2}}-\frac{bc^3\log(1+c^2x^2)}{2\pi^{3/2}}$$

```
[Out] -1/6*b*c/Pi^(3/2)/x^2-5/3*b*c^3*ln(x)/Pi^(3/2)-1/2*b*c^3*ln(c^2*x^2+1)/Pi^(
3/2)+1/3*(-a-b*arcsinh(c*x))/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)+4/3*c^2*(a+b*arcs
inh(c*x))/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^
2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {277, 197, 5804, 12, 1265, 907}

$$\frac{4c^2(a+b\sinh^{-1}(cx))}{3\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\sinh^{-1}(cx))}{3\pi\sqrt{\pi c^2x^2+\pi}}-\frac{5bc^3\log(x)}{3\pi^{3/2}}-\frac{bc^3\log(c^2x^2+1)}{2\pi^{3/2}}-\frac{bc}{6\pi^{3/2}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -1/6*(b*c)/(Pi^(3/2)*x^2) - (a + b*ArcSinh[c*x])/(3*Pi*x^3*Sqrt[Pi + c^2*Pi
*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]) + (8*c
^4*x*(a + b*ArcSinh[c*x]))/(3*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (5*b*c^3*Log[x])/
(3*Pi^(3/2)) - (b*c^3*Log[1 + c^2*x^2])/(2*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\int\\
&= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\int\\
&= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\mathrm{S}\\
&= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\mathrm{S}\\
&= -\frac{bc}{6\pi^{3/2}x^2}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx\right.}{3\pi^{3/2}\sqrt{1+c^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 127, normalized size = 0.83

$$\frac{-bcx\sqrt{1+c^2x^2}+2a(-1+4c^2x^2+8c^4x^4)+2b(-1+4c^2x^2+8c^4x^4)\sinh^{-1}(cx)}{6\pi^{3/2}x^3\sqrt{1+c^2x^2}}+\frac{-5bc^3\log(x)-\frac{3}{2}bc^3\log\left(1+c^2x^2\right)}{3\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) + 2*a*(-1 + 4*c^2*x^2 + 8*c^4*x^4) + 2*b*(-1 +
4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x])/(6*Pi^(3/2)*x^3*Sqrt[1 + c^2*x^2]) + (
-5*b*c^3*Log[x] - (3*b*c^3*Log[1 + c^2*x^2])/2)/(3*Pi^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(132) = 264$.
time = 5.36, size = 604, normalized size = 3.95

method	result
default	$a\left(-\frac{1}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2\left(-\frac{1}{\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{2c^2x}{\pi\sqrt{\pi c^2x^2+\pi}}\right)}{3}\right)+\frac{16b\,c^3\,\mathrm{arcsinh}(cx)}{3\pi^{\frac{3}{2}}}-\frac{32b\,x^8c^1}{3\pi^{\frac{3}{2}}(8c^2x^2-1)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)-4/3*c^2*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)
-2/Pi*c^2*x/(Pi*c^2*x^2+Pi)^(1/2)))+16/3*b*c^3/Pi^(3/2)*arcsinh(c*x)-32/3*b
/Pi^(3/2)/(8*c^2*x^2-1)*x^8/(c^2*x^2+1)*c^11+32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*
x^6*c^9-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^6/(c^2*x^2+1)*c^9+32/3*b/Pi^(3/2)/(
8*c^2*x^2-1)*x^4*c^7-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*arcsinh(
c*x)*c^7+64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c
^6-32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*c^7-56/3*b/Pi^(3/2)/(8*c^2
*x^2-1)*x^2/(c^2*x^2+1)*arcsinh(c*x)*c^5+8*b/Pi^(3/2)/(8*c^2*x^2-1)*x/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*c^4-4/3*b/Pi^(3/2)/(8*c^2*x^2-1)*c^3+8/3*b/Pi^(3/
2)/(8*c^2*x^2-1)/(c^2*x^2+1)*arcsinh(c*x)*c^3-4*b/Pi^(3/2)/(8*c^2*x^2-1)/x/
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2+1/6*b/Pi^(3/2)/(8*c^2*x^2-1)/x^2*c+1/3*b
/Pi^(3/2)/(8*c^2*x^2-1)/x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-5/3*b*c^3/Pi^(3/
2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-b*c^3/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/
2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(8*c^4*x/(pi*sqrt(pi + pi*c^2*x^2)) + 4*c^2/(pi*sqrt(pi + pi*c^2*x^2)*x
) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^
2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^8 + 2*pi^2*
c^2*x^6 + pi^2*x^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(3/2),x)
[Out] (Integral(a/(c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

3.100 $\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{bx^2}{4c^5\pi^{5/2}}-\frac{b}{6c^7\pi^{5/2}\left(1+c^2x^2\right)}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{5x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(\right.}{2c^6\pi^3}$$

```
[Out] -1/4*b*x^2/c^5/Pi^(5/2)-1/6*b/c^7/Pi^(5/2)/(c^2*x^2+1)-1/3*x^5*(a+b*arcsinh
(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-5/4*(a+b*arcsinh(c*x))^2/b/c^7/Pi^(5/2)
-7/6*b*ln(c^2*x^2+1)/c^7/Pi^(5/2)-5/3*x^3*(a+b*arcsinh(c*x))/c^4/Pi^2/(Pi*c
^2*x^2+Pi)^(1/2)+5/2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi^3
```

Rubi [A]
time = 0.27, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5810, 5812, 5783, 30, 272, 45}

$$-\frac{5\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{5/2}bc^7}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{5x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^3c^6}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2c^4\sqrt{\pi c^2x^2+\pi}}-\frac{bx^2}{4\pi^{5/2}c^5}-\frac{b}{6\pi^{5/2}c^7\left(c^2x^2+1\right)}-\frac{7b\log\left(c^2x^2+1\right)}{6\pi^{5/2}c^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/4*(b*x^2)/(c^5*Pi^(5/2)) - b/(6*c^7*Pi^(5/2)*(1 + c^2*x^2)) - (x^5*(a +
b*ArcSinh[c*x]))/(3*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh
[c*x]))/(3*c^4*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (5*x*Sqrt[Pi + c^2*Pi*x^2]*(a
+ b*ArcSinh[c*x]))/(2*c^6*Pi^3) - (5*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*Pi^(5
/2)) - (7*b*Log[1 + c^2*x^2])/(6*c^7*Pi^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^6 \left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{5\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{3c^2\pi} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^5}{\left(1+c^2x^2\right)^2}\,d}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{5\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^4\pi^2} + \frac{(5}{}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{5x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh\right.}{2c^6\pi^3}$$

$$= -\frac{b}{6c^7\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{13bx^2\sqrt{1+c^2x^2}}{12c^5\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}$$

$$= -\frac{b}{6c^7\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{bx^2\sqrt{1+c^2x^2}}{4c^5\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}$$

Mathematica [A]
time = 0.36, size = 202, normalized size = 1.05

$$\frac{60acx+80ac^3x^3+12ac^5x^5-7b\sqrt{1+c^2x^2}-9bc^2x^2\sqrt{1+c^2x^2}-6bc^4x^4\sqrt{1+c^2x^2}+4\left(-15a(1+c^2x^2)^{3/2}+bcx(15+20c^2x^2+3c^4x^4)\right)\sinh^{-1}(cx)-30b(1+c^2x^2)^{3/2}\sinh^{-1}(cx)^2-28b(1+c^2x^2)^{3/2}\log(1+c^2x^2)}{24c^7\pi^{5/2}(1+c^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (60*a*c*x + 80*a*c^3*x^3 + 12*a*c^5*x^5 - 7*b*Sqrt[1 + c^2*x^2] - 9*b*c^2*x
^2*Sqrt[1 + c^2*x^2] - 6*b*c^4*x^4*Sqrt[1 + c^2*x^2] + 4*(-15*a*(1 + c^2*x^
2)^(3/2) + b*c*x*(15 + 20*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] - 30*b*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]^2 - 28*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2
4*c^7*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. 2(164) = 328.
time = 8.03, size = 970, normalized size = 5.05

method	result
default	$\frac{385b\,\mathrm{arcsinh}(cx)x^5}{\pi^{\frac{5}{2}}(63c^4x^4+111c^2x^2+49)(c^2x^2+1)^{\frac{3}{2}}c^2} - \frac{2338b\,\mathrm{arcsinh}(cx)x^4}{3\pi^{\frac{5}{2}}(63c^4x^4+111c^2x^2+49)(c^2x^2+1)^2c^3} - \frac{49b}{6\pi^{\frac{5}{2}}(63c^4x^4+111c^2x^2+49)(c^2x^2+1)^2c^7} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^7-1463/3*b/Pi^
(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^5*arcsinh(c*x)*x^2-147*b/
Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^8-553*b
/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c*arcsinh(c*x)*x^6+1009
/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x
)*x^3+98*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^(3/2)/c^6*arcsi
nh(c*x)*x+385*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^(3/2)/c^2*
arcsinh(c*x)*x^5-2338/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^
2/c^3*arcsinh(c*x)*x^4-1/8*b/Pi^(5/2)/c^7+1/2*b/Pi^(5/2)/c^6*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)*x-49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^
2*c*x^8-98/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c*x^6-49*
b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^3*x^4-98/3*b/Pi^(5/2
)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^5*x^2-343/3*b/Pi^(5/2)/(63*c^
4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^7*arcsinh(c*x)+49/6*b/Pi^(5/2)/(63*c^
4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c*x^6+14*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x
^2+49)/(c^2*x^2+1)/c^3*x^4+6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^
2+1)/c^5*x^2+147*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^(3/2)*a
rcsinh(c*x)*x^7+5/2*a/c^6/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)-5/2*a/c^6/Pi^2*ln(Pi
*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+5/6*a/c^4*x^3/P
i/(Pi*c^2*x^2+Pi)^(3/2)+1/2*a*x^5/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-1/4*b*x^2/c^
5/Pi^(5/2)-7/3*b/c^7/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-5/4*b/c^7/Pi^
(5/2)*arcsinh(c*x)^2+14/3*b/c^7/Pi^(5/2)*arcsinh(c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a*(3*x^5/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 5*x*(3*x^2/(pi*(pi + pi*c^2
*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))/c^2 + 5*x/(pi^2*sqrt
(pi + pi*c^2*x^2)*c^6) - 15*arcsinh(c*x)/(pi^(5/2)*c^7)) + b*integrate(x^6*
log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^6*arcsinh(c*x) + a*x^6)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^6}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^6\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**6/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**6*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^6*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.101 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^3}-$$

```
[Out] -b*x/c^5/Pi^(5/2)+1/6*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x))/
c^6/Pi/(Pi*c^2*x^2+Pi)^(3/2)-11/6*b*arctan(c*x)/c^6/Pi^(5/2)+2*(a+b*arcsinh
(c*x))/c^6/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1
/2)/c^6/Pi^3
```

Rubi [A]
time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {272, 45, 5804, 12, 1171, 396, 209}

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi^3c^6}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{3\pi c^6\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{11b\text{ArcTan}(cx)}{6\pi^{5/2}c^6}-\frac{bx}{\pi^{5/2}c^5}+\frac{bx}{6\pi^{5/2}c^5\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -((b*x)/(c^5*Pi^(5/2))) + (b*x)/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*Arc
Sinh[c*x])/(3*c^6*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*(a + b*ArcSinh[c*x]))/(c
^6*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
))/(c^6*Pi^3) - (11*b*ArcTan[c*x])/(6*c^6*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{c^6\pi^{5/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{c^6\pi^{5/2}}$$

$$= \frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x}}{}$$

$$= -\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}$$

$$= -\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.12, size = 132, normalized size = 0.90

$$\frac{16a+24ac^2x^2+6ac^4x^4-5bcx\sqrt{1+c^2x^2}-6bc^3x^3\sqrt{1+c^2x^2}+2b(8+12c^2x^2+3c^4x^4)\sinh^{-1}(cx)-11b(1+c^2x^2)^{3/2}\,\mathrm{ArcTan}(cx)}{6c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (16*a + 24*a*c^2*x^2 + 6*a*c^4*x^4 - 5*b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 2*b*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - 11*b*
(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^6*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 9.84, size = 237, normalized size = 1.62

method	result
default	$a\left(\frac{x^4}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{\pi c^2\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{2}{3\pi c^4\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}}{\pi^{\frac{5}{2}}c^6}-\frac{bx}{c^5\pi^{\frac{5}{2}}}+\frac{2b\,\mathrm{arcsinh}}{\pi^{\frac{5}{2}}(c^2x^2-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-4/c^2*(-x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2
)-2/3/Pi/c^4/(Pi*c^2*x^2+Pi)^(3/2)))+b/Pi^(5/2)/c^6*arcsinh(c*x)*(c^2*x^2+1
```

```
)^(1/2)-b*x/c^5/Pi^(5/2)+2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)*x^
2+1/6*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+5/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^6*arc
sinh(c*x)+11/6*I*b/c^6/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-11/6*I*b/c^6/Pi
^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b*((3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))*log(c*x + sq
rt(c^2*x^2 + 1))/((pi^3*c^8*x^2 + pi^3*c^6)*sqrt(c^2*x^2 + 1)) + 3*integrat
e(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))/(pi^3*c^11*x^
6 + 2*pi^3*c^9*x^4 + pi^3*c^7*x^2 + (pi^3*c^10*x^5 + 2*pi^3*c^8*x^3 + pi^3*
c^6*x)*sqrt(c^2*x^2 + 1)), x) - 3*integrate(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sq
rt(pi)*c^2*x^2 + 8*sqrt(pi))/((pi^3*c^8*x^3 + pi^3*c^6*x)*sqrt(c^2*x^2 + 1)
), x)) + 1/3*a*(3*x^4/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 12*x^2/(pi*(pi + p
i*c^2*x^2)^(3/2)*c^4) + 8/(pi*(pi + pi*c^2*x^2)^(3/2)*c^6))
```

Fricas [A]
time = 0.44, size = 218, normalized size = 1.49

$$\frac{11\sqrt{\pi}\,(bc^4x^4+2\,bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}\,cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}\,(3\,bc^4x^4+12\,bc^2x^2+8\,b)\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{\pi+\pi c^2x^2}\left(6\,ac^4x^4+24\,ac^2x^2-(6\,bc^3x^3+5\,bcx)\sqrt{c^2x^2+1}+16\,a\right)}{12\,(\pi^3c^{10}x^4+2\,\pi^3c^8x^2+\pi^3c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/12*(11*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
+ pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*(3*b*c^4*x^4 + 12*b*c^2*x^2 + 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*s
qrt(pi + pi*c^2*x^2)*(6*a*c^4*x^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*
sqrt(c^2*x^2 + 1) + 16*a))/(pi^3*c^10*x^4 + 2*pi^3*c^8*x^2 + pi^3*c^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^5}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^5\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**5/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**5*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.102 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=139

$$\frac{b}{6c^5\pi^{5/2}(1+c^2x^2)}-\frac{x^3(a+b\sinh^{-1}(cx))}{3c^2\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{x(a+b\sinh^{-1}(cx))}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{(a+b\sinh^{-1}(cx))^2}{2bc^5\pi^{5/2}}+\frac{2b\log(1+c^2x^2)}{3c^5\pi^{5/2}}$$

```
[Out] 1/6*b/c^5/Pi^(5/2)/(c^2*x^2+1)-1/3*x^3*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^
2+Pi)^(3/2)+1/2*(a+b*arcsinh(c*x))^2/b/c^5/Pi^(5/2)+2/3*b*ln(c^2*x^2+1)/c^5
/Pi^(5/2)-x*(a+b*arcsinh(c*x))/c^4/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5810, 5783, 266, 272, 45}

$$\frac{(a+b\sinh^{-1}(cx))^2}{2\pi^{5/2}bc^5}-\frac{x^3(a+b\sinh^{-1}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}-\frac{x(a+b\sinh^{-1}(cx))}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{b}{6\pi^{5/2}c^5(c^2x^2+1)}+\frac{2b\log(c^2x^2+1)}{3\pi^{5/2}c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] b/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)) - (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*Pi*(Pi
 + c^2*Pi*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4*Pi^2*Sqrt[Pi + c^2*Pi
*x^2]) + (a + b*ArcSinh[c*x])^2/(2*b*c^5*Pi^(5/2)) + (2*b*Log[1 + c^2*x^2])
/(3*c^5*Pi^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{c^2\pi}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^3}{\left(1+c^2x^2\right)^2}\,d}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^4\pi^2}+\frac{\left(b\sqrt{1}\right.}{c}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5\pi^{5/2}}+\frac{b\sqrt{1}}{2}$$

$$= \frac{b}{6c^5\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A]
time = 0.25, size = 166, normalized size = 1.19

$$\frac{b+bc^2x^2-6acx\sqrt{1+c^2x^2}-8ac^3x^3\sqrt{1+c^2x^2}+2\left(3a\left(1+c^2x^2\right)^2-bcx\sqrt{1+c^2x^2}\left(3+4c^2x^2\right)\right)\sinh^{-1}(cx)+3b\left(1+c^2x^2\right)^2\sinh^{-1}(cx)^2+4b\left(1+c^2x^2\right)^2\log\left(1+c^2x^2\right)}{6c^5\pi^{5/2}\left(1+c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (b + b*c^2*x^2 - 6*a*c*x*Sqrt[1 + c^2*x^2] - 8*a*c^3*x^3*Sqrt[1 + c^2*x^2]
+ 2*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2))*ArcSinh
```

```
[c*x] + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]^2 + 4*b*(1 + c^2*x^2)^2*Log[1 + c^
2*x^2])/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(121) = 242$.
time = 6.90, size = 897, normalized size = 6.45

method	result
default	$-\frac{a x^3}{3\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{ax}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\pi^2c^4\sqrt{\pi c^2}}+\frac{b\,\mathrm{arcsinh}(cx)^2}{2c^5\pi^{\frac{5}{2}}}-\frac{8b\,\mathrm{arcsinh}(cx)}{3c^5\pi^{\frac{5}{2}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-a/Pi^2/c^4*x/(Pi*c^2*x^2+Pi)^(1/2)+
a/Pi^2/c^4*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)
+1/2*b/c^5/Pi^(5/2)*arcsinh(c*x)^2-8/3*b/c^5/Pi^(5/2)*arcsinh(c*x)+32*b/Pi^
(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*arcsinh(c*x)*x^8-32*b/Pi
^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*c^2*arcsinh(c*x)*x^7+8/
3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*x^8-8/3*b/Pi^(5/2
)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)*c*x^6+116*b/Pi^(5/2)/(24*c^4*x^4+3
9*c^2*x^2+16)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^6-76*b/Pi^(5/2)/(24*c^4*x^4+39
*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5+32/3*b/Pi^(5/2)/(24*c^4*x^4
+39*c^2*x^2+16)/(c^2*x^2+1)^2*c*x^6-4*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)
/(c^2*x^2+1)/c*x^4+472/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^
2/c*arcsinh(c*x)*x^4-181/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1
)^(3/2)/c^2*arcsinh(c*x)*x^3+16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*
x^2+1)^2/c*x^4-3/2*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)/c^3*x^
2+284/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^3*arcsinh(c*x
)*x^2-16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)/c^4*arcsin
h(c*x)*x+32/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^3*x^2+6
4/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^5*arcsinh(c*x)+8/
3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^5+4/3*b/c^5/Pi^(5/2
)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(x*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(
3/2)*c^4)) + x/(pi^2*sqrt(pi + pi*c^2*x^2)*c^4) - 3*arcsinh(c*x)/(pi^(5/2)*
c^5))*a + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5
/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^4\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**4*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.103 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)}+\frac{a+b\sinh^{-1}(cx)}{3c^4\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{5b\text{ArcTan}(cx)}{6c^4\pi^{5/2}}$$

```
[Out] -1/6*b*x/c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*(a+b*arcsinh(c*x))/c^4/Pi/(Pi*c^2*x^2
+Pi)^(3/2)+5/6*b*arctan(c*x)/c^4/Pi^(5/2)+(-a-b*arcsinh(c*x))/c^4/Pi^2/(Pi*
c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 45, 5804, 12, 393, 209}

$$-\frac{a+b\sinh^{-1}(cx)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\sinh^{-1}(cx)}{3\pi c^4\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{5b\text{ArcTan}(cx)}{6\pi^{5/2}c^4}-\frac{bx}{6\pi^{5/2}c^3\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/6*(b*x)/(c^3*Pi^(5/2)*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x])/(3*c^4*Pi*(P
i + c^2*Pi*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*Pi^2*Sqrt[Pi + c^2*Pi*x^
2]) + (5*b*ArcTan[c*x])/(6*c^4*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx &= \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}}-\frac{(bc)\int\frac{-2-3c^2x^2}{3c^4\left(1+c^2x^2\right)^2}\,dx}{\pi^{5/2}}\\ &= \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}}-\frac{b\int\frac{-2-3c^2x^2}{\left(1+c^2x^2\right)^2}\,dx}{3c^3\pi^{5/2}}\\ &= -\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)}+\frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{(5b)\int\frac{1}{1+c}\cdots}{6c^3\pi^{5}\cdots}\\ &= -\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)}+\frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{5b\tan^{-1}(\cdots}{6c^4\pi^{5/2}\cdots}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 0.89

$$\frac{-4a-6ac^2x^2-bcx\sqrt{1+c^2x^2}-2b(2+3c^2x^2)\sinh^{-1}(cx)+5b(1+c^2x^2)^{3/2}\operatorname{ArcTan}(cx)}{6c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (-4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*(2 + 3*c^2*x^2)*ArcSinh
[c*x] + 5*b*(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^4*Pi^(5/2)*(1 + c^2*x^2)^
(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 8.24, size = 176, normalized size = 1.68

method	result
default	$a\left(-\frac{x^{2}}{\pi c^{2}(\pi c^{2}x^{2}+\pi)^{\frac{3}{2}}}-\frac{2}{3\pi c^{4}(\pi c^{2}x^{2}+\pi)^{\frac{3}{2}}}\right)-\frac{b\operatorname{arcsinh}(cx)x^{2}}{\pi^{\frac{5}{2}}(c^{2}x^{2}+1)^{\frac{3}{2}}c^{2}}-\frac{bx}{6c^{3}\pi^{\frac{5}{2}}(c^{2}x^{2}+1)}-\frac{2b\operatorname{arcsinh}(cx)}{3\pi^{\frac{5}{2}}(c^{2}x^{2}+1)^{\frac{3}{2}}c^{4}}+\frac{5ib\ln\left(cx+\sqrt{\cdots}\right.}{6c\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-2/3/Pi/c^4/(Pi*c^2*x^2+Pi)^(3/2))-b/Pi
^(5/2)/(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)*x^2-1/6*b*x/c^3/Pi^(5/2)/(c^2*x^2
+1)-2/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)+5/6*I*b/c^4/Pi^(5/2)*
ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I*b/c^4/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I
)
```

Maxima [A]
time = 0.49, size = 138, normalized size = 1.31

$$-\frac{1}{6}\,bc\left(\frac{x}{\pi^{\frac{5}{2}}c^{6}x^{2}+\pi^{\frac{5}{2}}c^{4}}-\frac{5\,\arctan\left(cx\right)}{\pi^{\frac{5}{2}}c^{5}}\right)-\frac{1}{3}\,b\left(\frac{3\,x^{2}}{\pi(\pi+\pi c^{2}x^{2})^{\frac{3}{2}}c^{2}}+\frac{2}{\pi(\pi+\pi c^{2}x^{2})^{\frac{3}{2}}c^{4}}\right)\operatorname{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{3\,x^{2}}{\pi(\pi+\pi c^{2}x^{2})^{\frac{3}{2}}c^{2}}+\frac{2}{\pi(\pi+\pi c^{2}x^{2})^{\frac{3}{2}}c^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(x/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) - 5*arctan(c*x)/(pi^(5/2)*c^5
)) - 1/3*b*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2
)^(3/2)*c^4))*arcsinh(c*x) - 1/3*a*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
+ 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(91) = 182.
time = 0.42, size = 187, normalized size = 1.78

$$-\frac{5\,\sqrt{\pi}\,(bc^{4}x^{4}+2\,bc^{2}x^{2}+b)\arctan\left(-\frac{2\,\sqrt{\pi}\,\sqrt{\pi+\pi c^{2}x^{2}}\,\sqrt{c^{2}x^{2}+1}\,cx}{\pi-\pi c^{4}x^{4}}\right)+4\,\sqrt{\pi+\pi c^{2}x^{2}}\,(3\,bc^{2}x^{2}+2\,b)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+2\,\sqrt{\pi+\pi c^{2}x^{2}}\left(6\,ac^{2}x^{2}+\sqrt{c^{2}x^{2}+1}\,bcx+4\,a\right)}{12\,(\pi^{3}c^{8}x^{4}+2\,\pi^{3}c^{6}x^{2}+\pi^{3}c^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/12*(5*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
+ pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*(3*b*c^2*x^2 + 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2
*x^2)*(6*a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x + 4*a))/(pi^3*c^8*x^4 + 2*pi^3
*c^6*x^2 + pi^3*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^3\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**3*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.104 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{b}{6c^3\pi^{5/2}(1+c^2x^2)}+\frac{x^3(a+b\sinh^{-1}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{b\log(1+c^2x^2)}{6c^3\pi^{5/2}}$$

```
[Out] -1/6*b/c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*x^3*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+P
i)^(3/2)-1/6*b*ln(c^2*x^2+1)/c^3/Pi^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5800, 272, 45}

$$\frac{x^3(a+b\sinh^{-1}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}-\frac{b}{6\pi^{5/2}c^3(c^2x^2+1)}-\frac{b\log(c^2x^2+1)}{6\pi^{5/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/6*b/(c^3*Pi^(5/2)*(1 + c^2*x^2)) + (x^3*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi
+ c^2*Pi*x^2)^(3/2)) - (b*Log[1 + c^2*x^2])/(6*c^3*Pi^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
```

```
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x^3}{(1+c^2x^2)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$=\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{x}{(1+c^2x)^2}\,dx,x,x^2\right)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$=\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\left(-\frac{1}{c^2(1+c^2x)^2}+\frac{1}{c^2(1+c^2x)}\right)\,d\right.}{6\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$=-\frac{b}{6c^3\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{b\sqrt{1+c^2x^2}\,\log\left(\right.}{6c^3\pi^2\sqrt{\pi+c^2}}$$

Mathematica [A]
time = 0.10, size = 88, normalized size = 1.10

$$-\frac{-2ac^3x^3+b\sqrt{1+c^2x^2}-2bc^3x^3\sinh^{-1}(cx)+b\left(1+c^2x^2\right)^{3/2}\log\left(1+c^2x^2\right)}{6c^3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
[Out] -1/6*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] - 2*b*c^3*x^3*ArcSinh[c*x] + b*(1
+ c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(c^3*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(68) = 136$.
time = 6.52, size = 729, normalized size = 9.11

method	result
default	$a\left(-\frac{x}{2\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}}{2c^2}\right)+\frac{2b\,\mathrm{arcsinh}(cx)}{3c^3\pi^{\frac{5}{2}}}-\frac{b\,c^5\,\mathrm{arcsinh}(cx)x^8}{\pi^{\frac{5}{2}}(3c^4x^4+3c^2x^2+1)(c^2x^2+1)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
[Out] a*(-1/2*x/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+1/2/c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3
/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)))+2/3*b/c^3/Pi^(5/2)*arcsinh(c*x)-b/Pi
```

```
^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^5*arcsinh(c*x)*x^8+b/Pi^(5/2
)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*c^4*arcsinh(c*x)*x^7-1/6*b/Pi^(
5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^5*x^8+1/6*b/Pi^(5/2)/(3*c^4*x^
4+3*c^2*x^2+1)/(c^2*x^2+1)*c^3*x^6-3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^
2*x^2+1)^2*c^3*arcsinh(c*x)*x^6+b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2
+1)^(3/2)*c^2*arcsinh(c*x)*x^5-2/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*
x^2+1)^2*c^3*x^6-10/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c*ar
csinh(c*x)*x^4+1/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*arc
sinh(c*x)*x^3-b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c*x^4-5/3*b/
Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c*arcsinh(c*x)*x^2-2/3*b/Pi^
(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c*x^2-1/3*b/Pi^(5/2)/(3*c^4*x^4
+3*c^2*x^2+1)/(c^2*x^2+1)^2/c^3*arcsinh(c*x)-1/6*b/Pi^(5/2)/(3*c^4*x^4+3*c^
2*x^2+1)/(c^2*x^2+1)^2/c^3-1/3*b/c^3/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(68) = 136.
time = 0.29, size = 137, normalized size = 1.71

$$-\frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^6x^2+\pi^{\frac{5}{2}}c^4}+\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^4}\right)-\frac{1}{3}\,b\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}-\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}\,c^2}\right)\operatorname{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}-\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(1/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) + log(c^2*x^2 + 1)/(pi^(5/2)*
c^4)) - 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(pi^2*sqrt(pi + pi*c^
2*x^2)*c^2))*arcsinh(c*x) - 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(
pi^2*sqrt(pi + pi*c^2*x^2)*c^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^2\operatorname{asinh}\left(cx\right)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**2*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.105 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{bx}{6c\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{b\text{ArcTan}(cx)}{6c^2\pi^{5/2}}$$

```
[Out] 1/6*b*x/c/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+P
i)^(3/2)+1/6*b*arctan(c*x)/c^2/Pi^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5798, 205, 209}

$$-\frac{a+b\sinh^{-1}(cx)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b\text{ArcTan}(cx)}{6\pi^{5/2}c^2}+\frac{bx}{6\pi^{5/2}c\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (b*x)/(6*c*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(3*c^2*Pi*(Pi + c
^2*Pi*x^2)^(3/2)) + (b*ArcTan[c*x])/(6*c^2*Pi^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3c^2\pi\,(\pi+c^2\pi x^2)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{(1+c^2x^2)^2}\,dx}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$=\frac{bx}{6c\pi^2\sqrt{1+c^2x^2}\,\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2\pi\,(\pi+c^2\pi x^2)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x}$$

$$=\frac{bx}{6c\pi^2\sqrt{1+c^2x^2}\,\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2\pi\,(\pi+c^2\pi x^2)^{3/2}}+\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{6c^2\pi^2\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A]
time = 0.08, size = 72, normalized size = 0.96

$$\frac{-2a+bcx\sqrt{1+c^2x^2}-2b\sinh^{-1}(cx)+b(1+c^2x^2)^{3/2}\,\mathrm{ArcTan}(cx)}{6c^2\pi^{5/2}\,(1+c^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (-2*a + b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + b*(1 + c^2*x^2)^(3/2)*
ArcTan[c*x])/(6*c^2*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 3.73, size = 124, normalized size = 1.65

method	result
default	$-\frac{a}{3\pi\,c^2(\pi\,c^2x^2+\pi)^{\frac{3}{2}}}+\frac{bx}{6c\,\pi^{\frac{5}{2}}\,(c^2x^2+1)}-\frac{b\,\mathrm{arcsinh}(cx)}{3\pi^{\frac{5}{2}}\,(c^2x^2+1)^{\frac{3}{2}}\,c^2}+\frac{ib\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{6c^2\pi^{\frac{5}{2}}}-\frac{ib\ln\left(cx+\sqrt{c^2x^2+1}-i\right)}{6c^2\pi^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+1/6*b*x/c/Pi^(5/2)/(c^2*x^2+1)-1/3*b/Pi
^(5/2)/(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)+1/6*I*b/c^2/Pi^(5/2)*ln(c*x+(c^2*
x^2+1)^(1/2)+I)-1/6*I*b/c^2/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x) - 1/
3*a/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(63) = 126$.
time = 0.44, size = 165, normalized size = 2.20

$$-\frac{\sqrt{\pi}\left(bc^{4}x^{4}+2\,bc^{2}x^{2}+b\right)\arctan\left(-\frac{2\,\sqrt{\pi}\,\sqrt{\pi+\pi c^{2}x^{2}}\,\sqrt{c^{2}x^{2}+1}\,cx}{\pi-\pi c^{4}x^{4}}\right)+4\,\sqrt{\pi+\pi c^{2}x^{2}}\,b\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)-2\,\sqrt{\pi+\pi c^{2}x^{2}}\left(\sqrt{c^{2}x^{2}+1}\,bcx-2\,a\right)}{12\left(\pi^{3}c^{6}x^{4}+2\,\pi^{3}c^{4}x^{2}+\pi^{3}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi +
pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2*x
^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*(sqrt(c^2*x^2
+ 1)*b*c*x - 2*a))/(pi^3*c^6*x^4 + 2*pi^3*c^4*x^2 + pi^3*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax}{c^{4}x^{4}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\,dx+\int\frac{bx\operatorname{asinh}\left(cx\right)}{c^{4}x^{4}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 +
 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x*asinh(c*x)/(c**4*x**4*sqrt(c*
*2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/
pi**(5/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.106 $\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{b}{6c\pi^{5/2}\left(1+c^2x^2\right)}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{b\log\left(1+c^2x^2\right)}{3c\pi^{5/2}}$$

```
[Out] 1/6*b/c/Pi^(5/2)/(c^2*x^2+1)+1/3*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3
/2)-1/3*b*ln(c^2*x^2+1)/c/Pi^(5/2)+2/3*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^
2+Pi)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5788, 5787, 266, 267}

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b}{6\pi^{5/2}c\left(c^2x^2+1\right)}-\frac{b\log\left(c^2x^2+1\right)}{3\pi^{5/2}c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] b/(6*c*Pi^(5/2)*(1 + c^2*x^2)) + (x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*P
i*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) -
 (b*Log[1 + c^2*x^2])/(3*c*Pi^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2\int \frac{a+b\sinh^{-1}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx}{3\pi} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{(1+c^2x^2)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{b}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - $$

$$= \frac{b}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - $$

Mathematica [A]
time = 0.08, size = 100, normalized size = 0.93

$$\frac{6acx+4ac^3x^3+b\sqrt{1+c^2x^2}+2bcx(3+2c^2x^2)\sinh^{-1}(cx)-2b(1+c^2x^2)^{3/2}\log\left(1+c^2x^2\right)}{6c\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*c*x*(3 + 2*c^2*x^2)*ArcS
inh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(6*c*Pi^(5/2)*(1 + c^2
*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(92) = 184$.
time = 3.28, size = 619, normalized size = 5.73

method	result
default	$a\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)+\frac{4b\,\mathrm{arcsinh}(cx)}{3c\pi^{\frac{5}{2}}}+\frac{2b\,c^7x^8}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)^2}-\frac{2b\,c^5x^6}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)

[Out] a*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))+4/3*b/c /Pi^(5/2)*arcsinh(c*x)+2/3*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8-2 /3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6-2*b/Pi^(5/2)*c^5/(3*c^2*x^2 +4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6+2*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+ 1)^(3/2)*arcsinh(c*x)*x^5+8/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^ 6-2*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4-20/3*b/Pi^(5/2)*c^3/(3*c^2 *x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4+17/3*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c ^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3+4*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1) ^2*x^4-3/2*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2-22/3*b/Pi^(5/2)*c/(3* c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4*b/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x ^2+1)^(3/2)*arcsinh(c*x)*x+8/3*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2 -8/3*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)+2/3*b/Pi^(5/2)/c /(3*c^2*x^2+4)/(c^2*x^2+1)^2-2/3*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^ 2)

Maxima [A]
time = 0.29, size = 126, normalized size = 1.17

$$\frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2+\pi^{\frac{5}{2}}c^2}-\frac{2\,\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^2}\right)+\frac{1}{3}\,b\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}}+\frac{2\,x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}}+\frac{2\,x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")

[Out] 1/6*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/2) *c^2)) + 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi*c^2 *x^2)))*arcsinh(c*x) + 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sq rt(pi + pi*c^2*x^2)))

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^6 + 3*pi^3* c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1
) + sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x
**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**
(5/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.107 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{bcx}{6\pi^{5/2}\left(1+c^2x^2\right)}+\frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{7b\text{ArcTan}(cx)}{6\pi^{5/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}$$

```
[Out] -1/6*b*c*x/Pi^(5/2)/(c^2*x^2+1)+1/3*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(
3/2)-7/6*b*arctan(c*x)/Pi^(5/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1
)^(1/2))/Pi^(5/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(5/2)+b*polylog(2,
c*x+(c^2*x^2+1)^(1/2))/Pi^(5/2)+(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/
2)
```

Rubi [A]
time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5811, 5816, 4267, 2317, 2438, 209, 205}

$$\frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}}-\frac{7b\text{ArcTan}(cx)}{6\pi^{5/2}}-\frac{bcx}{6\pi^{5/2}\left(c^2x^2+1\right)}-\frac{b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}+\frac{b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c*x)/(Pi^(5/2)*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x])/(3*Pi*(Pi + c^
2*Pi*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (7*b
*ArcTan[c*x])/(6*Pi^(5/2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/Pi^(5/2) - (b*PolyLog[2, -E^ArcSinh[c*x]])/Pi^(5/2) + (b*PolyLog[2, E^Ar
cSinh[c*x]])/Pi^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx}{\pi} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{(1+c^2x^2)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{\int \frac{a+b\sinh}{x\sqrt{\pi+}}}{\pi}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c}}{6\pi^2\sqrt{}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c}}{6\pi^2\sqrt{}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c}}{6\pi^2\sqrt{}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c}}{6\pi^2\sqrt{}}$$

Mathematica [A]
time = 0.54, size = 209, normalized size = 1.41

$$\frac{\frac{2a}{(1+c^2x^2)^{3/2}} - \frac{bcx}{1+c^2x^2} + \frac{6a}{\sqrt{1+c^2x^2}} + \frac{8b\sinh^{-1}(cx)}{(1+c^2x^2)^{3/2}} + \frac{6bc^2x^2\sinh^{-1}(cx)}{(1+c^2x^2)^{3/2}} - 14b\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right) + 6b\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) - 6b\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) + 6a\log(x) - 6a\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) + 6b\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 6b\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)}{6\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] ((2*a)/(1 + c^2*x^2)^(3/2) - (b*c*x)/(1 + c^2*x^2) + (6*a)/Sqrt[1 + c^2*x^2
] + (8*b*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (6*b*c^2*x^2*ArcSinh[c*x])/(1
+ c^2*x^2)^(3/2) - 14*b*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*b*ArcSinh[c*x]*Log
[1 - E^(-ArcSinh[c*x])] - 6*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 6*a
*Log[x] - 6*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 6*b*PolyLog[2, -E^(-ArcSinh
[c*x])] - 6*b*PolyLog[2, E^(-ArcSinh[c*x])])/(6*Pi^(5/2))
```

Maple [A]
time = 4.18, size = 225, normalized size = 1.52

method	result

default	$a\left(\frac{1}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right)+\frac{b\,\mathrm{arcsinh}(cx)x^2c^2}{\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}}-\frac{bcx}{6\pi^{\frac{5}{2}}(c^2x^2+1)}+\frac{4b\,\mathrm{arc}}{3\pi^{\frac{5}{2}}(c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3/Pi/(Pi*c^2*x^2+Pi)^(3/2)+1/Pi*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2)
*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arc
sinh(c*x)*x^2*c^2-1/6*b*c*x/Pi^(5/2)/(c^2*x^2+1)+4/3*b/Pi^(5/2)/(c^2*x^2+1)
^(3/2)*arcsinh(c*x)-7/3*b/Pi^(5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))-b/Pi^(5/2)
*dilog(c*x+(c^2*x^2+1)^(1/2))-b/Pi^(5/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b/P
i^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(1/(c*abs(x)))/pi^(5/2) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)) -
 3/(pi^2*sqrt(pi + pi*c^2*x^2))) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))
/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^7 + 3*pi^3*
c^4*x^5 + 3*pi^3*c^2*x^3 + pi^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^4x^5\sqrt{c^2x^2+1}+2c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx+\int\frac{b\,\mathrm{asinh}\,(cx)}{c^4x^5\sqrt{c^2x^2+1}+2c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**5*sqrt(c**2*x**2 + 1) + 2*c**2*x**3*sqrt(c**2*x**2 + 1
) + x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**5*sqrt(c**2
*x**2 + 1) + 2*c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x))/
pi**(5/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

3.108 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{bc}{6\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{bc\log(x)}{\pi^{5/2}}+\frac{5bc\log}{}$$

```
[Out] -1/6*b*c/Pi^(5/2)/(c^2*x^2+1)+(-a-b*arcsinh(c*x))/Pi/x/(Pi*c^2*x^2+Pi)^(3/2
)-4/3*c^2*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)+b*c*ln(x)/Pi^(5/2)+
5/6*b*c*ln(c^2*x^2+1)/Pi^(5/2)-8/3*c^2*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^
2+Pi)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {277, 198, 197, 5804, 12, 1265, 907}

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc}{6\pi^{5/2}\left(c^2x^2+1\right)}+\frac{5bc\log\left(c^2x^2+1\right)}{6\pi^{5/2}}+\frac{bc\log(x)}{\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c)/(Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(Pi*x*(Pi + c^2*
Pi*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/
2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*
Log[x])/Pi^(5/2) + (5*b*c*Log[1 + c^2*x^2])/(6*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)}{}$$

$$= -\frac{bc}{6\pi^{5/2}\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x(a+b}{3\pi^{5/2}\sqrt{}}$$

Mathematica [A]
time = 0.13, size = 123, normalized size = 0.82

$$\frac{-bcx\sqrt{1+c^2x^2} - 2a(3+12c^2x^2+8c^4x^4) - 2b(3+12c^2x^2+8c^4x^4)\sinh^{-1}(cx)}{6\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} + \frac{3bc\log(x)+\frac{5}{2}bc\log\left(1+c^2x^2\right)}{3\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) - 2*a*(3 + 12*c^2*x^2 + 8*c^4*x^4) - 2*b*(3 + 1
2*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x])/(6*Pi^(5/2)*x*(1 + c^2*x^2)^(3/2)) + (
3*b*c*Log[x] + (5*b*c*Log[1 + c^2*x^2])/2)/(3*Pi^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(132) = 264.
time = 5.13, size = 778, normalized size = 5.19

method	result
default	$a\left(-\frac{1}{\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}} - 4c^2\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}} + \frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)\right) - \frac{16bc\,\mathrm{arcsinh}(cx)}{3\pi^{\frac{5}{2}}} + \frac{32b\,x^{10}c^{11}}{3\pi^{\frac{5}{2}}(8c^2x^2+9)(c^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)-4*c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/
Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)))-16/3*b*c/Pi^(5/2)*arcsinh(c*x)+32/3*b/Pi^(5/
```

```
2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^10*c^11-32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2
*x^2+1)*x^8*c^9+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^8*c^9-32*b/P
i^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^6*c^7+64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^
2*x^2+1)^2*x^6*arcsinh(c*x)*c^7-64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^(
3/2)*x^5*arcsinh(c*x)*c^6+64*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^6*c^7
-32*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^4*c^5+200/3*b/Pi^(5/2)/(8*c^2*x^
2+9)/(c^2*x^2+1)^2*x^4*arcsinh(c*x)*c^5-56*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)*x^3*arcsinh(c*x)*c^4+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^
2*x^4*c^5-12*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^2*c^3+208/3*b/Pi^(5/2)/
(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^2*arcsinh(c*x)*c^3-44*b/Pi^(5/2)/(8*c^2*x^2+9
)/(c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^2+32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x
^2+1)^2*x^2*c^3-3/2*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*c+24*b/Pi^(5/2)/(8
*c^2*x^2+9)/(c^2*x^2+1)^2*arcsinh(c*x)*c-9*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)/x*arcsinh(c*x)+b*c/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)+5/3*
b*c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*(4*c^2*x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 8*c^2*x/(pi^2*sqrt(pi + pi*c
^2*x^2)) + 3/(pi*(pi + pi*c^2*x^2)^(3/2)*x)) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^8 + 3*pi^3*
c^4*x^6 + 3*pi^3*c^2*x^4 + pi^3*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^6\sqrt{c^2x^2+1}+2c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^6\sqrt{c^2x^2+1}+2c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**6*sqrt(c**2*x**2 + 1) + 2*c**2*x**4*sqrt(c**2*x**2 + 1
) + x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**6*sqrt(c
**2*x**2 + 1) + 2*c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^2\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

3.109 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=247

$$-\frac{3bc}{4\pi^{5/2}x}+\frac{bc}{4\pi^{5/2}x\left(1+c^2x^2\right)}+\frac{5bc^3x}{12\pi^{5/2}\left(1+c^2x^2\right)}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5c^2\left(a+b\sin\right.}{2\pi^2\sqrt{\pi+}}$$

```
[Out] -3/4*b*c/Pi^(5/2)/x+1/4*b*c/Pi^(5/2)/x/(c^2*x^2+1)+5/12*b*c^3*x/Pi^(5/2)/(c
^2*x^2+1)-5/6*c^2*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)+1/2*(-a-b*arc
sinh(c*x))/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)+13/6*b*c^2*arctan(c*x)/Pi^(5/2)+5*c
^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(5/2)+5/2*b*c^2*pol
ylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(5/2)-5/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)
^(1/2))/Pi^(5/2)-5/2*c^2*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5809, 5811, 5816, 4267, 2317, 2438, 209, 205, 296, 331}

$$-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{5c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}}+\frac{13bc^2\text{ArcTan}(cx)}{6\pi^{5/2}}+\frac{5bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\pi^{5/2}}-\frac{5bc^2\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2\pi^{5/2}}+\frac{bc}{4\pi^{5/2}x\left(c^2x^2+1\right)}+\frac{5bc^3x}{12\pi^{5/2}\left(c^2x^2+1\right)}-\frac{3bc}{4\pi^{5/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-3*b*c)/(4*Pi^(5/2)*x) + (b*c)/(4*Pi^(5/2)*x*(1 + c^2*x^2)) + (5*b*c^3*x)/
(12*Pi^(5/2)*(1 + c^2*x^2)) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*Pi*(Pi + c^2*
Pi*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*(Pi + c^2*Pi*x^2)^(3/2)) -
(5*c^2*(a + b*ArcSinh[c*x]))/(2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (13*b*c^2*Arc
Tan[c*x])/(6*Pi^(5/2)) + (5*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/Pi^(5/2) + (5*b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Pi^(5/2)) - (5*b*c^2
*PolyLog[2, E^ArcSinh[c*x]])/(2*Pi^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2(1+c}}{2\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - $$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A]

time = 4.02, size = 331, normalized size = 1.34

$$\frac{-\frac{8ac^2}{(1+c^2x^2)^{3/2}}+\frac{4bc^3x}{1+c^2x^2}-\frac{48ac^2}{\sqrt{1+c^2x^2}}-\frac{12a\sqrt{1+c^2x^2}}{x^2}-\frac{56bc^2\sinh^{-1}(cx)}{(1+c^2x^2)^{3/2}}-\frac{48bc^4x^2\sinh^{-1}(cx)}{(1+c^2x^2)^{3/2}}+104bc^2\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)-6bc^2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-3bc^2\sinh^{-1}(cx)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)-60bc^2\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+60bc^2\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)-60ac^2\log(x)+60ac^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right)-60bc^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+60bc^2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+6bc^2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\frac{6bc\sinh^{-1}(cx)\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)}{x}}{24\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] ((-8*a*c^2)/(1 + c^2*x^2)^(3/2) + (4*b*c^3*x)/(1 + c^2*x^2) - (48*a*c^2)/Sq
rt[1 + c^2*x^2] - (12*a*Sqrt[1 + c^2*x^2])/x^2 - (56*b*c^2*ArcSinh[c*x])/(1
 + c^2*x^2)^(3/2) - (48*b*c^4*x^2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + 104*b
*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*b*c^2*Coth[ArcSinh[c*x]/2] - 3*b*c^2*
ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*b*c^2*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] + 60*b*c^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*a*c^2*L
og[x] + 60*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] - 60*b*c^2*PolyLog[2, -E^(
-ArcSinh[c*x])] + 60*b*c^2*PolyLog[2, E^(-ArcSinh[c*x])] + 6*b*c^2*Tanh[Arc
Sinh[c*x]/2] - (6*b*c*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2])/x)/(24*Pi^(5/2))
```

Maple [A]
time = 4.68, size = 314, normalized size = 1.27

method	result
default	$a\left(-\frac{1}{2\pi x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{5c^2\left(\frac{1}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right)}{2}\right)-\frac{5b\,x^2\operatorname{arcsinh}(cx}{2\pi^{\frac{5}{2}}(c^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)-5/2*c^2*(1/3/Pi/(Pi*c^2*x^2+Pi)^(3/2)+
1/Pi*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi
)^(1/2)))))-5/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*x^2*arcsinh(c*x)*c^4-1/3*b*c^3
*x/Pi^(5/2)/(c^2*x^2+1)-10/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*c^2-
1/2*b*c/Pi^(5/2)/x/(c^2*x^2+1)-1/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/x^2*arcsinh
(c*x)+13/3*b*c^2/Pi^(5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*
dilog(c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*dilog(1+c*x+(c^2*x^2+1)^(1/
2))+5/2*b*c^2/Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a*(15*c^2*arcsinh(1/(c*abs(x)))/pi^(5/2) - 5*c^2/(pi*(pi + pi*c^2*x^2)^
(3/2)) - 15*c^2/(pi^2*sqrt(pi + pi*c^2*x^2)) - 3/(pi*(pi + pi*c^2*x^2)^(3/2
)*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5/2)
*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^9 + 3*pi^3*
c^4*x^7 + 3*pi^3*c^2*x^5 + pi^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^7\sqrt{c^2x^2+1}+2c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^7\sqrt{c^2x^2+1}+2c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**7*sqrt(c**2*x**2 + 1) + 2*c**2*x**5*sqrt(c**2*x**2 + 1
) + x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**7*sqrt(c
**2*x**2 + 1) + 2*c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

3.110 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$-\frac{bc}{6\pi^{5/2}x^2}+\frac{bc^3}{6\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{16c^4x(\cdots}{3\pi^2\cdots}$$

```
[Out] -1/6*b*c/Pi^(5/2)/x^2+1/6*b*c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x)
)/Pi/x^3/(Pi*c^2*x^2+Pi)^(3/2)+2*c^2*(a+b*arcsinh(c*x))/Pi/x/(Pi*c^2*x^2+Pi
)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)-8/3*b*c^3*ln(
x)/Pi^(5/2)-4/3*b*c^3*ln(c^2*x^2+1)/Pi^(5/2)+16/3*c^4*x*(a+b*arcsinh(c*x))/
Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {277, 198, 197, 5804, 12, 1813, 1634}

$$\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{8bc^3\log(x)}{3\pi^{5/2}}+\frac{bc^3}{6\pi^{5/2}\left(c^2x^2+1\right)}-\frac{4bc^3\log\left(c^2x^2+1\right)}{3\pi^{5/2}}-\frac{bc}{6\pi^{5/2}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c)/(Pi^(5/2)*x^2) + (b*c^3)/(6*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*Arc
Sinh[c*x])/(3*Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x])
)/(Pi*x*(Pi + c^2*Pi*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi
 + c^2*Pi*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c
^2*Pi*x^2]) - (8*b*c^3*Log[x])/(3*Pi^(5/2)) - (4*b*c^3*Log[1 + c^2*x^2])/(3
*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} + \frac{16c^4}{3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} + \frac{16c^4}{3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} + \frac{16c^4}{3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} + \frac{16c^4}{3}$$

$$= -\frac{bc}{6\pi^{5/2}x^2} + \frac{bc^3}{6\pi^{5/2}\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} + \frac{8}{}$$

Mathematica [A]
time = 0.16, size = 142, normalized size = 0.68

$$\frac{-bcx\sqrt{1+c^2x^2}+2a(-1+6c^2x^2+24c^4x^4+16c^6x^6)+2b(-1+6c^2x^2+24c^4x^4+16c^6x^6)\sinh^{-1}(cx)}{6\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}} - \frac{8\left(bc^3\log(x)+\frac{1}{2}bc^3\log\left(1+c^2x^2\right)\right)}{3\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) + 2*a*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6
) + 2*b*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x])/(6*Pi^(5/2
)*x^3*(1 + c^2*x^2)^(3/2)) - (8*(b*c^3*Log[x] + (b*c^3*Log[1 + c^2*x^2])/2)
)/(3*Pi^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(181) = 362$.
time = 4.13, size = 1155, normalized size = 5.55

method	result	size
default	Expression too large to display	1155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/Pi/x^3/(Pi*c^2*x^2+Pi)^(3/2)-2*c^2*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)-4
*c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))))-64
*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^8*arcsinh(c*x)*c^11-2
```

```
*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^2*c^5+1/6*b/Pi^(5/2)/(1
2*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)/x^2*c+128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c
^2*x^2-1)/(c^2*x^2+1)*x^4*c^7+1/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2
*x^2+1)^(3/2)/x^3*arcsinh(c*x)+128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(
c^2*x^2+1)*x^10*c^13+128*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x
^8*c^11+128*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^6*c^9-2*b/Pi
^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*c^3-512/3*b/Pi^(5/2)/(12*c^4*x
^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^6*c^9-192*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^
2-1)/(c^2*x^2+1)^2*x^6*arcsinh(c*x)*c^9-560/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2
*x^2-1)/(c^2*x^2+1)^2*x^4*arcsinh(c*x)*c^7-160/3*b/Pi^(5/2)/(12*c^4*x^4+12*
c^2*x^2-1)/(c^2*x^2+1)^2*x^2*arcsinh(c*x)*c^5+64*b/Pi^(5/2)/(12*c^4*x^4+12*
c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^7*arcsinh(c*x)*c^10+32/3*b*c^3/Pi^(5/2)*arcs
inh(c*x)-8/3*b*c^3/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)+344/3*b/Pi^(5/2
)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^3*arcsinh(c*x)*c^6+12*b/Pi^
(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^4-6*b/Pi
^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/x*arcsinh(c*x)*c^2-512/3
*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^10*c^13-256*b/Pi^(5/2
)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^8*c^11+16/3*b/Pi^(5/2)/(12*c^4*
x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*arcsinh(c*x)*c^3+160*b/Pi^(5/2)/(12*c^4*x^4
+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^5*arcsinh(c*x)*c^8-128/3*b/Pi^(5/2)/(12*
c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^4*c^7-128/3*b/Pi^(5/2)/(12*c^4*x^4+12
*c^2*x^2-1)/(c^2*x^2+1)^2*x^12*c^15
```

Maxima [A]
time = 0.31, size = 236, normalized size = 1.13

$$-\frac{1}{6}\,bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}}+\frac{16\,c^2\log\left(x\right)}{\pi^{\frac{5}{2}}}+\frac{1}{\pi^{\frac{5}{2}}c^2x^4+\pi^{\frac{5}{2}}x^2}\right)+\frac{1}{3}\left(\frac{8\,c^4x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}}+\frac{16\,c^4x}{\pi^2\sqrt{\pi+\pi c^2x^2}}+\frac{6\,c^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x}-\frac{1}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x^3}\right)b\,\mathrm{arsinh}\left(cx\right)+\frac{1}{3}\left(\frac{8\,c^4x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}}+\frac{16\,c^4x}{\pi^2\sqrt{\pi+\pi c^2x^2}}+\frac{6\,c^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x}-\frac{1}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/pi^(5/2) + 16*c^2*log(x)/pi^(5/2) + 1/(pi^
(5/2)*c^2*x^4 + pi^(5/2)*x^2)) + 1/3*(8*c^4*x/(pi*(pi + pi*c^2*x^2)^(3/2))
+ 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(pi*(pi + pi*c^2*x^2)^(3/2)
*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*b*arcsinh(c*x) + 1/3*(8*c^4*x/(pi
*(pi + pi*c^2*x^2)^(3/2)) + 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(
pi*(pi + pi*c^2*x^2)^(3/2)*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^10 + 3*pi^3
*c^4*x^8 + 3*pi^3*c^2*x^6 + pi^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^8\sqrt{c^2x^2+1}+2c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^8\sqrt{c^2x^2+1}+2c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**8*sqrt(c**2*x**2 + 1) + 2*c**2*x**6*sqrt(c**2*x**2 + 1
) + x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**8*sqrt(c
**2*x**2 + 1) + 2*c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

3.111 $\int \frac{\sinh^{-1}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}}+\frac{2}{15ac^3\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}+\frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}}+\frac{4x\sinh^{-1}(ax)}{15c^2\left(c+a^2cx^2\right)^{3/2}}+\dots$$

```
[Out] 1/5*x*arcsinh(a*x)/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)/c^2/(a^2*c*x^2
+c)^(3/2)+1/20/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a
*x)/c^3/(a^2*c*x^2+c)^(1/2)+2/15/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-4/15*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5788, 5787, 266, 267}

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\,\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+\frac{4x\sinh^{-1}(ax)}{15c^2\left(a^2cx^2+c\right)^{3/2}}+\frac{x\sinh^{-1}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
```

```
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)}{5c(c+a^2cx^2)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx}{5c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\sinh^{-1}(ax)}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8\int \frac{\sinh}{(c+a^2}}{15}$$

$$= \frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c(c+a^2cx^2)^{5/2}}$$

$$= \frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c(c+a^2cx^2)^{5/2}}$$

Mathematica [A]
time = 0.12, size = 121, normalized size = 0.60

$$\frac{\sqrt{c+a^2cx^2}\left(4ax\sqrt{1+a^2x^2}(15+20a^2x^2+8a^4x^4)\sinh^{-1}(ax)-(1+a^2x^2)\left(-11-8a^2x^2+16(1+a^2x^2)^2\log(1+a^2x^2)\right)\right)}{60ac^4(1+a^2x^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4)
*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1 +
 a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(170) = 340.
time = 2.54, size = 363, normalized size = 1.82

method	result
default	$\frac{16\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{arcsinh}(ax)}{15\sqrt{a^{2}x^{2}+1}\,a\,c^{4}}+\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\left(8a^{5}x^{5}-8\sqrt{a^{2}x^{2}+1}\,a^{4}x^{4}+20a^{3}x^{3}-16\sqrt{a^{2}x^{2}+1}\,a^{2}x^{2}+15\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(a
^2*x^2+1))^(1/2)*(8*a^5*x^5-8*(a^2*x^2+1)^(1/2)*a^4*x^4+20*a^3*x^3-16*(a^2*
x^2+1)^(1/2)*a^2*x^2+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*a^8*x^8-64*(a^2*x^2+1
)^(1/2)*a^7*x^7-280*a^6*x^6-248*(a^2*x^2+1)^(1/2)*a^5*x^5+160*arcsinh(a*x)*
a^4*x^4-456*a^4*x^4-340*(a^2*x^2+1)^(1/2)*a^3*x^3+380*arcsinh(a*x)*a^2*x^2-
328*a^2*x^2-165*a*x*(a^2*x^2+1)^(1/2)+256*arcsinh(a*x)-88)/(40*a^10*x^10+21
5*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2*x^2+1)
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [A]
time = 0.32, size = 143, normalized size = 0.72

$$\frac{1}{60}\,a\left(\frac{3}{\left(a^{6}c^{\frac{5}{2}}x^{4}+2\,a^{4}c^{\frac{5}{2}}x^{2}+a^{2}c^{\frac{5}{2}}\right)c}+\frac{8}{\left(a^{4}c^{\frac{3}{2}}x^{2}+a^{2}c^{\frac{3}{2}}\right)c^{2}}-\frac{16\,\log\left(x^{2}+\frac{1}{a^{2}}\right)}{a^{2}c^{\frac{7}{2}}}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{a^{2}cx^{2}+c}\,c^{3}}+\frac{4\,x}{(a^{2}cx^{2}+c)^{\frac{3}{2}}c^{2}}+\frac{3\,x}{(a^{2}cx^{2}+c)^{\frac{5}{2}}c}\right)\operatorname{arsinh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/60*a*(3/((a^6*c^(5/2)*x^4 + 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) + 8/((a^4
*c^(3/2)*x^2 + a^2*c^(3/2))*c^2) - 16*log(x^2 + 1/a^2)/(a^2*c^(7/2))) + 1/1
5*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((
a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6*
a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(ax\right)}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)/(c*(a**2*x**2 + 1))**(7/2), x)
```

Giac [A]
time = 0.44, size = 124, normalized size = 0.62

$$-\frac{1}{60}\sqrt{c}\left(\frac{16\log\left(a^2x^2+1\right)}{ac^4}-\frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right)+\frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35)
/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x*l
og(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{asinh}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2), x)
```

3.112 $\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=86

$$\frac{3x^2}{16a^3}-\frac{x^4}{16a}-\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{16a^5}$$

```
[Out] 3/16*x^2/a^3-1/16*x^4/a+3/16*arcsinh(a*x)^2/a^5-3/8*x*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5812, 5783, 30}

$$\frac{3\sinh^{-1}(ax)^2}{16a^5}+\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{8a^4}-\frac{x^4}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\,dx}{4a}$$
$$= -\frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} + \frac{3\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^4}$$
$$= \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} + \frac{3\sinh^{-1}(ax)^2}{16a^5}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.73

$$\frac{3a^2x^2 - a^4x^4 + 2ax\sqrt{1+a^2x^2}\,(-3+2a^2x^2)\sinh^{-1}(ax) + 3\sinh^{-1}(ax)^2}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x
] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

Maple [A]
time = 2.25, size = 66, normalized size = 0.77

$$-\frac{-4\,\mathrm{arcsinh}\,(ax)\cosh\,(2\,\mathrm{arcsinh}\,(ax))\sinh\,(2\,\mathrm{arcsinh}\,(ax)) + \cosh^2\,(2\,\mathrm{arcsinh}\,(ax)) + 16\,\mathrm{arcsinh}\,(ax)\sinh\,(2}{64a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/64*(-4*arcsinh(a*x)*cosh(2*arcsinh(a*x))*sinh(2*arcsinh(a*x))+cosh(2*arc
sinh(a*x))^2+16*arcsinh(a*x)*sinh(2*arcsinh(a*x))-12*arcsinh(a*x)^2-8*cosh(
2*arcsinh(a*x)))/a^5
```

Maxima [A]
time = 0.29, size = 83, normalized size = 0.97

$$-\frac{1}{16}\left(\frac{x^4}{a^2} - \frac{3\,x^2}{a^4} + \frac{3\,\mathrm{arsinh}\,(ax)^2}{a^6}\right)a + \frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}\,x^3}{a^2} - \frac{3\,\sqrt{a^2x^2+1}\,x}{a^4} + \frac{3\,\mathrm{arsinh}\,(ax)}{a^5}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a*x)^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2
+ 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*arcsinh(a*x)
```

Fricas [A]
time = 0.35, size = 83, normalized size = 0.97

$$-\frac{a^4x^4 - 3\,a^2x^2 - 2\,(2\,a^3x^3 - 3\,ax)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right) - 3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

Sympy [A]
time = 0.46, size = 82, normalized size = 0.95

$$\begin{cases} -\frac{x^4}{16a} + \frac{x^3\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{4a^2} + \frac{3x^2}{16a^3} - \frac{3x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{8a^4} + \frac{3\operatorname{asinh}^2(ax)}{16a^5} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3*
x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*x)
**2/(16*a**5), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.113 $\int \frac{x^3 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=70

$$\frac{2x}{3a^3}-\frac{x^3}{9a}-\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2}$$

```
[Out] 2/3*x/a^3-1/9*x^3/a-2/3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(
a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5812, 5798, 8, 30}

$$\frac{2x}{3a^3}+\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^4}-\frac{x^3}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
```

```
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2}-\frac{2\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a^2}-\frac{\int x^2\,dx}{3a}\\ &= -\frac{x^3}{9a}-\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2}+\frac{2\int 1\,dx}{3a^3}\\ &= \frac{2x}{3a^3}-\frac{x^3}{9a}-\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.69

$$\frac{6ax-a^3x^3+3(-2+a^2x^2)\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4)
```

Maple [A]
time = 3.00, size = 50, normalized size = 0.71

$$\frac{-27\,\mathrm{arcsinh}\,(ax)\sqrt{a^2x^2+1}+3\,\mathrm{arcsinh}\,(ax)\cosh\left(3\,\mathrm{arcsinh}\,(ax)\right)+27ax-\sinh\left(3\,\mathrm{arcsinh}\,(ax)\right)}{36a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] 1/36*(-27*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+3*arcsinh(a*x)*cosh(3*arcsinh(a*x)
)+27*a*x-sinh(3*arcsinh(a*x)))/a^4
```

Maxima [A]
time = 0.29, size = 59, normalized size = 0.84

$$-\frac{1}{9}\,a\left(\frac{x^3}{a^2}-\frac{6\,x}{a^4}\right)+\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2}-\frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^
2 + 1)/a^4)*arcsinh(a*x)
```

Fricas [A]
time = 0.40, size = 55, normalized size = 0.79

$$-\frac{a^3x^3 - 3\sqrt{a^2x^2+1}\,(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right) - 6\,ax}{9\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
1)) - 6*a*x)/a^4
```

Sympy [A]
time = 0.33, size = 65, normalized size = 0.93

$$\begin{cases} -\frac{x^3}{9a} + \frac{x^2\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{3a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*x
/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, True)
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.114 $\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{x^2}{4a}+\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2}-\frac{\sinh^{-1}(ax)^2}{4a^3}$$

```
[Out] -1/4*x^2/a-1/4*arcsinh(a*x)^2/a^3+1/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5812, 5783, 30}

$$-\frac{\sinh^{-1}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a^2}-\frac{x^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2} - \frac{\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{\int x\,dx}{2a}$$
$$= -\frac{x^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2} - \frac{\sinh^{-1}(ax)^2}{4a^3}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.86

$$-\frac{a^2x^2 - 2ax\sqrt{1+a^2x^2}\,\sinh^{-1}(ax) + \sinh^{-1}(ax)^2}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] -1/4*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/a^3
```

Maple [A]
time = 3.13, size = 40, normalized size = 0.82

method	result	size
default	$-\frac{-2ax\,\mathrm{arcsinh}(ax)\sqrt{a^2x^2+1}+a^2x^2+\mathrm{arcsinh}(ax)^2+1}{4a^3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

Maxima [A]
time = 0.29, size = 55, normalized size = 1.12

$$-\frac{1}{4}\,a\left(\frac{x^2}{a^2} - \frac{\mathrm{arsinh}\,(ax)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}\,x}{a^2} - \frac{\mathrm{arsinh}\,(ax)}{a^3}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*a*(x^2/a^2 - arcsinh(a*x)^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*x/a^2 - arcs
inh(a*x)/a^3)*arcsinh(a*x)
```

Fricas [A]
time = 0.39, size = 62, normalized size = 1.27

$$-\frac{a^{2}x^{2}-2\sqrt{a^{2}x^{2}+1}\,ax\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)+\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}}{4\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log(
a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A]
time = 0.27, size = 42, normalized size = 0.86

$$\begin{cases} -\frac{x^2}{4a}+\frac{x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{2a^2}-\frac{\operatorname{asinh}^2(ax)}{4a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh(
a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.115 $\int \frac{x \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}$$

```
[Out] -x/a+arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5798, 8}

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}-\frac{\int 1\,dx}{a}\\ &= -\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$-\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Maple [A]
time = 2.42, size = 47, normalized size = 1.68

method	result	size
default	$\frac{\operatorname{arcsinh}(ax)a^2x^2+\operatorname{arcsinh}(ax)-ax\sqrt{a^2x^2+1}}{a^2\sqrt{a^2x^2+1}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)*a^2*x^2+arcsinh(a*x)-a*x*(a^2*x^2+1)^
(1/2))
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.93

$$-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

Fricas [A]
time = 0.38, size = 38, normalized size = 1.36

$$-\frac{ax-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A]
time = 0.21, size = 24, normalized size = 0.86

$$\begin{cases} -\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True)
)
```

Giac [**A**]
time = 0.40, size = 38, normalized size = 1.36

$$-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.116 $\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Rubi [A]

time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{5783\}$

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^2}{2a}$$

Mathematica [A]

time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Maple [A]
time = 0.25, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^2}{2a}$	12
default	$\frac{\operatorname{arcsinh}(ax)^2}{2a}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Maxima [A]
time = 0.30, size = 11, normalized size = 0.85

$$\frac{\operatorname{arsinh}(ax)^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.
time = 0.37, size = 23, normalized size = 1.77

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

Sympy [A]
time = 0.18, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [**B**]
time = 0.11, size = 11, normalized size = 0.85

$$\frac{\mathrm{asinh}(a\,x)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(a^2*x^2 + 1)^(1/2),x)
[Out] asinh(a*x)^2/(2*a)
```

3.117 $\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=34

$$-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(
1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A]
time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5816, 4267, 2317, 2438}

$$-\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
```

```
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx &= \text{Subst}\left(\int x\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + \text{Sub}\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(ax)}\right) + \text{Subst}\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + \text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 1.68

$$\sinh^{-1}(ax)\left(\log\left(1-e^{-\sinh^{-1}(ax)}\right)-\log\left(1+e^{-\sinh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + Po
lyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

Maple [A]
time = 2.24, size = 42, normalized size = 1.24

method	result	size
default	$2\,\text{dilog}\left(\frac{1}{ax+\sqrt{a^2x^2+1}}\right)-\frac{\text{dilog}\left(\frac{1}{\left(ax+\sqrt{a^2x^2+1}\right)^2}\right)}{2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*dilog(1/(a*x+(a^2*x^2+1)^(1/2)))-1/2*dilog(1/(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}(a\,x)}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.118 $\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\log(x)$$

```
[Out] a*ln(x)-arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5800, 29}

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\int \frac{1}{x}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\log(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.07

$$-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{x}+a\log(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.
time = 3.83, size = 56, normalized size = 2.07

method	result	size
default	$-2a\,\mathrm{arcsinh}\left(ax\right)+\frac{\left(ax-\sqrt{a^{2}x^{2}+1}\right)\mathrm{arcsinh}\left(ax\right)}{x}+a\ln\left(\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}-1\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^2
+1)^(1/2))^2-1)
```

Maxima [A]
time = 0.30, size = 25, normalized size = 0.93

$$a\log\left(x\right)-\frac{\sqrt{a^{2}x^{2}+1}\,\mathrm{arsinh}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

Fricas [A]
time = 0.37, size = 39, normalized size = 1.44

$$\frac{ax\log\left(x\right)-\sqrt{a^{2}x^{2}+1}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.
time = 0.40, size = 71, normalized size = 2.63

$$-a\log\left(-x|a|+\sqrt{a^2x^2+1}\right)+a\log\left(|x|\right)+\frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) + a*log(abs(x)) + 2*abs(a)*log(a*x +
sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{asinh}(a\,x)}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.119 $\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{PolyLo}$$

```
[Out] -1/2*a/x+a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,
-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsin
h(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5809, 5816, 4267, 2317, 2438, 30}

$$\frac{1}{2}a^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -1/2*a/x - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int\frac{1}{x^2}\,dx-\frac{1}{2}a^2\int\frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}-\frac{1}{2}a^2\text{Subst}\left(\int x\text{csch}(x)\,dx,x,\sinh^{-1}(ax)\right)\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{Subst}\Bigg(\int\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{Subst}\Bigg(\int\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{Li}_2\Big(-e^{\sin}
\end{aligned}
$$

Mathematica [A]
time = 0.46, size = 126, normalized size = 1.58

$$\frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(ax)\right)-\sinh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(ax)\right)-4\sinh^{-1}(ax)\log\left(1-e^{-\sinh^{-1}(ax)}\right)+4\sinh^{-1}(ax)\log\left(1+e^{-\sinh^{-1}(ax)}\right)-4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)+4\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)-\sinh^{-1}(ax)\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(ax)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

[Out] (a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*ArcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x])] - ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8

Maple [A]
time = 4.94, size = 150, normalized size = 1.88

method	result
default	$-\frac{\operatorname{arcsinh}(ax)a^2x^2+ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)}{2\sqrt{a^2x^2+1}\,x^2}-\frac{a^2\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)}{2}-\frac{a^2\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)*a^2*x^2+a*x*(a^2*x^2+1)^(1/2)+arcsinh(a*x))/x^2-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}{\left(ax \right)}}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{asinh}(a\,x)}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

3.120 $\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=175

$$\frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}+\frac{(d+c^2dx^2)^{5/2}\,(a}{5c^4}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^4/d+1/5*(c^2*d*x^2+d)^(5/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/15*b*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2
)-1/45*b*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/25*b*c*x^5*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45, 5804, 12}

$$\frac{(c^2dx^2+d)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d^2}-\frac{(c^2dx^2+d)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bcx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{bx^3\sqrt{c^2dx^2+d}}{45c\sqrt{c^2x^2+1}}+\frac{2bx\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*x*Sqrt[d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*x^3*Sqrt[d + c^
2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (b*c*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt
[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*d) + (
(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx = -\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-2+c^2x^2+3c^4x^4}{15c^4}\,dx}{\sqrt{1+c^2x^2}}+\left(a+b\sinh^{-1}(cx)\right)\int x^3$$

$$= -\frac{\left(b\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+3c^4x^4\right)\,dx}{15c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\sinh^{-1}(c\right.$$

$$= \frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a\right.$$

$$= \frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-\frac{(d+}{}$$

Mathematica [A]
time = 0.07, size = 120, normalized size = 0.69

$$\frac{\sqrt{d+c^2dx^2}\left(15a(1+c^2x^2)^2(-2+3c^2x^2)+bcx\sqrt{1+c^2x^2}\,(30-5c^2x^2-9c^4x^4)+15b(1+c^2x^2)^2(-2+3c^2x^2)\sinh^{-1}(cx)\right)}{225c^4\,(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) + b*c*x*Sqrt[1
+ c^2*x^2]*(30 - 5*c^2*x^2 - 9*c^4*x^4) + 15*b*(1 + c^2*x^2)^2*(-2 + 3*c^2*
x^2)*ArcSinh[c*x]))/(225*c^4*(1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(149) = 298$.
time = 1.80, size = 578, normalized size = 3.30

method	result

default	$a\left(\frac{x^{2}\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{5c^{2} d}-\frac{2\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{15 d\,c^{4}}\right)+b\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(16 x^{6} c^{6}+16\sqrt{c^{2} x^{2}+1}\,x^{5} c^{5}+28 c^{4} x^{4}+20\sqrt{c^{2} x^{2}+1}\right.}{800 c^{4}\left(c^{2} x^{2}+1\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] a*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*(1+5*arcsinh(c*x))/c^4/(c^2*x^2+1))

Maxima [A]
time = 0.28, size = 134, normalized size = 0.77

$$\frac{1}{15}\,b\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}(cx)+\frac{1}{15}\,a\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)-\frac{\left(9\,c^4\sqrt{d}\,x^5+5\,c^2\sqrt{d}\,x^3-30\,\sqrt{d}\,x\right)b}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/15*b*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4*d))*arcsinh(c*x) + 1/15*a*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4*d)) - 1/225*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x^3 - 30*sqrt(d)*x)*b/c^3

Fricas [A]
time = 0.36, size = 158, normalized size = 0.90

$$\frac{15\,(3\,bc^6x^6+4\,bc^4x^4-bc^2x^2-2\,b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6+60\,ac^4x^4-15\,ac^2x^2-(9\,bc^5x^5+5\,bc^3x^3-30\,bcx)\sqrt{c^2x^2+1}\,-30\,a\right)\sqrt{c^2dx^2+d}}{225\,(c^6x^2+c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")

[Out] 1/225*(15*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 + 60*a*c^4*x^4 - 15*a*c^2*x^2

```
 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*x^2 + 1) - 30*a)*sqrt(c^
2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.121 $\int x^2 \sqrt{d + c^2 d x^2} \left(a + b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=181

$$-\frac{bx^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{8c^2}+\frac{1}{4}x^3\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))$$

```
[Out] 1/8*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)-1/16*b*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/1
6*b*c*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*(a+b*arcsinh(c*x))^2*(
c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5806, 5812, 5783, 30}

$$\frac{x\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{8c^2}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))-\frac{\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bx^2\sqrt{c^2dx^2+d}}{16c\sqrt{c^2x^2+1}}-\frac{bcx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/16*(b*x^2*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (b*c*x^4*Sqrt[d +
 c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 - (Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
```

```
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{4}x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\int\frac{x^2(a+b\sinh^{-1}(}{\sqrt{1+c^2x}}}{4\sqrt{1+c^2x^2}}$$

$$=-\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{d}$$

$$=-\frac{bx^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sin\right.}{8c^2}$$

Mathematica [A]
time = 0.54, size = 129, normalized size = 0.71

$$-\frac{-16acx(1+2c^2x^2)\sqrt{d+c^2dx^2}+16a\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{b\sqrt{d+c^2dx^2}\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)}{\sqrt{1+c^2x^2}}}{128c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/128*(-16*a*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 16*a*Sqrt[d]*Log[c*
d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]
^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 +
c^2*x^2])/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(155) = 310$.
time = 2.39, size = 338, normalized size = 1.87

method	result
default	$\frac{ax(c^2d\,x^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{ax\sqrt{c^2d\,x^2+d}}{8c^2}-\frac{ad\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{16\sqrt{c^2x^2+1}\,c^3}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a/c^2
*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/16*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/256*(d*(c^2*x^2+1
))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x
^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^3/(c^2*x^2+1)+1/
256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3
-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))/c^
3/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{d\,(c^2x^2+1)}\,(a+b\operatorname{asinh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.122 $\int x\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=105

$$-\frac{bx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^2/d-1/3*b*x*(c^2*d*x^2+d)^(1/2
)/c/(c^2*x^2+1)^(1/2)-1/9*b*c*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5798}

$$\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{bx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}}-\frac{bcx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/3*(b*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (b*c*x^3*Sqrt[d + c^
2*d*x^2])/(9*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x
]))/(3*c^2*d)
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)\,dx &= \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{\left(b\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)\,d}{3c\sqrt{1+c^2x^2}}\\ &=-\frac{bx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-}\right.}{3c^2d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 0.88

$$\frac{\sqrt{d+c^2dx^2}\,\left(3a\left(1+c^2x^2\right)^2-bcx\sqrt{1+c^2x^2}\,\left(3+c^2x^2\right)+3b\left(1+c^2x^2\right)^2\sinh^{-1}(cx)\right)}{9c^2\left(1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^
2*x^2) + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]))/(9*c^2*(1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(89) = 178$.
time = 0.76, size = 321, normalized size = 3.06

method	result
default	$\frac{a(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\left(4c^4x^4+4\sqrt{c^2x^2+1}\,x^3c^3+5c^2x^2+3\sqrt{c^2x^2+1}\,cx+1\right)(-1+3\operatorname{arcsinh}(cx))}{72c^2(c^2x^2+1)}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a/c^2/d*(c^2*d*x^2+d)^(3/2)+b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*
(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsin
h(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2
)*c*x+1)*(arcsinh(c*x)-1)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^
2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/72*(d*(c^2*x^
2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^
(1/2)*c*x+1)*(1+3*arcsinh(c*x))/c^2/(c^2*x^2+1))
```

Maxima [A]
time = 0.31, size = 73, normalized size = 0.70

$$\frac{(c^2dx^2+d)^{\frac{3}{2}}b\operatorname{arsinh}(cx)}{3\,c^2d}-\frac{\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)b}{9\,cd}+\frac{(c^2dx^2+d)^{\frac{3}{2}}a}{3\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(c^2*d) - 1/9*(c^2*d^(3/2)*x^3 + 3
*d^(3/2)*x)*b/(c*d) + 1/3*(c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

Fricas [A]
time = 0.38, size = 127, normalized size = 1.21

$$\frac{3\,(bc^4x^4+2\,bc^2x^2+b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3\,ac^4x^4+6\,ac^2x^2-(bc^3x^3+3\,bcx)\sqrt{c^2x^2+1}+3\,a\right)\sqrt{c^2dx^2+d}}{9\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + (3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^
2 + 1) + 3*a)*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.123 $\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=111

$$-\frac{bcx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/4*b*c*x^2*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)+1/4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x
^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5785, 5783, 30}

$$\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/4*(b*c*x^2*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(
4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2d}}{}$$

Mathematica [A]
time = 0.31, size = 120, normalized size = 1.08

$$\frac{1}{8}\left(4ax\sqrt{d+c^2dx^2}+\frac{4a\sqrt{d}\,\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)}{c}+\frac{b\sqrt{d+c^2dx^2}\left(-\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{c\sqrt{1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (4*a*x*Sqrt[d + c^2*d*x^2] + (4*a*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/c + (b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]
*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2]))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(95) = 190$.
time = 1.52, size = 256, normalized size = 2.31

method	result
default	$\frac{ax\sqrt{c^2d\,x^2+d}}{2}+\frac{ad\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d\left(c^2x^2+1\right)}\left(2\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x*(c^2*d*x^2+d)^(1/2)+1/2*a*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+b*(1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsi
nh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)
+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/16*(d*(c^2*x^
2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))
*(1+2*arcsinh(c*x))/c/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\operatorname{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.124 $\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=177

$$-\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))-\frac{2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{1+c^2x^2}}-\frac{b}{}$$

```
[Out] (a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-b*c*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5806, 5816, 4267, 2317, 2438, 8}

$$\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))-\frac{2\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -((b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^Ar
cSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSi
nh[c*x]])/Sqrt[1 + c^2*x^2] + (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c
*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\int\frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} -$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}}{}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{d+c^2dx}}{}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{d+c^2dx}}{}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{d+c^2dx}}{}$$

Mathematica [A]

time = 0.32, size = 168, normalized size = 0.95

$$a\sqrt{d+c^2dx^2}+a\sqrt{d}\log(x)-a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{b\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\sinh^{-1}(cx)+\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] a*Sqrt[d + c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-
ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*
x])]))/Sqrt[1 + c^2*x^2]
```

Maple [A]
time = 1.81, size = 331, normalized size = 1.87

method	result
default	$-\sqrt{d}\ \ln\left(\frac{2d+2\sqrt{d}\ \sqrt{c^2 d\,x^2+d}}{x}\right) a+a\sqrt{c^2 d\,x^2+d}\ +\frac{b\sqrt{d\left(c^2 x^2+1\right)}\ \operatorname{arcsinh}(cx) x^2 c^2}{c^2 x^2+1}-\frac{b\sqrt{d\,(c^2x}}{\sqrt{c^2x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a+a*(c^2*d*x^2+d)^(1/2)+
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*x*c+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x
)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+
1)^(1/2))+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+
1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(
c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-
(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d))*a + b*integrate(sqrt
(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x, x)
```

3.125 $\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x}+\frac{c\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))^2}{2b\sqrt{1+c^2x^2}}+\frac{bc\sqrt{d+c^2dx^2}\,\log(x)}{\sqrt{1+c^2x^2}}$$

```
[Out] -(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x+1/2*c*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5805, 29, 5783}

$$\frac{c\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Lo
g[x])/Sqrt[1 + c^2*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
```

```
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x^2}\,dx = -\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x} + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1}{x}\,dx}{\sqrt{1+c^2x^2}} + \frac{(c$$

$$= -\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x} + \frac{c\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx)}{2b\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.21, size = 129, normalized size = 1.23

$$-\frac{a\sqrt{d(1+c^2x^2)}}{x} + \frac{bc\sqrt{d(1+c^2x^2)}\left(-\frac{2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{cx} + \sinh^{-1}(cx)^2 + 2\log(cx)\right)}{2\sqrt{1+c^2x^2}} + ac\sqrt{d}\log\left(cdx + \sqrt{d}\sqrt{d(1+c^2x^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
[Out] -((a*Sqrt[d*(1 + c^2*x^2)])/x) + (b*c*Sqrt[d*(1 + c^2*x^2)]*((-2*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x])/(c*x) + ArcSinh[c*x]^2 + 2*Log[c*x]))/(2*Sqrt[1 + c^2
*x^2]) + a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(93) = 186$.
time = 2.20, size = 263, normalized size = 2.50

method	result
default	$-\frac{a(c^2dx^2+d)^{\frac{3}{2}}}{dx} + ac^2x\sqrt{c^2dx^2+d} + \frac{ac^2d\ln\left(\frac{xc^2d}{\sqrt{c^2d}} + \sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}} + \frac{b\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2c}{2\sqrt{c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -a/d/x*(c^2*d*x^2+d)^(3/2)+a*c^2*x*(c^2*d*x^2+d)^(1/2)+a*c^2*d*ln(x*c^2*d/(
c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*c-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*x/(c^2*x^2+1)*c^2-b*
(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/x/(c^2*x^2+1)+b*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(d)*arcsinh(c*x) - sqrt(c^2*d*x^2 + d)/x)*a + b*integrate(sqrt(c^2*d
*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^2,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

3.126 $\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=201

$$-\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{1+c^2x^2}}-b \cdots$$

```
[Out] -1/2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2-1/2*b*c*(c^2*d*x^2+d)^(1/2)
/x/(c^2*x^2+1)^(1/2)-c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*b*c^2*polylog(2,c*x+(c^2*x
^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5805, 30, 5816, 4267, 2317, 2438}

$$-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{bc^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Sqrt[d + c^2*d*x^2])/(x*Sqrt[1 + c^2*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*c^2*Sqrt[d + c^2*d*x^2]
*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (b*c^2*Sqrt[d + c^2*d
*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1}{x^2}\,dx}{2\sqrt{1+c^2x^2}}+\frac{\left(c\right.}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{\left(c^2\sqrt{d+c^2d\right.}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{d+c^2dx}}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{d+c^2dx}}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{d+c^2dx}}{}$$

Mathematica [A]

time = 1.98, size = 223, normalized size = 1.11

$$\frac{1}{8}\left(-\frac{4a\sqrt{d+c^2dx^2}}{x^2}+4ac^2\sqrt{d}\log(x)-4ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bc^2\sqrt{d+c^2dx^2}\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\operatorname{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{\sqrt{1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2])/x^2 + 4*a*c^2*Sqrt[d]*Log[x] - 4*a*c^2*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*Sqrt[d + c^2*d*x^2]*(-2*Coth[
ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[
1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*Poly
Log[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]
*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

Maple [A]
time = 3.43, size = 377, normalized size = 1.88

method	result
default	$-\frac{a(c^2dx^2+d)^{\frac{3}{2}}}{2dx^2}-\frac{a\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)c^2}{2}+\frac{a\sqrt{c^2dx^2+d}\,c^2}{2}-\frac{b\sqrt{d(c^2x^2+1)}\operatorname{arcsinh}(cx)c^2}{2(c^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(3/2)-1/2*a*d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2
+d)^(1/2))/x)*c^2+1/2*a*(c^2*d*x^2+d)^(1/2)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/x/(c^2*x^2+1)^(1/
2)*c-1/2*b*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)*arcsinh(c*x)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c
^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^
(1/2))*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*po
lylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d)*c^2 + (c^2*d*
x^2 + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**3,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^3,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^3, x)
```

3.127 $\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=106

$$-\frac{bc\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{bc^3\sqrt{d+c^2dx^2}\,\log(x)}{3\sqrt{1+c^2x^2}}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/d/x^3-1/6*b*c*(c^2*d*x^2+d)^(1/
2)/x^2/(c^2*x^2+1)^(1/2)+1/3*b*c^3*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5800, 14}

$$-\frac{(c^2dx^2+d)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}}+\frac{bc^3\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[d + c^2*d*x^2])/(x^2*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (b*c^3*Sqrt[d + c^2*d*x^2]*Log[x])/(
3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x^4}\,dx = -\frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{3dx^3} + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{1+c^2x^2}{x^3}\,dx}{3\sqrt{1+c^2x^2}}$$
$$= -\frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{3dx^3} + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \left(\frac{1}{x^3}+\frac{c^2}{x}\right)}{3\sqrt{1+c^2x^2}}$$
$$= -\frac{bc\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{3dx^3} + \frac{bc^3\sqrt{d+c}}{3\sqrt{1}}$$

Mathematica [**A**]
time = 0.11, size = 131, normalized size = 1.24

$$-\frac{\sqrt{d+c^2dx^2}\left(2a(1+c^2x^2)^2+bcx\sqrt{1+c^2x^2}\,(1+3c^2x^2)+2b(1+c^2x^2)^2\sinh^{-1}(cx)\right)}{6x^3\,(1+c^2x^2)} + \frac{bc^3\sqrt{d\,(1+c^2x^2)}\,\log(x)}{3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(2*a*(1 + c^2*x^2)^2 + b*c*x*Sqrt[1 + c^2*x^2]*(1
 + 3*c^2*x^2) + 2*b*(1 + c^2*x^2)^2*ArcSinh[c*x]))/(x^3*(1 + c^2*x^2)) + (b
*c^3*Sqrt[d*(1 + c^2*x^2)]*Log[x])/(3*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. 2(90) = 180.
time = 3.66, size = 946, normalized size = 8.92

method	result
default	$-\frac{a(c^2dx^2+d)^{\frac{3}{2}}}{3dx^3} - \frac{2b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)c^3}{3\sqrt{c^2x^2+1}} - \frac{b\sqrt{d\,(c^2x^2+1)}\,x^5\operatorname{arcsinh}(cx)c^8}{(3c^4x^4+3c^2x^2+1)(c^2x^2+1)} + \frac{b\sqrt{d\,(c^2x^2+1)}\,x^4}{(3c^4x^4+3c^2x^2+1)\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(3/2)-2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*c^3-b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c
^2*x^2+1)*arcsinh(c*x)*c^8+b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*
x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+
3*c^2*x^2+1)*x^3*c^6-3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/
(c^2*x^2+1)*arcsinh(c*x)*c^6+b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1
)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
```

```
*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+3*c^2*x^2+1)*x*c^4-10/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1
)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c
^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-5/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)/
(3*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-1/3*b*(d*(c^2*x^2+1))^(1/2)
/(3*c^4*x^4+3*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+1/3*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

Maxima [A]
time = 0.27, size = 133, normalized size = 1.25

$$-\frac{\left(\left(-1\right)^{2\,c^{2}dx^{2}+2\,d}c^{2}d^{\frac{3}{2}}\log\left(2\,c^{2}d+\frac{2\,d}{x^{2}}\right)-c^{2}d^{\frac{3}{2}}\log\left(x^{2}+\frac{1}{c^{2}}\right)+\frac{\sqrt{c^{4}dx^{4}+2\,c^{2}dx^{2}+d}\,d}{x^{2}}\right)bc}{6\,d}-\frac{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}b\,\mathrm{arsinh}\left(cx\right)}{3\,dx^{3}}-\frac{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}a}{3\,dx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((-1)^(2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(2*c^2*d + 2*d/x^2) - c^2*d^(
3/2)*log(x^2 + 1/c^2) + sqrt(c^4*d*x^4 + 2*c^2*d*x^2 + d)*d/x^2)*b*c/d - 1/
3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(d*x^3) - 1/3*(c^2*d*x^2 + d)^(3/2)*
a/(d*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(90) = 180$.
time = 0.47, size = 217, normalized size = 2.05

$$-\frac{2\left(bc^{4}x^{4}+2\,bc^{2}x^{2}+b\right)\sqrt{c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)-\left(bc^{5}x^{5}+bc^{3}x^{3}\right)\sqrt{d}\,\log\left(\frac{c^{2}dx^{6}+c^{2}dx^{2}+dx^{4}+\sqrt{c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}+1}\,\left(x^{4}-1\right)\sqrt{d}+d}{c^{2}x^{4}+x^{2}}\right)+\left(2\,ac^{4}x^{4}+4\,ac^{2}x^{2}-\left(bcx^{3}-bcx\right)\sqrt{c^{2}x^{2}+1}+2\,a\right)\sqrt{c^{2}dx^{2}+d}}{6\left(c^{2}x^{5}+x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) - (b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 +
d*x^4 + sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x
^4 + x^2)) + (2*a*c^4*x^4 + 4*a*c^2*x^2 - (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 +
1) + 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*x^5 + x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^4, x)
```

3.128 $\int x^3(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=217

$$\frac{2bdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{bdx^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}}-\frac{8bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}}-\frac{bc^3dx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}\right.}{5c^4d}$$

```
[Out] -1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^4/d+1/7*(c^2*d*x^2+d)^(7/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/35*b*d*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1
/2)-1/105*b*d*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-8/175*b*c*d*x^5*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^3*d*x^7*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {272, 45, 5804, 12, 380}

$$\frac{(c^2dx^2+d)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d^2}-\frac{(c^2dx^2+d)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d}-\frac{8bcdx^5\sqrt{c^2dx^2+d}}{175\sqrt{c^2x^2+1}}-\frac{bdx^3\sqrt{c^2dx^2+d}}{105c\sqrt{c^2x^2+1}}+\frac{2bdx\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}-\frac{bc^3dx^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d*x*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^3*Sqrt[d
+ c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2]) - (8*b*c*d*x^5*Sqrt[d + c^2*d*x^2])
/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c
^2*x^2]) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d) + ((d + c
^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx &= -\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\frac{\left(1+c^2x^2\right)^2\left(-2+5c^2x^2\right)}{35c^4}\,dx}{\sqrt{1+c^2x^2}}+\left(a+b\sinh^{-1}(c\right.\\ &= -\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^2\left(-2+5c^2x^2\right)\,dx}{35c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\,\mathrm{s}\right.\\ &= -\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+8c^4x^4+5c^6x^6\right)\,dx}{35c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a\right.\\ &= \frac{2bdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{bdx^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}}-\frac{8bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 130, normalized size = 0.60

$$\frac{d\sqrt{d+c^2dx^2}\left(105a(1+c^2x^2)^3\left(-2+5c^2x^2\right)-bcx\sqrt{1+c^2x^2}\left(-210+35c^2x^2+168c^4x^4+75c^6x^6\right)+105b(1+c^2x^2)^3\left(-2+5c^2x^2\right)\sinh^{-1}(cx)\right)}{3675c^4\left(1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - b*c*x*Sqrt
[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c
^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]))/(3675*c^4*(1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(185) = 370$.
time = 2.05, size = 872, normalized size = 4.02

method	result
default	$a\left(\frac{x^2(c^2dx^2+d)^{\frac{5}{2}}}{7c^2d} - \frac{2(c^2dx^2+d)^{\frac{5}{2}}}{35dc^4}\right) + b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(64x^8c^8+64\sqrt{c^2x^2+1}\,x^7c^7+144x^6c^6+112\sqrt{c^2x^2+1}\right.}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b*(1/6
272*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*
c^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+
25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c
^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(
-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4
+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arc
sinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+
1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1
/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/
384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-
3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*(
c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c
^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*(1+5*arcsinh(
c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8-64*(c^2*x^
2+1)^(1/2)*x^7*c^7+144*x^6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4-56
*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x+1)*(1+7*arcsi
nh(c*x))*d/c^4/(c^2*x^2+1))
```

Maxima [A]
time = 0.30, size = 145, normalized size = 0.67

$$\frac{1}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d} - \frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)b\,\mathrm{arsinh}\,(cx) + \frac{1}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d} - \frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)a - \frac{\left(75\,c^6d^{\frac{3}{2}}x^7 + 168\,c^4d^{\frac{3}{2}}x^5 + 35\,c^2d^{\frac{3}{2}}x^3 - 210\,d^{\frac{3}{2}}x\right)b}{3675\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d)
)*b*arcsinh(c*x) + 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2
 + d)^(5/2)/(c^4*d))*a - 1/3675*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/2)*x^5 +
 35*c^2*d^(3/2)*x^3 - 210*d^(3/2)*x)*b/c^3
```

Fricas [A]
time = 0.37, size = 199, normalized size = 0.92

$$\frac{105\left(5\,bc^{8}dx^{8}+13\,bc^{6}dx^{6}+9\,bc^{4}dx^{4}-bc^{2}dx^{2}-2\,bd\right)\sqrt{c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(525\,ac^{8}dx^{8}+1365\,ac^{6}dx^{6}+945\,ac^{4}dx^{4}-105\,ac^{2}dx^{2}-210\,ad-\left(75\,bc^{7}dx^{7}+168\,bc^{5}dx^{5}+35\,bc^{3}dx^{3}-210\,bcdx\right)\sqrt{c^{2}x^{2}+1}\right)\sqrt{c^{2}dx^{2}+d}}{3675\left(c^{6}x^{2}+c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(105*(5*b*c^8*d*x^8 + 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 - b*c^2*d*x^2 -
 2*b*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (525*a*c^8*d*x^8
 + 1365*a*c^6*d*x^6 + 945*a*c^4*d*x^4 - 105*a*c^2*d*x^2 - 210*a*d - (75*b*c
^7*d*x^7 + 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 - 210*b*c*d*x)*sqrt(c^2*x^2 + 1
))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.129 $\int x^2(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=254

$$-\frac{bdx^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}-\frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}}+\frac{dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}+\frac{1}{8}dx^3\sqrt{d}$$

```
[Out] 1/6*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/16*d*x*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/c^2+1/8*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/
32*b*d*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/96*b*c*d*x^4*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/36*b*c^3*d*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-1/32*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^
(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5808, 5806, 5812, 5783, 30, 14}

$$\frac{dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}dx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc^3\sqrt{c^2x^2+1}}-\frac{bdx^2\sqrt{c^2dx^2+d}}{32c\sqrt{c^2x^2+1}}-\frac{7bcdx^4\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}-\frac{bc^3dx^6\sqrt{c^2dx^2+d}}{36\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(b*d*x^2*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sq
rt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*Sqrt[d + c^2*d*x^2
])/(36*Sqrt[1 + c^2*x^2]) + (d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/
(16*c^2) + (d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (x^3*(d + c
^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d\int x^2\sqrt{d+c^2dx^2}\,\left(a\right.$$

$$=\frac{1}{8}dx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\right.$$

$$=-\frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}}+\frac{dx\sqrt{d+c^2dx^2}\,(}{16}$$

$$=-\frac{bdx^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}-\frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.49, size = 251, normalized size = 0.99

$$\frac{48acdx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\,(3+14c^2x^2+8c^4x^4)-144ad^{3/2}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-18bd\sqrt{d+c^2dx^2}\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)+bd\sqrt{d+c^2dx^2}\left(72\sinh^{-1}(cx)^2+18\cosh\left(2\sinh^{-1}(cx)\right)+9\cosh\left(4\sinh^{-1}(cx)\right)-2\cosh\left(6\sinh^{-1}(cx)\right)+12\sinh^{-1}(cx)\left(-3\sinh\left(2\sinh^{-1}(cx)\right)-3\sinh\left(4\sinh^{-1}(cx)\right)+\sinh\left(6\sinh^{-1}(cx)\right)\right)\right)}{2304c^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(3 + 14*c^2*x^2 + 8*c^4*x
^4) - 144*a*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^
2]] - 18*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] -
 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) + b*d*Sqrt[d + c^2*d*x^2]*(72*ArcSinh
[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSi
nh[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]
] + Sinh[6*ArcSinh[c*x]])))/(2304*c^3*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(218) = 436$.
time = 2.43, size = 799, normalized size = 3.15

method	result
default	$\frac{ax\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{6c^2d}-\frac{ax\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{24c^2}-\frac{adx\sqrt{c^2d\,x^2+d}}{16c^2}-\frac{a\,d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{d\,(c^2x^2+}}{32\sqrt{c^2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a/c^2*x*(c^2*d*x^2+d)^(3/2)-1/16*a/c
^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a/c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x
```

```
^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/c^3*arcsinh(c*x)^2*d+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(
c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*
x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d/c^3/(c
^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2
)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arc
sinh(c*x))*d/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x
^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d/c^3/(c^
2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)
+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/512*(d*(c^
2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2
*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d/c^3/(c^2*x
^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)
+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^
(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^4 + a*d*x^2 + (b*c^2*d*x^4 + b*d*x^2)*arcsinh(c*x))*sqr
t(c^2*d*x^2 + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} \left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.130 $\int x(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=146

$$-\frac{bdx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{2bcdx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3dx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}$$

```
[Out] 1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^2/d-1/5*b*d*x*(c^2*d*x^2+d)^(1
/2)/c/(c^2*x^2+1)^(1/2)-2/15*b*c*d*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)-1/25*b*c^3*d*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5798, 200}

$$\frac{(c^2dx^2+d)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{bdx\sqrt{c^2dx^2+d}}{5c\sqrt{c^2x^2+1}}-\frac{2bcdx^3\sqrt{c^2dx^2+d}}{15\sqrt{c^2x^2+1}}-\frac{bc^3dx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/5*(b*d*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*x^3*Sqrt[
d + c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^5*Sqrt[d + c^2*d*x^2])/
(25*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^
2*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x\right.}{5c\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(1+2c^2\right.}{5c\sqrt{1+c^2x^2}}$$

$$=-\frac{bdx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{2bcdx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3dx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.08, size = 102, normalized size = 0.70

$$\frac{d\sqrt{d+c^2dx^2}\left(15a(1+c^2x^2)^3-bcx\sqrt{1+c^2x^2}\,(15+10c^2x^2+3c^4x^4)+15b(1+c^2x^2)^3\sinh^{-1}(cx)\right)}{75c^2\left(1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15
+ 10*c^2*x^2 + 3*c^4*x^4) + 15*b*(1 + c^2*x^2)^3*ArcSinh[c*x]))/(75*c^2*(1
+ c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(124)=248$.
time = 0.61, size = 559, normalized size = 3.83

method	result
default	$\frac{a\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{5c^2d}+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\left(16x^6c^6+16\sqrt{c^2x^2+1}\,x^5c^5+28c^4x^4+20\sqrt{c^2x^2+1}\,x^3c^3+13c^2x^2+5\sqrt{c^2x^2}\right.}{800c^2\left(c^2x^2+1\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a/c^2/d*(c^2*d*x^2+d)^(5/2)+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+
16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2
*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*
(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c
^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)*d/c^2/(c^2*
x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcs
```

```
inh(c*x))*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^
2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))*
d/c^2/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1
/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1
)^(1/2)*c*x+1)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

Maxima [A]
time = 0.30, size = 85, normalized size = 0.58

$$\frac{(c^2dx^2+d)^{\frac{5}{2}}b\operatorname{arsinh}(cx)}{5\,c^2d}+\frac{(c^2dx^2+d)^{\frac{5}{2}}a}{5\,c^2d}-\frac{\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)b}{75\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*d*x^2 + d)^(5/2)*b*arcsinh(c*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2
)*a/(c^2*d) - 1/75*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*
b/(c*d)
```

Fricas [A]
time = 0.38, size = 167, normalized size = 1.14

$$\frac{15\,(bc^6dx^6+3\,bc^4dx^4+3\,bc^2dx^2+bd)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(15\,ac^6dx^6+45\,ac^4dx^4+45\,ac^2dx^2+15\,ad-(3\,bc^5dx^5+10\,bc^3dx^3+15\,bcdx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{75\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*(b*c^6*d*x^6 + 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 + b*d)*sqrt(c^2*d*x^2
 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (15*a*c^6*d*x^6 + 45*a*c^4*d*x^4 + 45*
a*c^2*d*x^2 + 15*a*d - (3*b*c^5*d*x^5 + 10*b*c^3*d*x^3 + 15*b*c*d*x)*sqrt(c
^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.131 $\int \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=180

$$-\frac{5bcdx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bc^3dx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{3}{8}dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right.$$

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+3/8*d*x*(a+b*arcsinh(c*x))*(c^
2*d*x^2+d)^(1/2)-5/16*b*c*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*
b*c^3*d*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/16*d*(a+b*arcsinh(c*x))
^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5786, 5785, 5783, 30, 14}

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{5bcdx^2\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}-\frac{bc^3dx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*d*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^4*Sq
rt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]))/8 + (x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3
*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}(3d)\int \sqrt{d+c^2dx^2}\,\left(a+\right.$$

$$=\frac{3}{8}dx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh\right.$$

$$=-\frac{5bcdx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bc^3dx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{3}{8}dx\sqrt{d+c^2dx^2}\,\left(a\right.$$

Mathematica [A]
time = 0.53, size = 200, normalized size = 1.11

$$\frac{1}{8}adx\left(5+2c^2x^2\right)\sqrt{d+c^2dx^2}+\frac{3ad^{3/2}\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)}{8c}+\frac{bd\sqrt{d+c^2dx^2}\,\left(-\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{8c\sqrt{1+c^2x^2}}-\frac{bd\sqrt{d+c^2dx^2}\,\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)}{128c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*d*x*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/8 + (3*a*d^(3/2)*Log[c*d*x + Sq
rt[d]*Sqrt[d + c^2*d*x^2]])/(8*c) + (b*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSi
nh[c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt
[1 + c^2*x^2]) - (b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSin
h[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(154) = 308$.
time = 1.38, size = 496, normalized size = 2.76

method	result
default	$\frac{x(c^2dx^2+d)^{\frac{3}{2}}a}{4}+\frac{3adx\sqrt{c^2d\,x^2+d}}{8}+\frac{3a\,d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8\sqrt{c^2d}}+b\left(\frac{3\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{16\sqrt{c^2x^2+1}\,c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*a+3/8*a*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*d^2*ln(x*c^
2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(3/16*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d+1/256*(d*(c^2*x^2+1))^(1/2)*(8
*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)
+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*
x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2
))*(-1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^
3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d
/c/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)
^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4
*arcsinh(c*x))*d/c/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.132 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=249

$$-\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}\right.$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+d*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)-4/3*b*c*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c
^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*d*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*d*polylog(2,c*x+(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5808, 5806, 5816, 4267, 2317, 2438, 8}

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{2d\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{bd\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{4bcdx\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}-\frac{bc^3dx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-4*b*c*d*x*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[
d + c^2*d*x^2])/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - (2*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (
b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{x}\,dx = \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\sinh^{-1}(cx)) + d\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}\,(a+b$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}\,(a+$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}\,(a+$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}\,(a+$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}\,(a+$$

Mathematica [A]
time = 0.51, size = 248, normalized size = 1.00

$$\frac{1}{3}ad(4+c^2x^2)\sqrt{d+c^2dx^2} + \frac{bd\sqrt{d+c^2dx^2}\left(-cx(3+c^2x^2)+3(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\right)}{9\sqrt{1+c^2x^2}} + ad^{3/2}\log(x) - ad^{3/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bd\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\sinh^{-1}(cx)+\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (a*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 + (b*d*Sqrt[d + c^2*d*x^2]*(-(c*x
*(3 + c^2*x^2)) + 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]
) + a*d^(3/2)*Log[x] - a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*
d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*
x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + P
olyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^
2*x^2]
```

Maple [A]
time = 1.64, size = 428, normalized size = 1.72

method	result
default	$\frac{(c^2dx^2+d)^{\frac{3}{2}}a}{3} - a\,d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2dx^2+d}}{x}\right) + a\sqrt{c^2dx^2+d}\,d + \frac{b\sqrt{d\,(c^2x^2+1)}\,\operatorname{polylog}\left(2,cx+\right.}{\sqrt{c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/
x)+a*(c^2*d*x^2+d)^(1/2)*d+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylo
g(2,c*x+(c^2*x^2+1)^(1/2))*d+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d-1/9*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x
^2+1)^(1/2)*x^3*c^3+5/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*
x^2*c^2-4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*x*c+1/3*b*(d*(c^2*x
^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4+4/3*b*(d*(c^2*x^2+1))^(1/2)
*d/(c^2*x^2+1)*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polyl
og(2,-c*x-(c^2*x^2+1)^(1/2))*d-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2) - 3*sqrt(c^2*
d*x^2 + d)*d)*a + b*integrate((c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c^2*x^2
+ 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x, x)
```

3.133 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=177

$$-\frac{bc^3dx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{3cd\sqrt{d+c}}{}$$

```
[Out] -(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x+3/2*c^2*d*x*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)-1/4*b*c^3*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/4
*c*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*d*ln(
x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5807, 5785, 5783, 30, 14}

$$\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3cd\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bcd\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}-\frac{bc^3dx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -1/4*(b*c^3*d*x^2*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*d*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/x + (3*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt
[1 + c^2*x^2]) + (b*c*d*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx &= -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\left(3c^2d\right)\int \sqrt{d+c^2dx^2}\,\left(a+\right.\\ &= \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\,\mathrm{sinl}\right.}{x}\\ &= -\frac{bc^3dx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)-\end{aligned}$$

Mathematica [A]

time = 0.55, size = 200, normalized size = 1.13

$$\frac{1}{8}\left(\frac{4ad\left(-2+c^2x^2\right)\sqrt{d+c^2dx^2}}{x}+\frac{4bd\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)+cx\sinh^{-1}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}}+12acd^{3/2}\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)+\frac{bcd\sqrt{d+c^2dx^2}\left(-\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{\sqrt{1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] ((4*a*d*(-2 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/x + (4*b*d*Sqrt[d + c^2*d*x^2]*
(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/
(x*Sqrt[1 + c^2*x^2]) + 12*a*c*d^(3/2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x
^2]] + (b*c*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(
ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2])/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(155) = 310$.
time = 2.16, size = 392, normalized size = 2.21

method	result
default	$-\frac{a(c^2 d\,x^2+d)^{\frac{5}{2}}}{dx}+a\,c^2 x(c^2 d\,x^2+d)^{\frac{3}{2}}+\frac{3a\,c^2 dx\sqrt{c^2 d\,x^2+d}}{2}+\frac{3a\,c^2 d^2\ln\left(\frac{x\,c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{2\sqrt{c^2 d}}+\frac{3b\sqrt{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(5/2)+a*c^2*x*(c^2*d*x^2+d)^(3/2)+3/2*a*c^2*d*x*(c^2*d
*x^2+d)^(1/2)+3/2*a*c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)+3/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c
*d+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*
(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x^2-1/2*b*(d*(c^2*x^2+1))^(1/2)*
c^2*d/(c^2*x^2+1)*arcsinh(c*x)*x-b*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1/2)-b*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)*d/x/(c^2*x^2+1)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c*d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\operatorname{asinh}{(cx)})}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

3.134 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=270

$$-\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))-\frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(c}{2x^2}$$

```
[Out] -1/2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2+3/2*c^2*d*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)-1/2*b*c*d*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-b*c
^3*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*c^2*d*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/2*b*c^2
*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+
3/2*b*c^2*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5807, 5806, 5816, 4267, 2317, 2438, 8, 14}

$$\frac{3}{2}c^2d\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))-\frac{(c^2dx^2+d)^{3/2}\,(a+b\sinh^{-1}(cx))}{2x^2}-\frac{3c^2d\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2x^2+1}}-\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}-\frac{bcd\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}-\frac{bc^3dx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d*Sqrt[d + c^2*d*x^2])/(x*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x*Sqrt[d
+ c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (3*c^2
*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1
 + c^2*x^2] - (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(
2*Sqrt[1 + c^2*x^2]) + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[
c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))}{x^3}\,dx = -\frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))}{2x^2} + \frac{1}{2}(3c^2d)\int \frac{\sqrt{d+c^2dx^2}\,(a+}{x}$$

$$= \frac{3}{2}c^2d\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}}{2x^2}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}\,(a+b$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}\,(a+b$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}\,(a+b$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}\,(a+b$$

Mathematica [**A**]
time = 2.88, size = 352, normalized size = 1.30

$$a\left(c^2d-\frac{d}{2x^2}\right)\sqrt{d+c^2dx^2}+\frac{3}{2}ac^2d^{3/2}\log(x)-\frac{3}{2}ac^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bc^2d\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\sinh^{-1}(cx)+\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{1+c^2x^2}}+\frac{bc^2d\sqrt{d+c^2dx^2}\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{8\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] a*(c^2*d - d/(2*x^2))*Sqrt[d + c^2*d*x^2] + (3*a*c^2*d^(3/2)*Log[x])/2 - (3
*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/2 + (b*c^2*d*Sqrt[d +
c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 -
E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -
E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b
*c^2*d*Sqrt[d + c^2*d*x^2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[Arc
Sinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]
*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[
2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSin
h[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [**A**]
time = 3.29, size = 472, normalized size = 1.75

method	result

default	$-\frac{a(c^2dx^2+d)^{\frac{5}{2}}}{2dx^2}+\frac{ac^2(c^2dx^2+d)^{\frac{3}{2}}}{2}-\frac{3ac^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2dx^2+d}}{x}\right)d^{\frac{3}{2}}}{2}+\frac{3ac^2\sqrt{c^2dx^2+d}\,d}{2}+\frac{b\sqrt{d(c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(5/2)+1/2*a*c^2*(c^2*d*x^2+d)^(3/2)-3/2*a*c^2*ln
((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*d^(3/2)+3/2*a*c^2*(c^2*d*x^2+d)^(1/
2)*d+b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c^2*x
^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c
^2*x^2+1)*arcsinh(c*x)-1/2*b*(d*(c^2*x^2+1))^(1/2)*d/x/(c^2*x^2+1)^(1/2)*c-
1/2*b*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)*arcsinh(c*x)+3/2*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*
d+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(
1/2))*c^2*d-3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1
+c*x+(c^2*x^2+1)^(1/2))*c^2*d-3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*c^2*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2)*c^2 - 3*s
qrt(c^2*d*x^2 + d)*c^2*d + (c^2*d*x^2 + d)^(5/2)/(d*x^2))*a + b*integrate((
c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^3, x)
```

3.135 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=184

$$-\frac{bcd\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{c^2d\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x}-\frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{3x^3}+\frac{c^3d\sqrt{d+c^2dx^2}}{2b\sqrt{}}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3-c^2*d*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)/x-1/6*b*c*d*(c^2*d*x^2+d)^(1/2)/x^2/(c^2*x^2+1)^(1/2)+1/2
*c^3*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+4/3*b*c
^3*d*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5807, 5805, 29, 5783, 14}

$$-\frac{c^2d\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{x}-\frac{(c^2dx^2+d)^{3/2}\,(a+b\sinh^{-1}(cx))}{3x^3}+\frac{c^3d\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))^2}{2b\sqrt{c^2x^2+1}}-\frac{bcd\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}}+\frac{4bc^3d\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d*Sqrt[d + c^2*d*x^2])/(x^2*Sqrt[1 + c^2*x^2]) - (c^2*d*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/(3*x^3) + (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*Sq
rt[1 + c^2*x^2]) + (4*b*c^3*d*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x
^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx &= -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\left(c^2d\right)\int\frac{\sqrt{d+c^2dx^2}\,\left(a+b\right.}{x^2}\\ &= -\frac{c^2d\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-}\right.}{3x^3}\\ &= -\frac{bcd\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{c^2d\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(d+c^2\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 217, normalized size = 1.18

$$\frac{1}{6}\left(-\frac{2ad\left(1+4c^2x^2\right)\sqrt{d+c^2dx^2}}{x^3}-\frac{2b\left(d+c^2dx^2\right)^{3/2}\sinh^{-1}(cx)}{x^3}+\frac{3bc^3d\sqrt{d+c^2dx^2}\left(-\frac{2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{cx}+\sinh^{-1}(cx)^2+2\log(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{bcd\sqrt{d+c^2dx^2}\left(-1+2c^2x^2\log(cx)\right)}{x^2\sqrt{1+c^2x^2}}+6ac^3d^{3/2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d*(1 + 4*c^2*x^2)*Sqrt[d + c^2*d*x^2])/x^3 - (2*b*(d + c^2*d*x^2)^(3
/2)*ArcSinh[c*x])/x^3 + (3*b*c^3*d*Sqrt[d + c^2*d*x^2]*((-2*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x])/(c*x) + ArcSinh[c*x]^2 + 2*Log[c*x]))/Sqrt[1 + c^2*x^2] +
```

```
(b*c*d*Sqrt[d + c^2*d*x^2]*(-1 + 2*c^2*x^2*Log[c*x]))/(x^2*Sqrt[1 + c^2*x^2
]) + 6*a*c^3*d^(3/2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1106 vs. $2(160) = 320$.
time = 3.67, size = 1107, normalized size = 6.02

method	result
default	$-\frac{a(c^2dx^2+d)^{\frac{5}{2}}}{3dx^3} - \frac{2ac^2(c^2dx^2+d)^{\frac{5}{2}}}{3dx} + \frac{2ac^4x(c^2dx^2+d)^{\frac{3}{2}}}{3} + ac^4dx\sqrt{c^2dx^2+d} + \frac{ac^4d^2\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2}\right.}{\sqrt{c^2d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(5/2)-2/3*a*c^2/d/x*(c^2*d*x^2+d)^(5/2)+2/3*a*c^
4*x*(c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(c^2*d*x^2+d)^(1/2)+a*c^4*d^2*ln(x*c^2*d/
(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d-8/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c^3*d-32*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c
^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8+32*b*(d*(c^2*x^2+1))^(1/2)*d/(24
*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7-8/3*b*(d*(c^2*
x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8+8/3*b*(d*(c^2*
x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3*c^6-52*b*(d*(c^2*x^2+1))^(1/2)
*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6+12*b*(d*(c^2*x
^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
c^5-10/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)
*c^6-4*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(
1/2)*c^5+2/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x*c^4-73/3*
b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*
x)*c^4+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(
1/2)*arcsinh(c*x)*c^3-2/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1
)*x/(c^2*x^2+1)*c^4-3/2*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/
(c^2*x^2+1)^(1/2)*c^3-14/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+
1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4
+9*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-1/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4
*x^4+9*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+4/3*b*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3*d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^4, x)
```

3.136 $\int x^3(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=266

$$\frac{2bd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{bd^2x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}}-\frac{bcd^2x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}}-\frac{19bc^3d^2x^7\sqrt{d+c^2dx^2}}{441\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^9\sqrt{d+c^2dx^2}}{81\sqrt{1+c^2x^2}}$$

```
[Out] -1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^4/d+1/9*(c^2*d*x^2+d)^(9/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/63*b*d^2*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^
(1/2)-1/189*b*d^2*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/21*b*c*d^2*
x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*x^7*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-1/81*b*c^5*d^2*x^9*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {272, 45, 5804, 12, 380}

$$\frac{(c^2dx^2+d)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9c^4d^2}-\frac{(c^2dx^2+d)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d}-\frac{bcd^2x^5\sqrt{c^2dx^2+d}}{21\sqrt{c^2x^2+1}}-\frac{bd^2x^3\sqrt{c^2dx^2+d}}{189c\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^9\sqrt{c^2dx^2+d}}{81\sqrt{c^2x^2+1}}+\frac{2bd^2x\sqrt{c^2dx^2+d}}{63c^3\sqrt{c^2x^2+1}}-\frac{19bc^3d^2x^7\sqrt{c^2dx^2+d}}{441\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d^2*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (b*d^2*x^3*Sqr
t[d + c^2*d*x^2])/(189*c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^5*Sqrt[d + c^2*d*x
^2])/(21*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2])/(441*S
qrt[1 + c^2*x^2]) - (b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2])/(81*Sqrt[1 + c^2*x^
2]) - ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d) + ((d + c^2*d*
x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(9*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx &= -\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\frac{\left(1+c^2x^2\right)^3\left(-2+7c^2x^2\right)}{63c^4}\,dx}{\sqrt{1+c^2x^2}}+\left(a+b\sinh^{-1}(cx\right. \\
&= -\frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^3\left(-2+7c^2x^2\right)\,dx}{63c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\,\mathrm{s}\right. \\
&= -\frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+15c^4x^4+19c^6x^6+7c^8x^8\right)\,d}{63c^3\sqrt{1+c^2x^2}} \\
&= \frac{2bd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{bd^2x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}}-\frac{bcd^2x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 140, normalized size = 0.53

$$\frac{d^2\sqrt{d+c^2dx^2}\left(63a\left(1+c^2x^2\right)^4\left(-2+7c^2x^2\right)-bcx\sqrt{1+c^2x^2}\left(-126+21c^2x^2+189c^4x^4+171c^6x^6+49c^8x^8\right)+63b\left(1+c^2x^2\right)^4\left(-2+7c^2x^2\right)\sinh^{-1}(cx)\right)}{3969c^4\left(1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - b*c*x*Sqr
t[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8)
```

```
 + 63*b*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4*(1 + c^2*
x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(228) = 456$.
time = 1.58, size = 996, normalized size = 3.74

method	result
default	$a\left(\frac{x^{2}\left(c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{9c^{2}d}-\frac{2\left(c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{63d\,c^{4}}\right)+b\left(\frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(256x^{10}c^{10}+256\sqrt{c^{2}x^{2}+1}\,x^{9}c^{9}+704x^{8}c^{8}+576\sqrt{c^{2}x}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b*(1/4
1472*(d*(c^2*x^2+1))^(1/2)*(256*x^10*c^10+256*(c^2*x^2+1)^(1/2)*x^9*c^9+704
*x^8*c^8+576*(c^2*x^2+1)^(1/2)*x^7*c^7+688*x^6*c^6+432*(c^2*x^2+1)^(1/2)*x^
5*c^5+280*c^4*x^4+120*(c^2*x^2+1)^(1/2)*x^3*c^3+41*c^2*x^2+9*(c^2*x^2+1)^(1
/2)*c*x+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^
(1/2)*(64*x^8*c^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6+112*(c^2*x^2+1)^
(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2+7*(c^2*x^
2+1)^(1/2)*c*x+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2
+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(
1/2)*c*x+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)*d^2/c^4/(c^2*x^2+1)
-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c
*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+
1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))*d^
2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8-64*(c^2*x^2+1)^
(1/2)*x^7*c^7+144*x^6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4-56*(c^2
*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x+1)*(1+7*arcsinh(c*
x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*x^10*c^10-256*(c
^2*x^2+1)^(1/2)*x^9*c^9+704*x^8*c^8-576*(c^2*x^2+1)^(1/2)*x^7*c^7+688*x^6*c
^6-432*(c^2*x^2+1)^(1/2)*x^5*c^5+280*c^4*x^4-120*(c^2*x^2+1)^(1/2)*x^3*c^3+
41*c^2*x^2-9*(c^2*x^2+1)^(1/2)*c*x+1)*(1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1
))
```

Maxima [A]
time = 0.30, size = 156, normalized size = 0.59

$$\frac{1}{63}\left(\frac{7\left(c^{2}dx^{2}+d\right)^{\frac{7}{2}}x^{2}}{c^{2}d}-\frac{2\left(c^{2}dx^{2}+d\right)^{\frac{7}{2}}}{c^{4}d}\right)b\operatorname{arsinh}\left(cx\right)+\frac{1}{63}\left(\frac{7\left(c^{2}dx^{2}+d\right)^{\frac{7}{2}}x^{2}}{c^{2}d}-\frac{2\left(c^{2}dx^{2}+d\right)^{\frac{7}{2}}}{c^{4}d}\right)a-\frac{\left(49\,c^{8}d^{\frac{5}{2}}x^{9}+171\,c^{6}d^{\frac{5}{2}}x^{7}+189\,c^{4}d^{\frac{5}{2}}x^{5}+21\,c^{2}d^{\frac{5}{2}}x^{3}-126\,d^{\frac{5}{2}}x\right)b}{3969\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d)
)*b*arcsinh(c*x) + 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2
 + d)^(7/2)/(c^4*d))*a - 1/3969*(49*c^8*d^(5/2)*x^9 + 171*c^6*d^(5/2)*x^7 +
 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*b/c^3
```

Fricas [**A**]

time = 0.36, size = 263, normalized size = 0.99

$$\frac{63\left(7\,bc^{10}d^2x^{10}+26\,bc^8d^2x^8+34\,bc^6d^2x^6+16\,bc^4d^2x^4-bc^2d^2x^2-2\,bd^2\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(441\,ac^{10}d^2x^{10}+1638\,ac^8d^2x^8+2142\,ac^6d^2x^6+1008\,ac^4d^2x^4-63\,ac^2d^2x^2-126\,ad^2-\left(49\,bc^9d^2x^9+171\,bc^7d^2x^7+189\,bc^5d^2x^5+21\,bc^3d^2x^3-126\,bcd^2x\right)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{3969\,(c^6x^2+c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3969*(63*(7*b*c^10*d^2*x^10 + 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 + 16*b*
c^4*d^2*x^4 - b*c^2*d^2*x^2 - 2*b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 + 1)) + (441*a*c^10*d^2*x^10 + 1638*a*c^8*d^2*x^8 + 2142*a*c^6*d^2*x
^6 + 1008*a*c^4*d^2*x^4 - 63*a*c^2*d^2*x^2 - 126*a*d^2 - (49*b*c^9*d^2*x^9
+ 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 + 21*b*c^3*d^2*x^3 - 126*b*c*d^2*x)
*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.137 $\int x^2(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=337

$$-\frac{5bd^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}}+\frac{5d^2x\sqrt{d+c^2}}{}$$

```
[Out] 5/48*d*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/8*x^3*(c^2*d*x^2+d)^(5/
2)*(a+b*arcsinh(c*x))+5/128*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^
2+5/64*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-5/256*b*d^2*x^2*(c^2*
d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-59/768*b*c*d^2*x^4*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-17/288*b*c^3*d^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
-1/64*b*c^5*d^2*x^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/256*d^2*(a+b*ar
csinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5808, 5806, 5812, 5783, 30, 14, 272, 45}

$$\frac{5d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{128c^2}+\frac{5}{64}d^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}dx^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{256bc^3\sqrt{c^2x^2+1}}-\frac{5bd^2x^2\sqrt{c^2dx^2+d}}{256c\sqrt{c^2x^2+1}}-\frac{59bcd^2x^4\sqrt{c^2dx^2+d}}{768\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^8\sqrt{c^2dx^2+d}}{64\sqrt{c^2x^2+1}}-\frac{17bc^3d^2x^6\sqrt{c^2dx^2+d}}{288\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*d^2*x^2*Sqrt[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(768*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^2*x^6*Sqrt[d
 + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]
)/(64*Sqrt[1 + c^2*x^2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(128*c^2) + (5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5
*d*x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48 + (x^3*(d + c^2*d*x^2
)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
```

```
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}(5d)\int x^2\left(d+c^2dx^2\right.$$

$$=\frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}\left(\right.$$

$$=\frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}$$

$$=-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^8\sqrt{}}{64\sqrt{1}}$$

$$=-\frac{5bd^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3d^2x^6\sqrt{c}}{288\sqrt{1+}}$$

Mathematica [A]
time = 0.65, size = 388, normalized size = 1.15

$$\frac{d^2\left(2880acx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+22656ac^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+26112ac^5x^5\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+9216ac^7x^7\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}-1440b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)^2+576b\sqrt{d+c^2dx^2}\cosh\left(2\sinh^{-1}(cx)\right)-144b\sqrt{d+c^2dx^2}\cosh\left(4\sinh^{-1}(cx)\right)-64b\sqrt{d+c^2dx^2}\cosh\left(6\sinh^{-1}(cx)\right)-9b\sqrt{d+c^2dx^2}\cosh\left(8\sinh^{-1}(cx)\right)-2880a\sqrt{d}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+24b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)\left(-48\sinh\left(2\sinh^{-1}(cx)\right)+24\sinh\left(4\sinh^{-1}(cx)\right)+16\sinh\left(6\sinh^{-1}(cx)\right)+3\sinh\left(8\sinh^{-1}(cx)\right)\right)\right)}{73728c^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(2880*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 22656*a*c^3*x^3*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqr
t[d + c^2*d*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 1
440*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 + 576*b*Sqrt[d + c^2*d*x^2]*Cosh[2
*ArcSinh[c*x]] - 144*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 64*b*Sqrt
[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 9*b*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSi
nh[c*x]] - 2880*a*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^
2*d*x^2]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(-48*Sinh[2*ArcSinh[c*x]]
 + 24*Sinh[4*ArcSinh[c*x]] + 16*Sinh[6*ArcSinh[c*x]] + 3*Sinh[8*ArcSinh[c*x
]])))/(73728*c^3*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1164 vs. 2(291) = 582.
time = 2.31, size = 1165, normalized size = 3.46

method	result
default	$\frac{ax\left(c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{8 c^{2} d}-\frac{ax\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{48 c^{2}}-\frac{5 a d x\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{192 c^{2}}-\frac{5 a\,d^{2} x\sqrt{c^{2} d\,x^{2}+d}}{128 c^{2}}-\frac{5 a\,d^{3} \ln\left(\frac{x\,c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{128 c^{2} \sqrt{c^{2} d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a/c^2*x*(c^2*d*x^2+d)^(5/2)-5/192*a/
c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-5/128*a/c
^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-5/25
6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d^2+1/16384*(d
*(c^2*x^2+1))^(1/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^7+
256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+1)^(1/2)+88*
c^3*x^3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*(-1+8*arcsinh
(c*x))*d^2/c^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*
x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18
*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2
/c^3/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2
+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(
-1+4*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x
^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))
*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*
x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1
)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^
3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x
))*d^2/c^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*
(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2
*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c^3/
(c^2*x^2+1)+1/16384*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9-128*c^8*x^8*(c^2*x^2
+1)^(1/2)+320*c^7*x^7-256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5-160*c^4*x^4
*(c^2*x^2+1)^(1/2)+88*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1
)^(1/2))*(1+8*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((a*c^4*d^2*x^6 + 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 + 2*
b*c^2*d^2*x^4 + b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.138 $\int x(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=193

$$-\frac{bd^2x\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{bcd^2x^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{3bc^3d^2x^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{7/2}\,(a+}{7c^2d}$$

```
[Out] 1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^2/d-1/7*b*d^2*x*(c^2*d*x^2+d)^
(1/2)/c/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-3/35*b*c^3*d^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^5*d^
2*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5798, 200}

$$\frac{(c^2dx^2+d)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2d}-\frac{bd^2x\sqrt{c^2dx^2+d}}{7c\sqrt{c^2x^2+1}}-\frac{bcd^2x^3\sqrt{c^2dx^2+d}}{7\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}-\frac{3bc^3d^2x^5\sqrt{c^2dx^2+d}}{35\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/7*(b*d^2*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^3*Sqr
t[d + c^2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x
^2])/(35*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[
1 + c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(d+c^2dx^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2d} - \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int (1+c$$

$$= \frac{\left(d+c^2dx^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2d} - \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int (1+3}{7c\sqrt{1+}}$$

$$= -\frac{bd^2x\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{bcd^2x^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{3bc^3d^2x^5\sqrt{d+c^2d}}{35\sqrt{1+c^2x^2}}$$

Mathematica [**A**]
time = 0.09, size = 112, normalized size = 0.58

$$\frac{d^2\sqrt{d+c^2dx^2}\left(35a(1+c^2x^2)^4 - bcx\sqrt{1+c^2x^2}\,(35+35c^2x^2+21c^4x^4+5c^6x^6) + 35b(1+c^2x^2)^4\sinh^{-1}(cx)\right)}{245c^2\,(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(35*a*(1 + c^2*x^2)^4 - b*c*x*Sqrt[1 + c^2*x^2]*(3
5 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^4*ArcSinh[c*x
]))/(245*c^2*(1 + c^2*x^2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 862 vs. 2(165) = 330.
time = 0.64, size = 863, normalized size = 4.47

method	result
default	$\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{7c^2d} + b\left(\frac{\sqrt{d\,(c^2x^2+1)}\,\left(64x^8c^8+64\sqrt{c^2x^2+1}\,x^7c^7+144x^6c^6+112\sqrt{c^2x^2+1}\,x^5c^5+104c^4x^4+56\sqrt{c}\right.}{6272c^2(c^2x^2+1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a/c^2/d*(c^2*d*x^2+d)^(7/2)+b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8
+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+104
*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*c*x+1)
*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*x^
6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+
13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+
1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2
*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/1
```

```
28*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)
*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)
*c*x+1)*(1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4
*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(
1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c
^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*
c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1
/6272*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8-64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^
6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4-56*(c^2*x^2+1)^(1/2)*x^3*c^
3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x+1)*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2
+1))
```

Maxima [A]
time = 0.29, size = 96, normalized size = 0.50

$$\frac{(c^2dx^2+d)^{\frac{7}{2}}b\operatorname{arsinh}(cx)}{7\,c^2d}+\frac{(c^2dx^2+d)^{\frac{7}{2}}a}{7\,c^2d}-\frac{\left(5\,c^6d^{\frac{7}{2}}x^7+21\,c^4d^{\frac{7}{2}}x^5+35\,c^2d^{\frac{7}{2}}x^3+35\,d^{\frac{7}{2}}x\right)b}{245\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*(c^2*d*x^2 + d)^(7/2)*b*arcsinh(c*x)/(c^2*d) + 1/7*(c^2*d*x^2 + d)^(7/2
)*a/(c^2*d) - 1/245*(5*c^6*d^(7/2)*x^7 + 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2
)*x^3 + 35*d^(7/2)*x)*b/(c*d)
```

Fricas [A]
time = 0.38, size = 225, normalized size = 1.17

$$\frac{35\,(bc^8d^2x^8+4\,bc^6d^2x^6+6\,bc^4d^2x^4+4\,bc^2d^2x^2+bd^2)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(35\,ac^8d^2x^8+140\,ac^6d^2x^6+210\,ac^4d^2x^4+140\,ac^2d^2x^2+35\,ad^2-(5\,bc^7d^2x^7+21\,bc^5d^2x^5+35\,bc^3d^2x^3+35\,bcd^2x)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{245\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*(b*c^8*d^2*x^8 + 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 + 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (35*a*c^8*d
^2*x^8 + 140*a*c^6*d^2*x^6 + 210*a*c^4*d^2*x^4 + 140*a*c^2*d^2*x^2 + 35*a*d
^2 - (5*b*c^7*d^2*x^7 + 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 + 35*b*c*d^2*x)
*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.139 $\int \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=254

$$-\frac{25bcd^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bc^3d^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{16}d^2x\sqrt{d+c^2dx^2}\left(a+b\sin\right.$$

[Out] 5/24*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))-1/36*b*d^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-25/96*b*c*d^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5786, 5785, 5783, 30, 14, 267}

$$\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}dx\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{25bcd^2x^2\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}-\frac{5bc^3d^2x^4\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]

[Out] (-25*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*d^2*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}(5d)\int \left(d+c^2dx^2\right)^{3/2}\left(a\right.$$

$$=-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh\right.$$

$$=-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{16}d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh\right.$$

$$=-\frac{25bcd^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bc^3d^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bd^2\left(1+c^2x^2\right)}{}$$

Mathematica [A]
time = 0.47, size = 317, normalized size = 1.25

$$\frac{d^2\left(1584acx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+1248ac^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+384ac^5x^5\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+360b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)^2-270b\sqrt{d+c^2dx^2}\cosh\left(2\sinh^{-1}(cx)\right)-27b\sqrt{d+c^2dx^2}\cosh\left(4\sinh^{-1}(cx)\right)-2b\sqrt{d+c^2dx^2}\cosh\left(6\sinh^{-1}(cx)\right)+720a\sqrt{d}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+12b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)\left(45\sinh\left(2\sinh^{-1}(cx)\right)+9\sinh\left(4\sinh^{-1}(cx)\right)+\sinh\left(6\sinh^{-1}(cx)\right)\right)\right)}{2304c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(1584*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 1248*a*c^3*x^3*Sqr
t[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 384*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 360*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 - 270*b*Sqrt[d + c
^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 27*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*
x]] - 2*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 720*a*Sqrt[d]*Sqrt[1 +
 c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 12*b*Sqrt[d + c^2*d*x^
2]*ArcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + Sinh[6*
ArcSinh[c*x]])))/(2304*c*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(218) = 436$.
time = 1.46, size = 801, normalized size = 3.15

method	result
default	$\frac{x(c^2dx^2+d)^{\frac{5}{2}}a}{6}+\frac{5adx(c^2dx^2+d)^{\frac{3}{2}}}{24}+\frac{5ad^2x\sqrt{c^2dx^2+d}}{16}+\frac{5ad^3\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{16\sqrt{c^2d}}+b\left(\frac{5\sqrt{d(c^2x}}{32}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(c^2*d*x^2+d)^(5/2)*a+5/24*a*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(c^
2*d*x^2+d)^(1/2)+5/16*a*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)+b*(5/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)
^2*d^2+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2
)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)
^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+3/512
*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*
c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))*d^2/
c/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)
^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+15/25
6*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x
^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+3/512*(d*(c^2*x^2+1))^(1/
2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^
(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/2304*
(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-4
8*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(
c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.140 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=329

$$-\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}$$

```
[Out] 1/3*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/5*(c^2*d*x^2+d)^(5/2)*(a+b*a
rcsinh(c*x))+d^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-23/15*b*c*d^2*x*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-11/45*b*c^3*d^2*x^3*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)-1/25*b*c^5*d^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-
2*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-b*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)+b*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5808, 5806, 5816, 4267, 2317, 2438, 8, 200}

$$d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{2d^2\sqrt{c^2dx^2+d}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{23bcd^2x\sqrt{c^2dx^2+d}}{15\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{11bc^3d^2x^3\sqrt{c^2dx^2+d}}{45\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-23*b*c*d^2*x*Sqrt[d + c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (11*b*c^3*d^2*
x^3*Sqrt[d + c^2*d*x^2])/(45*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c
^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) + d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]) + (d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((d + c^2*d*x^2)
^(5/2)*(a + b*ArcSinh[c*x]))/5 - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*d^2*Sqrt[d + c^2*d*x^
2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b*d^2*Sqrt[d + c^2*d*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{5}\left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) + d\int \frac{\left(d+c^2dx^2\right)^{3/2} \left(a+b\sin\right.}{x}$$

$$= \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh\right.$$

$$= -\frac{8bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2}}{25\sqrt{1+c^2x^2}}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c}}{25\sqrt{1+c^2x}}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c}}{25\sqrt{1+c^2x}}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c}}{25\sqrt{1+c^2x}}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c}}{25\sqrt{1+c^2x}}$$

Mathematica [A]
time = 0.91, size = 353, normalized size = 1.07

$$\frac{1}{225}d^2\left(-\frac{40bcx(3+c^2x^2)\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3bc^3x^3(5+3c^2x^2)\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + 15a\sqrt{d+c^2dx^2}\,(23+11c^2x^2+3c^4x^4) + 150b(1+c^2x^2)\sqrt{d+c^2dx^2}\sinh^{-1}(cx) + 15b(1+c^2x^2)(-2+3c^2x^2)\sqrt{d+c^2dx^2}\sinh^{-1}(cx) + 225a\sqrt{d}\log(x) - 225a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{225b\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\sinh^{-1}(cx)+\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^2*((-40*b*c*x*(3 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] - (3*
b*c^3*x^3*(5 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + 15*a*Sqr
t[d + c^2*d*x^2]*(23 + 11*c^2*x^2 + 3*c^4*x^4) + 150*b*(1 + c^2*x^2)*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x] + 15*b*(1 + c^2*x^2)*(-2 + 3*c^2*x^2)*Sqrt[d + c
^2*d*x^2]*ArcSinh[c*x] + 225*a*Sqrt[d]*Log[x] - 225*a*Sqrt[d]*Log[d + Sqrt[
d]*Sqrt[d + c^2*d*x^2]] + (225*b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2
*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]
*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E
^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]))/225
```

Maple [A]
time = 1.67, size = 540, normalized size = 1.64

method	result
default	$\frac{(c^2 d\,x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d (c^2 d\,x^2+d)^{\frac{3}{2}}}{3}-a\,d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)+a\sqrt{c^2 d\,x^2+d}\,d^2-\frac{b\sqrt{d\,(c^2x^2+}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)

[Out] 1/5*(c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+a*(c^2*d*x^2+d)^(1/2)*d^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2+1/5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^6*c^6-1/25*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*x^5*c^5+14/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4-11/45*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*x^3*c^3+34/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*x*c+23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")

[Out] -1/15*(15*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2) - 5*(c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(c^2*d*x^2 + d)*d^2)*a + b*integrate((c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")

[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x, x)
```

3.141 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=257

$$-\frac{9bc^3d^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3}$$

```
[Out] 5/4*c^2*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-(c^2*d*x^2+d)^(5/2)*(a+b
*arcsinh(c*x))/x+15/8*c^2*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-9/16
*b*c^3*d^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^5*d^2*x^4*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+15/16*c*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*
x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*d^2*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5807, 5786, 5785, 5783, 30, 14, 272, 45}

$$\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{15cd^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16b\sqrt{c^2x^2+1}}+\frac{5}{4}c^2dx\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bcd^2\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}-\frac{9bc^3d^2x^2\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-9*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (15*c^2*d^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*
ArcSinh[c*x]))/4 - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x + (15*c*d
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*Sqrt[1 + c^2*x^2]) + (
b*c*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x} + (5c^2d)\int (d+c^2dx^2)^{3/2}\,(a$$

$$= \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{(d+c^2dx^2)^{5/2}\left(a+b\,\mathrm{s}\right.}{x}$$

$$= \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}(a$$

$$= -\frac{9bc^3d^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{15}{8}c^2d^2x\sqrt{d+c}$$

Mathematica [A]
time = 0.96, size = 270, normalized size = 1.05

$$\frac{1}{128}d^2\left(\frac{16a\sqrt{d+c^2dx^2}\,(-8+9c^2x^2+2c^4x^4)}{x} + \frac{64b\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\sinh^{-1}(cx)+cx\sinh^{-1}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}} + 240ac\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{32bc\sqrt{d+c^2dx^2}\left(-\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{\sqrt{1+c^2x^2}} - \frac{bc\sqrt{d+c^2dx^2}\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)}{\sqrt{1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^2*((16*a*Sqrt[d + c^2*d*x^2]*(-8 + 9*c^2*x^2 + 2*c^4*x^4))/x + (64*b*Sqr
t[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 +
2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x^2]) + 240*a*c*Sqrt[d]*Log[c*d*x + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] + (32*b*c*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]]
 + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2]
 - (b*c*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*Ar
cSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2]))/128
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(225) = 450$.
time = 3.09, size = 506, normalized size = 1.97

method	result
default	$-\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{dx} + a\,c^2x(c^2dx^2+d)^{\frac{5}{2}} + \frac{5a\,c^2dx(c^2dx^2+d)^{\frac{3}{2}}}{4} + \frac{15a\,c^2d^2x\sqrt{c^2dx^2+d}}{8} + \frac{15a\,c^2d^3\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\right.}{8\sqrt{c^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(7/2)+a*c^2*x*(c^2*d*x^2+d)^(5/2)+5/4*a*c^2*d*x*(c^2*d
*x^2+d)^(3/2)+15/8*a*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+15/8*a*c^2*d^3*ln(x*c^2*
```

```
d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+15/16*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c*d^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c*d^2-33/128*b*(d*(c^2*x^2+1))^(1/2)*c*d^2/(c^2*x^
2+1)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1
/2))^2-1)*c*d^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x
)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*x^4+11/8*b*(d*
(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-9/16*b*(d*(c^2*x^2+
1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)*x^2+1/8*b*(d*(c^2*x^2+1))^(1/2)*c^2*d^2
/(c^2*x^2+1)*arcsinh(c*x)*x-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*d^2/x/(c^2
*x^2+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\operatorname{asinh}(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

3.142 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=355

$$-\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{6}c^2$$

```
[Out] 5/6*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-1/2*(c^2*d*x^2+d)^(5/2)*(a
+b*arcsinh(c*x))/x^2+5/2*c^2*d^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/2
*b*c*d^2*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-7/3*b*c^3*d^2*x*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c^5*d^2*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-5*c^2*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/2*b*c^2*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/2*b*c^2*d^2*polylog(2,c*x+(c^2
*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5807, 5808, 5806, 5816, 4267, 2317, 2438, 8, 276}

$$\frac{5}{2}c^2d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{5c^2d^2\sqrt{c^2dx^2+d}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{5}{6}c^2d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}-\frac{bcd^2\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}-\frac{7bc^3d^2x\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^2*Sqrt[d + c^2*d*x^2])/(x*Sqrt[1 + c^2*x^2]) - (7*b*c^3*d^2*x*S
qrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^3*Sqrt[d + c^2*d*x
^2])/(9*Sqrt[1 + c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/2 + (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - ((d + c
^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b*
c^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^
2]) + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[
1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
```

```
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a\right.}{x}$$

$$= \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh\right.}{2x^2}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}+\frac{bc^3d^2x\sqrt{d+c^2dx^2}}{6\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 5.36, size = 424, normalized size = 1.19

$$\frac{1}{72}d^2\left(\frac{12a\sqrt{d+c^2dx^2}\left(-3+14c^2x^2+2c^4x^4\right)}{x^2}-\frac{8bc^2\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3\left(1+c^2x^2\right)^{3/2}\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+180ac^2\sqrt{d}\log(x)-180ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{144bc^2\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\sinh^{-1}(cx)+\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{1+c^2x^2}}+\frac{9bc^2\sqrt{d+c^2dx^2}\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{\sqrt{1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^2*((12*a*Sqrt[d + c^2*d*x^2]*(-3 + 14*c^2*x^2 + 2*c^4*x^4))/x^2 - (8*b*c
^2*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x
```

```
]))/Sqrt[1 + c^2*x^2] + 180*a*c^2*Sqrt[d]*Log[x] - 180*a*c^2*Sqrt[d]*Log[d
+ Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (144*b*c^2*Sqrt[d + c^2*d*x^2]*(-(c*x) + S
qrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - A
rcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - P
olyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (9*b*c^2*Sqrt[d + c^2*d*
x^2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c
*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])]
- ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c
^2*x^2]))/72
```

Maple [A]
time = 3.43, size = 588, normalized size = 1.66

method	result
default	$-\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{2dx^2}+\frac{ac^2(c^2dx^2+d)^{\frac{5}{2}}}{2}+\frac{5ac^2d(c^2dx^2+d)^{\frac{3}{2}}}{6}-\frac{5ac^2d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2dx^2+d}}{x}\right)}{2}+\frac{5ac^2\sqrt{c^2dx^2}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(7/2)+1/2*a*c^2*(c^2*d*x^2+d)^(5/2)+5/6*a*c^2*d*
(c^2*d*x^2+d)^(3/2)-5/2*a*c^2*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2)
)/x)+5/2*a*c^2*(c^2*d*x^2+d)^(1/2)*d^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d^2-1/2*b*(d*(c^2*x^2+1))^(1
/2)*d^2/x/(c^2*x^2+1)^(1/2)*c+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2-5/2*b*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2+11/6*b*(d*(c
^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh(c*x)-5/2*b*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d^2-1/2*b
*(d*(c^2*x^2+1))^(1/2)*d^2/x^2/(c^2*x^2+1)*arcsinh(c*x)+1/3*b*(d*(c^2*x^2+1
))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-1/9*b*(d*(c^2*x^2+1))^(1/2)*c
^5*d^2/(c^2*x^2+1)^(1/2)*x^3+8/3*b*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1
)*arcsinh(c*x)*x^2-7/3*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*c^2*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2)*c^2 -
5*(c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(c^2*d*x^2 + d)*c^2*d^2 + 3*(c^2*d*x
```

```
^2 + d)^(7/2)/(d*x^2))*a + b*integrate((c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt
(c^2*x^2 + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^3, x)
```

3.143 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=266

$$-\frac{bcd^2\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{5c^2d(d+c^2dx^2)^{3/2}\left(a+\right.}{3x}$$

```
[Out] -5/3*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x-1/3*(c^2*d*x^2+d)^(5/2)
*(a+b*arcsinh(c*x))/x^3+5/2*c^4*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)-1/6*b*c*d^2*(c^2*d*x^2+d)^(1/2)/x^2/(c^2*x^2+1)^(1/2)-1/4*b*c^5*d^2*x^2*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/4*c^3*d^2*(a+b*arcsinh(c*x))^2*(c^2*
d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*ln(x)*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5807, 5785, 5783, 30, 14, 272, 45}

$$-\frac{5c^2d(c^2dx^2+d)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x}-\frac{(c^2dx^2+d)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5c^3d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{bcd^2\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}+\frac{7bc^3d^2\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^2*Sqrt[d + c^2*d*x^2])/(x^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^2
*Sqrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x]))/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(3*x^3) + (5*c
^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt[1 + c^2*x^2])
+ (7*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 5783

Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && NeQ[n, -1]

Rule 5785

Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0]

Rule 5807

Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]

Rubi steps

$$\int \frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3} + \frac{1}{3}(5c^2d)\int \frac{(d+c^2dx^2)^{3/2}}{}$$

$$= -\frac{5c^2d(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x} - \frac{(d+c^2dx^2)^{5/2}\left(a+b\,\mathrm{s}\right.}{3x^3}$$

$$= \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{5c^2d(d+c^2dx^2)^{3/2}\left(a+\right.}{3x}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}$$

Mathematica [A]
time = 0.67, size = 286, normalized size = 1.08

$$\frac{d^2\left(4a\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(-2-14c^2x^2+3c^4x^4)+24bc^2x^2\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\sinh^{-1}(cx)+cx\sinh^{-1}(cx)^2+2cx\log(cx)\right)-4b\sqrt{d+c^2dx^2}\left(cx+2(1+c^2x^2)^{3/2}\sinh^{-1}(cx)-2c^3x^3\log(cx)\right)+60ac^3\sqrt{d}\,x^3\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-3bc^3x^3\sqrt{d+c^2dx^2}\left(\cosh\left(2\sinh^{-1}(cx)\right)-2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)\right)}{24x^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^2*(4*a*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-2 - 14*c^2*x^2 + 3*c^4*x^
4) + 24*b*c^2*x^2*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]) - 4*b*Sqrt[d + c^2*d*x^2]*(c*x + 2*(1
+ c^2*x^2)^(3/2)*ArcSinh[c*x] - 2*c^3*x^3*Log[c*x]) + 60*a*c^3*Sqrt[d]*x^3*
Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 3*b*c^3*x^3*Sq
rt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Si
nh[2*ArcSinh[c*x]]))))/(24*x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(230) = 460$.
time = 3.70, size = 1316, normalized size = 4.95

method	result	size
default	Expression too large to display	1316

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -190/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*
arcsinh(c*x)*c^4-23/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)
/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-147*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4
```

```
+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-1/8*b*(d*(c^2*x^2+1))^(1/2)
*d^2*c^3/(c^2*x^2+1)^(1/2)-203*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c
^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6+5/2*a*c^4*d^3*ln(x*c^2*d/(c^2*d)
^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/3*a/d/x^3*(c^2*d*x^2+d)^(7/2)+4
/3*a*c^4*x*(c^2*d*x^2+d)^(5/2)+5/3*a*c^4*d*x*(c^2*d*x^2+d)^(3/2)+5/2*a*c^4*
d^2*x*(c^2*d*x^2+d)^(1/2)-4/3*a*c^2/d/x*(c^2*d*x^2+d)^(7/2)+147*b*(d*(c^2*x
^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*
x)*c^7+35*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*c^5+1/2*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)
*arcsinh(c*x)*x^3-1/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)
/x^2/(c^2*x^2+1)^(1/2)*c-21/2*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^
2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*
x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+1/2*b*(d*(c^2*x^2+1))^
(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x-49/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(
63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-28/3*b*(d*(c^2*x^2+1))^(1/2)*d
^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-7/6*b*(d*(c^2*x^2+1))^(1/2
)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/3*b*(d*(c^2*x^2+1))^(1/
2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)-14/3*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*d^2*c^3+5/4*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*d^2*c^3+7/3*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*d^2*c^3-1/4*b*(d*(c^2*x
^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^2+49/6*b*(d*(c^2*x^2+1))^(1/2)*d^2
/(63*c^4*x^4+15*c^2*x^2+1)*x^3*c^6+7/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*
x^4+15*c^2*x^2+1)*x*c^4-5/2*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*
x^2+1)/(c^2*x^2+1)^(1/2)*c^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^4, x)
```

3.144 $\int \sqrt{1+x^2} \sinh^{-1}(x)\,dx$

Optimal. Leaf size=32

$$-\frac{x^2}{4}+\frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

[Out] -1/4*x^2+1/4*arcsinh(x)^2+1/2*x*arcsinh(x)*(x^2+1)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5785, 5783, 30}

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}\,x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2]*ArcSinh[x],x]

[Out] -1/4*x^2 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{1+x^2}\,\sinh^{-1}(x)\,dx = \frac{1}{2}x\sqrt{1+x^2}\,\sinh^{-1}(x) - \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx$$
$$= -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1+x^2}\,\sinh^{-1}(x) + \frac{1}{4}\sinh^{-1}(x)^2$$

Mathematica [**A**]
time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-x^2 + 2x\sqrt{1+x^2}\,\sinh^{-1}(x) + \sinh^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] (-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

Maple [**A**]
time = 0.90, size = 26, normalized size = 0.81

method	result	size
default	$\frac{x \operatorname{arcsinh}(x)\sqrt{x^2+1}}{2} + \frac{\operatorname{arcsinh}(x)^2}{4} - \frac{x^2}{4} - \frac{1}{4}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x)*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

Maxima [**A**]
time = 0.48, size = 28, normalized size = 0.88

$$-\frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{x^2+1}\,x + \operatorname{arsinh}(x)\right)\operatorname{arsinh}(x) - \frac{1}{4}\operatorname{arsinh}(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^2
```

Fricas [**A**]
time = 0.38, size = 40, normalized size = 1.25

$$\frac{1}{2}\sqrt{x^2+1}\,x\log\left(x+\sqrt{x^2+1}\right) - \frac{1}{4}x^2 + \frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^2
 + 1))^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\operatorname{asinh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x)*(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*asinh(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{asinh}(x)\,\sqrt{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x)*(x^2 + 1)^(1/2),x)
```

```
[Out] int(asinh(x)*(x^2 + 1)^(1/2), x)
```

3.145 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=215

$$-\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}}+\frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}}-\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^6d}-\frac{4x^2\sqrt{d+c^2dx}}{}$$

```
[Out] -8/15*b*x*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)+4/45*b*x^3*(c^2*x^2+1)^
(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/25*b*x^5*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^
(1/2)+8/15*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d-4/15*x^2*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+1/5*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/c^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5812, 5798, 8, 30}

$$\frac{x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}+\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{15c^6d}-\frac{4x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{15c^4d}-\frac{bx^5\sqrt{c^2x^2+1}}{25c\sqrt{c^2dx^2+d}}-\frac{8bx\sqrt{c^2x^2+1}}{15c^5\sqrt{c^2dx^2+d}}+\frac{4bx^3\sqrt{c^2x^2+1}}{45c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-8*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (4*b*x^3*Sqrt[1 +
 c^2*x^2])/(45*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^5*Sqrt[1 + c^2*x^2])/(25*c*S
qrt[d + c^2*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^6*
d) - (4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*d) + (x^4*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d} - \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{5c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int}{5c\sqrt{d+c^2dx}}$$

$$= -\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} - \frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}\left(a+\right.}{5c^2d}$$

$$= \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^6d} - \frac{4x^2}{}$$

$$= -\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}\left(a\right.}{15c}$$

Mathematica [A]
time = 0.10, size = 119, normalized size = 0.55

$$\frac{bcx\sqrt{1+c^2x^2}\left(-120+20c^2x^2-9c^4x^4\right)+15a\left(8+4c^2x^2-c^4x^4+3c^6x^6\right)+15b\left(8+4c^2x^2-c^4x^4+3c^6x^6\right)\sinh^{-1}(cx)}{225c^6\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4*x^4) + 15*a*(8 + 4*c^2*
x^2 - c^4*x^4 + 3*c^6*x^6) + 15*b*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*Arc
Sinh[c*x])/(225*c^6*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(185) = 370$.
time = 3.27, size = 625, normalized size = 2.91

method	result

default	$a\left(\frac{x^4\sqrt{c^2d\,x^2+d}}{5c^2d}-\frac{4\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)}{5c^2}\right)+b\left(\frac{\sqrt{d\,(c^2x^2+1)}\,\left(16x^6c^6+16\sqrt{c^2x^2}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(
1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6
*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+1
3*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1)-
5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^
2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/16*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)/c^6/d
/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(
1+arcsinh(c*x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*
(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh
(c*x))/c^6/d/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^
2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^
2*x^2+1)^(1/2)*c*x+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

Maxima [A]
time = 0.31, size = 174, normalized size = 0.81

$$\frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)b\,\mathrm{arsinh}\,(cx)+\frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)a-\frac{(9\,c^4x^5-20\,c^2x^3+120\,x)b}{225\,c^5\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d)
 + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*b*arcsinh(c*x) + 1/15*(3*sqrt(c^2*d*x^2 +
 d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2 + d)
/(c^6*d))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*b/(c^5*sqrt(d))
```

Fricas [A]
time = 0.35, size = 161, normalized size = 0.75

$$\frac{15\,(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6-15\,ac^4x^4+60\,ac^2x^2-(9\,bc^5x^5-20\,bc^3x^3+120\,bcx)\sqrt{c^2x^2+1}+120\,a\right)\sqrt{c^2dx^2+d}}{225\,(c^8dx^2+c^6d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 - 15*a*c^4*x^4 + 60*a*c^2*x^2
```

```
 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^2*x^2 + 1) + 120*a)*sqrt
(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b \operatorname{asinh}(cx))}{\sqrt{d(c^2x^2+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,(a + b\,\operatorname{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.146 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=192

$$\frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}}-\frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}}-\frac{3x\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{8c^4d}+\frac{x^3\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{4c^2d}$$

```
[Out] 3/16*b*x^2*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/16*b*x^4*(c^2*x^2+1)
^(1/2)/c/(c^2*d*x^2+d)^(1/2)+3/16*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c^5/(c^2*d*x^2+d)^(1/2)-3/8*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+
1/4*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5812, 5783, 30}

$$\frac{x^3\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{4c^2d}+\frac{3\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))^2}{16bc^5\sqrt{c^2dx^2+d}}-\frac{3x\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{8c^4d}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{c^2dx^2+d}}+\frac{3bx^2\sqrt{c^2x^2+1}}{16c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (3*b*x^2*Sqrt[1 + c^2*x^2])/(16*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 +
c^2*x^2])/(16*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x]))/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*c^
2*d) + (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[d + c^2*
d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
```

```
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{4c^2d} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int}{4c\sqrt{d+c^2dx}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}\left(a+\right.}{4c^2d}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3}{}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3}{}$$

Mathematica [A]
time = 0.40, size = 151, normalized size = 0.79

$$\frac{\frac{16acx(-3+2c^2x^2)\sqrt{d+c^2dx^2}}{d}+\frac{48a\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}}+\frac{b\sqrt{1+c^2x^2}\left(16\cosh\left(2\sinh^{-1}(cx)\right)-\cosh\left(4\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\left(6\sinh^{-1}(cx)-8\sinh\left(2\sinh^{-1}(cx)\right)+\sinh\left(4\sinh^{-1}(cx)\right)\right)\right)}{\sqrt{d+c^2dx^2}}}{128c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] ((16*a*c*x*(-3 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/d + (48*a*Log[c*d*x + Sqrt
[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh
[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*A
rcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/(128*c^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(166) = 332$.
time = 4.06, size = 519, normalized size = 2.70

method	result

default	$\frac{a\,x^{3}\sqrt{c^{2}d\,x^{2}+d}}{4c^{2}d}-\frac{3ax\sqrt{c^{2}d\,x^{2}+d}}{8c^{4}d}+\frac{3a\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{8c^{4}\sqrt{c^{2}d}}+b\left(\frac{3\sqrt{d\,(c^{2}x^{2}+1)}\ \mathrm{arcsinh}(cx}{16\sqrt{c^{2}x^{2}+1}\ c^{5}d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3/8*a
/c^4*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(3/16*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^2+1/256*(d*(c^2*x^
2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^
2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+
1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+
(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1
))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1
+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8
*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^
2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{4}(a+b\,\mathrm{asinh}\,(cx))}{\sqrt{d\,(c^{2}x^{2}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.147 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=142

$$\frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}-\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}}-\frac{2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{3c^4d}+\frac{x^2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{3c^2d}$$

```
[Out] 2/3*b*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/9*b*x^3*(c^2*x^2+1)^(1/
2)/c/(c^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+1
/3*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5812, 5798, 8, 30}

$$\frac{x^2\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{3c^4d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}+\frac{2bx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (2*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^3*Sqrt[1 + c^2
*x^2])/(9*c*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/(3*c^4*d) + (x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{3c^2d} - \frac{2\int \frac{x(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{3c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int}{3c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}\,(a+b\,\mathrm{s}}{3c^2d}$$

$$= \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{3c^4d} + \frac{x^2\sqrt{d}}{}$$

Mathematica [A]
time = 0.08, size = 93, normalized size = 0.65

$$\frac{bcx(6-c^2x^2)\sqrt{1+c^2x^2} + 3a(-2-c^2x^2+c^4x^4) + 3b(-2-c^2x^2+c^4x^4)\sinh^{-1}(cx)}{9c^4\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
[Out] (b*c*x*(6 - c^2*x^2)*Sqrt[1 + c^2*x^2] + 3*a*(-2 - c^2*x^2 + c^4*x^4) + 3*b
*(-2 - c^2*x^2 + c^4*x^4)*ArcSinh[c*x])/(9*c^4*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(122) = 244.
time = 2.65, size = 358, normalized size = 2.52

method	result
default	$a\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d} - \frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right) + b\left(\frac{\sqrt{d\,(c^2x^2+1)}\,\left(4c^4x^4+4\sqrt{c^2x^2+1}\,x^3c^3+5c^2x^2+3\sqrt{c^2x^2+}\right.}{72c^4d(c^2x^2+1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b*(1/72
*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(
c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)/c^4/d/(c^2*
x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsi
nh(c*x))/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2
+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))/c
^4/d/(c^2*x^2+1))
```

Maxima [**A**]
time = 0.29, size = 117, normalized size = 0.82

$$\frac{1}{3}\,b\left(\frac{\sqrt{c^2dx^2+d}\,x^2}{c^2d}-\frac{2\,\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{\sqrt{c^2dx^2+d}\,x^2}{c^2d}-\frac{2\,\sqrt{c^2dx^2+d}}{c^4d}\right)-\frac{(c^2x^3-6\,x)b}{9\,c^3\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/(c^4*d))*arc
sinh(c*x) + 1/3*a*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/
(c^4*d)) - 1/9*(c^2*x^3 - 6*x)*b/(c^3*sqrt(d))
```

Fricas [**A**]
time = 0.40, size = 132, normalized size = 0.93

$$\frac{3\,(bc^4x^4-bc^2x^2-2\,b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3\,ac^4x^4-3\,ac^2x^2-(bc^3x^3-6\,bcx)\sqrt{c^2x^2+1}-6\,a\right)\sqrt{c^2dx^2+d}}{9\,(c^6dx^2+c^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + (3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^3 - 6*b*c*x)*sqrt(c^2*x^
2 + 1) - 6*a)*sqrt(c^2*d*x^2 + d))/(c^6*d*x^2 + c^4*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\operatorname{asinh}\left(cx\right))}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 (a + b \operatorname{asinh}(c x))}{\sqrt{d c^2 x^2 + d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.148 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=119

$$-\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{d+c^2dx^2}}$$

```
[Out] -1/4*b*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*(a+b*arcsinh(c*x))^2
*(c^2*x^2+1)^(1/2)/b/c^3/(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5812, 5783, 30}

$$\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -1/4*(b*x^2*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
```

```
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{2c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x\,d\text{x}}{2c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(}{\sqrt{1+c^2}}}{2c^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{4bc^3\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.50, size = 121, normalized size = 1.02

$$-\frac{-\frac{4acx\sqrt{d+c^2dx^2}}{d}+\frac{4a\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}}+\frac{b\sqrt{1+c^2x^2}\left(\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{\sqrt{d+c^2dx^2}}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -1/8*((-4*a*c*x*Sqrt[d + c^2*d*x^2])/d + (4*a*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[c*x]] + 2*ArcSi
nh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(103) = 206.
time = 3.65, size = 273, normalized size = 2.29

method	result
default	$\frac{ax\sqrt{c^2d\,x^2+d}}{2c^2d} - \frac{a\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d\,(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c^3d} + \frac{\sqrt{d\,(c^2x^2+1)}\,(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a/c^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d
*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
```

```
)/c^3/d*arcsinh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2
*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1
)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(
c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{asinh}(cx))}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^2/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.149 $\int \frac{x(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=64

$$-\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{c^2d}$$

```
[Out] -b*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5798, 8}

$$\frac{\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{c^2d}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{d+c^2dx^2}}\\ &= -\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{c^2d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 74, normalized size = 1.16

$$\frac{\sqrt{d+c^2dx^2}\left(-bcx+a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\sinh^{-1}(cx)\right)}{c^2d\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]))/(c^2*d*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(58) = 116.
time = 1.09, size = 148, normalized size = 2.31

method	result
default	$\frac{a\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d\,(c^2x^2+1)}\left(c^2x^2+\sqrt{c^2x^2+1}\,cx+1\right)(\mathrm{arcsinh}(cx)-1)}{2c^2d(c^2x^2+1)}+\frac{\sqrt{d\,(c^2x^2+1)}\left(c^2x^2-\sqrt{\right.}}{2c^2d(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/c^2/d*(c^2*d*x^2+d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+
1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/2
)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))/c^2/d/(c^2*x^2+1))
```

Maxima [A]
time = 0.33, size = 55, normalized size = 0.86

$$-\frac{bx}{c\sqrt{d}}+\frac{\sqrt{c^2dx^2+d}\,b\,\mathrm{arsinh}\left(cx\right)}{c^2d}+\frac{\sqrt{c^2dx^2+d}\,a}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*
x^2 + d)*a/(c^2*d)
```

Fricas [A]
time = 0.37, size = 96, normalized size = 1.50

$$\frac{(bc^2x^2+b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2x^2-\sqrt{c^2x^2+1}\,bcx+a\right)\sqrt{c^2dx^2+d}}{c^4dx^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*
x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{asinh}(cx))}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/sqrt(c^2*d*x^2 + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,(a + b\operatorname{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.150 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\ &= \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.02

$$\frac{\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)\left(2a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(2*a + b*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d
*x^2])
```

Maple [A]
time = 0.35, size = 77, normalized size = 1.64

method	result	size
default	$\frac{a \ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d\, x^2+d}\right)}{\sqrt{c^2 d}}+\frac{b \sqrt{d\left(c^2 x^2+1\right)}\, \operatorname{arcsinh}(c x)^2}{2 \sqrt{c^2 x^2+1}\, c d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.60

$$\frac{b \operatorname{arsinh}(cx)^2}{2\, c\sqrt{d}}+\frac{a \operatorname{arsinh}(cx)}{c\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b \operatorname{asinh}(cx)}{\sqrt{d\left(c^2x^2+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

3.151 $\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\,\mathrm{I}}{\sqrt{}}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2
*d*x^2+d)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*
d*x^2+d)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*
x^2+d)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5816, 4267, 2317, 2438}

$$-\frac{2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d
+ c^2*d*x^2] - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[d + c
^2*d*x^2] + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d
*x^2]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \text{log}\ldots\right.}{\sqrt{d+\ldots}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\text{lo}\ldots}{\ldots}\right.}{\sqrt{d+c^2\ldots}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\text{Li}_2\left(-e^{\sinh^{-1}(c\ldots}\right.}{\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.24, size = 129, normalized size = 1.06

$$\frac{a\log(x)}{\sqrt{d}} - \frac{a\log\left(d+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{1+c^2x^2}\left(\sinh^{-1}(cx)\left(\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\log\left(1+e^{-\sinh^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{d(1+c^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/Sqrt[d] + (
b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(
-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[
c*x])]))/Sqrt[d*(1 + c^2*x^2)]
```

Maple [A]
time = 1.89, size = 234, normalized size = 1.92

method	result
default	$-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{\sqrt{d}}+\frac{b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}+\frac{b\sqrt{d\,(c^2x^2+1)}}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x), x) - a*ar
csinh(1/(c*abs(x)))/sqrt(d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{asinh}(cx)}{x\sqrt{d\,(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(1/2)), x)
```

3.152 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{dx}+\frac{bc\sqrt{1+c^2x^2}\,\log(x)}{\sqrt{d+c^2dx^2}}$$

```
[Out] b*c*ln(x)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))*(c^2*d*x
^2+d)^(1/2)/d/x
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5800, 29}

$$\frac{bc\sqrt{c^2x^2+1}\,\log(x)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[1 + c^2*x^2
]*Log[x])/Sqrt[d + c^2*d*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx&=-\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{dx}+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x}\,dx}{\sqrt{d+c^2dx^2}}\\&=-\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{dx}+\frac{bc\sqrt{1+c^2x^2}\,\log(x)}{\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 1.06

$$-\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{dx}+\frac{bc\sqrt{d(1+c^2x^2)}\,\log(x)}{d\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[d*(1 + c^2*
x^2)]*Log[x])/(d*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(57) = 114.
time = 1.97, size = 183, normalized size = 2.90

method	result
default	$-\frac{a\sqrt{c^2d\,x^2+d}}{dx}-\frac{b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)c}{\sqrt{c^2x^2+1}\,d}-\frac{b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x\,c^2}{(c^2x^2+1)d}-\frac{b\sqrt{d\,(c^2x^2+1)}\,\operatorname{ar}}{(c^2x^2+1)xd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
[Out] -a/d/x*(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcs
inh(c*x)*c-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)*x/d*c^2-b*(d*(c
^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/x/d+b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [A]
time = 0.35, size = 101, normalized size = 1.60

$$-\frac{\left((-1)^{2\,c^2dx^2+2\,d}\,\sqrt{d}\,\log\left(2\,c^2d+\frac{2\,d}{x^2}\right)-\sqrt{d}\,\log\left(x^2+\frac{1}{c^2}\right)\right)bc}{2\,d}-\frac{\sqrt{c^2dx^2+d}\,b\,\operatorname{arsinh}(cx)}{dx}-\frac{\sqrt{c^2dx^2+d}\,a}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] -1/2*((-1)^(2*c^2*d*x^2 + 2*d)*sqrt(d)*log(2*c^2*d + 2*d/x^2) - sqrt(d)*log
(x^2 + 1/c^2))*b*c/d - sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(d*x) - sqrt(c^2*
d*x^2 + d)*a/(d*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(57) = 114.
time = 0.38, size = 132, normalized size = 2.10

$$\frac{bc\sqrt{d}\,x\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,(x^4-1)\sqrt{d}+d}{c^2x^4+x^2}\right)-2\sqrt{c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2dx^2+d}\,a}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*sqrt(d)*x*log((c^2*d*x^6 + c^2*d*x^2 + d*x^4 + sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x^4 + x^2)) - 2*sqrt(c^2*d*x
^2 + d)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*a)/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{x^2 \sqrt{d(c^2x^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^2\,\sqrt{d\,c^2\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(1/2)), x)
```

3.153 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=203

$$-\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{2dx^2}+\frac{c^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}}+\frac{bc}{}$$

```
[Out] -1/2*b*c*(c^2*x^2+1)^(1/2)/x/(c^2*d*x^2+d)^(1/2)+c^2*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/2*b*c^2
*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/
2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1
/2)-1/2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A]

time = 0.16, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5809, 5816, 4267, 2317, 2438, 30}

$$-\frac{\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{2dx^2}+\frac{c^2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2dx^2+d}}+\frac{bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -1/2*(b*c*Sqrt[1 + c^2*x^2])/(x*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x]))/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b*c^2*Sqrt[1 + c^2*x^2]
*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2]) - (b*c^2*Sqrt[1 + c^2
*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx &= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}-\frac{1}{2}c^2\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right) \cdots}{2\sqrt{d+c^2d \cdots}}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}-\frac{\left(c^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-} \cdots}{x\sqrt{1+c \cdots}}}{2\sqrt{d+c^2dx^2}}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}-\frac{\left(c^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int (\cdots\right.}{2\sqrt{\cdots}}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}+\frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1} \cdots\right.}{\sqrt{d+\cdots}}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}+\frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1} \cdots\right.}{\sqrt{d+\cdots}}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}+\frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1} \cdots\right.}{\sqrt{d+\cdots}}
\end{aligned}$$

Mathematica [A]
time = 2.00, size = 229, normalized size = 1.13

$$\frac{-\frac{4a\sqrt{d+c^2dx^2}}{x^2}-4ac^2\sqrt{d}\log(x)+4ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bc^2d^2(1+c^2x^2)^{3/2}\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+4\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)-4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\operatorname{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{(d+c^2dx^2)^{3/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2])/x^2 - 4*a*c^2*Sqrt[d]*Log[x] + 4*a*c^2*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-2*C
oth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]*
Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 4*
PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[
c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2
))/(8*d)
```

Maple [A]
time = 3.49, size = 380, normalized size = 1.87

method	result
default	$-\frac{a\sqrt{c^2d\,x^2+d}}{2d\,x^2}+\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}-\frac{b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)c^2}{2d(c^2x^2+1)}-\frac{b\sqrt{d\,(c^2x^2+1)}}{2dx\sqrt{c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d
*x^2+d)^(1/2))/x)-1/2*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*c^
2-1/2*b*(d*(c^2*x^2+1))^(1/2)/d/x/(c^2*x^2+1)^(1/2)*c-1/2*b*(d*(c^2*x^2+1))
^(1/2)/d/x^2/(c^2*x^2+1)*arcsinh(c*x)-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2+1/2*b*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1
/2))*c^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(d) - sqrt(c^2*d*x^2 + d)/(d*x^2))*a + b
*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{x^3 \sqrt{d (c^2x^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^3\,\sqrt{d\,c^2\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(1/2)), x)
```

3.154 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=141

$$-\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{2c^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3dx}-\frac{2bc^3\sqrt{1+c^2x^2}\,\log}{3\sqrt{d+c^2dx^2}}$$

```
[Out] -1/6*b*c*(c^2*x^2+1)^(1/2)/x^2/(c^2*d*x^2+d)^(1/2)-2/3*b*c^3*ln(x)*(c^2*x^2
+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/
x^3+2/3*c^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A]
time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5809, 5800, 29, 30}

$$\frac{2c^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3dx}-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}}-\frac{2bc^3\sqrt{c^2x^2+1}\,\log(x)}{3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -1/6*(b*c*Sqrt[1 + c^2*x^2])/(x^2*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x]))/(3*d*x) - (2*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*Sqrt[d + c^2*d*x
^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3dx^3} - \frac{1}{3}\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x\right.}{3\sqrt{d+}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh\right.}{3dx}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh\right.}{3dx}$$

Mathematica [A]
time = 0.13, size = 135, normalized size = 0.96

$$\frac{bcx\sqrt{1+c^2x^2}\,(-1+6c^2x^2)+2a(-1+c^2x^2+2c^4x^4)+2b(-1+c^2x^2+2c^4x^4)\sinh^{-1}(cx)}{6x^3\sqrt{d+c^2dx^2}} - \frac{2bc^3\sqrt{d\,(1+c^2x^2)}\,\log(x)}{3d\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (b*c*x*Sqrt[1 + c^2*x^2]*(-1 + 6*c^2*x^2) + 2*a*(-1 + c^2*x^2 + 2*c^4*x^4)
+ 2*b*(-1 + c^2*x^2 + 2*c^4*x^4)*ArcSinh[c*x])/(6*x^3*Sqrt[d + c^2*d*x^2])
- (2*b*c^3*Sqrt[d*(1 + c^2*x^2)]*Log[x])/(3*d*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(121) = 242$.
time = 4.65, size = 792, normalized size = 5.62

method	result
default	$a\left(-\frac{\sqrt{c^2d\,x^2+d}}{3d\,x^3}+\frac{2c^2\sqrt{c^2d\,x^2+d}}{3dx}\right)+\frac{4b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)c^3}{3\sqrt{c^2x^2+1}\,d}+\frac{2b\sqrt{d\,(c^2x^2+1)}\,x^5c^8}{3(3c^4x^4+2c^2x^2-1)d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/d/x^3*(c^2*d*x^2+d)^(1/2)+2/3*c^2/d/x*(c^2*d*x^2+d)^(1/2))+4/3*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*c^3+2/3*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^5/d*c^8-2/3*b*(d*(c^2*x^2+1))^(1/2)/(3*
c^4*x^4+2*c^2*x^2-1)*x^3/d*(c^2*x^2+1)*c^6+2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+2*c^2*x^2-1)*x^3/d*arcsinh(c*x)*c^6-2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)*x^2/d*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+1/3*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^3/d*c^6+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*
c^4*x^4+2*c^2*x^2-1)*x/d*(c^2*x^2+1)*c^4+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+2*c^2*x^2-1)*x/d*arcsinh(c*x)*c^4+2/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)/d*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/3*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x/d*c^4-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
4+2*c^2*x^2-1)/d*(c^2*x^2+1)^(1/2)*c^3-4/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)/x/d*arcsinh(c*x)*c^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4
+2*c^2*x^2-1)/x^2/d*(c^2*x^2+1)^(1/2)*c+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+2*c^2*x^2-1)/x^3/d*arcsinh(c*x)-2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

Maxima [A]
time = 0.28, size = 121, normalized size = 0.86

$$-\frac{1}{6}\left(\frac{4\,c^2\log\left(x\right)}{\sqrt{d}}+\frac{1}{\sqrt{d}\,x^2}\right)bc+\frac{1}{3}\,b\left(\frac{2\,\sqrt{c^2dx^2+d}\,c^2}{dx}-\frac{\sqrt{c^2dx^2+d}}{dx^3}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{2\,\sqrt{c^2dx^2+d}\,c^2}{dx}-\frac{\sqrt{c^2dx^2+d}}{dx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(4*c^2*log(x)/sqrt(d) + 1/(sqrt(d)*x^2))*b*c + 1/3*b*(2*sqrt(c^2*d*x^2
 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))*arcsinh(c*x) + 1/3*a*(2*sqrt
(c^2*d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))
```

Fricas [A]
time = 0.43, size = 222, normalized size = 1.57

$$\frac{2\left(2\,bc^4x^4+bc^2x^2-b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(bc^5x^5+bc^3x^3\right)\sqrt{d}\,\log\left(\frac{c^2dx^6+c^2dx^2+dx^4-\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,(x^4-1)\sqrt{d}+d}{c^2x^4+x^2}\right)+\left(4\,ac^4x^4+2\,ac^2x^2+\left(bcx^3-bcx\right)\sqrt{c^2x^2+1}-2\,a\right)\sqrt{c^2dx^2+d}}{6\left(c^2dx^5+dx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*b*c^4*x^4 + b*c^2*x^2 - b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*(b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 +
```

```
 d*x^4 - sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*
x^4 + x^2)) + (4*a*c^4*x^4 + 2*a*c^2*x^2 + (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 +
 1) - 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*d*x^5 + d*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{x^{4} \sqrt{d \left(c^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^4\,\sqrt{d\,c^2\,x^2+d}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(1/2)), x)
```

3.155 $\int \frac{x^5(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{5bx\sqrt{d+c^2dx^2}}{3c^5d^2\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{9c^3d^2\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{c^6d\sqrt{d+c^2dx^2}}-\frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6d^2}+\frac{(d+c^2dx^2)^{3/2}\left(a\right.}{3c^6}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^6/d^3+(-a-b*arcsinh(c*x))/c^6/
d/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^2+5/3*
b*x*(c^2*d*x^2+d)^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-1/9*b*x^3*(c^2*d*x^2+d)^(
1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)+b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^6/d^2/(c
^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 45, 5804, 12, 1167, 209}

$$\frac{(c^2dx^2+d)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^6d^3}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^6d^2}-\frac{a+b\sinh^{-1}(cx)}{c^6d\sqrt{c^2dx^2+d}}+\frac{b\text{ArcTan}(cx)\sqrt{c^2dx^2+d}}{c^6d^2\sqrt{c^2x^2+1}}+\frac{5bx\sqrt{c^2dx^2+d}}{3c^5d^2\sqrt{c^2x^2+1}}-\frac{bx^3\sqrt{c^2dx^2+d}}{9c^3d^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (5*b*x*Sqrt[d + c^2*d*x^2])/(3*c^5*d^2*Sqrt[1 + c^2*x^2]) - (b*x^3*Sqrt[d +
 c^2*d*x^2])/(9*c^3*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(c^6*d*Sq
rt[d + c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^6*d^2)
 + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6*d^3) + (b*Sqrt[d + c
^2*d*x^2]*ArcTan[c*x])/(c^6*d^2*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{4\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^4}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2} - \frac{8\int\frac{x\left(a+b\sinh^{-1}\right.}{\sqrt{d+c^2}}}{3c^4d}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{c^5d\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{8\sqrt{d+c^2dx}}{}$$

$$= \frac{5bx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{8\sqrt{d+c^2dx}}{}$$

Mathematica [A]
time = 0.16, size = 148, normalized size = 0.70

$$\frac{\sqrt{d+c^2dx^2}\left(bcx(15-c^2x^2)\sqrt{1+c^2x^2}+3a(-8-4c^2x^2+c^4x^4)+3b(-8-4c^2x^2+c^4x^4)\sinh^{-1}(cx)\right)}{9c^6d^2(1+c^2x^2)} + \frac{b\sqrt{d(1+c^2x^2)}\,\mathrm{ArcTan}(cx)}{c^6d^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]

[Out] (Sqrt[d + c^2*d*x^2]*(b*c*x*(15 - c^2*x^2)*Sqrt[1 + c^2*x^2] + 3*a*(-8 - 4*c^2*x^2 + c^4*x^4) + 3*b*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x]))/(9*c^6*d^2*(1 + c^2*x^2)) + (b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^6*d^2*Sqrt[1 + c^2*x^2])

Maple [C] Result contains complex when optimal does not.
time = 3.17, size = 367, normalized size = 1.73

method	result
default	$a\left(\frac{x^4}{3c^2d\sqrt{c^2d\,x^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)}{3c^2}\right)-\frac{ib\sqrt{d\,(c^2x^2+1)}\,\ln\left(cx+\sqrt{c^2x^2+}\right.}{\sqrt{c^2x^2+1}\,c^6d^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] a*(1/3*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3/c^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2/d/c^4/(c^2*d*x^2+d)^(1/2)))-I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-I)+I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)+I)+1/3*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-1/9*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x^3-4/3*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+5/3*b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*x-8/3*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^2/(c^2*x^2+1)*arcsinh(c*x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/3*a*(x^4/(sqrt(c^2*d*x^2 + d)*c^2*d) - 4*x^2/(sqrt(c^2*d*x^2 + d)*c^4*d) - 8/(sqrt(c^2*d*x^2 + d)*c^6*d)) + 1/3*b*((c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*x^2 - 8*sqrt(d))*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^6*d^2) - integrate((c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*x^2 - 8*sqrt(d))/(sqrt(c^2*x^2 + 1)*x), x)/(c^6*d^2) + 3*integrate(1/3*(c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*x^2 - 8*sqrt(d))/(c^9*d^2*x^4 + c^7*d^2*x^2 + (c^8*d^2*x^3 + c^6*d^2*x)*sqrt(c^2*x^2 + 1)), x))

Fricas [A]
time = 0.45, size = 197, normalized size = 0.93

$$-\frac{9\,(bc^2x^2+b)\sqrt{d}\,\arctan\left(\frac{2\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)-6\,(bc^4x^4-4\,bc^2x^2-8\,b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)-2\left(3\,ac^4x^4-12\,ac^2x^2-(bc^3x^3-15\,bcx)\sqrt{c^2x^2+1}-24\,a\right)\sqrt{c^2dx^2+d}}{18\,(c^8d^2x^2+c^6d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18*(9*(b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2
+ 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 6*(b*c^4*x^4 - 4*b*c^2*x^2 - 8*b)*sqrt(
c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(3*a*c^4*x^4 - 12*a*c^2*x^2
 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1) - 24*a)*sqrt(c^2*d*x^2 + d))/(c
^8*d^2*x^2 + c^6*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5(a+b\,\mathrm{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^5\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.156 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{4bc^5d\sqrt{d+c^2dx^2}}$$

```
[Out] -x^3*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)-1/4*b*x^2*(c^2*x^2+1)^(1/
2)/c^3/d/(c^2*d*x^2+d)^(1/2)-3/4*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c
^5/d/(c^2*d*x^2+d)^(1/2)-1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d
*x^2+d)^(1/2)+3/2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5810, 5812, 5783, 30, 272, 45}

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^5d\sqrt{c^2dx^2+d}}+\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2c^5d\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c^3d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -1/4*(b*x^2*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*Ar
cSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x]))/(2*c^4*d^2) - (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(
4*b*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*
c^5*d*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^3}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2} - \frac{3\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{2c^4d}$$

$$= -\frac{3bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}$$

Mathematica [A]
time = 0.30, size = 161, normalized size = 0.78

$$\frac{4ac\sqrt{d}\,x(3+c^2x^2)-12a\sqrt{d+c^2dx^2}\,\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)+b\sqrt{d}\left(8cx\sinh^{-1}(cx)-\sqrt{1+c^2x^2}\,\left(6\sinh^{-1}(cx)^2+\cosh\left(2\sinh^{-1}(cx)\right)+4\log\left(1+c^2x^2\right)-2\sinh^{-1}(cx)\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{8c^5d^{3/2}\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[
d]*Sqrt[d + c^2*d*x^2]] + b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]
*(6*ArcSinh[c*x]^2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[
c*x]*Sinh[2*ArcSinh[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(182) = 364$.
time = 4.09, size = 366, normalized size = 1.78

method	result
default	$\frac{a\,x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3ax}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{3b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c^5d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3/2*a
/c^4/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-3/4*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)^2+1/2*b*(d*(c^2*
x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1
/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*
x^2+1)*arcsinh(c*x)*x+b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^3/(sqrt(c^2*d*x^2 + d)*c^2*d) + 3*x/(sqrt(c^2*d*x^2 + d)*c^4*d) -
3*arcsinh(c*x)/(c^5*d^(3/2))) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1)
)/(c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{asinh}(cx))}{(d(c^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.157 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{bx\sqrt{d+c^2dx^2}}{c^3d^2\sqrt{1+c^2x^2}}+\frac{a+b\sinh^{-1}(cx)}{c^4d\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{c^4d^2}-\frac{b\sqrt{d+c^2dx^2}\,\mathrm{ArcTan}(cx)}{c^4d^2\sqrt{1+c^2x^2}}$$

```
[Out] (a+b*arcsinh(c*x))/c^4/d/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c^4/d^2-b*x*(c^2*d*x^2+d)^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)-b*arctan
(c*x)*(c^2*d*x^2+d)^(1/2)/c^4/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 45, 5804, 12, 396, 209}

$$\frac{\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{c^4d^2}+\frac{a+b\sinh^{-1}(cx)}{c^4d\sqrt{c^2dx^2+d}}-\frac{b\mathrm{ArcTan}(cx)\sqrt{c^2dx^2+d}}{c^4d^2\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{c^3d^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((b*x*Sqrt[d + c^2*d*x^2])/(c^3*d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c
*x])/(c^4*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(c^4*d^2) - (b*Sqrt[d + c^2*d*x^2]*ArcTan[c*x])/(c^4*d^2*Sqrt[1 + c^2*x^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^2}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{bx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}$$

Mathematica [A]
time = 0.15, size = 143, normalized size = 1.05

$$\frac{\sqrt{d+c^2dx^2}\left(a\sqrt{1+c^2x^2}\,(2+c^2x^2)-b(cx+c^3x^3)+b\sqrt{1+c^2x^2}\,(2+c^2x^2)\sinh^{-1}(cx)\right)}{c^4d^2\left(1+c^2x^2\right)^{3/2}} - \frac{b\sqrt{d\left(1+c^2x^2\right)}\,\mathrm{ArcTan}(cx)}{c^4d^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a*Sqrt[1 + c^2*x^2]*(2 + c^2*x^2) - b*(c*x + c^3*x^3)
 + b*Sqrt[1 + c^2*x^2]*(2 + c^2*x^2)*ArcSinh[c*x]))/(c^4*d^2*(1 + c^2*x^2)^
(3/2)) - (b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^4*d^2*Sqrt[1 + c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 2.61, size = 261, normalized size = 1.92

method	result
default	$a\left(\frac{x^2}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2 d\,x^2+d}}\right)+\frac{b\sqrt{d\,(c^2x^2+1)}\;\mathrm{arcsinh}(cx)x^2}{c^2d^2(c^2x^2+1)}-\frac{b\sqrt{d\,(c^2x^2+1)}\;x}{c^3d^2\sqrt{c^2x^2+1}}+\frac{2b\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2/d/c^4/(c^2*d*x^2+d)^(1/2))+b*(d*(c^2*x^2
+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)/c^3
/d^2/(c^2*x^2+1)^(1/2)*x+2*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcs
inh(c*x)+I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*ln(c*x+(c^2*x^
2+1)^(1/2)-I)-I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*ln(c*x+(c
^2*x^2+1)^(1/2)+I)
```

Maxima [A]
time = 0.54, size = 119, normalized size = 0.88

$$-bc\left(\frac{x}{c^4d^{\frac{3}{2}}}+\frac{\arctan(cx)}{c^5d^{\frac{3}{2}}}\right)+b\left(\frac{x^2}{\sqrt{c^2dx^2+d}\,c^2d}+\frac{2}{\sqrt{c^2dx^2+d}\,c^4d}\right)\mathrm{arsinh}(cx)+a\left(\frac{x^2}{\sqrt{c^2dx^2+d}\,c^2d}+\frac{2}{\sqrt{c^2dx^2+d}\,c^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -b*c*(x/(c^4*d^(3/2)) + arctan(c*x)/(c^5*d^(3/2))) + b*(x^2/(sqrt(c^2*d*x^2
 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))*arcsinh(c*x) + a*(x^2/(sqrt(c
^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))
```

Fricas [A]
time = 0.41, size = 166, normalized size = 1.22

$$\frac{(bc^2x^2+b)\sqrt{d}\,\arctan\left(\frac{2\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+2\,(bc^2x^2+2\,b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(ac^2x^2-\sqrt{c^2x^2+1}\,bcx+2\,a\right)\sqrt{c^2dx^2+d}}{2\,(c^6d^2x^2+c^4d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)
*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*(b*c^2*x^2 + 2*b)*sqrt(c^2*d*x^2 + d)*log
```

```
(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + 2*a)*s
qrt(c^2*d*x^2 + d))/(c^6*d^2*x^2 + c^4*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{asinh}\,(cx))}{(d\,(c^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a + b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2 + d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.158 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{x(a+b\sinh^{-1}(cx))}{c^2d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^2}{2bc^3d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\,\log(1+c^2x^2)}{2c^3d\sqrt{d+c^2dx^2}}$$

```
[Out] -x*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c
^2*x^2+1)^(1/2)/b/c^3/d/(c^2*d*x^2+d)^(1/2)+1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)
^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5810, 5783, 266}

$$-\frac{x(a+b\sinh^{-1}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))^2}{2bc^3d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\log(c^2x^2+1)}{2c^3d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2])) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^2)/(2*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2])/(2*c^3*d*Sqrt[d + c^2*d*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
```

```
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx=-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^3d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}}{\sqrt{1+c^2}}}{c^2d\sqrt{d+c^2dx^2}}$$

$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\log(1}{2c^3d\sqrt{d+c^2}}$$

Mathematica [A]
time = 0.12, size = 146, normalized size = 1.12

$$-\frac{ax\sqrt{d\left(1+c^2x^2\right)}}{c^2d^2\left(1+c^2x^2\right)}+\frac{b\left(-2cx\sinh^{-1}(cx)+\sqrt{1+c^2x^2}\left(\sinh^{-1}(cx)^2+2\log\left(\sqrt{1+c^2x^2}\right)\right)\right)}{2c^3d\sqrt{d\left(1+c^2x^2\right)}}+\frac{a\log\left(cdx+\sqrt{d}\sqrt{d\left(1+c^2x^2\right)}\right)}{c^3d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a*x*Sqrt[d*(1 + c^2*x^2)])/(c^2*d^2*(1 + c^2*x^2))) + (b*(-2*c*x*ArcSinh
[c*x] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]^2 + 2*Log[Sqrt[1 + c^2*x^2]])))/(2*
c^3*d*Sqrt[d*(1 + c^2*x^2)]) + (a*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]
])/(c^3*d^(3/2))
```

Maple [A]
time = 3.56, size = 232, normalized size = 1.78

method	result
default	$-\frac{ax}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{a\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^2d\sqrt{c^2d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b\sqrt{d\left(c^2x^2+1\right)}}{\sqrt{c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a/c^2/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+
d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d
```

```
^2*arcsinh(c*x)^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh
(c*x)-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2/(c^2*x^2+1)*x+b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d) - arcsinh(c*x)/(c^3*d^(3/2))) + b*integra
te(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.159 $\int \frac{x(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=70

$$-\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\,\text{ArcTan}(cx)}{c^2d\sqrt{d+c^2dx^2}}$$

```
[Out] (-a-b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)+b*arctan(c*x)*(c^2*x^2+1)^(1/
2)/c^2/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5798, 209}

$$\frac{b\sqrt{c^2x^2+1}\,\text{ArcTan}(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])) + (b*Sqrt[1 + c^2*x^2]*
ArcTan[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 82, normalized size = 1.17

$$-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{c^2d^2\left(1+c^2x^2\right)}+\frac{b\sqrt{d\left(1+c^2x^2\right)}\,\text{ArcTan}(cx)}{c^2d^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^2*d^2*(1 + c^2*x^2))) + (b*
Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^2*d^2*Sqrt[1 + c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.92, size = 164, normalized size = 2.34

method	result
default	$-\frac{a}{c^2d\sqrt{c^2d\,x^2+d}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)}{c^2d^2(c^2x^2+1)}+\frac{ib\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{\sqrt{c^2x^2+1}\,c^2d^2}-\frac{ib\sqrt{d(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/c^2/d/(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*ar
csinh(c*x)+I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+(c^2*
x^2+1)^(1/2)+I)-I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+
(c^2*x^2+1)^(1/2)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(c^2*d^(3/2)) - log(c*x + sqrt(c^2
*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^2*d^(3/2)) - integrate(1/(c^5*d^(3/2)*x^4 +
 c^3*d^(3/2)*x^2 + (c^4*d^(3/2)*x^3 + c^2*d^(3/2)*x)*sqrt(c^2*x^2 + 1)), x)
) - a/(sqrt(c^2*d*x^2 + d)*c^2*d)
```

Fricas [A]
time = 0.45, size = 128, normalized size = 1.83

$$-\frac{(bc^2x^2+b)\sqrt{d}\,\arctan\left(\frac{2\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+2\sqrt{c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{c^2dx^2+d}\,a}{2\left(c^4d^2x^2+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1
)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*sqrt(c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\operatorname{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.160 $\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{2cd\sqrt{d+c^2dx^2}}$$

[Out] x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5787, 266}

$$\frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]

[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d*x^2])

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{3/2}}\,dx&=\frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}\\&=\frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{2cd\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 100, normalized size = 1.32

$$\frac{\sqrt{d+c^2dx^2}\left(2acx\sqrt{1+c^2x^2}+2bcx\sqrt{1+c^2x^2}\sinh^{-1}(cx)-(b+bc^2x^2)\log(1+c^2x^2)\right)}{2cd^2(1+c^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(2*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x] - (b + b*c^2*x^2)*Log[1 + c^2*x^2]))/(2*c*d^2*(1 + c^2*x^2)^(
3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(68) = 136.
time = 1.38, size = 143, normalized size = 1.88

method	result
default	$\frac{ax}{d\sqrt{c^2d\,x^2+d}}+\frac{b\sqrt{d\,(c^2x^2+1)}\;\mathrm{arcsinh}(cx)}{\sqrt{c^2x^2+1}\;c\,d^2}+\frac{b\sqrt{d\,(c^2x^2+1)}\;\mathrm{arcsinh}(cx)x}{d^2(c^2x^2+1)}-\frac{b\sqrt{d\,(c^2x^2+1)}\;\ln\left(1+\left(\right.\right.}{\sqrt{c^2x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x/d/(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*a
rcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^2*x^2+1)*x-b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.76

$$\frac{bx\,\mathrm{arsinh}\,(cx)}{\sqrt{c^2dx^2+d}\,d}+\frac{ax}{\sqrt{c^2dx^2+d}\,d}-\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{2\,cd^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a*x/(sqrt(c^2*d*x^2 + d)*d) - 1/
2*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*
x^2 + d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{(d(c^2x^2 + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{{(d\,c^2\,x^2 + d)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2), x)
```

3.161 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\,\text{ArcTan}(cx)}{d\sqrt{d+c^2dx^2}}-\frac{2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x}}{}$$

```
[Out] (a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-b*arctan(c*x)*(c^2*x^2+1)^(1/2)/d/
(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^
2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5811, 5816, 4267, 2317, 2438, 209}

$$\frac{a+b\sinh^{-1}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\text{ArcTan}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcTan[
c*x])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*A
rcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Poly
Log[2, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*Pol
yLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)\mathrm{csch}(x)\,d\right.}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\tanh^{-}}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\tanh^{-}}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\tanh^{-}}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.62, size = 231, normalized size = 1.19

$$\frac{\frac{a\sqrt{d+c^2dx^2}}{1+c^2x^2} + a\sqrt{d}\,\log(x) - a\sqrt{d}\,\log\left(d+\sqrt{d}\,\sqrt{d+c^2dx^2}\right) + \frac{bd\left(\sinh^{-1}(cx) - 2\sqrt{1+c^2x^2}\,\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right) + \sqrt{1+c^2x^2}\,\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) - \sqrt{1+c^2x^2}\,\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) + \sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - \sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{d+c^2dx^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] ((a*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2) + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d
 + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d*(ArcSinh[c*x] - 2*Sqrt[1 + c^2*x^2]*
ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-A
rcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] +
 Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*PolyL
og[2, E^(-ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2])/d^2
```

Maple [A]
time = 1.65, size = 274, normalized size = 1.41

method	result
default	$\frac{a}{d\sqrt{c^2 d\,x^2+d}} - \frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}} + \frac{b\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}(cx)}{d^2\left(c^2x^2+1\right)} - \frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arctan}}{\sqrt{c^2x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/d/(c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)
+b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^2+1)*arcsinh(c*x)-2*b*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d^2*arctan(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(1/(c*abs(x)))/d^(3/2) - 1/(sqrt(c^2*d*x^2 + d)*d)) + b*integrat
e(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*
x^3 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{x \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asinh}(c\,x)}{x\,\left(d\,c^2\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(3/2)), x)
```

3.162 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}}+\frac{bc\sqrt{d+c^2dx^2}\,\log(x)}{d^2\sqrt{1+c^2x^2}}+\frac{bc\sqrt{d+c^2dx^2}\,\log\left(1+c^2x^2\right)}{2d^2\sqrt{1+c^2x^2}}$$

```
[Out] (-a-b*arcsinh(c*x))/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))/d/(c
^2*d*x^2+d)^(1/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)+1/2*b
*c*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {277, 197, 5804, 12, 457, 78}

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{c^2dx^2+d}}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{d^2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\,\log\left(c^2x^2+1\right)}{2d^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSinh
[c*x]))/(d*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[x])/(d^2*Sqr
t[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(2*d^2*Sqrt[1
+ c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}}-\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x(1+c^2x^2)}\,dx}{d\sqrt{d+c^2dx^2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}}+\frac{\left(bc\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{x(1+c^2x)}\right.}{2d\sqrt{d+c^2dx^2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}}+\frac{bc\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{d\sqrt{d+c^2dx^2}}+\\ &= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}}+\frac{bc\sqrt{1+c^2x^2}\,\log(x)}{d\sqrt{d+c^2dx^2}}+\end{aligned}$$

Mathematica [A]

time = 0.19, size = 163, normalized size = 1.14

$$-\frac{\sqrt{d+c^2dx^2}\left(2a\sqrt{1+c^2x^2}+4ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}\left(1+2c^2x^2\right)\sinh^{-1}(cx)+bcx\left(1+c^2x^2\right)\log\left(1+\frac{1}{c^2x^2}\right)-2bcx\log\left(1+c^2x^2\right)-2bc^3x^3\log\left(1+c^2x^2\right)\right)}{2d^2x\left(1+c^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[d + c^2*d*x^2]*(2*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2
*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2)*ArcSinh[c*x] + b*c*x*(1 + c^2
*x^2)*Log[1 + 1/(c^2*x^2)] - 2*b*c*x*Log[1 + c^2*x^2] - 2*b*c^3*x^3*Log[1 +
 c^2*x^2]))/(d^2*x*(1 + c^2*x^2)^(3/2))
```

Maple [A]
time = 1.90, size = 206, normalized size = 1.44

method	result
default	$a\left(-\frac{1}{dx\sqrt{c^2 d\,x^2+d}}-\frac{2c^2x}{d\sqrt{c^2 d\,x^2+d}}\right)-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)c}{\sqrt{c^2x^2+1}\,d^2}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(c}{\left(c^2x^2+1\right)d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/2))-2*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c-2*b*(d*(c^2*x^2+1))^(1/2)*
arcsinh(c*x)/(c^2*x^2+1)*x/d^2*c^2-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^
2*x^2+1)/x/d^2+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x
^2+1)^(1/2))^4-1)*c
```

Maxima [A]
time = 0.30, size = 119, normalized size = 0.83

$$\frac{1}{2}\,bc\left(\frac{\log\left(c^2x^2+1\right)}{d^{\frac{3}{2}}}+\frac{2\,\log\left(x\right)}{d^{\frac{3}{2}}}\right)-\left(\frac{2\,c^2x}{\sqrt{c^2dx^2+d}\,d}+\frac{1}{\sqrt{c^2dx^2+d}\,dx}\right)b\operatorname{arsinh}\left(cx\right)-\left(\frac{2\,c^2x}{\sqrt{c^2dx^2+d}\,d}+\frac{1}{\sqrt{c^2dx^2+d}\,dx}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*c*(log(c^2*x^2 + 1)/d^(3/2) + 2*log(x)/d^(3/2)) - (2*c^2*x/(sqrt(c^2*
d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*b*arcsinh(c*x) - (2*c^2*x/(sqr
t(c^2*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*
x^4 + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\,(cx)}{x^2\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(3/2)), x)
```

3.163 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=287

$$-\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}}-\frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}}+\frac{bc^2\sqrt{1+c^2x^2}\,\mathrm{ArcTan}(cx)}{d\sqrt{d+c^2dx^2}}+\frac{3c^2\sqrt{1+c^2x^2}}{}$$

```
[Out] -3/2*c^2*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)+1/2*(-a-b*arcsinh(c*x))/d
/x^2/(c^2*d*x^2+d)^(1/2)-1/2*b*c*(c^2*x^2+1)^(1/2)/d/x/(c^2*d*x^2+d)^(1/2)+
b*c^2*arctan(c*x)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arcsin
h(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1
/2)+3/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*
x^2+d)^(1/2)-3/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d
/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5809, 5811, 5816, 4267, 2317, 2438, 209, 331}

$$-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{3c^2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{bc^2\sqrt{c^2x^2+1}\,\mathrm{ArcTan}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{3bc^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}}{2dx\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(b*c*Sqrt[1 + c^2*x^2])/(d*x*Sqrt[d + c^2*d*x^2]) - (3*c^2*(a + b*ArcS
inh[c*x]))/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])/(2*d*x^2*Sqrt[d
 + c^2*d*x^2]) + (b*c^2*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(d*Sqrt[d + c^2*d*x^
2]) + (3*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]]
)/(d*Sqrt[d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSin
h[c*x]])/(2*d*Sqrt[d + c^2*d*x^2]) - (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2,
E^ArcSinh[c*x]])/(2*d*Sqrt[d + c^2*d*x^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
```

```
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2(1+c^2 \ldots}}{2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} - \frac{\left(3c^2\right)\int \frac{a+b\sin\ldots}{x\sqrt{d+\ldots}}}{2d\ldots}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\ldots}{d\sqrt{d+c\ldots}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\ldots}{d\sqrt{d+c\ldots}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\ldots}{d\sqrt{d+c\ldots}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\ldots}{d\sqrt{d+c\ldots}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\ldots}{d\sqrt{d+c\ldots}}$$

Mathematica [A]

time = 5.66, size = 369, normalized size = 1.29

$$\frac{-\frac{4a\left(1+3c^2x^2\right)\sqrt{d+c^2dx^2}}{x^2+c^2x^4} - 12ac^2\sqrt{d}\log(x) + 12ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bc^2d\left(-8\sinh^{-1}(cx)+16\sqrt{1+c^2x^2}\,\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)-2\sqrt{1+c^2x^2}\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sqrt{1+c^2x^2}\sinh^{-1}(cx)\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)-12\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+12\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)-12\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+12\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sqrt{1+c^2x^2}\sinh^{-1}(cx)\operatorname{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+2\sqrt{1+c^2x^2}\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{\sqrt{d+c^2dx^2}}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] ((-4*a*(1 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(x^2 + c^2*x^4) - 12*a*c^2*Sqrt
[d]*Log[x] + 12*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2
*d*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 2
*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Cs
ch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSin
h[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 1
2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*P
olyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[
```

```
c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/Sqrt[d + c^2*d*x^2])
/(8*d^2)
```

Maple [A]
time = 3.23, size = 389, normalized size = 1.36

method	result
default	$-\frac{a}{2d\,x^2\sqrt{c^2d\,x^2+d}}-\frac{3a\,c^2}{2d\sqrt{c^2d\,x^2+d}}+\frac{3a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2d^{\frac{3}{2}}}-\frac{3b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(c}{2d^2(c^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2/(c^2*d*x^2+d)^(1/2)-3/2*a*c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a*c^2/
d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)-3/2*b*(d*(c^2*x^2+1))^(1/
2)/d^2/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^
2+1)^(1/2)/x*c-1/2*b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^2+1)/x^2*arcsinh(c*x)
+2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arctan(c*x+(c^2*x^2+1)^(1/
2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(c*x+(c^2*x^
2+1)^(1/2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(1+c
*x+(c^2*x^2+1)^(1/2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2
*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/d^(3/2) - 3*c^2/(sqrt(c^2*d*x^2 + d)*d) -
1/(sqrt(c^2*d*x^2 + d)*d*x^2))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))
/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*
x^5 + d^2*x^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{x^{3} \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^3\,{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(3/2)), x)
```

3.164 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=228

$$-\frac{bc\sqrt{d+c^2dx^2}}{6d^2x^2\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{d+c^2dx^2}}-\frac{5bc^3\sqrt{d+c^2dx^2}\ \mathrm{lo}}{3d^2\sqrt{1+c^2x^2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3/(c^2*d*x^2+d)^(1/2)+4/3*c^2*(a+b*arcsinh(c*x)
)/d/x/(c^2*d*x^2+d)^(1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2
)-1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2+1)^(1/2)-5/3*b*c^3*ln(x)*(c^
2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)-1/2*b*c^3*ln(c^2*x^2+1)*(c^2*d*x^2+d
)^(1/2)/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {277, 197, 5804, 12, 1265, 907}

$$\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^2x^2\sqrt{c^2x^2+1}}-\frac{5bc^3\log(x)\sqrt{c^2dx^2+d}}{3d^2\sqrt{c^2x^2+1}}-\frac{bc^3\sqrt{c^2dx^2+d}\,\log\left(c^2x^2+1\right)}{2d^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[d + c^2*d*x^2])/(d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh
[c*x])/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*d*x*
Sqrt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*Sqrt[d + c^2*d*x
^2]) - (5*b*c^3*Sqrt[d + c^2*d*x^2]*Log[x])/(3*d^2*Sqrt[1 + c^2*x^2]) - (b*
c^3*Sqrt[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(2*d^2*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} - \frac{1}{3}\left(4c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^3(1+c^2 \cdots}}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}} + \frac{1}{3}\left(8c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx + \cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1 \cdots\right.}{\cdots}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x\left(a+b\,\mathrm{si}\cdots\right.}{3d\sqrt{d+\cdots}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x\left(a+b\,\mathrm{si}\cdots\right.}{3d\sqrt{d+\cdots}}$$

Mathematica [A]

time = 0.21, size = 216, normalized size = 0.95

$$\frac{\sqrt{d+c^2dx^2}\left(-bcx-bc^3x^3-2a\sqrt{1+c^2x^2}+8ac^2x^2\sqrt{1+c^2x^2}+16ac^4x^4\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}\left(-1+4c^2x^2+8c^4x^4\right)\sinh^{-1}(cx)+5bc^3x^3(1+c^2x^2)\log\left(1+\frac{1}{c^2x^2}\right)-8bc^3x^3\log\left(1+c^2x^2\right)-8bc^5x^5\log\left(1+c^2x^2\right)\right)}{6d^2x^3\left(1+c^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 8*a*c^
2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2
*x^2]*(-1 + 4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 5*b*c^3*x^3*(1 + c^2*x^2)
*Log[1 + 1/(c^2*x^2)] - 8*b*c^3*x^3*Log[1 + c^2*x^2] - 8*b*c^5*x^5*Log[1 +
c^2*x^2]))/(6*d^2*x^3*(1 + c^2*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(201) = 402$.
time = 3.33, size = 968, normalized size = 4.25

method	result
default	$a\left(-\frac{1}{3d\,x^3\sqrt{c^2d\,x^2+d}}-\frac{4c^2\left(-\frac{1}{dx\sqrt{c^2d\,x^2+d}}-\frac{2c^2x}{d\sqrt{c^2d\,x^2+d}}\right)}{3}\right)+\frac{16b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)c^3}{3\sqrt{c^2x^2+1}\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/d/x^3/(c^2*d*x^2+d)^(1/2)-4/3*c^2*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2
/d*x/(c^2*d*x^2+d)^(1/2)))+16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
^2*arcsinh(c*x)*c^3+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^
2*x^7*c^10-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^
2*x^2+1)*c^8+16*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8
-16/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c
^6+64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x
)*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)*c^5+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1
)/d^2*x^3*c^6+4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*(c^
2*x^2+1)*c^4+8*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsin
h(c*x)*c^4+8/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)*c^3-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2
-1)/d^2*x*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*(c^2*
x^2+1)^(1/2)*c^3-4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*ar
csinh(c*x)*c^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^2*
(c^2*x^2+1)^(1/2)*c+1/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2
/x^3*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln(1+(c*x+(
```

```
c^2*x^2+1)^(1/2))^2)*c^3-5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*
ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(8*c^4*x/(sqrt(c^2*d*x^2 + d)*d) + 4*c^2/(sqrt(c^2*d*x^2 + d)*d*x) - 1/
(sqrt(c^2*d*x^2 + d)*d*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(
(c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*
x^6 + d^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{x^{4} \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**4*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(3/2)), x)
```

3.165 $\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{bx^2\sqrt{1+c^2x^2}}{4c^5d^2\sqrt{d+c^2dx^2}}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{5x\sqrt{d}}{}$$

```
[Out] -1/3*x^5*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)-5/3*x^3*(a+b*arcsinh(
c*x))/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/6*b/c^7/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^
2+d)^(1/2)-1/4*b*x^2*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-5/4*(a+b
*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^7/d^2/(c^2*d*x^2+d)^(1/2)-7/6*b*ln(c
^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^7/d^2/(c^2*d*x^2+d)^(1/2)+5/2*x*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^3
```

Rubi [A]
time = 0.28, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5810, 5812, 5783, 30, 272, 45}

$$-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{5\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^7d^2\sqrt{c^2dx^2+d}}+\frac{5x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{2c^6d^3}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{b}{6c^7d^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}-\frac{7b\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)}{6c^7d^2\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c^5d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*b/(c^7*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^
2*x^2])/(4*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (x^5*(a + b*ArcSinh[c*x]))/(3*c^2
*d*(d + c^2*d*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh[c*x]))/(3*c^4*d^2*Sqrt[d
+ c^2*d*x^2]) + (5*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*c^6*d^3)
- (5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*d^2*Sqrt[d + c^2*d*
x^2]) - (7*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(6*c^7*d^2*Sqrt[d + c^2*d*
x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^6(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{x^5(a+b\sinh^{-1}(cx))}{3c^2d(d+c^2dx^2)^{3/2}} + \frac{5\int \frac{x^4(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}}dx}{3c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^5}{(1+c^2x^2)^2}}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^5(a+b\sinh^{-1}(cx))}{3c^2d(d+c^2dx^2)^{3/2}} - \frac{5x^3(a+b\sinh^{-1}(cx))}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5\int \frac{x^2(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}dx}{c^4d^2} + \cdots$$

$$= -\frac{x^5(a+b\sinh^{-1}(cx))}{3c^2d(d+c^2dx^2)^{3/2}} - \frac{5x^3(a+b\sinh^{-1}(cx))}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5x\sqrt{d+c^2dx^2}(a+b\sinh}{2c^6d^3}$$

$$= -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{13bx^2\sqrt{1+c^2x^2}}{12c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^5(a+b\sinh^{-1}(cx)}{3c^2d(d+c^2dx^2)^{3/2}}$$

$$= -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}}{4c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^5(a+b\sinh^{-1}(cx))}{3c^2d(d+c^2dx^2)^{3/2}}$$

Mathematica [A]
time = 0.65, size = 222, normalized size = 0.79

$$\frac{4acdx(15+20c^2x^2+3c^4x^4)+bd\left(4cx(15+20c^2x^2+3c^4x^4)\sinh^{-1}(cx)-30(1+c^2x^2)^{3/2}\sinh^{-1}(cx)^2-\sqrt{1+c^2x^2}\,(7+9c^2x^2+6c^4x^4+28(1+c^2x^2)\log(1+c^2x^2))\right)-60a\sqrt{d}\,(1+c^2x^2)\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{24c^7d^3(1+c^2x^2)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a*c*d*x*(15 + 20*c^2*x^2 + 3*c^4*x^4) + b*d*(4*c*x*(15 + 20*c^2*x^2 + 3*
c^4*x^4)*ArcSinh[c*x] - 30*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - Sqrt[1 + c^
2*x^2]*(7 + 9*c^2*x^2 + 6*c^4*x^4 + 28*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - 6
0*a*Sqrt[d]*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]])/(24*c^7*d^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1606 vs. $2(245) = 490$.
time = 4.68, size = 1607, normalized size = 5.72

method	result	size
default	Expression too large to display	1607

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/6*a/c^4*x^3/d/(c^2*d*x^2+d)^(3/2)+5/2*a/c^6/d^2*x/(c^2*d*x^2+d)^(1/2)-5/2
*a/c^6/d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/8*
```

```
b*(d*(c^2*x^2+1))^(1/2)/c^7/d^3/(c^2*x^2+1)^(1/2)-406*b*(d*(c^2*x^2+1))^(1/
2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^3/d^3*(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*x^4+1/2*a*x^5/c^2/d/(c^2*d*x^2+d)^(3/2)+147*b*(d*(c^2*x^
2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/d^3*arcsinh
(c*x)*x^7+14/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^7/d^3*arcsinh(c*
x)+70/3*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2
*x^2+49)/c^2/d^3*x^5+133/6*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+
334*c^4*x^4+209*c^2*x^2+49)/c^4/d^3*x^3+7*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x
^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^6/d^3*x-49/6*b*(d*(c^2*x^2+1))
^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^7/d^3*(c^2*x^2
+1)^(1/2)-7/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^7/d^3*ln(1+(c*x+(
c^2*x^2+1)^(1/2))^2)-5/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^7/d^3*
arcsinh(c*x)^2-1/4*b*(d*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x^2-14
7*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+4
9)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^6-1120/3*b*(d*(c^2*x^2+1))^(1/2)/
(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^5/d^3*(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*x^2-49/6*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+33
4*c^4*x^4+209*c^2*x^2+49)/c^2/d^3*(c^2*x^2+1)*x^5+385*b*(d*(c^2*x^2+1))^(1/
2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^2/d^3*arcsinh(c*x)
*x^5-21/2*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c
^2*x^2+49)/c^3/d^3*(c^2*x^2+1)^(1/2)*x^4-91/6*b*(d*(c^2*x^2+1))^(1/2)/(63*c
^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^4/d^3*(c^2*x^2+1)*x^3+1009
/3*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+
49)/c^4/d^3*arcsinh(c*x)*x^3-37/2*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c
^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^5/d^3*(c^2*x^2+1)^(1/2)*x^2-7*b*(d*(c^
2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^6/d^3
*(c^2*x^2+1)*x+98*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x
^4+209*c^2*x^2+49)/c^6/d^3*arcsinh(c*x)*x-343/3*b*(d*(c^2*x^2+1))^(1/2)/(63
*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^7/d^3*(c^2*x^2+1)^(1/2)*
arcsinh(c*x)+1/2*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(c*x)*x
^3+1/2*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)*x+49/6*b*(d
*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/d^3
*x^7
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(3*x^5/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 5*x*(3*x^2/((c^2*d*x^2 + d)^(3
/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4*d))/c^2 + 5*x/(sqrt(c^2*d*x^2 + d
)*c^6*d^2) - 15*arcsinh(c*x)/(c^7*d^(5/2))) + b*integrate(x^6*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6*arcsinh(c*x) + a*x^6)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(a + b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**6*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,(a + b\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^6*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.166 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=210

$$\frac{bx\sqrt{d+c^2dx^2}}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{bx\sqrt{d+c^2dx^2}}{c^5d^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^6d\left(d+c^2dx^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6d^2\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{c^6d^3}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/c^6/d/(c^2*d*x^2+d)^(3/2)+2*(a+b*arcsinh(c*x))/c^6/
d^2/(c^2*d*x^2+d)^(1/2)+1/6*b*x*(c^2*d*x^2+d)^(1/2)/c^5/d^3/(c^2*x^2+1)^(3/
2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^3-b*x*(c^2*d*x^2+d)^(1/2)/c
^5/d^3/(c^2*x^2+1)^(1/2)-11/6*b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^6/d^3/(c^
2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {272, 45, 5804, 12, 1171, 396, 209}

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^6d^3}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{3c^6d\left(c^2dx^2+d\right)^{3/2}}-\frac{11b\text{ArcTan}(cx)\sqrt{c^2dx^2+d}}{6c^6d^3\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{c^5d^3\sqrt{c^2x^2+1}}+\frac{bx\sqrt{c^2dx^2+d}}{6c^5d^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x*Sqrt[d + c^2*d*x^2])/(6*c^5*d^3*(1 + c^2*x^2)^(3/2)) - (b*x*Sqrt[d + c
^2*d*x^2])/(c^5*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*c^6*d*(d +
 c^2*d*x^2)^(3/2)) + (2*(a + b*ArcSinh[c*x]))/(c^6*d^2*Sqrt[d + c^2*d*x^2])
 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^6*d^3) - (11*b*Sqrt[d + c^
2*d*x^2]*ArcTan[c*x])/(6*c^6*d^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{4\int \frac{x^3(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}}\,dx}{3c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^4}{(1+c^2x^2)^2}\,d}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{4x^2\left(a+b\sinh^{-1}(cx\right.}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} + \frac{11bx\sqrt{1+c^2x^2}}{6c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\,(d+c^2dx^2)^{3/2}} -$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{5bx\sqrt{1+c^2x^2}}{6c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\,(d+c^2dx^2)^{3/2}} -$$

Mathematica [A]
time = 0.16, size = 154, normalized size = 0.73

$$\frac{\sqrt{d+c^2dx^2}\left(-bcx\sqrt{1+c^2x^2}\,(5+6c^2x^2)+2a(8+12c^2x^2+3c^4x^4)+2b(8+12c^2x^2+3c^4x^4)\sinh^{-1}(cx)\right)}{6c^6d^3\,(1+c^2x^2)^2} - \frac{11b\sqrt{d\,(1+c^2x^2)}\,\mathrm{ArcTan}(cx)}{6c^6d^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x*Sqrt[1 + c^2*x^2]*(5 + 6*c^2*x^2)) + 2*a*(8 +
 12*c^2*x^2 + 3*c^4*x^4) + 2*b*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]))/
(6*c^6*d^3*(1 + c^2*x^2)^2) - (11*b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(6*c
^6*d^3*Sqrt[1 + c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 3.14, size = 400, normalized size = 1.90

method	result
default	$a\left(\frac{x^4}{c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{c^2d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2}{3d\,c^4\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}(cx)x^2}{c^4d^3(c^2x^2+1)}-\frac{b\sqrt{d\left(c^2x^2+\right.}}{c^5d^3\sqrt{c^2x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(x^4/c^2/d/(c^2*d*x^2+d)^(3/2)-4/c^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3/
d/c^4/(c^2*d*x^2+d)^(3/2)))+b*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arc
sinh(c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x+b*(d*(c^2
```

```
*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)+2*b*(d*(c^2*x^2+1))^(1/2)/d
^3/(c^2*x^2+1)^2/c^4*arcsinh(c*x)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*
x^2+1)^(3/2)/c^5*x+5/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsin
h(c*x)+11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2
*x^2+1)^(1/2)-I)-11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*l
n(c*x+(c^2*x^2+1)^(1/2)+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*((3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))*log(c*x + sqrt(
c^2*x^2 + 1))/((c^8*d^3*x^2 + c^6*d^3)*sqrt(c^2*x^2 + 1)) + 3*integrate(1/3
*(3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))/(c^11*d^3*x^6 + 2*c^9
*d^3*x^4 + c^7*d^3*x^2 + (c^10*d^3*x^5 + 2*c^8*d^3*x^3 + c^6*d^3*x)*sqrt(c^
2*x^2 + 1)), x) - 3*integrate(1/3*(3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 +
 8*sqrt(d))/((c^8*d^3*x^3 + c^6*d^3*x)*sqrt(c^2*x^2 + 1)), x)) + 1/3*a*(3*x
^4/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 12*x^2/((c^2*d*x^2 + d)^(3/2)*c^4*d) + 8
/((c^2*d*x^2 + d)^(3/2)*c^6*d))
```

Fricas [A]
time = 0.44, size = 219, normalized size = 1.04

$$\frac{11\,(bc^4x^4+2\,bc^2x^2+b)\sqrt{d}\,\arctan\left(\frac{2\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+4\,(3\,bc^4x^4+12\,bc^2x^2+8\,b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(6\,ac^4x^4+24\,ac^2x^2-(6\,bc^3x^3+5\,bcx)\sqrt{c^2x^2+1}+16\,a\right)\sqrt{c^2dx^2+d}}{12\,(c^{10}d^3x^4+2\,c^8d^3x^2+c^6d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(11*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^4*x^4 + 12*b*c^2
*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^4*x
^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*sqrt(c^2*x^2 + 1) + 16*a)*sqrt(
c^2*d*x^2 + d))/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+b\operatorname{asinh}(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.167 $\int \frac{x^4 (a+b \sinh^{-1}(cx))}{(d+c^2 d x^2)^{5/2}} \, dx$

Optimal. Leaf size=203

$$\frac{b}{6c^5d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{d+c^2dx^2}}$$

```
[Out] -1/3*x^3*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)-x*(a+b*arcsinh(c*x))/
c^4/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b/c^5/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(
1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^5/d^2/(c^2*d*x^2+d)^(1/
2)+2/3*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5810, 5783, 266, 272, 45}

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{b}{6c^5d^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)}{3c^5d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b/(6*c^5*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c
*x]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4*d^2*S
qrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx &= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}} \\
&= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{c^4d^2}+\frac{\left(b\sqrt{1+\right.}{c^3d} \\
&= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{2c^5d^2\sqrt{d+c^2dx^2}}+ \\
&= \frac{b}{6c^5d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}}+
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 191, normalized size = 0.94

$$
\frac{-2ac\sqrt{d}\,x(3+4c^2x^2)+b\sqrt{d}\left(\sqrt{1+c^2x^2}+2cx\sinh^{-1}(cx)-8cx(1+c^2x^2)\sinh^{-1}(cx)+(1+c^2x^2)^{3/2}\left(3\sinh^{-1}(cx)^2+4\log\left(1+c^2x^2\right)\right)\right)+6a(1+c^2x^2)\sqrt{d+c^2dx^2}\,\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)}{6c^5d^{5/2}\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-2*a*c*Sqrt[d]*x*(3 + 4*c^2*x^2) + b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*x*Ar
cSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*ArcS
inh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 6*a*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*L
og[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(6*c^5*d^(5/2)*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1429 vs. $2(179) = 358$.
time = 4.05, size = 1430, normalized size = 7.04

method	result	size
default	Expression too large to display	1430

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*x^3/c^2/d/(c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a/c^4/
d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)^2-8/3*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)-32*b*(d*(c^2*x^2+1))^(1/2
)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*arcsinh(c*x)*x^
7+32*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+
16)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6-8/3*b*(d*(c^2*x^2+1))^(1/2)/(2
4*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*x^7+8/3*b*(d*(c^2*x
^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*(c^2*x^2
+1)*x^5-76*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^
2*x^2+16)/d^3*arcsinh(c*x)*x^5+84*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^
6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-2
2/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+1
6)/d^3*x^5+4*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*
c^2*x^2+16)/c/d^3*(c^2*x^2+1)^(1/2)*x^4+14/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^
8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*(c^2*x^2+1)*x^3-181/3*b
*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^
2/d^3*arcsinh(c*x)*x^3+220/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6
+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2-20/3
*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/
c^2/d^3*x^3+13/2*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4
+71*c^2*x^2+16)/c^3/d^3*(c^2*x^2+1)^(1/2)*x^2+2*b*(d*(c^2*x^2+1))^(1/2)/(24
*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*(c^2*x^2+1)*x-16*b*(
d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/
d^3*arcsinh(c*x)*x+64/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*
c^4*x^4+71*c^2*x^2+16)/c^5/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2*b*(d*(c^2*x
^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*x+8/
3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)
```

```
/c^5/d^3*(c^2*x^2+1)^(1/2)+4/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^
5/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d)) + x/(sqrt(c^2*d*x^2 + d)*c^4*d^2) - 3*arcsinh(c*x)/(c^5*d^(5/2)))*a +
b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{asinh}(cx))}{(d(c^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(a+b\,\mathrm{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.168 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{bx\sqrt{d+c^2dx^2}}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{3c^4d\left(d+c^2dx^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{c^4d^2\sqrt{d+c^2dx^2}}+\frac{5b\sqrt{d+c^2dx^2}\,\text{ArcTan}(cx)}{6c^4d^3\sqrt{1+c^2x^2}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))/c^4/d/(c^2*d*x^2+d)^(3/2)+(-a-b*arcsinh(c*x))/c^4/d^
2/(c^2*d*x^2+d)^(1/2)-1/6*b*x*(c^2*d*x^2+d)^(1/2)/c^3/d^3/(c^2*x^2+1)^(3/2)
+5/6*b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^4/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 45, 5804, 12, 393, 209}

$$-\frac{a+b\sinh^{-1}(cx)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{3c^4d\left(c^2dx^2+d\right)^{3/2}}+\frac{5b\text{ArcTan}(cx)\sqrt{c^2dx^2+d}}{6c^4d^3\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{6c^3d^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*(b*x*Sqrt[d + c^2*d*x^2])/(c^3*d^3*(1 + c^2*x^2)^(3/2)) + (a + b*ArcSi
nh[c*x])/(3*c^4*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*d^2*Sq
rt[d + c^2*d*x^2]) + (5*b*Sqrt[d + c^2*d*x^2]*ArcTan[c*x])/(6*c^4*d^3*Sqrt[
1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^2}{\left(1+c^2x^2\right)^2}\,d}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx}{6c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx}{6c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.15, size = 151, normalized size = 1.05

$$-\frac{\sqrt{d+c^2dx^2}\left(2a\sqrt{1+c^2x^2}\left(2+3c^2x^2\right)+b\left(cx+c^3x^3\right)+2b\sqrt{1+c^2x^2}\left(2+3c^2x^2\right)\sinh^{-1}(cx)\right)}{6c^4d^3\left(1+c^2x^2\right)^{5/2}}+\frac{5b\sqrt{d\left(1+c^2x^2\right)}\,\mathrm{ArcTan}(cx)}{6c^4d^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(2*a*Sqrt[1 + c^2*x^2]*(2 + 3*c^2*x^2) + b*(c*x +
 c^3*x^3) + 2*b*Sqrt[1 + c^2*x^2]*(2 + 3*c^2*x^2)*ArcSinh[c*x]))/(c^4*d^3*(
1 + c^2*x^2)^(5/2)) + (5*b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(6*c^4*d^3*Sq
rt[1 + c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 3.37, size = 263, normalized size = 1.83

method	result
default	$a\left(-\frac{x^2}{c^2 d(c^2 d\,x^2+d)^{\frac{3}{2}}}-\frac{2}{3d\,c^4(c^2 d\,x^2+d)^{\frac{3}{2}}}\right)-\frac{b\sqrt{d\,(c^2x^2+1)}\ \mathrm{arcsinh}(cx)x^2}{d^3(c^2x^2+1)^2c^2}-\frac{b\sqrt{d\,(c^2x^2+1)}\ x}{6d^3(c^2x^2+1)^{\frac{3}{2}}c^3}-\frac{2b\sqrt{d\,(c^2}{3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(c^2*d*x^2+d)^(3/2))-b*(d*(c^2*
x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)*x^2-1/6*b*(d*(c^2*x^2+1))^
(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*x-2/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+
1)^2/c^4*arcsinh(c*x)+5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d
^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [A]
time = 0.50, size = 138, normalized size = 0.96

$$-\frac{1}{6}\,bc\left(\frac{x}{c^6 d^{\frac{5}{2}} x^2 + c^4 d^{\frac{5}{2}}} - \frac{5\,\arctan\left(cx\right)}{c^5 d^{\frac{5}{2}}}\right) - \frac{1}{3}\,b\left(\frac{3\,x^2}{(c^2dx^2+d)^{\frac{3}{2}}c^2d} + \frac{2}{(c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)\mathrm{arsinh}\left(cx\right) - \frac{1}{3}\,a\left(\frac{3\,x^2}{(c^2dx^2+d)^{\frac{3}{2}}c^2d} + \frac{2}{(c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2)))
- 1/3*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d))*arcsinh(c*x) - 1/3*a*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*
x^2 + d)^(3/2)*c^4*d))
```

Fricas [A]
time = 0.42, size = 188, normalized size = 1.31

$$-\frac{5\,(bc^4x^4+2\,bc^2x^2+b)\sqrt{d}\,\arctan\left(\frac{2\,\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+4\,(3\,bc^2x^2+2\,b)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(6\,ac^2x^2+\sqrt{c^2x^2+1}\,bcx+4\,a\right)\sqrt{c^2dx^2+d}}{12\,(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(5*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^2*x^2 + 2*b)*sqr
```

```
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^2*x^2 + sqrt(c^2*x
^2 + 1)*b*c*x + 4*a)*sqrt(c^2*d*x^2 + d))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^
4*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(a + b\operatorname{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.169 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{b}{6c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\,(d+c^2dx^2)^{3/2}}-\frac{b\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{6c^3d^2\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)-1/6*b/c^3/d^2/(c^2*x^2+1)^
(1/2)/(c^2*d*x^2+d)^(1/2)-1/6*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^
2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5800, 272, 45}

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\,(c^2dx^2+d)^{3/2}}-\frac{b}{6c^3d^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)}{6c^3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*b/(c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSin
h[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2
])/(6*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
```

```
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x^3}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$
$$=\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{x}{(1+c^2x)^2}\,dx,x,x^2\right)}{6d^2\sqrt{d+c^2dx^2}}$$
$$=\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\left(-\frac{1}{c^2(1+c^2x)^2}+\frac{1}{c^2(1+c^2x)}\right)\,d\right.}{6d^2\sqrt{d+c^2dx^2}}$$
$$=-\frac{b}{6c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{b\sqrt{1+c^2x^2}\,\log\left(1\right.}{6c^3d^2\sqrt{d+c^2}}$$

Mathematica [A]
time = 0.12, size = 118, normalized size = 0.99

$$-\frac{\sqrt{d+c^2dx^2}\left(b+bc^2x^2-2ac^3x^3\sqrt{1+c^2x^2}-2bc^3x^3\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)+b(1+c^2x^2)^2\log\left(1+c^2x^2\right)\right)}{6c^3d^3\left(1+c^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 - 2*a*c^3*x^3*Sqrt[1 + c^2*x^2] -
2*b*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(1 + c^2*x^2)^2*Log[1 + c^2*
x^2]))/(c^3*d^3*(1 + c^2*x^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(103) = 206$.
time = 3.41, size = 1175, normalized size = 9.87

method	result
default	$a\left(-\frac{x}{2c^2d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}}{2c^2}\right)+\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)}{3\sqrt{c^2x^2+1}\,c^3d^3}+\frac{b\sqrt{d\left(c^2x^2+\right.}}{(3x^8c^8+9x^6c^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2*x/c^2/d/(c^2*d*x^2+d)^(3/2)+1/2/c^2*(1/3*x/d/(c^2*d*x^2+d)^(3/2)+2/
3/d^2*x/(c^2*d*x^2+d)^(1/2)))+2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/c^3/d^3*arcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x
^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*x)*x^7-b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8
+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^3/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x
^6+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
*c^4/d^3*x^7-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*
c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*x^5+b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6
*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)*x^5-2*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*x^4+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4
+5*c^2*x^2+1)*c^2/d^3*x^5-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+
10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*x^4-1/6*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x^3+1/3*b
*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arc
sinh(c*x)*x^3-4/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5
*c^2*x^2+1)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+1/6*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*x^3-1/2*b*(d*(c^2*x^
2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*(c^2*x^2+1)^
(1/2)*x^2-1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2
*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/6*b*(d*(c^2*x^2+1))^(1/2)/
(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)-1/3*
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/
2))^2)
```

Maxima [A]
time = 0.29, size = 137, normalized size = 1.15

$$-\frac{1}{6}\,bc\left(\frac{1}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}+\frac{\log\left(c^2x^2+1\right)}{c^4d^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{x}{\sqrt{c^2dx^2+d}\,c^2d^2}-\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{x}{\sqrt{c^2dx^2+d}\,c^2d^2}-\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/2)
)) + 1/3*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*c^2*
d))*arcsinh(c*x) + 1/3*a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 +
 d)^(3/2)*c^2*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.170 $\int \frac{x\left(a+b \sinh ^{-1}(c x)\right)}{\left(d+c^2 d x^2\right)^{5/2}} d x$

Optimal. Leaf size=114

$$\frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{b\sqrt{1+c^2x^2}\,\text{ArcTan}(cx)}{6c^2d^2\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*x/c/d^2/(c^2*x^2+1)
^(1/2)/(c^2*d*x^2+d)^(1/2)+1/6*b*arctan(c*x)*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2
*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5798, 205, 209}

$$-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\,\text{ArcTan}(cx)}{6c^2d^2\sqrt{c^2dx^2+d}}+\frac{bx}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x)/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x]
)/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) + (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^
2*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}\\ &= \frac{bx}{6cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}}{6cd^2\sqrt{d+c^2dx^2}}\\ &= \frac{bx}{6cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{6c^2d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [**A**]
time = 0.12, size = 130, normalized size = 1.14

$$\frac{\sqrt{d+c^2dx^2}\left(bcx+bc^3x^3-2a\sqrt{1+c^2x^2}-2b\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)\right)}{6c^2d^3\left(1+c^2x^2\right)^{5/2}}+\frac{b\sqrt{d\left(1+c^2x^2\right)}\,\mathrm{ArcTan}(cx)}{6c^2d^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] - 2*b*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x]))/(6*c^2*d^3*(1 + c^2*x^2)^(5/2)) + (b*Sqrt[d*(1
+ c^2*x^2)]*ArcTan[c*x])/(6*c^2*d^3*Sqrt[1 + c^2*x^2])
```

Maple [**C**] Result contains complex when optimal does not.
time = 1.01, size = 198, normalized size = 1.74

method	result
default	$-\frac{a}{3c^2d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,x}{6d^3(c^2x^2+1)^{\frac{3}{2}}c}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}(cx)}{3d^3(c^2x^2+1)^2c^2}+\frac{ib\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(cx+\sqrt{c^2x^2}\right.}{6\sqrt{c^2x^2+1}\,c^2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1
)^(3/2)/c*x-1/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)+
1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^
```

```
(1/2)+I)-1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^
2*x^2+1)^(1/2)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x) - 1/3*
a/((c^2*d*x^2 + d)^(3/2)*c^2*d)
```

Fricas [A]
time = 0.44, size = 166, normalized size = 1.46

$$-\frac{(bc^4x^4+2\,bc^2x^2+b)\sqrt{d}\,\arctan\left(\frac{2\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+4\sqrt{c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2dx^2+d}\left(\sqrt{c^2x^2+1}\,bcx-2\,a\right)}{12\,(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*((b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*s
qrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*sqrt(c^2*d*x^2 + d)*b*log
(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*(sqrt(c^2*x^2 + 1)*b*c*x
- 2*a))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.171 $\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{5/2}}\,dx$

<u>**Optimal**</u>. Leaf size=147

$$\frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{3cd^2\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arcsinh(c*x))/d^2
/(c^2*d*x^2+d)^(1/2)+1/6*b/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/3*
b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)
```

<u>**Rubi**</u> [A]

time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5788, 5787, 266, 267}

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{3cd^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])
)/(3*d*(d + c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d +
c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c*d^2*Sqrt[d + c^2*
d*x^2])
```

<u>Rule 266</u>

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

<u>Rule 267</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

<u>Rule 5787</u>

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{x(a+b\sinh^{-1}(cx))}{3d(d+c^2dx^2)^{3/2}} + \frac{2\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{3/2}}\,dx}{3d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x(a+b\sinh^{-1}(cx))}{3d(d+c^2dx^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+c^2dx^2}} - \frac{(2}{}$$

$$= \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x(a+b\sinh^{-1}(cx))}{3d(d+c^2dx^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+c^2dx^2}} - \frac{b\sqrt{}}{}$$

Mathematica [A]
time = 0.13, size = 143, normalized size = 0.97

$$\frac{\sqrt{d+c^2dx^2}\left(b+bc^2x^2+6acx\sqrt{1+c^2x^2}+4ac^3x^3\sqrt{1+c^2x^2}+2bcx\sqrt{1+c^2x^2}(3+2c^2x^2)\sinh^{-1}(cx)-2b(1+c^2x^2)^2\log(1+c^2x^2)\right)}{6cd^3(1+c^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 + 6*a*c*x*Sqrt[1 + c^2*x^2] + 4*a*c^3*x
^3*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2)*ArcSinh[c*
x] - 2*b*(1 + c^2*x^2)^2*Log[1 + c^2*x^2]))/(6*c*d^3*(1 + c^2*x^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(127) = 254$.
time = 1.57, size = 1006, normalized size = 6.84

method	result
default	$a\left(\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}\right)+\frac{4b\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)}{3\sqrt{c^2x^2+1}\,c\,d^3}-\frac{2b\sqrt{d(c^2x^2+1)}\,c^6x^7}{3(3x^6c^6+10c^4x^4+11c^2x^2+4)d^3}+\frac{2b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x/d/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))+4/3*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^3*arcsinh(c*x)-2/3*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*x^7+2/3*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*(c^2*x^2+1)*x^5+2*b*(d*(c
^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x
^5-2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d^3*(c
^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-7/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*
c^4*x^4+11*c^2*x^2+4)*c^4/d^3*x^5+5/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10
*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*x^3+17/3*b*(d*(c^2*x^2+1))^(1/2)
/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*x^3-14/3*b*(d*(c^
2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*(c^2*x^2+1)^(1/2)
*arcsinh(c*x)*x^2-8/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*
x^2+4)*c^2/d^3*x^3+1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2
*x^2+4)*c/d^3*(c^2*x^2+1)^(1/2)*x^2+b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c
^4*x^4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*x+4*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6
+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*x-8/3*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)-b*(d
*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x+2/3*b*(d*(c^2
*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)-
2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^3*ln(1+(c*x+(c^2*x^2+1)^(
1/2))^2)
```

Maxima [A]
time = 0.30, size = 126, normalized size = 0.86

$$\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5/2
))) + 1/3*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d))*a
rcsinh(c*x) + 1/3*a*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/
2)*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{(d(c^2x^2 + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2), x)
```

3.172 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{bcx}{6d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}+\frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}}-\frac{7b\sqrt{1+c^2x^2}\,\text{ArcTan}(cx)}{6d^2\sqrt{d+c^2dx^2}}-\frac{2\sqrt{1+c^2x^2}}{}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+(a+b*arcsinh(c*x))/d^2/(c^2*d*
x^2+d)^(1/2)-1/6*b*c*x/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-7/6*b*arct
an(c*x)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*arct
anh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-b*poly
log(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+b*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5811, 5816, 4267, 2317, 2438, 209, 205}

$$\frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{7b\sqrt{c^2x^2+1}\,\text{ArcTan}(cx)}{6d^2\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{bcx}{6d^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c*x)/(d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c
*x])/(3*d*(d + c^2*d*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(d^2*Sqrt[d + c^2*d
*x^2]) - (7*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) -
(2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqr
t[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*
Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(d^
2*Sqrt[d + c^2*d*x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x(d+c^2dx^2)^{3/2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\sinh^{-1}}{x\sqrt{d+c}}}{d^2}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2}}{6d^2\sqrt{d}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2}}{6d^2\sqrt{d}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2}}{6d^2\sqrt{d}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2}}{6d^2\sqrt{d}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2}}{6d^2\sqrt{d}}$$

Mathematica [A]
time = 0.79, size = 247, normalized size = 0.94

$$\frac{\frac{2a(4+3c^2x^2)\sqrt{d+c^2dx^2}}{(1+c^2x^2)^2} + 6a\sqrt{d}\,\log(x) - 6a\sqrt{d}\,\log\left(d+\sqrt{d}\,\sqrt{d+c^2dx^2}\right) + \frac{bd^2(1+c^2x^2)^{3/2}\left(-\frac{cx}{1+c^2x^2} + \frac{2\sinh^{-1}(cx)}{(1+c^2x^2)^{3/2}} + \frac{6\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} - 14\,\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right) + 6\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right) - 6\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right) + 6\,\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 6\,\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{(d+c^2dx^2)^{3/2}}}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((2*a*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 6*a*Sqrt[d]*Lo
g[x] - 6*a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d^2*(1 + c^2*x
^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) +
(6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[
c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSinh[c*x])]
))/(d + c^2*d*x^2)^(3/2))/(6*d^3)
```

Maple [A]
time = 1.74, size = 364, normalized size = 1.39

method	result
default	$\frac{a}{3d(c^2 d\,x^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{c^2 d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{d^{\frac{5}{2}}}+\frac{b\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x^2c^2}{d^3(c^2x^2+1)^2}-\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a/d/(c^2*d*x^2+d)^(3/2)+a/d^2/(c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+2*d
^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2*ar
csinh(c*x)*x^2*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)*x*c+4/
3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2*arcsinh(c*x)-7/3*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(1/(c*abs(x)))/d^(5/2) - 3/(sqrt(c^2*d*x^2 + d)*d^2) - 1/(
(c^2*d*x^2 + d)^(3/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*
d*x^2 + d)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*
x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{x \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(5/2)), x)
```

3.173 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{bc\sqrt{d+c^2dx^2}}{6d^3\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}+\frac{bc\sqrt{d+c^2dx^2}\,\mathrm{lo}}{d^3\sqrt{1+c^2x^2}}$$

```
[Out] (-a-b*arcsinh(c*x))/d/x/(c^2*d*x^2+d)^(3/2)-4/3*c^2*x*(a+b*arcsinh(c*x))/d/
(c^2*d*x^2+d)^(3/2)-8/3*c^2*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-1/
6*b*c*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(3/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/
2)/d^3/(c^2*x^2+1)^(1/2)+5/6*b*c*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2
*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {277, 198, 197, 5804, 12, 1265, 907}

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^3\left(c^2x^2+1\right)^{3/2}}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{d^3\sqrt{c^2x^2+1}}+\frac{5bc\sqrt{c^2dx^2+d}\,\log\left(c^2x^2+1\right)}{6d^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[d + c^2*d*x^2])/(d^3*(1 + c^2*x^2)^(3/2)) - (a + b*ArcSinh[c
*x])/(d*x*(d + c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*d*(d +
 c^2*d*x^2)^(3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x
^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[x])/(d^3*Sqrt[1 + c^2*x^2]) + (5*b*c*Sq
rt[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(6*d^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \left(4c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x(1+c^2x^2)^2}}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(8c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{3/2}}\,dx}{3d} + \frac{\left(bc\sqrt{1\right.}}{}$$

$$= -\frac{2bc}{3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{8}{}$$

$$= -\frac{bc}{6d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{8}{}$$

Mathematica [A]
time = 0.23, size = 227, normalized size = 1.06

$$-\frac{\sqrt{d+c^2dx^2}\left(bcx+bc^3x^3+6a\sqrt{1+c^2x^2}+24ac^2x^2\sqrt{1+c^2x^2}+16ac^4x^4\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}\left(3+12c^2x^2+8c^4x^4\right)\sinh^{-1}(cx)+3bcx\left(1+c^2x^2\right)^2\log\left(1+\frac{1}{c^2x^2}\right)-8bcx\log\left(1+c^2x^2\right)-16bc^3x^3\log\left(1+c^2x^2\right)-8bc^5x^5\log\left(1+c^2x^2\right)\right)}{6d^3x\left(1+c^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 + 6*a*Sqrt[1 + c^2*x^2] + 24*a
*c^2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 +
c^2*x^2]*(3 + 12*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 3*b*c*x*(1 + c^2*x^2)^
2*Log[1 + 1/(c^2*x^2)] - 8*b*c*x*Log[1 + c^2*x^2] - 16*b*c^3*x^3*Log[1 + c^
2*x^2] - 8*b*c^5*x^5*Log[1 + c^2*x^2]))/(d^3*x*(1 + c^2*x^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. $2(190) = 380$.
time = 2.02, size = 1257, normalized size = 5.87

method	result
default	$a\left(-\frac{1}{dx\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2d\,x^2+d}}\right)\right)-\frac{16b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)c}{3\sqrt{c^2x^2+1}\,d^3}-\frac{32b}{3(8x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d/x/(c^2*d*x^2+d)^(3/2)-4*c^2*(1/3*x/d/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/
(c^2*d*x^2+d)^(1/2)))-16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ar
csinh(c*x)*c-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+
```

```
9)/d^3*x^9*c^10+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x
^2+9)/d^3*x^7*(c^2*x^2+1)*c^8-112/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c
^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8+80/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)
/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6+64/3*b*(d*(c^
2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*c^5-140/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^5*c^6+20*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-56*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4+136/3*b*(d*(c^2*x^2+1))^(1/
2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*c^3-24*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x
^3*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*
x^2*(c^2*x^2+1)^(1/2)*c^3+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+2
6*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-44*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+24*b*(d*(c^2*x^2+1))^(1/2)/(8
*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-4*b*
(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-3/2*b*(
d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*(c^2*x^2+1)^(1
/2)*c-9*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*a
rcsinh(c*x)+5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)*c+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x+
(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(8*c^2*x/(sqrt(c^2*d*x^2 + d)*d^2) + 4*c^2*x/((c^2*d*x^2 + d)^(3/2)*
d) + 3/((c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2 +
1))/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*
x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{x^2 (d (c^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^2\,(d\,c^2\,x^2 + d)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(5/2)), x)
```

3.174 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=400

$$\frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6d\left(d+c^2dx^2\right)^{3/2}}-\frac{a}{2dx}$$

```
[Out] -5/6*c^2*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+1/2*(-a-b*arcsinh(c*x))/d
/x^2/(c^2*d*x^2+d)^(3/2)-5/2*c^2*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)
+1/4*b*c/d^2/x/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+5/12*b*c^3*x/d^2/(c^2*
x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-3/4*b*c*(c^2*x^2+1)^(1/2)/d^2/x/(c^2*d*x^2
+d)^(1/2)+13/6*b*c^2*arctan(c*x)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+
5*c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d
^2/(c^2*d*x^2+d)^(1/2)+5/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2
+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-5/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5809, 5811, 5816, 4267, 2317, 2438, 209, 205, 296, 331}

$$-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^2\sqrt{c^2dx^2+d}}+\frac{5c^2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{13bc^2\sqrt{c^2x^2+1}\,\text{ArcTan}(cx)}{6d^2\sqrt{c^2dx^2+d}}+\frac{5bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{3bc\sqrt{c^2x^2+1}}{4d^2x\sqrt{c^2dx^2+d}}+\frac{bc}{4d^2x\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{5bc^3x}{12d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c)/(4*d^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (5*b*c^3*x)/(12*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (3*b*c*Sqrt[1 + c^2*x^2])/(4*d^2*
x*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*d*(d + c^2*d*x^2)^
(3/2)) - (a + b*ArcSinh[c*x])/(2*d*x^2*(d + c^2*d*x^2)^(3/2)) - (5*c^2*(a +
 b*ArcSinh[c*x]))/(2*d^2*Sqrt[d + c^2*d*x^2]) + (13*b*c^2*Sqrt[1 + c^2*x^2]
*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^
2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*d^2*Sqrt[d + c^2*d*x^2]
) - (5*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*d^2*Sqrt[d +
c^2*d*x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
```

```
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2(1+}}{2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{(}{}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 4.69, size = 409, normalized size = 1.02

$$\frac{-\frac{4a\sqrt{d+c^2dx^2}\left(3+20c^2x^2+15c^4x^4\right)}{\left(x+c^2x^3\right)^2} - 60ac^2\sqrt{d}\log(x) + 60ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bc^2d\left(\cdots\right)}{\sqrt{d+c^2dx^2}}}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2]*(3 + 20*c^2*x^2 + 15*c^4*x^4))/(x + c^2*x^3)^2 -
 60*a*c^2*Sqrt[d]*Log[x] + 60*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*
x^2]] + (b*c^2*d*((4*c*x)/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[
c*x])/(1 + c^2*x^2) + 104*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] -
6*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
*Csch[ArcSinh[c*x]/2]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])]
- 60*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2
]*PolyLog[2, E^(-ArcSinh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[Arc
```

Sinh[c*x]/2]^2 + 6*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/Sqrt[d + c^2*d*x^2])/(24*d^3)

Maple [A]
time = 3.32, size = 546, normalized size = 1.36

method	result
default	$-\frac{a}{2d\,x^2(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{5a\,c^2}{6d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{5a\,c^2}{2d^2\sqrt{c^2d\,x^2+d}}+\frac{5a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}-\frac{5b\sqrt{d\,(c^2x^2}{2(c^4x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2/(c^2*d*x^2+d)^(3/2)-5/6*a*c^2/d/(c^2*d*x^2+d)^(3/2)-5/2*a*c^2/
d^2/(c^2*d*x^2+d)^(1/2)+5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(
1/2))/x)-5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*x^2*arcsinh(
c*x)*c^4-1/3*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*x*(c^2*x^2+1
)^(1/2)*c^3-10/3*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*arcsinh(
c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3/x*(c^2*x^2+1
)^(1/2)*c-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3/x^2*arcsinh
(c*x)+13/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^
2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x
+(c^2*x^2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*d
ilog(1+c*x+(c^2*x^2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(15*c^2*arcsinh(1/(c*abs(x)))/d^(5/2) - 15*c^2/(sqrt(c^2*d*x^2 + d)*d
^2) - 5*c^2/((c^2*d*x^2 + d)^(3/2)*d) - 3/((c^2*d*x^2 + d)^(3/2)*d*x^2)) +
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*
x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{x^3 \left(d \left(c^2x^2 + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**3*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^3 \left(d\,c^2\,x^2 + d\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(5/2)), x)
```

3.175 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=297

$$\frac{bc^3\sqrt{d+c^2dx^2}}{6d^3(1+c^2x^2)^{3/2}}-\frac{bc\sqrt{d+c^2dx^2}}{6d^3x^2\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3(d+c^2dx^2)^{3/2}}+\frac{2c^2(a+b\sinh^{-1}(cx))}{dx(d+c^2dx^2)^{3/2}}+\frac{8c^4x(a+b\sinh^{-1}(cx))}{3d(d+c^2dx^2)^{3/2}}+$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3/(c^2*d*x^2+d)^(3/2)+2*c^2*(a+b*arcsinh(c*x))/
d/x/(c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+
16/3*c^4*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b*c^3*(c^2*d*x^2+
d)^(1/2)/d^3/(c^2*x^2+1)^(3/2)-1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^3/x^2/(c^2*x^2
+1)^(1/2)-8/3*b*c^3*ln(x)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(1/2)-4/3*b*c
^3*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {277, 198, 197, 5804, 12, 1813, 1634}

$$\frac{2c^2(a+b\sinh^{-1}(cx))}{dx(c^2dx^2+d)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3(c^2dx^2+d)^{3/2}}+\frac{16c^4x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\sinh^{-1}(cx))}{3d(c^2dx^2+d)^{3/2}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^3x^2\sqrt{c^2x^2+1}}+\frac{bc^3\sqrt{c^2dx^2+d}}{6d^3(c^2x^2+1)^{3/2}}-\frac{8bc^3\log(x)\sqrt{c^2dx^2+d}}{3d^3\sqrt{c^2x^2+1}}-\frac{4bc^3\sqrt{c^2dx^2+d}\log(c^2x^2+1)}{3d^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c^3*Sqrt[d + c^2*d*x^2])/(6*d^3*(1 + c^2*x^2)^(3/2)) - (b*c*Sqrt[d + c^2
*d*x^2])/(6*d^3*x^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*d*x^3*(d +
 c^2*d*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x]))/(d*x*(d + c^2*d*x^2)^(3/2
)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x
*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b*c^3*Sqrt[d + c^2*
d*x^2]*Log[x])/(3*d^3*Sqrt[1 + c^2*x^2]) - (4*b*c^3*Sqrt[d + c^2*d*x^2]*Log
[1 + c^2*x^2])/(3*d^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} - \left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^3(1+c^2x}\cdots}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(d+c^2dx^2\right)^{3/2}} + \left(8c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx + \cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(d+c^2dx^2\right)^{3/2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(16c^4\right)\cdots}{\cdots}$$

$$= \frac{7bc^3}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}}{6d^2x^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+\cdots\right.}{dx\left(d\cdots\right.}$$

$$= \frac{bc^3}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}}{6d^2x^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+\cdots\right.}{dx\left(d\cdots\right.}$$

Mathematica [**A**]
time = 0.24, size = 267, normalized size = 0.90

$$\frac{\sqrt{d+c^2dx^2}\left(-bcx-bc^3x^3-2a\sqrt{1+c^2x^2}+12ac^2x^2\sqrt{1+c^2x^2}+48ac^4x^4\sqrt{1+c^2x^2}+32ac^6x^6\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}\left(-1+6c^2x^2+24c^4x^4+16c^6x^6\right)\sinh^{-1}(cx)+8bc^3x^3\left(1+c^2x^2\right)^2\log\left(1+\frac{1}{c^2x^2}\right)-16bc^3x^3\log\left(1+c^2x^2\right)-32bc^5x^5\log\left(1+c^2x^2\right)-16bc^7x^7\log\left(1+c^2x^2\right)\right)}{6d^3x^3\left(1+c^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 12*a*c
^2*x^2*Sqrt[1 + c^2*x^2] + 48*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 32*a*c^6*x^6*Sq
rt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c
^6*x^6)*ArcSinh[c*x] + 8*b*c^3*x^3*(1 + c^2*x^2)^2*Log[1 + 1/(c^2*x^2)] - 1
6*b*c^3*x^3*Log[1 + c^2*x^2] - 32*b*c^5*x^5*Log[1 + c^2*x^2] - 16*b*c^7*x^7
*Log[1 + c^2*x^2]))/(6*d^3*x^3*(1 + c^2*x^2)^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1791 vs. 2(262) = 524.
time = 3.36, size = 1792, normalized size = 6.03

method	result	size
default	Expression too large to display	1792

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 344/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-
1)/d^3*x^3*arcsinh(c*x)*c^6-2*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^
```

```
6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*(c^2*x^2+1)^(1/2)*c^5+8/3*b*(d*(c^2*x^2+
1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*(c^2*x^2+1)
*c^4+12*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^
2-1)/d^3*x*arcsinh(c*x)*c^4+16/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6
*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+1/6*b*
(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x
^2*(c^2*x^2+1)^(1/2)*c-128/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6
+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2*x^2+1)*c^12-320/3*b*(d*(c^2*x^2+1))^
(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*(c^2*x^2+1)*c
^10+64*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2
-1)/d^3*x^7*arcsinh(c*x)*c^10-80*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6
*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*(c^2*x^2+1)*c^8+160*b*(d*(c^2*x^2+1))
^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*x)
*c^8-2*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2
-1)/d^3*(c^2*x^2+1)^(1/2)*c^3+32/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)/d^3*arcsinh(c*x)*c^3+560/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6
+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*c^10+280/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^
8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*c^8+32/3*b*(d*(c^2*x^2+1)
)^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*c^6-8/3*b*(
d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*
c^4+1/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^
2-1)/d^3/x^3*arcsinh(c*x)-8/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3
*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*c^3+128/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*
x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*c^14+448/3*b*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*c^12-6*b*(
d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*
arcsinh(c*x)*c^2-40/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4
*x^4+10*c^2*x^2-1)/d^3*x^3*(c^2*x^2+1)*c^6-176/3*b*(d*(c^2*x^2+1))^(1/2)/(1
2*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*c^5-64*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+
10*c^2*x^2-1)/d^3*x^6*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^9-128*b*(d*(c^2*x^2+
1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^4*(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*c^7+a*(-1/3/d/x^3/(c^2*d*x^2+d)^(3/2)-2*c^2*(-1/d/x/(
c^2*d*x^2+d)^(3/2)-4*c^2*(1/3*x/d/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+
d)^(1/2))))
```

Maxima [A]
time = 0.31, size = 236, normalized size = 0.79

$$-\frac{1}{6}\,bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{d^{\frac{5}{2}}}+\frac{16\,c^2\log\left(x\right)}{d^{\frac{5}{2}}}+\frac{1}{c^2d^{\frac{5}{2}}x^4+d^{\frac{5}{2}}x^2}\right)+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{8\,c^4x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}+\frac{6\,c^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}dx}-\frac{1}{\left(c^2dx^2+d\right)^{\frac{3}{2}}dx^3}\right)b\,\mathrm{arsinh}\left(cx\right)+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{8\,c^4x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}+\frac{6\,c^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}dx}-\frac{1}{\left(c^2dx^2+d\right)^{\frac{3}{2}}dx^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2*d
^(5/2)*x^4 + d^(5/2)*x^2)) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^2 + d)*d^2) + 8*c^
```

```
4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*d*x) - 1/((c^2
*d*x^2 + d)^(3/2)*d*x^3))*b*arcsinh(c*x) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^2 +
d)*d^2) + 8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*
d*x) - 1/((c^2*d*x^2 + d)^(3/2)*d*x^3))*a
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3
*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{x^4 (d (c^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))/(x**4*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^4 \left(d\,c^2\,x^2 + d\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(5/2)), x)
```

3.176 $\int \frac{\sinh^{-1}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}}+\frac{2}{15ac^3\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}+\frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}}+\frac{4x\sinh^{-1}(ax)}{15c^2\left(c+a^2cx^2\right)^{3/2}}+$$

```
[Out] 1/5*x*arcsinh(a*x)/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)/c^2/(a^2*c*x^2
+c)^(3/2)+1/20/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a
*x)/c^3/(a^2*c*x^2+c)^(1/2)+2/15/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-4/15*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5788, 5787, 266, 267}

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\,\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+\frac{4x\sinh^{-1}(ax)}{15c^2\left(a^2cx^2+c\right)^{3/2}}+\frac{x\sinh^{-1}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
```

```
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)}{5c\,(c+a^2cx^2)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx}{5c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{20ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\,(c+a^2cx^2)^{5/2}} + \frac{4x\sinh^{-1}(ax)}{15c^2\,(c+a^2cx^2)^{3/2}} + \frac{8\int \frac{\sinh}{(c+a^2}}{15}$$

$$= \frac{1}{20ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\,(c+a^2cx^2)^{5/2}}$$

$$= \frac{1}{20ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\,(c+a^2cx^2)^{5/2}}$$

Mathematica [A]
time = 0.10, size = 121, normalized size = 0.60

$$\frac{\sqrt{c+a^2cx^2}\left(4ax\sqrt{1+a^2x^2}\,(15+20a^2x^2+8a^4x^4)\sinh^{-1}(ax)-(1+a^2x^2)\left(-11-8a^2x^2+16(1+a^2x^2)^2\log\left(1+a^2x^2\right)\right)\right)}{60ac^4\,(1+a^2x^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4)
*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1 +
 a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(170) = 340$.
time = 2.59, size = 363, normalized size = 1.82

method	result
default	$\frac{16\sqrt{c\left(a^{2}x^{2}+1\right)}\,\mathrm{arcsinh}(ax)}{15\sqrt{a^{2}x^{2}+1}\,a\,c^{4}}+\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\left(8a^{5}x^{5}-8a^{4}\sqrt{a^{2}x^{2}+1}\,x^{4}+20a^{3}x^{3}-16\sqrt{a^{2}x^{2}+1}\,a^{2}x^{2}+15\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] 16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(a ^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*(a^2*x^2+1)^(1/2)*x^4+20*a^3*x^3-16*(a^2* x^2+1)^(1/2)*a^2*x^2+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*a^8*x^8-64*(a^2*x^2+1)^(1/2)*a^7*x^7-280*a^6*x^6-248*(a^2*x^2+1)^(1/2)*a^5*x^5+160*arcsinh(a*x)* a^4*x^4-456*a^4*x^4-340*(a^2*x^2+1)^(1/2)*a^3*x^3+380*arcsinh(a*x)*a^2*x^2- 328*a^2*x^2-165*(a^2*x^2+1)^(1/2)*a*x+256*arcsinh(a*x)-88)/(40*a^10*x^10+21 5*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)

Maxima [A]
time = 0.32, size = 143, normalized size = 0.72

$$\frac{1}{60}\,a\left(\frac{3}{\left(a^{6}c^{\frac{5}{2}}x^{4}+2\,a^{4}c^{\frac{5}{2}}x^{2}+a^{2}c^{\frac{5}{2}}\right)c}+\frac{8}{\left(a^{4}c^{\frac{3}{2}}x^{2}+a^{2}c^{\frac{3}{2}}\right)c^{2}}-\frac{16\,\log\left(x^{2}+\frac{1}{a^{2}}\right)}{a^{2}c^{\frac{7}{2}}}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{a^{2}cx^{2}+c}\;c^{3}}+\frac{4\,x}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}c^{2}}+\frac{3\,x}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}c}\right)\mathrm{arsinh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")

[Out] 1/60*a*(3/((a^6*c^(5/2)*x^4 + 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) + 8/((a^4 *c^(3/2)*x^2 + a^2*c^(3/2))*c^2) - 16*log(x^2 + 1/a^2)/(a^2*c^(7/2))) + 1/1 5*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6* a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{asinh}\left(ax\right)}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)/(c*(a**2*x**2 + 1))**(7/2), x)
```

Giac [A]
time = 0.45, size = 124, normalized size = 0.62

$$-\frac{1}{60}\sqrt{c}\left(\frac{16\log\left(a^2x^2+1\right)}{ac^4}-\frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right)+\frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35)
/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x*l
og(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2), x)
```

3.177 $\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=86

$$\frac{3x^2}{16a^3}-\frac{x^4}{16a}-\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{16a^5}$$

```
[Out] 3/16*x^2/a^3-1/16*x^4/a+3/16*arcsinh(a*x)^2/a^5-3/8*x*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5812, 5783, 30}

$$\frac{3\sinh^{-1}(ax)^2}{16a^5}+\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{8a^4}-\frac{x^4}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\,dx}{4a}$$
$$= -\frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} + \frac{3\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^4}$$
$$= \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} + \frac{3\sinh^{-1}(ax)^2}{16a^5}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.73

$$\frac{3a^2x^2 - a^4x^4 + 2ax\sqrt{1+a^2x^2}\,(-3+2a^2x^2)\sinh^{-1}(ax) + 3\sinh^{-1}(ax)^2}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x
] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

Maple [A]
time = 2.48, size = 68, normalized size = 0.79

$$\frac{4\operatorname{arcsinh}(ax)\cosh(2\operatorname{arcsinh}(ax))\sinh(2\operatorname{arcsinh}(ax)) + 12\operatorname{arcsinh}(ax)^2 - 16\operatorname{arcsinh}(ax)\sinh(2\operatorname{arcsinh}(a}{64a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/64*(4*arcsinh(a*x)*cosh(2*arcsinh(a*x))*sinh(2*arcsinh(a*x))+12*arcsinh(a
*x)^2-16*arcsinh(a*x)*sinh(2*arcsinh(a*x))-cosh(2*arcsinh(a*x))^2+8*cosh(2*
arcsinh(a*x)))/a^5
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.97

$$-\frac{1}{16}\left(\frac{x^4}{a^2} - \frac{3x^2}{a^4} + \frac{3\operatorname{arsinh}(ax)^2}{a^6}\right)a + \frac{1}{8}\left(\frac{2\sqrt{a^2x^2+1}\,x^3}{a^2} - \frac{3\sqrt{a^2x^2+1}\,x}{a^4} + \frac{3\operatorname{arsinh}(ax)}{a^5}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a*x)^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2
+ 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*arcsinh(a*x)
```

Fricas [A]
time = 0.39, size = 83, normalized size = 0.97

$$-\frac{a^4x^4-3\,a^2x^2-2\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

Sympy [A]
time = 0.44, size = 82, normalized size = 0.95

$$\begin{cases} -\frac{x^4}{16a}+\frac{x^3\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{4a^2}+\frac{3x^2}{16a^3}-\frac{3x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{8a^4}+\frac{3\operatorname{asinh}^2(ax)}{16a^5} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3*
x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*x)
**2/(16*a**5), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.178 $\int \frac{x^3 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=70

$$\frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2}$$

```
[Out] 2/3*x/a^3-1/9*x^3/a-2/3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(
a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5812, 5798, 8, 30}

$$\frac{2x}{3a^3} + \frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^2} - \frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^4} - \frac{x^3}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
```

```
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\,dx}{3a}\\ &= -\frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2} + \frac{2\int 1\,dx}{3a^3}\\ &= \frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.69

$$\frac{6ax - a^3x^3 + 3(-2+a^2x^2)\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4)
```

Maple [A]
time = 3.24, size = 50, normalized size = 0.71

$$\frac{-27\,\mathrm{arcsinh}\,(ax)\,\sqrt{a^2x^2+1} + 3\,\mathrm{arcsinh}\,(ax)\cosh\left(3\,\mathrm{arcsinh}\,(ax)\right) + 27ax - \sinh\left(3\,\mathrm{arcsinh}\,(ax)\right)}{36a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] 1/36*(-27*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+3*arcsinh(a*x)*cosh(3*arcsinh(a*x)
)+27*a*x-sinh(3*arcsinh(a*x)))/a^4
```

Maxima [A]
time = 0.30, size = 59, normalized size = 0.84

$$-\frac{1}{9}\,a\left(\frac{x^3}{a^2} - \frac{6\,x}{a^4}\right) + \frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2} - \frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^
2 + 1)/a^4)*arcsinh(a*x)
```

Fricas [A]
time = 0.38, size = 55, normalized size = 0.79

$$-\frac{a^3x^3 - 3\sqrt{a^2x^2+1}\,(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right) - 6\,ax}{9\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
1)) - 6*a*x)/a^4
```

Sympy [A]
time = 0.34, size = 65, normalized size = 0.93

$$\begin{cases} -\frac{x^3}{9a} + \frac{x^2\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{3a^4} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*x
/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, True)
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.179 $\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=49

$$-\frac{x^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2} - \frac{\sinh^{-1}(ax)^2}{4a^3}$$

```
[Out] -1/4*x^2/a-1/4*arcsinh(a*x)^2/a^3+1/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5812, 5783, 30}

$$-\frac{\sinh^{-1}(ax)^2}{4a^3} + \frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a^2} - \frac{x^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2} - \frac{\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{\int x\,dx}{2a}$$
$$= -\frac{x^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2} - \frac{\sinh^{-1}(ax)^2}{4a^3}$$

Mathematica [**A**]
time = 0.03, size = 42, normalized size = 0.86

$$-\frac{a^2x^2 - 2ax\sqrt{1+a^2x^2}\,\sinh^{-1}(ax) + \sinh^{-1}(ax)^2}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] -1/4*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/a^3
```

Maple [**A**]
time = 1.60, size = 40, normalized size = 0.82

method	result	size
default	$-\frac{-2\,\mathrm{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+a^2x^2+\mathrm{arcsinh}(ax)^2+1}{4a^3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

Maxima [**A**]
time = 0.31, size = 55, normalized size = 1.12

$$-\frac{1}{4}\,a\left(\frac{x^2}{a^2} - \frac{\mathrm{arsinh}\,(ax)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}\,x}{a^2} - \frac{\mathrm{arsinh}\,(ax)}{a^3}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*a*(x^2/a^2 - arcsinh(a*x)^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*x/a^2 - arcs
inh(a*x)/a^3)*arcsinh(a*x)
```

Fricas [A]
time = 0.36, size = 62, normalized size = 1.27

$$-\frac{a^2x^2 - 2\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log(
a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A]
time = 0.27, size = 42, normalized size = 0.86

$$\begin{cases} -\frac{x^2}{4a} + \frac{x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{2a^2} - \frac{\operatorname{asinh}^2(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh(
a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.180 $\int \frac{x \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=28

$$-\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}$$

```
[Out] -x/a+arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5798, 8}

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}-\frac{\int 1\,dx}{a}\\ &= -\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$-\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Maple [A]
time = 2.09, size = 47, normalized size = 1.68

method	result	size
default	$\frac{\operatorname{arcsinh}(ax)a^2x^2+\operatorname{arcsinh}(ax)-\sqrt{a^2x^2+1}\,ax}{a^2\sqrt{a^2x^2+1}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)*a^2*x^2+arcsinh(a*x)-(a^2*x^2+1)^(1/2
)*a*x)
```

Maxima [A]
time = 0.32, size = 26, normalized size = 0.93

$$-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

Fricas [A]
time = 0.35, size = 38, normalized size = 1.36

$$-\frac{ax-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A]
time = 0.20, size = 24, normalized size = 0.86

$$\begin{cases} -\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True)
)
```

Giac [**A**]
time = 0.43, size = 38, normalized size = 1.36

$$-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.181 $\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{5783\}$

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^2}{2a}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Maple [A]
time = 0.29, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^2}{2a}$	12
default	$\frac{\operatorname{arcsinh}(ax)^2}{2a}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Maxima [A]
time = 0.31, size = 11, normalized size = 0.85

$$\frac{\operatorname{arsinh}(ax)^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 0.36, size = 23, normalized size = 1.77

$$\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

Sympy [A]
time = 0.18, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [B]
time = 0.10, size = 11, normalized size = 0.85

$$\frac{\mathrm{asinh}(a\,x)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(a^2*x^2 + 1)^(1/2),x)
[Out] asinh(a*x)^2/(2*a)
```

3.182 $\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=34

$$-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(
1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A]
time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5816, 4267, 2317, 2438}

$$-\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
```

```
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + \text{Sub}$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(ax)}\right) + \text{Subst}$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + \text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)$$

Mathematica [A]
time = 0.08, size = 57, normalized size = 1.68

$$\sinh^{-1}(ax)\left(\log\left(1-e^{-\sinh^{-1}(ax)}\right)-\log\left(1+e^{-\sinh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
[Out] ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + Po
lyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

Maple [A]
time = 1.32, size = 42, normalized size = 1.24

method	result	size
default	$2\,\text{dilog}\left(\frac{1}{ax+\sqrt{a^2x^2+1}}\right)-\frac{\text{dilog}\left(\frac{1}{\left(ax+\sqrt{a^2x^2+1}\right)^2}\right)}{2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*dilog(1/(a*x+(a^2*x^2+1)^(1/2)))-1/2*dilog(1/(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}(a\,x)}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.183 $\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\log(x)$$

```
[Out] a*ln(x)-arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5800, 29}

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\int \frac{1}{x}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\log(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.07

$$-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{x}+a\log(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(25) = 50$.
time = 3.90, size = 56, normalized size = 2.07

method	result	size
default	$-2a\,\mathrm{arcsinh}\,(ax)+\frac{\left(ax-\sqrt{a^2x^2+1}\right)\mathrm{arcsinh}(ax)}{x}+a\ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2-1\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^2
+1)^(1/2))^2-1)
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.93

$$a\log\left(x\right)-\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

Fricas [A]
time = 0.37, size = 39, normalized size = 1.44

$$\frac{ax\log\left(x\right)-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.
time = 0.41, size = 71, normalized size = 2.63

$$-a\log\left(-x|a|+\sqrt{a^2x^2+1}\right)+a\log(|x|)+\frac{2|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) + a*log(abs(x)) + 2*abs(a)*log(a*x +
sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{asinh}(a\,x)}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.184 $\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{PolyLo}$$

```
[Out] -1/2*a/x+a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,
-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsin
h(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]

time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5809, 5816, 4267, 2317, 2438, 30}

$$\frac{1}{2}a^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -1/2*a/x - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int \frac{1}{x^2}\,dx-\frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}-\frac{1}{2}a^2\text{Subst}\left(\int x\text{csch}(x)\,dx,x,\sinh^{-1}(ax)\right)\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{Subst}\left(\int\right.\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{Subst}\left(\int\right.\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{Li}_2\left(-e^{\sin}\right.
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 126, normalized size = 1.58

$$\frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(ax)\right)-\sinh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(ax)\right)-4\sinh^{-1}(ax)\log\left(1-e^{-\sinh^{-1}(ax)}\right)+4\sinh^{-1}(ax)\log\left(1+e^{-\sinh^{-1}(ax)}\right)-4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)+4\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)-\sinh^{-1}(ax)\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(ax)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

[Out] (a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*ArcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x])] - ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8

Maple [A]
time = 5.02, size = 150, normalized size = 1.88

method	result
default	$-\frac{\operatorname{arcsinh}(ax)a^2x^2+\sqrt{a^2x^2+1}\,ax+\operatorname{arcsinh}(ax)}{2\sqrt{a^2x^2+1}\,x^2}-\frac{a^2\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)}{2}-\frac{a^2\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)*a^2*x^2+(a^2*x^2+1)^(1/2)*a*x+arcsinh(a*x))/x^2-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

3.185 $\int x^m(d+c^2dx^2)^3\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=313

$$-\frac{bcd^3(2271+1329m+284m^2+27m^3+m^4)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2}-\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2}-b$$

```
[Out] d^3*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+3*c^2*d^3*x^(3+m)*(a+b*arcsinh(c*x))/(
3+m)+3*c^4*d^3*x^(5+m)*(a+b*arcsinh(c*x))/(5+m)+c^6*d^3*x^(7+m)*(a+b*arcsin
h(c*x))/(7+m)-3*b*c*d^3*(35*m^3+455*m^2+1813*m+2161)*x^(2+m)*hypergeom([1/2
, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(m^2+3*m+2)/(m^3+15*m^2+71*m+105)^2-b*c*d^3*
(m^4+27*m^3+284*m^2+1329*m+2271)*x^(2+m)*(c^2*x^2+1)^(1/2)/(7+m)^2/(m^2+8*m
+15)^2-b*c^3*d^3*(9+m)*(13+2*m)*x^(4+m)*(c^2*x^2+1)^(1/2)/(5+m)^2/(7+m)^2-b
*c^5*d^3*x^(6+m)*(c^2*x^2+1)^(1/2)/(7+m)^2
```

Rubi [A]
time = 1.31, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.292$, Rules used = {276, 5803, 12, 1823, 1281, 470, 371}

$$\frac{c^6d^3x^{m+7}\left(a+b\sinh^{-1}(cx)\right)}{m+7}+\frac{3c^4d^3x^{m+5}\left(a+b\sinh^{-1}(cx)\right)}{m+5}+\frac{3c^2d^3x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{d^3x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{m+1}-\frac{3bcd^3(35m^3+455m^2+1813m+2161)\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)^2(m+7)^2}-\frac{bcd^3(m^4+27m^3+284m^2+1329m+2271)\sqrt{c^2x^2+1}\,x^{m+2}}{(m+3)^2(m+5)^2(m+7)^2}-\frac{bc^5d^3\sqrt{c^2x^2+1}\,x^{m+6}}{(m+7)^2}-\frac{bc^3d^3(m+9)(2m+13)\sqrt{c^2x^2+1}\,x^{m+4}}{(m+5)^2(m+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*x^(2 + m)*Sqrt[1 + c^2*
x^2])/((3 + m)^2*(5 + m)^2*(7 + m)^2)) - (b*c^3*d^3*(9 + m)*(13 + 2*m)*x^(4
 + m)*Sqrt[1 + c^2*x^2])/((5 + m)^2*(7 + m)^2) - (b*c^5*d^3*x^(6 + m)*Sqrt[
1 + c^2*x^2])/(7 + m)^2 + (d^3*x^(1 + m)*(a + b*ArcSinh[c*x]))/(1 + m) + (3
*c^2*d^3*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) + (3*c^4*d^3*x^(5 + m)*(a
+ b*ArcSinh[c*x]))/(5 + m) + (c^6*d^3*x^(7 + m)*(a + b*ArcSinh[c*x]))/(7 +
m) - (3*b*c*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*x^(2 + m)*Hypergeometric
2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 +
 m)^2*(7 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^m\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{3c^2d^3x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}+\frac{3c^4}{}$$

$$=\frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{3c^2d^3x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}+\frac{3c^4}{}$$

$$=-\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2}+\frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{3c^2d^3x^{3+}}{}$$

$$=-\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2}-\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x}}{(7+m)^2}$$

$$=-\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2}-$$

$$=-\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2}-$$

Mathematica [A]
time = 0.33, size = 257, normalized size = 0.82

$$\frac{x^{1+m}\left(\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^3x\,{}_2F_1\left(-\frac{5}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)}{2+m}+\frac{6d\left(\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2x\,{}_2F_1\left(-\frac{3}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)}{2+m}+\frac{4d^2\left((2+m)\left(3+m+c^2x^2+c^2mx^2\right)\left(a+b\sinh^{-1}(cx)\right)-bc(1+m)x\,{}_2F_1\left(-\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)-2bcx\,{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)\right)}{(1+m)(2+m)(3+m)}\right)}{5+m}\right)}{7+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]) - (b*c*d^3*x*Hypergeomet
ric2F1[-5/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (6*d*((d + c^2*d*x^2)
^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeometric2F1[-3/2, 1 + m/2, 2 + m
/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a
 + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2
, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)
]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)))/(7 + m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^m\left(c^2d\,x^2+d\right)^3\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] a*c^6*d^3*x^(m + 7)/(m + 7) + 3*a*c^4*d^3*x^(m + 5)/(m + 5) + 3*a*c^2*d^3*x
^(m + 3)/(m + 3) + a*d^3*x^(m + 1)/(m + 1) + ((m^3 + 9*m^2 + 23*m + 15)*b*c
^6*d^3*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^4*d^3*x^5 + 3*(m^3 + 13*m^2 +
 47*m + 35)*b*c^2*d^3*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*x)*x^m*log(c*
x + sqrt(c^2*x^2 + 1))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) - integrate(((
m^3 + 9*m^2 + 23*m + 15)*b*c^7*d^3*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^5
*d^3*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^3*d^3*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*b*c*d^3*x)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*x^3 + (m
^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*sqrt(c^2*x^2 + 1)), x)
 - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^8*d^3*x^8 + 3*(m^3 + 11*m^2 + 3
1*m + 21)*b*c^6*d^3*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^4*d^3*x^4 + (m^3
 + 15*m^2 + 71*m + 105)*b*c^2*d^3*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))*x^m, x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int ax^{m}\, dx + \int bx^{m} \operatorname{asinh}{\left(cx \right)}\, dx + \int 3ac^{2}x^{2}x^{m}\, dx + \int 3ac^{4}x^{4}x^{m}\, dx + \int ac^{6}x^{6}x^{m}\, dx + \int 3bc^{2}x^{2}x^{m} \operatorname{asinh}{\left(cx \right)}\, dx + \int 3bc^{4}x^{4}x^{m} \operatorname{asinh}{\left(cx \right)}\, dx + \int bc^{6}x^{6}x^{m} \operatorname{asinh}{\left(cx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] d**3*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(3*a*c
**2*x**2*x**m, x) + Integral(3*a*c**4*x**4*x**m, x) + Integral(a*c**6*x**6*
x**m, x) + Integral(3*b*c**2*x**2*x**m*asinh(c*x), x) + Integral(3*b*c**4*x
**4*x**m*asinh(c*x), x) + Integral(b*c**6*x**6*x**m*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.186 $\int x^m(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=217

$$-\frac{bcd^2(38+13m+m^2)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2}-\frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2}+\frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{2c^2d^2x^{3+m}\left(a\right.}{3}$$

```
[Out] d^2*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+2*c^2*d^2*x^(3+m)*(a+b*arcsinh(c*x))/(
3+m)+c^4*d^2*x^(5+m)*(a+b*arcsinh(c*x))/(5+m)-b*c*d^2*(15*m^2+100*m+149)*x^
(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(m^2+3*m+2)/(m^2+8*m+15)
^2-b*c*d^2*(m^2+13*m+38)*x^(2+m)*(c^2*x^2+1)^(1/2)/(3+m)^2/(5+m)^2-b*c^3*d^
2*x^(4+m)*(c^2*x^2+1)^(1/2)/(5+m)^2
```

Rubi [A]
time = 0.22, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {276, 5803, 12, 1281, 470, 371}

$$\frac{c^4d^2x^{m+5}\left(a+b\sinh^{-1}(cx)\right)}{m+5}+\frac{2c^2d^2x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{d^2x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{m+1}-\frac{bcd^2(15m^2+100m+149)\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)^2}-\frac{bcd^2(m^2+13m+38)\sqrt{c^2x^2+1}\,x^{m+2}}{(m+3)^2(m+5)^2}-\frac{bc^3d^2\sqrt{c^2x^2+1}\,x^{m+4}}{(m+5)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*x^(2 + m)*Sqrt[1 + c^2*x^2])/((3 + m)^2*(5 + m
)^2)) - (b*c^3*d^2*x^(4 + m)*Sqrt[1 + c^2*x^2])/(5 + m)^2 + (d^2*x^(1 + m)*
(a + b*ArcSinh[c*x]))/(1 + m) + (2*c^2*d^2*x^(3 + m)*(a + b*ArcSinh[c*x]))/
(3 + m) + (c^4*d^2*x^(5 + m)*(a + b*ArcSinh[c*x]))/(5 + m) - (b*c*d^2*(149
+ 100*m + 15*m^2)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(
c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{c^4}{}$$

$$= \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{c^4}{}$$

$$= -\frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^3}{}$$

$$= -\frac{bcd^2(38+13m+m^2)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} - \frac{bc^3d^2x^{4+m}\sqrt{1+c^2x}}{(5+m)^2}$$

$$= -\frac{bcd^2(38+13m+m^2)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} - \frac{bc^3d^2x^{4+m}\sqrt{1+c^2x}}{(5+m)^2}$$

Mathematica [A]
time = 0.01, size = 188, normalized size = 0.87

$$\frac{x^{1+m}\left(\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2x\,{}_2F_1\left(-\frac{3}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)}{2+m} + \frac{4d^2\left((2+m)(3+m+c^2x^2+c^2mx^2)(a+b\sinh^{-1}(cx)) - bc(1+m)x\,{}_2F_1\left(-\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right) - 2bcx\,{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)\right)}{(1+m)(2+m)(3+m)}\right)}{5+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeomet
ric2F1[-3/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 +
m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometri
c2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1
 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^m \left(c^2d\,x^2+d\right)^2 \left(a+b\,\mathrm{arcsinh}\,(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x^(m + 5)/(m + 5) + 2*a*c^2*d^2*x^(m + 3)/(m + 3) + a*d^2*x^(m +
1)/(m + 1) + ((m^2 + 4*m + 3)*b*c^4*d^2*x^5 + 2*(m^2 + 6*m + 5)*b*c^2*d^2*x
^3 + (m^2 + 8*m + 15)*b*d^2*x)*x^m*log(c*x + sqrt(c^2*x^2 + 1))/(m^3 + 9*m^
2 + 23*m + 15) - integrate(((m^2 + 4*m + 3)*b*c^5*d^2*x^5 + 2*(m^2 + 6*m +
5)*b*c^3*d^2*x^3 + (m^2 + 8*m + 15)*b*c*d^2*x)*x^m/((m^3 + 9*m^2 + 23*m + 1
5)*c^3*x^3 + (m^3 + 9*m^2 + 23*m + 15)*c*x + ((m^3 + 9*m^2 + 23*m + 15)*c^2
*x^2 + m^3 + 9*m^2 + 23*m + 15)*sqrt(c^2*x^2 + 1)), x) - integrate(((m^2 +
4*m + 3)*b*c^6*d^2*x^6 + 2*(m^2 + 6*m + 5)*b*c^4*d^2*x^4 + (m^2 + 8*m + 15)
*b*c^2*d^2*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m
 + 15), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int ax^{m}\,dx+\int bx^{m}\operatorname{asinh}\left(cx\right)dx+\int 2ac^{2}x^{2}x^{m}\,dx+\int ac^{4}x^{4}x^{m}\,dx+\int 2bc^{2}x^{2}x^{m}\operatorname{asinh}\left(cx\right)dx+\int bc^{4}x^{4}x^{m}\operatorname{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] d**2*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(2*a*c
**2*x**2*x**m, x) + Integral(a*c**4*x**4*x**m, x) + Integral(2*b*c**2*x**2*
x**m*asinh(c*x), x) + Integral(b*c**4*x**4*x**m*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.187 $\int x^m(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=128

$$-\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}-\frac{bcd(7+3m)x^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2}\right.}{(1+m)(2+m)(3+}$$

```
[Out] d*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+c^2*d*x^(3+m)*(a+b*arcsinh(c*x))/(3+m)-b
*c*d*(7+3*m)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(3+m)^2/(
m^2+3*m+2)-b*c*d*x^(2+m)*(c^2*x^2+1)^(1/2)/(3+m)^2
```

Rubi [A]
time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {14, 5803, 12, 470, 371}

$$\frac{c^2dx^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{dx^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{m+1}-\frac{bcd(3m+7)x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{(m+1)(m+2)(m+3)^2}-\frac{bcd\sqrt{c^2x^2+1}\,x^{m+2}}{(m+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d*x^(2 + m)*Sqrt[1 + c^2*x^2])/(3 + m)^2) + (d*x^(1 + m)*(a + b*ArcS
inh[c*x]))/(1 + m) + (c^2*d*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) - (b*c*
d*(7 + 3*m)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^
2)])/((1 + m)*(2 + m)*(3 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx &= \frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}-(bc)\int\\ &= \frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}-(bcd)\\ &= -\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a\right.}{3}\\ &= -\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a\right.}{3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 118, normalized size = 0.92

$$\frac{dx^{1+m}\left((2+m)\left(3+m+c^2x^2+c^2mx^2\right)\left(a+b\sinh^{-1}(cx)\right)-bc(1+m)x\,{}_2F_1\left(-\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)-2bcx\,{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)\right)}{(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*x^(1 + m)*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) -
b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c
*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*
(3 + m))
```

Maple [F]
time = 4.56, size = 0, normalized size = 0.00

$$\int x^m\left(c^2d\,x^2+d\right)\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] a*c^2*d*x^(m + 3)/(m + 3) + a*d*x^(m + 1)/(m + 1) + (b*c^2*d*(m + 1)*x^3 +
b*d*(m + 3)*x)*x^m*log(c*x + sqrt(c^2*x^2 + 1))/(m^2 + 4*m + 3) - integrate
((b*c^3*d*(m + 1)*x^3 + b*c*d*(m + 3)*x)*x^m/((m^2 + 4*m + 3)*c^3*x^3 + (m^
2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3)*sqrt(c^2*x^2 +
 1)), x) - integrate((b*c^4*d*(m + 1)*x^4 + b*c^2*d*(m + 3)*x^2)*x^m/((m^2
+ 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int ax^m\,dx + \int bx^m \operatorname{asinh}(cx)\,dx + \int ac^2x^2x^m\,dx + \int bc^2x^2x^m \operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] d*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(a*c**2*x
**2*x**m, x) + Integral(b*c**2*x**2*x**m*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.188 $\int \frac{x^m(a+b\sinh^{-1}(cx))}{d+c^2dx^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx=\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Mathematica [A]
time = 2.63, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+b\,\mathrm{arcsinh}\,(cx))}{c^2d\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^m}{c^2x^2+1}\,dx + \int \frac{bx^m \operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**m/(c**2*x**2 + 1), x) + Integral(b*x**m*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,(a+b\,\mathrm{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.189 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=116

$$\frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}-\frac{bcx^{2+m}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{2d^2(2+m)}+\frac{(1-m)\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2},x\right)}{2d}$$

```
[Out] 1/2*x^(1+m)*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)-1/2*b*c*x^(2+m)*hypergeom([3
/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/d^2/(2+m)+1/2*(1-m)*Unintegrable(x^m*(a+b*
arcsinh(c*x))/(c^2*d*x^2+d),x)/d
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (b*c*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(2*d^2*(2 + m)) + ((
1 - m)*Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x])/(2*d)
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx &= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}-\frac{(bc)\int\frac{x^{1+m}}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2}+\frac{(1-m)\int\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,}{2d}\\ &= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}-\frac{bcx^{2+m}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{2d^2(2+m)}+\frac{(1-m)\int\frac{x^m(}{}}{2}\end{aligned}$$

Mathematica [A]
time = 3.89, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} (a +b \,\mathrm{arcsinh}\left(c x \right))}{\left(c^{2} d \,x^{2}+d \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a x^{m}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx + \int \frac{b x^{m} \operatorname{asinh}{\left(c x \right)}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a*x**m/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**m*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.190 $\int \frac{x^m(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=207

$$\frac{x^{1+m}(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2}+\frac{(3-m)x^{1+m}(a+b\sinh^{-1}(cx))}{8d^3(1+c^2x^2)}-\frac{bc(3-m)x^{2+m}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{8d^3(2+m)}-\frac{bcx^{2+m}}{}$$

```
[Out] 1/4*x^(1+m)*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/8*(3-m)*x^(1+m)*(a+b*arc
sinh(c*x))/d^3/(c^2*x^2+1)-1/8*b*c*(3-m)*x^(2+m)*hypergeom([3/2, 1+1/2*m],[
2+1/2*m],-c^2*x^2)/d^3/(2+m)-1/4*b*c*x^(2+m)*hypergeom([5/2, 1+1/2*m],[2+1/
2*m],-c^2*x^2)/d^3/(2+m)+1/8*(1-m)*(3-m)*Unintegrable(x^m*(a+b*arcsinh(c*x)
)/(c^2*d*x^2+d),x)/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + ((3 - m)*x^(1 +
m)*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) - (b*c*(3 - m)*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(8*d^3*(2 + m)) - (b
*c*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(4*d
^3*(2 + m)) + ((1 - m)*(3 - m)*Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c
^2*d*x^2), x])/(8*d^2)
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^3}\,dx &= \frac{x^{1+m}(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2}-\frac{(bc)\int \frac{x^{1+m}}{(1+c^2x^2)^{5/2}}\,dx}{4d^3}+\frac{(3-m)\int \frac{x^m(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^2}\,dx}{4d}\\ &= \frac{x^{1+m}(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2}+\frac{(3-m)x^{1+m}(a+b\sinh^{-1}(cx))}{8d^3(1+c^2x^2)}-\frac{bcx^{2+m}\,{}_2F_1\left(\frac{5}{2},\right.}{4d^3(2}\\ &= \frac{x^{1+m}(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2}+\frac{(3-m)x^{1+m}(a+b\sinh^{-1}(cx))}{8d^3(1+c^2x^2)}-\frac{bc(3-m)x^{2+m}}{8}\end{aligned}$$

Mathematica [A]
time = 4.14, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(a+b \sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (a+b \,\mathrm{arcsinh}\,(cx))}{\left(c^2 d\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*
x^2 + d^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^m}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^m \operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**m/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**m*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.191 $\int x^m(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=618

$$-\frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}}-\frac{5bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(6+m)\left(8+6m+m^2\right)\sqrt{1+c^2x^2}}-\frac{bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{\left(12+8m+m^2\right)\sqrt{1+c^2x^2}}-$$

```
[Out] 5*d*x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(4+m)/(6+m)+x^(1+m)*(c^2
*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(6+m)+15*d^2*x^(1+m)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)-15*b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/2
)/(2+m)^2/(4+m)/(6+m)/(c^2*x^2+1)^(1/2)-5*b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/
2)/(6+m)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/2)/
(m^2+8*m+12)/(c^2*x^2+1)^(1/2)-5*b*c^3*d^2*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m
)^2/(6+m)/(c^2*x^2+1)^(1/2)-2*b*c^3*d^2*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m)/(
6+m)/(c^2*x^2+1)^(1/2)-b*c^5*d^2*x^(6+m)*(c^2*d*x^2+d)^(1/2)/(6+m)^2/(c^2*x
^2+1)^(1/2)+15*d^2*x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3
/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^3+7*m^2+14*m+8)/(c^2*x^2+1
)^(1/2)-15*b*c*d^2*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/
2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(6+m)/(m^2+5*m+4)/(c^2*x^2+1)^(1
/2)
```

Rubi [A]
time = 0.37, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5808, 5806, 5817, 30, 14, 276}

$$-\frac{15bcd^2x^{m+2}\sqrt{c^2dx^2+d}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)(m+6)\sqrt{c^2x^2+1}}+\frac{15d^2x^{m+1}\sqrt{c^2dx^2+d}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(m+6)\left(m^3+7m^2+14m+8\right)\sqrt{c^2x^2+1}}+\frac{15d^2x^{m+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{(m+6)\left(m^2+6m+8\right)}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{m+6}+\frac{5dx^{m+1}\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(m+4)(m+6)}-\frac{5bcd^2x^{m+2}\sqrt{c^2dx^2+d}}{(m+6)\left(m^2+6m+8\right)\sqrt{c^2x^2+1}}-\frac{bcd^2x^{m+2}\sqrt{c^2dx^2+d}}{\left(m^2+8m+12\right)\sqrt{c^2x^2+1}}-\frac{15bcd^2x^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2(m+4)(m+6)\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^{m+6}\sqrt{c^2dx^2+d}}{(m+6)^2\sqrt{c^2x^2+1}}-\frac{2bc^3d^2x^{m+4}\sqrt{c^2dx^2+d}}{(m+4)(m+6)\sqrt{c^2x^2+1}}-\frac{5bc^3d^2x^{m+4}\sqrt{c^2dx^2+d}}{(m+4)^2(m+6)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*(6 + m)*Sqrt
[1 + c^2*x^2]) - (5*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((6 + m)*(8 + 6*
m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((12
+ 8*m + m^2)*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*x^(4 + m)*Sqrt[d + c^2*d*x^2
])/((4 + m)^2*(6 + m)*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d^2*x^(4 + m)*Sqrt[d +
c^2*d*x^2])/((4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^(6 + m)*Sqrt
[d + c^2*d*x^2])/((6 + m)^2*Sqrt[1 + c^2*x^2]) + (15*d^2*x^(1 + m)*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((6 + m)*(8 + 6*m + m^2)) + (5*d*x^(1 + m
)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((4 + m)*(6 + m)) + (x^(1 + m
)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(6 + m) + (15*d^2*x^(1 + m)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, -(c^2*x^2)])/((6 + m)*(8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2])
 - (15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
```

```
 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*(6
 + m)*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
```

```
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\int x^m\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{x^{1+m}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{6+m}+\frac{(5d)\int x^m\left(d+c^2dx^2\right.}{}$$

$$=\frac{5dx^{1+m}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(4+m)(6+m)}+\frac{x^{1+m}\left(d+c^2dx^2\right)^{5}}{6}$$

$$=-\frac{bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{\left(12+8m+m^2\right)\sqrt{1+c^2x^2}}-\frac{2bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)(6+m)\sqrt{1+c^2x^2}}$$

$$=-\frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}}-\frac{5bcd^2x^{2+m}\sqrt{d}}{(2+m)(4+m)(6+}$$

Mathematica [**A**]
time = 0.84, size = 332, normalized size = 0.54

$$\frac{d^2x^{1+m}\sqrt{d+c^2dx^2}\left(-\frac{bcx\left((4+m)(6+m)+2c^2(2+m)(6+m)x^2+c^4(2+m)(4+m)x^4\right)}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}}+\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)-\frac{5\left(bc(1+m)(2+m)x\left(4+m+c^2(2+m)x^2\right)-(1+m)(2+m)^2(4+m)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+3(4+m)\left(bc(1+m)x-(1+m)(2+m)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-(2+m)\left(a+b\sinh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)+bcx\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)\right)\right)}{(1+m)(2+m)^2(4+m)^2\sqrt{1+c^2x^2}}\right)}{6+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*((4 + m)*(6 + m) + 2*c^2*(2 +
m)*(6 + m)*x^2 + c^4*(2 + m)*(4 + m)*x^4))/((2 + m)*(4 + m)*(6 + m)*Sqrt[1
+ c^2*x^2])) + (1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]) - (5*(b*c*(1 + m)*(2 +
m)*x*(4 + m + c^2*(2 + m)*x^2) - (1 + m)*(2 + m)^2*(4 + m)*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]) + 3*(4 + m)*(b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2 + m)*(a + b*ArcSinh[c*x])*Hypergeome
tric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] + b*c*x*HypergeometricPFQ[{1
, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])))/((1 + m)*(2 + m)^
2*(4 + m)^2*Sqrt[1 + c^2*x^2])))/(6 + m)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^m\left(c^2d\,x^2+d\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
[Out] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.192 $\int x^m(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=390

$$-\frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)\sqrt{1+c^2x^2}}-\frac{bcdx^{2+m}\sqrt{d+c^2dx^2}}{(8+6m+m^2)\sqrt{1+c^2x^2}}-\frac{bc^3dx^{4+m}\sqrt{d+c^2dx^2}}{(4+m)^2\sqrt{1+c^2x^2}}+\frac{3dx^{1+m}\sqrt{d+c^2dx^2}\,(a}{8+6m+}$$

```
[Out] x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(4+m)+3*d*x^(1+m)*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(m^2+6*m+8)-3*b*c*d*x^(2+m)*(c^2*d*x^2+d)^(1/2
)/(2+m)^2/(4+m)/(c^2*x^2+1)^(1/2)-b*c*d*x^(2+m)*(c^2*d*x^2+d)^(1/2)/(m^2+6*
m+8)/(c^2*x^2+1)^(1/2)-b*c^3*d*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(c^2*x^2
+1)^(1/2)+3*d*x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(m^3+7*m^2+14*m+8)/(c^2*x^2+1)^(1/2)-3*b
*c*d*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)
*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(m^2+5*m+4)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5808, 5806, 5817, 30, 14}

$$-\frac{3bcdx^{m+2}\sqrt{c^2dx^2+d}\;{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)\sqrt{c^2x^2+1}}+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(m^3+7m^2+14m+8)\sqrt{c^2x^2+1}}+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{m^2+6m+8}+\frac{x^{m+1}(c^2dx^2+d)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{m+4}-\frac{bcdx^{m+2}\sqrt{c^2dx^2+d}}{(m^2+6m+8)\sqrt{c^2x^2+1}}-\frac{3bcdx^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2(m+4)\sqrt{c^2x^2+1}}-\frac{bc^3dx^{m+4}\sqrt{c^2dx^2+d}}{(m+4)^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*Sqrt[1 + c^2*x^
2]) - (b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((8 + 6*m + m^2)*Sqrt[1 + c^2*x
^2]) - (b*c^3*d*x^(4 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^2*Sqrt[1 + c^2*x^2]
) + (3*d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8 + 6*m + m^2
) + (x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(4 + m) + (3*d*x
^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2
]) - (3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*Sqr
t[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) \, dx = \frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)}{4+m} + \frac{(3d)\int x^m \sqrt{d+c^2dx}}{4}$$

$$= \frac{3dx^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8+6m+m^2} + \frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}}{4+}$$

$$= -\frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)\sqrt{1+c^2x^2}} - \frac{bcdx^{2+m}\sqrt{d+c^2dx^2}}{\left(8+6m+m^2\right)\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.36, size = 233, normalized size = 0.60

$$\frac{dx^{1+m}\sqrt{d+c^2dx^2}\left(-\frac{bcx(4+m+c^2(2+m)x^2)}{(2+m)(4+m)\sqrt{1+c^2x^2}}+(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)-\frac{3\left(bc(1+m)x-(1+m)(2+m)\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))-(2+m)(a+b\sinh^{-1}(cx))\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)+bcx\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)\right)}{(1+m)(2+m)^2\sqrt{1+c^2x^2}}\right)}{4+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*(4 + m + c^2*(2 + m)*x^2))/((2 +
 m)*(4 + m)*Sqrt[1 + c^2*x^2])) + (1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (3*(
b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2
 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c
^2*x^2)] + b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])))/(4 + m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{m}\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)*x^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.193 $\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=240

$$-\frac{bcx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2+m}+\frac{x^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1-}{}\right.}{(2+3m+m^2)\sqrt{1+c^2x^2}}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)-b*c*x^(2+m)*(c^2*d*x^2
+d)^(1/2)/(2+m)^2/(c^2*x^2+1)^(1/2)+x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1
/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(m^2+3*m+2)/(c^2*x
^2+1)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m
],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(1+m)/(2+m)^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5806, 5817, 30}

$$-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(m^2+3m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{m+2}-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*Sqrt[1 + c^2*x^2])) + (x^(
1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2 + m) + (x^(1 + m)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*x^(2 + m)*Sqr
t[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{x^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2+m}+\frac{\sqrt{d+c^2dx^2}\int\frac{x^m(a+b\sinh}{\sqrt{1+c}}}{(2+m)\sqrt{1+c^2x}}$$
$$=-\frac{bcx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2+m}+$$

Mathematica [A]
time = 0.05, size = 179, normalized size = 0.75

$$\frac{x^{1+m}\sqrt{d+c^2dx^2}\left((1+m)\left(-bcx+a(2+m)\sqrt{1+c^2x^2}+b(2+m)\sqrt{1+c^2x^2}\sinh^{-1}(cx)\right)+(2+m)\left(a+b\sinh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)-bcx\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)\right)}{(1+m)(2+m)^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[d + c^2*d*x^2]*((1 + m)*(-(b*c*x) + a*(2 + m)*Sqrt[1 + c^2*
x^2] + b*(2 + m)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]) + (2 + m)*(a + b*ArcSinh[c
*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*Hyper
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1
 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^m\sqrt{c^2d\,x^2+d}\,\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
[Out] Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.194 $\int \frac{x^m(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=161

$$\frac{x^{1+m}\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)}{(1+m)\sqrt{d+c^2dx^2}}-\frac{bcx^{2+m}\sqrt{1+c^2x^2}\;{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\right.}{(2+3m+m^2)\sqrt{d+c^2dx^2}}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)
*(c^2*x^2+1)^(1/2)/(1+m)/(c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/
2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/(m^2+3*m+2)/
(c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {5817}

$$\frac{\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)(a+b\sinh^{-1}(cx))}{(m+1)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m^2+3m+2)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2
+ m)*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2,
 2 + m/2}, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{x^m(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\ &= \frac{x^{1+m}\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)}{(1+m)\sqrt{d+c^2dx^2}}-\frac{bcx^{2+m}\sqrt{1+}}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 129, normalized size = 0.80

$$\frac{x^{1+m}\sqrt{1+c^2x^2}\left((2+m)\left(a+b\sinh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)-bcx\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)\right)}{(1+m)(2+m)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 +
m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)*Sqrt[d
+ c^2*d*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+b\,\mathrm{arcsinh}\,(cx))}{\sqrt{c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (a + b\,\mathrm{asinh}\,(cx))}{\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a + b\,\mathrm{asinh}(c\,x)\right)}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.195 $\int \frac{x^m (a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}} \, dx$

Optimal. Leaf size=268

$$\frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}}-\frac{mx^{1+m}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)}{d(1+m)\sqrt{d+c^2dx^2}}-\frac{bcx^{2+m}\sqrt{1+c^2x^2}}{d(2+}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-m*x^(1+m)*(a+b*arcsinh(c*x
))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d/(1+
m)/(c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-c^2*x^
2)*(c^2*x^2+1)^(1/2)/d/(2+m)/(c^2*d*x^2+d)^(1/2)+b*c*m*x^(2+m)*hypergeom([1
, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d/(m^2
+3*m+2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5811, 5817, 371}

$$\frac{bcm\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{d\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{m\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{d(m+1)\sqrt{c^2dx^2+d}}+\frac{x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\,x^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{d(m+2)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (m*x^(1 + m)*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/(d*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1
+ c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d*(2 +
m)*Sqrt[d + c^2*d*x^2]) + (b*c*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric
PFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(d*(2 + 3*m +
 m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
```

```
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{m\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x^{1+}}{1+c^2}}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\ {}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{d(2+m)\sqrt{d+c^2dx^2}} - \frac{(n}{}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{mx^{1+m}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\right.}{d(1+m)\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.16, size = 206, normalized size = 0.77

$$\frac{x^{1+m}\left(-m(2+m)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)+(1+m)\left((2+m)\left(a+b\sinh^{-1}(cx)\right)-bcx\sqrt{1+c^2x^2}\ {}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)\right)+bcmx\sqrt{1+c^2x^2}\ {}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)\right)}{d(1+m)(2+m)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(-(m*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)]) + (1 + m)*((2 + m)*(a + b*Arc
Sinh[c*x]) - b*c*x*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2,
 -(c^2*x^2)]) + b*c*m*x*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1
+ m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/(d*(1 + m)*(2 + m)*Sqrt[d + c^2
*d*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}(a+b\,\mathrm{arcsinh}\,(cx))}{(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*
d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}(a+b\,\mathrm{asinh}\,(cx))}{(d\,(c^{2}x^{2}+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.196 $\int \frac{x^m(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=402

$$\frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}-\frac{(2-m)mx^{1+m}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,{}_2F_1}{3d^2(1+m)\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*x^(1+m)*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+1/3*(2-m)*x^(1+m)*(a+b
*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-1/3*(2-m)*m*x^(1+m)*(a+b*arcsinh(c*x
))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(
1+m)/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(2-m)*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/
2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(2+m)/(c^2*d*x^2+d)^(1/2)-1/3*b*c*x^(2
+m)*hypergeom([2, 1+1/2*m],[2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(2+m)/
(c^2*d*x^2+d)^(1/2)+1/3*b*c*(2-m)*m*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m]
,[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(m^2+3*m+2)/(c^2*d*x^
2+d)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5811, 5817, 371}

$$\frac{bc(2-m)m\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{3d^2\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{(2-m)m\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3d^2(m+1)\sqrt{c^2dx^2+d}}+\frac{(2-m)x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc(2-m)\sqrt{c^2x^2+1}\,x^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{3d^2(m+2)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\,x^{m+2}\,{}_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{3d^2(m+2)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + ((2 - m)*x^(
1 + m)*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - ((2 - m)*m*x^(1
+ m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)])/(3*d^2*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*(2 -
m)*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -
(c^2*x^2)])/(3*d^2*(2 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 + c
^2*x^2]*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(3*d^2*(2 +
 m)*Sqrt[d + c^2*d*x^2]) + (b*c*(2 - m)*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hyper
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(3*d
^2*(2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(2-m)\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)}{3d^2\sqrt{d+}}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c}}{3d^2(}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(2-m)x^{2+m}}{3}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{(2-m)mx^{1+m}}{}$$

Mathematica [A]
time = 0.29, size = 286, normalized size = 0.71

$$\frac{x^{1+m}\left((1+m)(2+m)\left(a+b\sinh^{-1}(cx)\right)-bc(1+m)x\left(1+c^2x^2\right)^{3/2}{}_2F_1\left(2,1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)+(2-m)\left(1+c^2x^2\right)\left((1+m)(2+m)\left(a+b\sinh^{-1}(cx)\right)-bc(1+m)x\sqrt{1+c^2x^2}\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)-m\sqrt{1+c^2x^2}\left((2+m)\left(a+b\sinh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)-bcx\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)\right)\right)\right)}{3d^2(1+m)(2+m)\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*(1 + c^2*x
^2)^(3/2)*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, -(c^2*x^2)] + (2 - m)*(1 +
 c^2*x^2)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Sqrt[1 + c^
2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(c^2*x^2)] - m*Sqrt[1 + c^2*
x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m
)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/
2, 2 + m/2}, -(c^2*x^2)]))))/(3*d^2*(1 + m)*(2 + m)*(1 + c^2*x^2)*Sqrt[d +
c^2*d*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}(a+b\,\mathrm{arcsinh}\,(c x))}{(c^{2} d\,x^{2}+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*
d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,(a+b\,\mathrm{asinh}(c\,x))}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.197 $\int \frac{x^m \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=102

$$\frac{x^{1+m}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}-\frac{ax^{2+m}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-a^2x^2\right)}{2+3m+m^2}$$

```
[Out] x^(1+m)*arcsinh(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)
-a*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-a^2*x^2)/(
m^2+3*m+2)
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5817}

$$\frac{x^{m+1}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-a^2x^2\right)}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*
x^2)])/(1 + m) - (a*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2
 + m/2, 2 + m/2}, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{x^m\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{x^{1+m}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}-\frac{ax^{2+m}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\right.}{2+3m+m^2}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 0.95

$$\frac{x^{1+m}\left((2+m)\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)-ax\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-a^2x^2\right)\right)}{(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x^(1 + m)*((2 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, -(a^2*x^2)] - a*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2
 + m/2}, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{arcsinh}\left(a x\right)}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{asinh}\left(a x\right)}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)/sqrt(a**2*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.198 $\int x^4(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=283

$$\frac{304b^2dx}{3675c^4}-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7-\frac{32bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{525c^5}+\frac{16bdx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{525c^3}$$

```
[Out] 304/3675*b^2*d*x/c^4-152/11025*b^2*d*x^3/c^2+38/6125*b^2*d*x^5+2/343*b^2*c^
2*d*x^7-2/21*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^5+4/35*b*d*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))/c^5-2/49*b*d*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*
x))/c^5+2/35*d*x^5*(a+b*arcsinh(c*x))^2+1/7*d*x^5*(c^2*x^2+1)*(a+b*arcsinh(
c*x))^2-32/525*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+16/525*b*d*x^2*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-4/175*b*d*x^4*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.30, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12}

$$\frac{1}{7}dx^5(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{4bdx^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{175c}-\frac{2bd(c^2x^2+1)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c^5}+\frac{4bd(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{35c^5}-\frac{2bd(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{21c^5}-\frac{32bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{525c^5}+\frac{16bdx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{525c^3}+\frac{2}{35}dx^5\left(a+b\sinh^{-1}(cx)\right)^2+\frac{304b^2dx}{3675c^4}+\frac{2}{343}b^2c^2dx^7-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (304*b^2*d*x)/(3675*c^4) - (152*b^2*d*x^3)/(11025*c^2) + (38*b^2*d*x^5)/612
5 + (2*b^2*c^2*d*x^7)/343 - (32*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(525*c^5) + (16*b*d*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(525*c^3)
- (4*b*d*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d*(1 +
c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(21*c^5) + (4*b*d*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x]))/(35*c^5) - (2*b*d*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[
c*x]))/(49*c^5) + (2*d*x^5*(a + b*ArcSinh[c*x])^2)/35 + (d*x^5*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^4\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{7}dx^5\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}(2d)\int x^4\left(a+b\sinh^{-1}(\right.$$

$$=-\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{21c^5}+\frac{4bd\left(1+c^2x^2\right)^{5/2}\left(a+b\right.}{35c^5}$$

$$=-\frac{4bdx^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{175c}-\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{21c^5}$$

$$=\frac{16b^2dx}{735c^4}-\frac{8b^2dx^3}{2205c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7+\frac{16bdx^2\sqrt{1+c^2x^2}}{52}$$

$$=\frac{16b^2dx}{735c^4}-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7-\frac{32bd\sqrt{1+c^2x^2}}{5}$$

$$=\frac{304b^2dx}{3675c^4}-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7-\frac{32bd\sqrt{1+c^2x^2}}{}$$

Mathematica [A]
time = 0.17, size = 201, normalized size = 0.71

$$\frac{d\left(11025a^2c^5x^5(7+5c^2x^2)-210ab\sqrt{1+c^2x^2}\,(152-76c^2x^2+57c^4x^4+75c^6x^6)+b^2(31920cx-5320c^3x^3+2394c^5x^5+2250c^7x^7)-210b\left(-105ac^5x^5(7+5c^2x^2)+b\sqrt{1+c^2x^2}\,(152-76c^2x^2+57c^4x^4+75c^6x^6)\right)\sinh^{-1}(cx)+11025b^2c^5x^5(7+5c^2x^2)\sinh^{-1}(cx)^2\right)}{385875c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(11025*a^2*c^5*x^5*(7 + 5*c^2*x^2) - 210*a*b*Sqrt[1 + c^2*x^2]*(152 - 76
*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6) + b^2*(31920*c*x - 5320*c^3*x^3 + 2394*
c^5*x^5 + 2250*c^7*x^7) - 210*b*(-105*a*c^5*x^5*(7 + 5*c^2*x^2) + b*Sqrt[1
+ c^2*x^2]*(152 - 76*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 110
25*b^2*c^5*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^5)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^4\left(c^2d\,x^2+d\right)\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.31, size = 441, normalized size = 1.56

$$\frac{1}{7}\,b^2c^2dx^7\,\mathrm{arsinh}\left(cx\right)^2+\frac{1}{7}\,a^2c^2dx^7+\frac{1}{5}\,b^2dx^5\,\mathrm{arsinh}\left(cx\right)^2+\frac{1}{5}\,a^2dx^5+\frac{2}{245}\left(35\,x^7\,\mathrm{arsinh}\left(cx\right)-\left(\frac{5\,\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^2d-\frac{2}{25725}\left(105\left(\frac{5\,\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\,\mathrm{arsinh}\left(cx\right)-\frac{75\,c^6x^7-126\,c^4x^5+280\,c^2x^3-1680\,x}{c^6}\right)b^2c^2d+\frac{2}{75}\left(15\,x^5\,\mathrm{arsinh}\left(cx\right)-\left(\frac{3\,\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)abd-\frac{2}{1125}\left(15\left(\frac{3\,\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\,\mathrm{arsinh}\left(cx\right)-\frac{9\,c^4x^5-20\,c^2x^3+120\,x}{c^4}\right)b^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^2*d*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^2*d*x^7 + 1/5*b^2*d*x^5*arcsin
h(c*x)^2 + 1/5*a^2*d*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1
)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*
sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6
/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(
c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3
- 1680*x)/c^6)*b^2*c^2*d + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)
*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d
- 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c
^4)*b^2*d
```

Fricas [A]
time = 0.38, size = 260, normalized size = 0.92

$$\frac{1125\,(49\,a^2+2\,b^2)c^7dx^7+63\,(1225\,a^2+38\,b^2)c^5dx^5-5320\,b^2c^3dx^3+31920\,b^2cdx+11025\,(5\,b^2c^7dx^7+7\,b^2c^5dx^5)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+210\left(525\,abc^7dx^7+735\,abc^5dx^5-(75\,b^2c^6dx^6+57\,b^2c^4dx^4-76\,b^2c^2dx^2+152\,b^2d)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-210\,(75\,abc^6dx^6+57\,abc^4dx^4-76\,abc^2dx^2+152\,abd)\sqrt{c^2x^2+1}}{385875\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d*x^7 + 63*(1225*a^2 + 38*b^2)*c^5*d*x^
5 - 5320*b^2*c^3*d*x^3 + 31920*b^2*c*d*x + 11025*(5*b^2*c^7*d*x^7 + 7*b^2*c
^5*d*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d*x^7 + 735*a*b
*c^5*d*x^5 - (75*b^2*c^6*d*x^6 + 57*b^2*c^4*d*x^4 - 76*b^2*c^2*d*x^2 + 152*
b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d*
x^6 + 57*a*b*c^4*d*x^4 - 76*a*b*c^2*d*x^2 + 152*a*b*d)*sqrt(c^2*x^2 + 1))/c
^5
```

Sympy [A]
time = 1.11, size = 388, normalized size = 1.37

$$\begin{cases} \frac{a^{2}c^{2}dx^{7}}{7} + \frac{a^{2}dx^{5}}{5} + \frac{2abc^{2}dx^{7}\operatorname{asinh}{\left(cx \right)}}{7} - \frac{2abcdx^{6}\sqrt{c^{2}x^{2}+1}}{49} + \frac{2abdx^{5}\operatorname{asinh}{\left(cx \right)}}{5} - \frac{38abdx^{4}\sqrt{c^{2}x^{2}+1}}{1225c} + \frac{152abdx^{2}\sqrt{c^{2}x^{2}+1}}{3675c^{3}} - \frac{304abd\sqrt{c^{2}x^{2}+1}}{3675c^{5}} + \frac{b^{2}c^{2}dx^{7}\operatorname{asinh}^{2}{\left(cx \right)}}{7} + \frac{2b^{2}c^{2}dx^{7}}{343} - \frac{2b^{2}cdx^{6}\sqrt{c^{2}x^{2}+1}\operatorname{asinh}{\left(cx \right)}}{49} + \frac{b^{2}dx^{5}\operatorname{asinh}^{2}{\left(cx \right)}}{5} + \frac{38b^{2}dx^{5}}{6125} - \frac{38b^{2}dx^{4}\sqrt{c^{2}x^{2}+1}\operatorname{asinh}{\left(cx \right)}}{1225c} - \frac{152b^{2}dx^{3}}{11025c^{2}} + \frac{152b^{2}dx^{2}\sqrt{c^{2}x^{2}+1}\operatorname{asinh}{\left(cx \right)}}{3675c^{3}} + \frac{304b^{2}dx}{3675c^{4}} - \frac{304b^{2}d\sqrt{c^{2}x^{2}+1}\operatorname{asinh}{\left(cx \right)}}{3675c^{5}} & \text{for}\: c \neq 0 \\ \frac{a^{2}dx^{5}}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**7/7 + a**2*d*x**5/5 + 2*a*b*c**2*d*x**7*asinh(c*x
)/7 - 2*a*b*c*d*x**6*sqrt(c**2*x**2 + 1)/49 + 2*a*b*d*x**5*asinh(c*x)/5 - 3
8*a*b*d*x**4*sqrt(c**2*x**2 + 1)/(1225*c) + 152*a*b*d*x**2*sqrt(c**2*x**2 +
 1)/(3675*c**3) - 304*a*b*d*sqrt(c**2*x**2 + 1)/(3675*c**5) + b**2*c**2*d*x
**7*asinh(c*x)**2/7 + 2*b**2*c**2*d*x**7/343 - 2*b**2*c*d*x**6*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/49 + b**2*d*x**5*asinh(c*x)**2/5 + 38*b**2*d*x**5/6125 -
 38*b**2*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1225*c) - 152*b**2*d*x**3/(
11025*c**2) + 152*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*c**3) +
304*b**2*d*x/(3675*c**4) - 304*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*
c**5), Ne(c, 0)), (a**2*d*x**5/5, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\left(d\,c^2\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

3.199 $\int x^3(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=198

$$-\frac{b^2dx^2}{24c^2}+\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6+\frac{bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{12c^3}-\frac{bdx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{1}{18}b$$

```
[Out] -1/24*b^2*d*x^2/c^2+1/72*b^2*d*x^4+1/108*b^2*c^2*d*x^6-1/24*d*(a+b*arcsinh(
c*x))^2/c^4+1/12*d*x^4*(a+b*arcsinh(c*x))^2+1/6*d*x^4*(c^2*x^2+1)*(a+b*arcs
inh(c*x))^2+1/12*b*d*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-1/18*b*d*x^
3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-1/18*b*c*d*x^5*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {5808, 5776, 5812, 5783, 30, 5806}

$$-\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{24c^4}-\frac{1}{18}bcdx^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}dx^4\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bdx^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{18c}+\frac{bdx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{12c^3}+\frac{1}{12}dx^4\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{108}b^2c^2dx^6-\frac{b^2dx^2}{24c^2}+\frac{1}{72}b^2dx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/24*(b^2*d*x^2)/c^2 + (b^2*d*x^4)/72 + (b^2*c^2*d*x^6)/108 + (b*d*x*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(12*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(18*c) - (b*c*d*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x]))/18 - (d*(a + b*ArcSinh[c*x])^2)/(24*c^4) + (d*x^4*(a + b*ArcSinh[c*x])
^2)/12 + (d*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{6}dx^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}d\int x^3\left(a+b\sinh^{-1}(cx)\right)$$

$$=-\frac{1}{18}bcdx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{12}dx^4\left(a+b\sinh^{-1}(cx\right)$$

$$=\frac{1}{108}b^2c^2dx^6-\frac{bdx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{1}{18}bcdx^5\sqrt{1+}$$

$$=\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6+\frac{bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{12c^3}-bdx$$

$$=-\frac{b^2dx^2}{24c^2}+\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6+\frac{bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\right)}{12c^3}$$

Mathematica [**A**]
time = 0.16, size = 186, normalized size = 0.94

$$\frac{d\left(cx\left(18a^2c^3x^3(3+2c^2x^2)-6ab\sqrt{1+c^2x^2}\,(-3+2c^2x^2+2c^4x^4)+b^2cx(-9+3c^2x^2+2c^4x^4)\right)+6b\left(bcx\sqrt{1+c^2x^2}\,(3-2c^2x^2-2c^4x^4)+3a(-1+6c^4x^4+4c^6x^6)\right)\sinh^{-1}(cx)+9b^2(-1+6c^4x^4+4c^6x^6)\sinh^{-1}(cx)^2\right)}{216c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(c*x*(18*a^2*c^3*x^3*(3 + 2*c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-3 + 2*c
^2*x^2 + 2*c^4*x^4) + b^2*c*x*(-9 + 3*c^2*x^2 + 2*c^4*x^4)) + 6*b*(b*c*x*Sq
rt[1 + c^2*x^2]*(3 - 2*c^2*x^2 - 2*c^4*x^4) + 3*a*(-1 + 6*c^4*x^4 + 4*c^6*x
^6))*ArcSinh[c*x] + 9*b^2*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]^2))/(21
6*c^4)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3\left(c^2d\,x^2+d\right)\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(174) = 348.
time = 0.34, size = 442, normalized size = 2.23

$$\frac{1}{6}b^2c^2dx^6\,\mathrm{arsinh}\,(cx)^2+\frac{1}{6}a^2c^2dx^6+\frac{1}{4}b^2dx^4\,\mathrm{arsinh}\,(cx)^2+\frac{1}{4}a^2dx^4+\frac{1}{144}\left(48x^6\,\mathrm{arsinh}\,(cx)-\left(\frac{8\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\,\mathrm{arsinh}\,(cx)}{c^7}\right)c\right)abc^2d+\frac{1}{864}\left(\left(\frac{8x^6}{c^2}-\frac{15x^4}{c^4}+\frac{45x^2}{c^6}-\frac{45\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^8}\right)c^2-6\left(\frac{8\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\,\mathrm{arsinh}\,(cx)}{c^7}\right)c\,\mathrm{arsinh}\,(cx)\right)b^2c^2d+\frac{1}{16}\left(8x^4\,\mathrm{arsinh}\,(cx)-\left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\,\mathrm{arsinh}\,(cx)}{c^5}\right)c\right)abd+\frac{1}{32}\left(\left(\frac{x^4}{c^2}-\frac{3x^2}{c^4}+\frac{3\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2-2\left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\,\mathrm{arsinh}\,(cx)}{c^5}\right)c\,\mathrm{arsinh}\,(cx)\right)b^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*c^2*d*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^2*d*x^6 + 1/4*b^2*d*x^4*arcsin
h(c*x)^2 + 1/4*a^2*d*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1
)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*
arcsinh(c*x)/c^7)*c)*a*b*c^2*d + 1/864*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^
6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5
/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsi
nh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^2*d + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt
(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*
a*b*d + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*
c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsin
h(c*x)/c^5)*c*arcsinh(c*x))*b^2*d
```

Fricas [A]
time = 0.39, size = 240, normalized size = 1.21

$$\frac{2\,(18\,a^2+b^2)c^6dx^6+3\,(18\,a^2+b^2)c^4dx^4-9\,b^2c^2dx^2+9\,(4\,b^2c^6dx^6+6\,b^2c^4dx^4-b^2d)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(12\,abc^6dx^6+18\,abc^4dx^4-3\,abd-(2\,b^2c^5dx^5+2\,b^2c^3dx^3-3\,b^2cdx)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-6\,(2\,abc^5dx^5+2\,abc^3dx^3-3\,abcdx)\sqrt{c^2x^2+1}}{216\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/216*(2*(18*a^2 + b^2)*c^6*d*x^6 + 3*(18*a^2 + b^2)*c^4*d*x^4 - 9*b^2*c^2*
d*x^2 + 9*(4*b^2*c^6*d*x^6 + 6*b^2*c^4*d*x^4 - b^2*d)*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 6*(12*a*b*c^6*d*x^6 + 18*a*b*c^4*d*x^4 - 3*a*b*d - (2*b^2*c^5*d
*x^5 + 2*b^2*c^3*d*x^3 - 3*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) - 6*(2*a*b*c^5*d*x^5 + 2*a*b*c^3*d*x^3 - 3*a*b*c*d*x)*sqrt(c^2*x
^2 + 1))/c^4
```

Sympy [A]
time = 0.80, size = 332, normalized size = 1.68

$$\begin{cases} \frac{a^2c^2dx^6}{6}+\frac{a^2dx^4}{4}+\frac{abc^2dx^6\operatorname{asinh}(cx)}{3}-\frac{abcdx^5\sqrt{c^2x^2+1}}{18}+\frac{abdx^4\operatorname{asinh}(cx)}{2}-\frac{abdx^3\sqrt{c^2x^2+1}}{18c}+\frac{abdx\sqrt{c^2x^2+1}}{12c^3}-\frac{abd\operatorname{asinh}(cx)}{12c^4}+\frac{b^2c^2dx^6\operatorname{asinh}^2(cx)}{6}+\frac{b^2c^2dx^6}{108}-\frac{b^2cdx^5\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{18}+\frac{b^2dx^4\operatorname{asinh}^2(cx)}{4}+\frac{b^2dx^4}{72}-\frac{b^2dx^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{18c}-\frac{b^2dx^2}{24c^2}+\frac{b^2dx\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{12c^3}-\frac{b^2d\operatorname{asinh}^2(cx)}{24c^4} & \text{for}\: c \neq 0 \\ \frac{a^2dx^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**6/6 + a**2*d*x**4/4 + a*b*c**2*d*x**6*asinh(c*x)/
3 - a*b*c*d*x**5*sqrt(c**2*x**2 + 1)/18 + a*b*d*x**4*asinh(c*x)/2 - a*b*d*x
**3*sqrt(c**2*x**2 + 1)/(18*c) + a*b*d*x*sqrt(c**2*x**2 + 1)/(12*c**3) - a*
b*d*asinh(c*x)/(12*c**4) + b**2*c**2*d*x**6*asinh(c*x)**2/6 + b**2*c**2*d*x
**6/108 - b**2*c*d*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/18 + b**2*d*x**4*asi
nh(c*x)**2/4 + b**2*d*x**4/72 - b**2*d*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/
(18*c) - b**2*d*x**2/(24*c**2) + b**2*d*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
2*c**3) - b**2*d*asinh(c*x)**2/(24*c**4), Ne(c, 0)), (a**2*d*x**4/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (a + b\,\mathrm{asinh}(c\,x))^2 \left(d\,c^2\,x^2 + d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

3.200 $\int x^2(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=206

$$-\frac{52b^2dx}{225c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5+\frac{8bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3}-\frac{4bdx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c}$$

```
[Out] -52/225*b^2*d*x/c^2+26/675*b^2*d*x^3+2/125*b^2*c^2*d*x^5+2/15*b*d*(c^2*x^2+
1)^(3/2)*(a+b*arcsinh(c*x))/c^3-2/25*b*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x
))/c^3+2/15*d*x^3*(a+b*arcsinh(c*x))^2+1/5*d*x^3*(c^2*x^2+1)*(a+b*arcsinh(c
*x))^2+8/45*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-4/45*b*d*x^2*(a+b*
arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.23, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12}

$$-\frac{4bdx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{45c}+\frac{1}{5}dx^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c^3}+\frac{2bd(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15c^3}+\frac{8bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{45c^3}+\frac{2}{15}dx^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{125}b^2c^2dx^5-\frac{52b^2dx}{225c^2}+\frac{26}{675}b^2dx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-52*b^2*d*x)/(225*c^2) + (26*b^2*d*x^3)/675 + (2*b^2*c^2*d*x^5)/125 + (8*b
*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3) - (4*b*d*x^2*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c) + (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/(15*c^3) - (2*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2
5*c^3) + (2*d*x^3*(a + b*ArcSinh[c*x])^2)/15 + (d*x^3*(1 + c^2*x^2)*(a + b*
ArcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{5}dx^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(2d)\int x^2\left(a+b\sinh^{-1}(\right.$$

$$=\frac{2bd(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15c^3}-\frac{2bd(1+c^2x^2)^{5/2}\left(a+b\sin\right.}{25c^3}$$

$$=-\frac{4bdx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c}+\frac{2bd(1+c^2x^2)^{3/2}\left(a+b\right.}{15c^3}$$

$$=-\frac{4b^2dx}{75c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5+\frac{8bd\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{45c^3}$$

$$=-\frac{52b^2dx}{225c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5+\frac{8bd\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{45c^3}$$

Mathematica [A]
time = 0.15, size = 177, normalized size = 0.86

$$\frac{d\left(225a^2c^3x^3(5+3c^2x^2)-30ab\sqrt{1+c^2x^2}\,(-26+13c^2x^2+9c^4x^4)+2b^2cx(-390+65c^2x^2+27c^4x^4)-30b\left(-15ac^3x^3(5+3c^2x^2)+b\sqrt{1+c^2x^2}\,(-26+13c^2x^2+9c^4x^4)\right)\sinh^{-1}(cx)+225b^2c^3x^3(5+3c^2x^2)\sinh^{-1}(cx)^2\right)}{3375c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(225*a^2*c^3*x^3*(5 + 3*c^2*x^2) - 30*a*b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^
2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(-390 + 65*c^2*x^2 + 27*c^4*x^4) - 30*b*(-15
*a*c^3*x^3*(5 + 3*c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^2*x^2 + 9*c^4*
x^4))*ArcSinh[c*x] + 225*b^2*c^3*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]^2))/(3375
*c^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(c^2d\,x^2+d\right)\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.30, size = 346, normalized size = 1.68

$$\frac{1}{5}\,b^{2}c^{2}dx^{5}\operatorname{arsinh}\left(cx\right)^{2}+\frac{1}{5}\,a^{2}c^{2}dx^{5}+\frac{1}{3}\,b^{2}dx^{3}\operatorname{arsinh}\left(cx\right)^{2}+\frac{2}{75}\left(15\,x^{5}\operatorname{arsinh}\left(cx\right)-\left(\frac{3\,\sqrt{c^{2}x^{2}+1}\,x^{4}}{c^{2}}-\frac{4\,\sqrt{c^{2}x^{2}+1}\,x^{2}}{c^{4}}+\frac{8\,\sqrt{c^{2}x^{2}+1}}{c^{6}}\right)c\right)abc^{2}d-\frac{2}{1125}\left(15\left(\frac{3\,\sqrt{c^{2}x^{2}+1}\,x^{4}}{c^{2}}-\frac{4\,\sqrt{c^{2}x^{2}+1}\,x^{2}}{c^{4}}+\frac{8\,\sqrt{c^{2}x^{2}+1}}{c^{6}}\right)c\operatorname{arsinh}\left(cx\right)-\frac{9\,c^{4}x^{5}-20\,c^{2}x^{3}+120\,x}{c^{4}}\right)b^{2}c^{2}d+\frac{1}{3}\,a^{2}dx^{3}+\frac{2}{9}\left(3\,x^{3}\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^{2}x^{2}+1}\,x^{2}}{c^{2}}-\frac{2\,\sqrt{c^{2}x^{2}+1}}{c^{4}}\right)\right)abd-\frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^{2}x^{2}+1}\,x^{2}}{c^{2}}-\frac{2\,\sqrt{c^{2}x^{2}+1}}{c^{4}}\right)\operatorname{arsinh}\left(cx\right)-\frac{c^{2}x^{3}-6\,x}{c^{2}}\right)b^{2}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*c^2*d*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^2*d*x^5 + 1/3*b^2*d*x^3*arcsin
h(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqr
t(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d - 2/1125*(15
*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*
d + 1/3*a^2*d*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d
```

Fricas [A]
time = 0.37, size = 225, normalized size = 1.09

$$\frac{27\,(25\,a^{2}+2\,b^{2})c^{5}dx^{5}+5\,(225\,a^{2}+26\,b^{2})c^{3}dx^{3}-780\,b^{2}cdx+225\,(3\,b^{2}c^{5}dx^{5}+5\,b^{2}c^{3}dx^{3})\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}+30\left(45\,abc^{5}dx^{5}+75\,abc^{3}dx^{3}-(9\,b^{2}c^{4}dx^{4}+13\,b^{2}c^{2}dx^{2}-26\,b^{2}d)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)-30\,(9\,abc^{4}dx^{4}+13\,abc^{2}dx^{2}-26\,abd)\sqrt{c^{2}x^{2}+1}}{3375\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*d*x^5 + 5*(225*a^2 + 26*b^2)*c^3*d*x^3 - 78
0*b^2*c*d*x + 225*(3*b^2*c^5*d*x^5 + 5*b^2*c^3*d*x^3)*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 30*(45*a*b*c^5*d*x^5 + 75*a*b*c^3*d*x^3 - (9*b^2*c^4*d*x^4 + 13
*b^2*c^2*d*x^2 - 26*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 30*(9*a*b*c^4*d*x^4 + 13*a*b*c^2*d*x^2 - 26*a*b*d)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.54, size = 313, normalized size = 1.52

$$\begin{cases}\frac{a^{2}c^{2}dx^{5}}{5}+\frac{a^{2}dx^{3}}{3}+\frac{2abc^{2}dx^{5}\operatorname{asinh}\left(cx\right)}{5}-\frac{2abcdx^{4}\sqrt{c^{2}x^{2}+1}}{25}+\frac{2abdx^{3}\operatorname{asinh}\left(cx\right)}{3}-\frac{26abdx^{2}\sqrt{c^{2}x^{2}+1}}{225c}+\frac{52abd\sqrt{c^{2}x^{2}+1}}{225c^{3}}+\frac{b^{2}c^{2}dx^{5}\operatorname{asinh}^{2}\left(cx\right)}{5}+\frac{2b^{2}c^{2}dx^{5}}{125}-\frac{2b^{2}cdx^{4}\sqrt{c^{2}x^{2}+1}\operatorname{asinh}\left(cx\right)}{25}+\frac{b^{2}dx^{3}\operatorname{asinh}^{2}\left(cx\right)}{3}+\frac{26b^{2}dx^{3}}{675}-\frac{26b^{2}dx^{2}\sqrt{c^{2}x^{2}+1}\operatorname{asinh}\left(cx\right)}{225c}-\frac{52b^{2}dx}{225c^{2}}+\frac{52b^{2}d\sqrt{c^{2}x^{2}+1}\operatorname{asinh}\left(cx\right)}{225c^{3}} & \text{for}\: c \neq 0 \\ \frac{a^{2}dx^{3}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**5/5 + a**2*d*x**3/3 + 2*a*b*c**2*d*x**5*asinh(c*x
)/5 - 2*a*b*c*d*x**4*sqrt(c**2*x**2 + 1)/25 + 2*a*b*d*x**3*asinh(c*x)/3 - 2
```

```
6*a*b*d*x**2*sqrt(c**2*x**2 + 1)/(225*c) + 52*a*b*d*sqrt(c**2*x**2 + 1)/(22
5*c**3) + b**2*c**2*d*x**5*asinh(c*x)**2/5 + 2*b**2*c**2*d*x**5/125 - 2*b**
2*c*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + b**2*d*x**3*asinh(c*x)**2/3
+ 26*b**2*d*x**3/675 - 26*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c
) - 52*b**2*d*x/(225*c**2) + 52*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*
c**3), Ne(c, 0)), (a**2*d*x**3/3, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

3.201 $\int x(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=135

$$\frac{5}{32}b^2dx^2+\frac{1}{32}b^2c^2dx^4-\frac{3bdx\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{16c}-\frac{bdx(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3d\left(a+b\sin\right.}{32c}$$

```
[Out] 5/32*b^2*d*x^2+1/32*b^2*c^2*d*x^4-1/8*b*d*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(
c*x))/c-3/32*d*(a+b*arcsinh(c*x))^2/c^2+1/4*d*(c^2*x^2+1)^2*(a+b*arcsinh(c*
x))^2/c^2-3/16*b*d*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {5798, 5786, 5785, 5783, 30, 14}

$$-\frac{bdx(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3bdx\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{16c}+\frac{d(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{3d\left(a+b\sinh^{-1}(cx)\right)^2}{32c^2}+\frac{1}{32}b^2c^2dx^4+\frac{5}{32}b^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (5*b^2*d*x^2)/32 + (b^2*c^2*d*x^4)/32 - (3*b*d*x*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/(16*c) - (b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(8*
c) - (3*d*(a + b*ArcSinh[c*x])^2)/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcS
inh[c*x])^2)/(4*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx &= \frac{d(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{(bd)\int(1+c^2x^2)^{3/2}\left(a+b\sin\right.}{2c} \\
&= -\frac{bdx(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}+\frac{d(1+c^2x^2)^2\left(a+b\sinh^{-1}\right.}{4c^2} \\
&= -\frac{3bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{16c}-\frac{bdx(1+c^2x^2)^{3/2}\left(a+b\sin\right.}{8c} \\
&= \frac{5}{32}b^2dx^2+\frac{1}{32}b^2c^2dx^4-\frac{3bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{16c}-\frac{bd}{}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 155, normalized size = 1.15

$$\frac{d\left(cx\left(8a^2cx(2+c^2x^2)+b^2cx(5+c^2x^2)-2ab\sqrt{1+c^2x^2}\,(5+2c^2x^2)\right)+2b\left(-bcx\sqrt{1+c^2x^2}\,(5+2c^2x^2)+a(5+16c^2x^2+8c^4x^4)\right)\sinh^{-1}(cx)+b^2(5+16c^2x^2+8c^4x^4)\sinh^{-1}(cx)^2\right)}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(c*x*(8*a^2*c*x*(2 + c^2*x^2) + b^2*c*x*(5 + c^2*x^2) - 2*a*b*Sqrt[1 + c
^2*x^2]*(5 + 2*c^2*x^2)) + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2))
+ a*(5 + 16*c^2*x^2 + 8*c^4*x^4))*ArcSinh[c*x] + b^2*(5 + 16*c^2*x^2 + 8*c^
4*x^4)*ArcSinh[c*x]^2))/(32*c^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x \left(c^{2} d \,x^{2}+d \right) \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(119) = 238$.
time = 0.33, size = 347, normalized size = 2.57

$$\frac{1}{4} b^{2} c^{2} d x^{4} \operatorname{arsinh}\left(c x\right)^{2} + \frac{1}{4} a^{2} c^{2} d x^{4} + \frac{1}{2} b^{2} d x^{2} \operatorname{arsinh}\left(c x\right)^{2} + \frac{1}{16} \left(8 x^{4} \operatorname{arsinh}\left(c x\right) - \left(\frac{2 \sqrt{c^{2} x^{2} + 1} x^{3}}{c^{2}} - \frac{3 \sqrt{c^{2} x^{2} + 1} x}{c^{4}} + \frac{3 \operatorname{arsinh}\left(c x\right)}{c^{5}}\right) c\right) a b c^{2} d + \frac{1}{32} \left(\left(\frac{x^{4}}{c^{2}} - \frac{3 x^{2}}{c^{4}} + \frac{3 \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2}}{c^{6}}\right) c^{2} - 2 \left(\frac{2 \sqrt{c^{2} x^{2} + 1} x^{3}}{c^{2}} - \frac{3 \sqrt{c^{2} x^{2} + 1} x}{c^{4}} + \frac{3 \operatorname{arsinh}\left(c x\right)}{c^{5}}\right) c \operatorname{arsinh}\left(c x\right)\right) b^{2} c^{2} d + \frac{1}{2} a^{2} d x^{2} + \frac{1}{2} \left(2 x^{2} \operatorname{arsinh}\left(c x\right) - c \left(\frac{\sqrt{c^{2} x^{2} + 1} x}{c^{2}} - \frac{\operatorname{arsinh}\left(c x\right)}{c^{3}}\right)\right) a b d + \frac{1}{4} \left(c^{2} \left(\frac{x^{2}}{c^{2}} - \frac{\log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2}}{c^{4}}\right) - 2 c \left(\frac{\sqrt{c^{2} x^{2} + 1} x}{c^{2}} - \frac{\operatorname{arsinh}\left(c x\right)}{c^{3}}\right) \operatorname{arsinh}\left(c x\right)\right) b^{2} d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*c^2*d*x^4*arcsinh(c*x)^2 + 1/4*a^2*c^2*d*x^4 + 1/2*b^2*d*x^2*arcsin
h(c*x)^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt
(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2*d + 1/32*((x^4/c^2 - 3
*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 +
1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x)
)*b^2*c^2*d + 1/2*a^2*d*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d + 1/4*(c^2*(x^2/c^2 - log(c*x + sqrt(c^2
*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcsin
h(c*x))*b^2*d
```

Fricas [A]
time = 0.35, size = 204, normalized size = 1.51

$$\frac{{\left(8 \, a^{2} + b^{2}\right)} c^{4} d x^{4} + {\left(16 \, a^{2} + 5 \, b^{2}\right)} c^{2} d x^{2} + {\left(8 \, b^{2} c^{4} d x^{4} + 16 \, b^{2} c^{2} d x^{2} + 5 \, b^{2} d\right)} \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2} + 2 \, {\left(8 \, a b c^{4} d x^{4} + 16 \, a b c^{2} d x^{2} + 5 \, a b d - {\left(2 \, b^{2} c^{3} d x^{3} + 5 \, b^{2} c d x\right)} \sqrt{c^{2} x^{2} + 1}\right)} \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right) - 2 \, {\left(2 \, a b c^{3} d x^{3} + 5 \, a b c d x\right)} \sqrt{c^{2} x^{2} + 1}}{32 \, c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*c^4*d*x^4 + (16*a^2 + 5*b^2)*c^2*d*x^2 + (8*b^2*c^4*d*x
^4 + 16*b^2*c^2*d*x^2 + 5*b^2*d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(8*a*b*
```

```
c^4*d*x^4 + 16*a*b*c^2*d*x^2 + 5*a*b*d - (2*b^2*c^3*d*x^3 + 5*b^2*c*d*x)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(2*a*b*c^3*d*x^3 + 5*a*b*
c*d*x)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(129) = 258$.
time = 0.38, size = 269, normalized size = 1.99

$$\begin{cases} \frac{a^2c^2dx^4}{4} + \frac{a^2dx^2}{2} + \frac{abc^2dx^4\operatorname{asinh}(cx)}{2} - \frac{abcdx^3\sqrt{c^2x^2+1}}{8} + abdx^2\operatorname{asinh}(cx) - \frac{5abdx\sqrt{c^2x^2+1}}{16c} + \frac{5abd\operatorname{asinh}(cx)}{16c^2} + \frac{b^2c^2dx^4\operatorname{asinh}^2(cx)}{4} + \frac{b^2c^2dx^4}{32} - \frac{b^2cdx^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{8} + \frac{b^2dx^2\operatorname{asinh}^2(cx)}{2} + \frac{5b^2dx^2}{32} - \frac{5b^2dx\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{16c} + \frac{5b^2d\operatorname{asinh}^2(cx)}{32c^2} & \text{for } c \neq 0 \\ \frac{a^2dx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**4/4 + a**2*d*x**2/2 + a*b*c**2*d*x**4*asinh(c*x)/
2 - a*b*c*d*x**3*sqrt(c**2*x**2 + 1)/8 + a*b*d*x**2*asinh(c*x) - 5*a*b*d*x*
sqrt(c**2*x**2 + 1)/(16*c) + 5*a*b*d*asinh(c*x)/(16*c**2) + b**2*c**2*d*x**
4*asinh(c*x)**2/4 + b**2*c**2*d*x**4/32 - b**2*c*d*x**3*sqrt(c**2*x**2 + 1)
*asinh(c*x)/8 + b**2*d*x**2*asinh(c*x)**2/2 + 5*b**2*d*x**2/32 - 5*b**2*d*x
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c) + 5*b**2*d*asinh(c*x)**2/(32*c**2),
Ne(c, 0)), (a**2*d*x**2/2, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\operatorname{asinh}(c\,x)\right)^2\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

3.202 $\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{2bd(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{2}{3}dx\big(a+b\sinh$$

```
[Out] 14/9*b^2*d*x+2/27*b^2*c^2*d*x^3-2/9*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x)
)/c+2/3*d*x*(a+b*arcsinh(c*x))^2+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2-4
/3*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5786, 5772, 5798, 8}

$$\frac{1}{3}dx\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}-\frac{4bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{2}{3}dx\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{27}b^2c^2dx^3+\frac{14}{9}b^2dx$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (14*b^2*d*x)/9 + (2*b^2*c^2*d*x^3)/27 - (4*b*d*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(3*c) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(9*c)
+ (2*d*x*(a + b*ArcSinh[c*x])^2)/3 + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx &= \frac{1}{3}dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}(2d)\int \left(a+b\sinh^{-1}(cx)\right)^2 \\ &= -\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{2}{3}dx\left(a+b\sinh^{-1}(cx)\right)^2+ \\ &= \frac{2}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{2bd(1+ \\ &= \frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{2bd(1\end{aligned}$$

Mathematica [A]
time = 0.11, size = 135, normalized size = 1.08

$$\frac{d\left(9a^2cx(3+c^2x^2)-6ab\sqrt{1+c^2x^2}\,(7+c^2x^2)+2b^2cx(21+c^2x^2)-6b\left(-3acx(3+c^2x^2)+b\sqrt{1+c^2x^2}\,(7+c^2x^2)\right)\sinh^{-1}(cx)+9b^2cx(3+c^2x^2)\sinh^{-1}(cx)^2\right)}{27c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(9*a^2*c*x*(3 + c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(7 + c^2*x^2) + 2*b^2
*c*x*(21 + c^2*x^2) - 6*b*(-3*a*c*x*(3 + c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(7
+ c^2*x^2))*ArcSinh[c*x] + 9*b^2*c*x*(3 + c^2*x^2)*ArcSinh[c*x]^2))/(27*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(c^2d\,x^2+d\right)\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(109) = 218$.
time = 0.29, size = 230, normalized size = 1.84

$$\frac{1}{3}b^2c^2dx^3\operatorname{arsinh}(cx)^2+\frac{1}{3}a^2c^2dx^3+\frac{2}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d-\frac{2}{27}\left(3c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx)-\frac{c^2x^3-6x}{c^2}\right)b^2c^2d+b^2dx\operatorname{arsinh}(cx)^2+2b^2d\left(x-\frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right)+a^2dx+\frac{2\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)abd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*c^2*d*x^3*arcsinh(c*x)^2 + 1/3*a^2*c^2*d*x^3 + 2/9*(3*x^3*arcsinh(c
*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d -
2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x
) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arcsinh(c*x)^2 + 2*b^2*d*(x -
sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^
2*x^2 + 1))*a*b*d/c
```

Fricas [A]
time = 0.36, size = 178, normalized size = 1.42

$$\frac{(9a^2+2b^2)c^3dx^3+3(9a^2+14b^2)cdx+9(b^2c^3dx^3+3b^2cdx)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3abc^3dx^3+9abcdx-(b^2c^2dx^2+7b^2d)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-6(abc^2dx^2+7abd)\sqrt{c^2x^2+1}}{27c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*d*x^3 + 3*(9*a^2 + 14*b^2)*c*d*x + 9*(b^2*c^3*d*x
^3 + 3*b^2*c*d*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^3*d*x^3 + 9*a
*b*c*d*x - (b^2*c^2*d*x^2 + 7*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) - 6*(a*b*c^2*d*x^2 + 7*a*b*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A]
time = 0.24, size = 224, normalized size = 1.79

$$\begin{cases}\frac{a^2c^2dx^3}{3}+a^2dx+\frac{2abc^2dx^3\operatorname{asinh}(cx)}{3}-\frac{2abcdx^2\sqrt{c^2x^2+1}}{9}+2abdx\operatorname{asinh}(cx)-\frac{14abd\sqrt{c^2x^2+1}}{9c}+\frac{b^2c^2dx^3\operatorname{asinh}^2(cx)}{3}+\frac{2b^2c^2dx^3}{27}-\frac{2b^2cdx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{9}+b^2dx\operatorname{asinh}^2(cx)+\frac{14b^2dx}{9}-\frac{14b^2d\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{9c} & \text{for } c\neq 0\\ a^2dx & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**3/3 + a**2*d*x + 2*a*b*c**2*d*x**3*asinh(c*x)/3 -
 2*a*b*c*d*x**2*sqrt(c**2*x**2 + 1)/9 + 2*a*b*d*x*asinh(c*x) - 14*a*b*d*sqr
t(c**2*x**2 + 1)/(9*c) + b**2*c**2*d*x**3*asinh(c*x)**2/3 + 2*b**2*c**2*d*x
**3/27 - 2*b**2*c*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/9 + b**2*d*x*asinh(
c*x)**2 + 14*b**2*d*x/9 - 14*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c), N
e(c, 0)), (a**2*d*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

3.203 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=166

$$\frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))-\frac{1}{4}d(a+b\sinh^{-1}(cx))^2+\frac{1}{2}d(1+c^2x^2)\,(a+b\sinh^{-1}(cx))^2+$$

```
[Out] 1/4*b^2*c^2*d*x^2-1/4*d*(a+b*arcsinh(c*x))^2+1/2*d*(c^2*x^2+1)*(a+b*arcsinh
(c*x))^2+1/3*d*(a+b*arcsinh(c*x))^3/b+d*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c
^2*x^2+1)^(1/2))^2)-b*d*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/
2))^2)-1/2*b^2*d*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*c*d*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30}

$$\frac{1}{2}d(c^2x^2+1)\,(a+b\sinh^{-1}(cx))^2-\frac{1}{2}bcdx\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))-bd\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))+\frac{d(a+b\sinh^{-1}(cx))^3}{3b}-\frac{1}{4}d(a+b\sinh^{-1}(cx))^2+d\log\left(1-e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2+\frac{1}{4}b^2c^2dx^2-\frac{1}{2}b^2d\,\text{Li}_3\left(e^{-2\sinh^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (b^2*c^2*d*x^2)/4 - (b*c*d*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (d
*(a + b*ArcSinh[c*x])^2)/4 + (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 + (
d*(a + b*ArcSinh[c*x])^3)/(3*b) + d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Ar
cSinh[c*x])] - b*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])] - (
b^2*d*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+d\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx-(b$$

$$=-\frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(\right.$$

$$=\frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}d\left(a+b\sinh^{-1}(\right.$$

$$=\frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}d\left(a+b\sinh^{-1}(\right.$$

$$=\frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}d\left(a+b\sinh^{-1}(\right.$$

$$=\frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}d\left(a+b\sinh^{-1}(\right.$$

$$=\frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}d\left(a+b\sinh^{-1}(\right.$$

Mathematica [A]
time = 0.27, size = 222, normalized size = 1.34

$$\frac{1}{8}d\left(4a^2c^2x^2+8abc^2x^2\sinh^{-1}(cx)-4ab\left(cx\sqrt{1+c^2x^2}-\tanh^{-1}\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)+b^2\left(1+2\sinh^{-1}(cx)^2\right)\cosh\left(2\sinh^{-1}(cx)\right)+8ab\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)+8a^2\log(x)-8ab\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+8b^2\left(-\frac{1}{3}\sinh^{-1}(cx)^3+\sinh^{-1}(cx)^2\log\left(1-e^{2\sinh^{-1}(cx)}\right)+\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)\right)-2b^2\sinh^{-1}(cx)\sinh\left(2\sinh^{-1}(cx)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d*(4*a^2*c^2*x^2 + 8*a*b*c^2*x^2*ArcSinh[c*x] - 4*a*b*(c*x*Sqrt[1 + c^2*x^
2] - ArcTanh[(c*x)/Sqrt[1 + c^2*x^2]]) + b^2*(1 + 2*ArcSinh[c*x]^2)*Cosh[2*
```

```
ArcSinh[c*x]] + 8*a*b*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1 - E^(-2*ArcSinh[
c*x])]) + 8*a^2*Log[x] - 8*a*b*PolyLog[2, E^(-2*ArcSinh[c*x])] + 8*b^2*(-1/
3*ArcSinh[c*x]^3 + ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + ArcSinh[c*x
]*PolyLog[2, E^(2*ArcSinh[c*x])] - PolyLog[3, E^(2*ArcSinh[c*x])]/2) - 2*b^
2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(179) = 358$.
time = 3.69, size = 425, normalized size = 2.56

method	result
derivativedivides	$\frac{a^2c^2dx^2}{2}+a^2d\ln(cx)-\frac{b^2d\,\text{arcsinh}(cx)^3}{3}+\frac{b^2d\,\text{arcsinh}(cx)^2c^2x^2}{2}-\frac{b^2d\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{2}+\frac{b^2}{}$
default	$\frac{a^2c^2dx^2}{2}+a^2d\ln(cx)-\frac{b^2d\,\text{arcsinh}(cx)^3}{3}+\frac{b^2d\,\text{arcsinh}(cx)^2c^2x^2}{2}-\frac{b^2d\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{2}+\frac{b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*c^2*d*x^2+a^2*d*ln(c*x)-1/3*b^2*d*arcsinh(c*x)^3+1/2*b^2*d*arcsinh(
c*x)^2*c^2*x^2-1/2*b^2*d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1/4*b^2*d*arcsi
nh(c*x)^2+1/4*d*b^2*c^2*x^2+1/8*b^2*d+b^2*d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^
2+1)^(1/2))+2*b^2*d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*d*p
olylog(3,c*x+(c^2*x^2+1)^(1/2))+b^2*d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(
1/2))+2*b^2*d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*b^2*d*polylo
g(3,-c*x-(c^2*x^2+1)^(1/2))-b*d*a*arcsinh(c*x)^2+b*d*a*arcsinh(c*x)*c^2*x^2
-1/2*b*d*a*c*x*(c^2*x^2+1)^(1/2)+1/2*b*d*a*arcsinh(c*x)+2*b*d*a*arcsinh(c*x
)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b*d*a*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*b*
d*a*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*b*d*a*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^2*d*x^2 + a^2*d*log(x) + integrate(b^2*c^2*d*x*log(c*x + sqrt(c^2
*x^2 + 1))^2 + 2*a*b*c^2*d*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d*log(c*x +
 sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a^2}{x}\,dx + \int a^2c^2x\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x}\,dx + \int b^2c^2x\operatorname{asinh}^2(cx)\,dx + \int 2abc^2x\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d*(Integral(a**2/x, x) + Integral(a**2*c**2*x, x) + Integral(b**2*asinh(c*x
)**2/x, x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(b**2*c**2*x*asinh(c
*x)**2, x) + Integral(2*a*b*c**2*x*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\left(d\,c^2\,x^2+d\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x, x)
```

3.204 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=131

$$2b^2c^2dx-2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2-\frac{d(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{x}-4$$

```
[Out] 2*b^2*c^2*d*x+2*c^2*d*x*(a+b*arcsinh(c*x))^2-d*(c^2*x^2+1)*(a+b*arcsinh(c*x
))^2/x-4*b*c*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)-2*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5807, 5772, 5798, 8, 5806, 5816, 4267, 2317, 2438}

$$-2bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{d(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{x}+2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2-4bcd\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+2b^2c^2dx-2b^2cd\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+2b^2cd\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] 2*b^2*c^2*d*x - 2*b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + 2*c^2*d*x*
(a + b*ArcSinh[c*x])^2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x - 4*b*c
*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d*PolyLog[2, -E^A
rcSinh[c*x]] + 2*b^2*c*d*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{x} + (2bcd)\int \frac{\sqrt{1+c^2x^2}\,(a+b\sin$$

$$= 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d}{-}$$

$$= -2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh\right.$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}\right.$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}\right.$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}\right.$$

Mathematica [A]
time = 0.28, size = 192, normalized size = 1.47

$$\frac{d\left(-a^2+a^2c^2x^2+2abcx\left(-\sqrt{1+c^2x^2}+cx\sinh^{-1}(cx)\right)+b^2cx\left(2cx-2\sqrt{1+c^2x^2}\sinh^{-1}(cx)+cx\sinh^{-1}(cx)^2\right)-2ab\left(\sinh^{-1}(cx)+cx\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)\right)-b^2\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2cx\left(-\log\left(1-e^{-\sinh^{-1}(cx)}\right)+\log\left(1+e^{-\sinh^{-1}(cx)}\right)\right)\right)-2cx\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+2cx\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d*(-a^2 + a^2*c^2*x^2 + 2*a*b*c*x*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x])
+ b^2*c*x*(2*c*x - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2) -
 2*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]) - b^2*(ArcSinh[c*x]*
(ArcSinh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[c*
x])])) - 2*c*x*PolyLog[2, -E^(-ArcSinh[c*x])] + 2*c*x*PolyLog[2, E^(-ArcSin
h[c*x])])))/x
```

Maple [A]
time = 4.11, size = 239, normalized size = 1.82

method	result
derivativedivides	$c\left(a^2d\left(cx-\frac{1}{cx}\right)+b^2d\operatorname{arcsinh}(cx)^2cx-2b^2d\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2b^2cdx-\frac{b^2d\operatorname{ar}}{}\right.$
default	$c\left(a^2d\left(cx-\frac{1}{cx}\right)+b^2d\operatorname{arcsinh}(cx)^2cx-2b^2d\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2b^2cdx-\frac{b^2d\operatorname{ar}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a^2*d*(c*x-1/c/x)+b^2*d*arcsinh(c*x)^2*c*x-2*b^2*d*arcsinh(c*x)*(c^2*x^2
+1)^(1/2)+2*b^2*c*d*x-b^2*d*arcsinh(c*x)^2/c/x+2*b^2*d*arcsinh(c*x)*ln(1-c*
x-(c^2*x^2+1)^(1/2))+2*b^2*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*d*arcsi
nh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))+2*b*d*a*(arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)-arctanh(1/(
c^2*x^2+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] b^2*c^2*d*x*arcsinh(c*x)^2 + 2*b^2*c^2*d*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x
)/c) + a^2*c^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d - 2*(
c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*d - b^2*d*(log(c*x + sqrt(c^2
*x^2 + 1))^2/x - integrate(2*(c^3*x^2 + sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*
x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)),
 x)) - a^2*d/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^{2}c^{2}\,dx + \int \frac{a^{2}}{x^{2}}\,dx + \int b^{2}c^{2}\operatorname{asinh}^{2}{\left(cx \right)}\,dx + \int \frac{b^{2}\operatorname{asinh}^{2}{\left(cx \right)}}{x^{2}}\,dx + \int 2abc^{2}\operatorname{asinh}{\left(cx \right)}\,dx + \int \frac{2ab\operatorname{asinh}{\left(cx \right)}}{x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(b**2*c**2*asi
nh(c*x)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2
*asinh(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^2, x)
```

3.205 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=180

$$-\frac{bcd\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{x}+\frac{1}{2}c^2d(a+b\sinh^{-1}(cx))^2-\frac{d(1+c^2x^2)\,(a+b\sinh^{-1}(cx))^2}{2x^2}+\frac{c^2d(a+b\,\mathrm{si}\ldots}{3\ldots}$$

```
[Out] 1/2*c^2*d*(a+b*arcsinh(c*x))^2-1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/x^2+1
/3*c^2*d*(a+b*arcsinh(c*x))^3/b+c^2*d*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2
*x^2+1)^(1/2))^2)+b^2*c^2*d*ln(x)-b*c^2*d*(a+b*arcsinh(c*x))*polylog(2,1/(c
*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*c^2*d*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^
2)-b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x
```

Rubi [A]

time = 0.24, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5807, 5775, 3797, 2221, 2611, 2320, 6724, 5805, 29, 5783}

$$-bc^2d\,\mathrm{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))-\frac{d(c^2x^2+1)\,(a+b\sinh^{-1}(cx))^2}{2x^2}-\frac{bcd\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{x}+\frac{c^2d(a+b\sinh^{-1}(cx))^3}{3b}+\frac{1}{2}c^2d(a+b\sinh^{-1}(cx))^2+c^2d\log\left(1-e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2-\frac{1}{2}b^2c^2d\,\mathrm{Li}_3\left(e^{-2\sinh^{-1}(cx)}\right)+b^2c^2d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/x) + (c^2*d*(a + b*ArcSinh
[c*x])^2)/2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (c^2*d*(a
+ b*ArcSinh[c*x])^3)/(3*b) + c^2*d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Arc
Sinh[c*x])] + b^2*c^2*d*Log[x] - b*c^2*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^
(-2*ArcSinh[c*x])] - (b^2*c^2*d*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{d(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + (bcd)\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\right.}{x^2}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x} - \frac{d(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(}{}$$

Mathematica [A]
time = 0.21, size = 212, normalized size = 1.18

$$\frac{1}{2}d\left(-\frac{a^2}{x^2} - \frac{2ab\left(cx\sqrt{1+c^2x^2}+\sinh^{-1}(cx)\right)}{x^2} + 2a^2c^2\log(x) - \frac{b^2\left(2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)+\sinh^{-1}(cx)^2-2c^2x^2\log(cx)\right)}{x^2} + 2abc^2\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right) - \frac{1}{3}b^2c^2\left(2\sinh^{-1}(cx)^2\left(\sinh^{-1}(cx)-3\log\left(1-e^{2\sinh^{-1}(cx)}\right)\right)-6\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+3\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d*(-(a^2/x^2) - (2*a*b*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + 2*a^2
*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 -
```

```
 2*c^2*x^2*Log[c*x]))/x^2 + 2*a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1
 - E^(-2*ArcSinh[c*x])]) - PolyLog[2, E^(-2*ArcSinh[c*x])]) - (b^2*c^2*(2*A
rcSinh[c*x]^2*(ArcSinh[c*x] - 3*Log[1 - E^(2*ArcSinh[c*x])]) - 6*ArcSinh[c*
x]*PolyLog[2, E^(2*ArcSinh[c*x])] + 3*PolyLog[3, E^(2*ArcSinh[c*x])]))/3))/
2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(197) = 394$.
time = 6.48, size = 478, normalized size = 2.66

method	result
derivativedivides	$c^2\left(-\frac{a^2 d}{2c^2x^2} + a^2 d \ln(cx) - \frac{b^2 d \operatorname{arcsinh}(cx)^3}{3} - \frac{b^2 d \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{cx} + b^2 d \operatorname{arcsinh}(cx) - \right.$
default	$c^2\left(-\frac{a^2 d}{2c^2x^2} + a^2 d \ln(cx) - \frac{b^2 d \operatorname{arcsinh}(cx)^3}{3} - \frac{b^2 d \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{cx} + b^2 d \operatorname{arcsinh}(cx) - \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^2*d/c^2/x^2+a^2*d*ln(c*x)-1/3*b^2*d*arcsinh(c*x)^3-b^2*d*arcsin
h(c*x)/c/x*(c^2*x^2+1)^(1/2)+b^2*d*arcsinh(c*x)-1/2*b^2*d*arcsinh(c*x)^2/c^
2/x^2+b^2*d*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*d*ln(c*x+(c^2*x^2+1)^(1/2))+b
^2*d*ln(c*x+(c^2*x^2+1)^(1/2)-1)+b^2*d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^
(1/2))+2*b^2*d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*b^2*d*polyl
og(3,-c*x-(c^2*x^2+1)^(1/2))+b^2*d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2
))+2*b^2*d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*d*polylog(3,
c*x+(c^2*x^2+1)^(1/2))-b*d*a*arcsinh(c*x)^2-b*d*a/c/x*(c^2*x^2+1)^(1/2)+b*d
*a-b*d*a*arcsinh(c*x)/c^2/x^2+2*b*d*a*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))+2*b*d*a*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b*d*a*arcsinh(c*x)*ln(1-c*x
-(c^2*x^2+1)^(1/2))+2*b*d*a*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d*log(x) - a*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a
^2*d/x^2 + integrate(b^2*c^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*c^2
*d*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x^
3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a^{2}}{x^{3}}\, dx + \int \frac{a^{2} c^{2}}{x}\, dx + \int \frac{b^{2} \operatorname{asinh}^{2}{\left(cx \right)}}{x^{3}}\, dx + \int \frac{2ab \operatorname{asinh}{\left(cx \right)}}{x^{3}}\, dx + \int \frac{b^{2} c^{2} \operatorname{asinh}^{2}{\left(cx \right)}}{x}\, dx + \int \frac{2abc^{2} \operatorname{asinh}{\left(cx \right)}}{x}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d*(Integral(a**2/x**3, x) + Integral(a**2*c**2/x, x) + Integral(b**2*asinh(
c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(b**2*c**2*
asinh(c*x)**2/x, x) + Integral(2*a*b*c**2*asinh(c*x)/x, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\left(d\,c^2\,x^2+d\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^3, x)
```

3.206 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=158

$$-\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{3x^2}-\frac{2c^2d(a+b\sinh^{-1}(cx))^2}{3x}-\frac{d(1+c^2x^2)\,(a+b\sinh^{-1}(cx))^2}{3x^3}-\frac{10}{3}$$

```
[Out] -1/3*b^2*c^2*d/x-2/3*c^2*d*(a+b*arcsinh(c*x))^2/x-1/3*d*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2/x^3-10/3*b*c^3*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(
1/2))-5/3*b^2*c^3*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/3*b^2*c^3*d*polylog
(2,c*x+(c^2*x^2+1)^(1/2))-1/3*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x^
2
```

Rubi [A]
time = 0.26, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5807, 5776, 5816, 4267, 2317, 2438, 5805, 30}

$$-\frac{10}{3}bc^3d\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{3x^2}-\frac{d(c^2x^2+1)\,(a+b\sinh^{-1}(cx))^2}{3x^3}-\frac{2c^2d(a+b\sinh^{-1}(cx))^2}{3x}-\frac{5}{3}b^2c^3d\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\frac{5}{3}b^2c^3d\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)-\frac{b^2c^2d}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d)/x - (b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2)
 - (2*c^2*d*(a + b*ArcSinh[c*x])^2)/(3*x) - (d*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x])^2)/(3*x^3) - (10*b*c^3*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/3 - (5*b^2*c^3*d*PolyLog[2, -E^ArcSinh[c*x]])/3 + (5*b^2*c^3*d*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3} + \frac{1}{3}(2bcd)\int \frac{\sqrt{1+c^2x^2}\,(a+b\,\mathrm{s}}{x^3}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{3x} - d$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{2c^2d\left(a+b\sinh^{-1}(c\right.}{3x}$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{2c^2d\left(a+b\sinh^{-1}(c\right.}{3x}$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{2c^2d\left(a+b\sinh^{-1}(c\right.}{3x}$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{2c^2d\left(a+b\sinh^{-1}(c\right.}{3x}$$

Mathematica [A]
time = 0.49, size = 245, normalized size = 1.55

$$-\frac{d\left(a^2+3a^2c^2x^2+b^2c^2x^2+abcx\sqrt{1+c^2x^2}+2ab\sinh^{-1}(cx)+6abc^2x^2\sinh^{-1}(cx)+b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)+b^2\sinh^{-1}(cx)^2+3b^2c^2x^2\sinh^{-1}(cx)^2+5abc^3x^3\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)-5b^2c^3x^3\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+5b^2c^3x^3\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)-5b^2c^3x^3\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+5b^2c^3x^3\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(d*(a^2 + 3*a^2*c^2*x^2 + b^2*c^2*x^2 + a*b*c*x*Sqrt[1 + c^2*x^2] + 2*
a*b*ArcSinh[c*x] + 6*a*b*c^2*x^2*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*A
rcSinh[c*x] + b^2*ArcSinh[c*x]^2 + 3*b^2*c^2*x^2*ArcSinh[c*x]^2 + 5*a*b*c^3
*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] - 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] + 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 5*b^
2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] + 5*b^2*c^3*x^3*PolyLog[2, E^(-Arc
Sinh[c*x])]))/x^3
```

Maple [A]
time = 5.94, size = 272, normalized size = 1.72

method	result
derivativedivides	$c^3\left(a^2d\left(-\frac{1}{cx}-\frac{1}{3c^3x^3}\right)-\frac{b^2d\,\mathrm{arcsinh}(cx)^2}{cx}-\frac{b^2d\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}}{3c^2x^2}-\frac{b^2d\,\mathrm{arcsinh}(cx)^2}{3c^3x^3}-\frac{b^2d}{3cx}-\right.$

default	$c^3\left(a^2d\left(-\frac{1}{cx}-\frac{1}{3c^3x^3}\right)-\frac{b^2d\operatorname{arcsinh}(cx)^2}{cx}-\frac{b^2d\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{3c^2x^2}-\frac{b^2d\operatorname{arcsinh}(cx)^2}{3c^3x^3}-\frac{b^2d}{3cx}-\frac{5}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)

[Out] c^3*(a^2*d*(-1/c/x-1/3/c^3/x^3)-b^2*d*arcsinh(c*x)^2/c/x-1/3*b^2*d/c^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-1/3*b^2*d/c^3/x^3*arcsinh(c*x)^2-1/3*b^2*d/c/x-5/3*b^2*d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-5/3*b^2*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/3*b^2*d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+5/3*b^2*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*b*d*a*(-arcsinh(c*x)/c/x-1/3*arcsinh(c*x)/c^3/x^3-5/6*arctanh(1/(c^2*x^2+1)^(1/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")

[Out] -2*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d + 1/3*((c^2*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d - a^2*c^2*d/x - 1/3*a^2*d/x^3 - 1/3*(3*b^2*c^2*d*x^2 + b^2*d)*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + integrate(2/3*(3*b^2*c^5*d*x^4 + 4*b^2*c^3*d*x^2 + b^2*c*d + (3*b^2*c^4*d*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")

[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 + 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^4, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a^2}{x^4}\,dx+\int\frac{a^2c^2}{x^2}\,dx+\int\frac{b^2\operatorname{asinh}^2(cx)}{x^4}\,dx+\int\frac{2ab\operatorname{asinh}(cx)}{x^4}\,dx+\int\frac{b^2c^2\operatorname{asinh}^2(cx)}{x^2}\,dx+\int\frac{2abc^2\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d*(Integral(a**2/x**4, x) + Integral(a**2*c**2/x**2, x) + Integral(b**2*asi
nh(c*x)**2/x**4, x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(b**2*c*
*2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2*asinh(c*x)/x**2, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^4, x)
```

3.207 $\int x^4(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=386

$$\frac{4208b^2d^2x}{99225c^4}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{526b^2d^2x^5}{165375}+\frac{212b^2c^2d^2x^7}{27783}+\frac{2}{729}b^2c^4d^2x^9-\frac{128bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{4725c^5}+\frac{64b}{}$$

```
[Out] 4208/99225*b^2*d^2*x/c^4-2104/297675*b^2*d^2*x^3/c^2+526/165375*b^2*d^2*x^5
+212/27783*b^2*c^2*d^2*x^7+2/729*b^2*c^4*d^2*x^9-8/189*b*d^2*(c^2*x^2+1)^(3
/2)*(a+b*arcsinh(c*x))/c^5+2/315*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))
/c^5+20/441*b*d^2*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^5-2/81*b*d^2*(c^2*
x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^5+8/315*d^2*x^5*(a+b*arcsinh(c*x))^2+4/63
*d^2*x^5*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/9*d^2*x^5*(c^2*x^2+1)^2*(a+b*ar
csinh(c*x))^2-128/4725*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+64/47
25*b*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/1575*b*d^2*x^4*(a+
b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.51, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.423$, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12, 1167}

$$\frac{1}{9}d^2x^5(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{63}d^2x^5(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{16bd^2x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{1575c}-\frac{2bd^2(c^2x^2+1)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{81c^5}+\frac{20bd^2(c^2x^2+1)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{441c^5}+\frac{2bd^2(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{315c^5}-\frac{8bd^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{189c^5}-\frac{128bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{4725c^5}+\frac{64bd^2x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{4725c^3}+\frac{8}{315}d^2x^5\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{729}b^2c^4d^2x^9+\frac{4208b^2d^2x}{99225c^4}+\frac{212b^2c^2d^2x^7}{27783}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{526b^2d^2x^5}{165375}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4208*b^2*d^2*x)/(99225*c^4) - (2104*b^2*d^2*x^3)/(297675*c^2) + (526*b^2*d
^2*x^5)/165375 + (212*b^2*c^2*d^2*x^7)/27783 + (2*b^2*c^4*d^2*x^9)/729 - (1
28*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^5) + (64*b*d^2*x^2
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^3) - (16*b*d^2*x^4*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(1575*c) - (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a
 + b*ArcSinh[c*x]))/(189*c^5) + (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh
[c*x]))/(315*c^5) + (20*b*d^2*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(44
1*c^5) - (2*b*d^2*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^5) + (8*d
^2*x^5*(a + b*ArcSinh[c*x])^2)/315 + (4*d^2*x^5*(1 + c^2*x^2)*(a + b*ArcSin
h[c*x])^2)/63 + (d^2*x^5*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/9
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
```

```
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^4\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{9}d^2x^5\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{9}(4d)\int x^4\left(d+c^2dx^2\right)\left(\right.$$

$$=-\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{45c^5}+\frac{4bd^2\left(1+c^2x^2\right)^{7/2}\left(a+\right.}{63c^5}$$

$$=-\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{189c^5}+\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+\right.}{315c^5}$$

$$=-\frac{16bd^2x^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{1575c}-\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+\right.}{189c^5}$$

$$=\frac{304b^2d^2x}{19845c^4}-\frac{152b^2d^2x^3}{59535c^2}+\frac{526b^2d^2x^5}{165375}+\frac{212b^2c^2d^2x^7}{27783}+\frac{2}{729}b^2c^4d^2$$

$$=\frac{304b^2d^2x}{19845c^4}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{526b^2d^2x^5}{165375}+\frac{212b^2c^2d^2x^7}{27783}+\frac{2}{729}b^2c^4d$$

$$=\frac{4208b^2d^2x}{99225c^4}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{526b^2d^2x^5}{165375}+\frac{212b^2c^2d^2x^7}{27783}+\frac{2}{729}b^2c^4$$

Mathematica [A]
time = 0.22, size = 251, normalized size = 0.65

$$\frac{d^2\left(99225a^2c^5x^5(63+90c^2x^2+35c^4x^4)-630ab\sqrt{1+c^2x^2}\,(2104-1052c^2x^2+789c^4x^4+2650c^6x^6+1225c^8x^8)+2b^2cx(662760-110460c^2x^2+49707c^4x^4+119250c^6x^6+42875c^8x^8)-630b\left(-315ac^5x^5(63+90c^2x^2+35c^4x^4)+b\sqrt{1+c^2x^2}\,(2104-1052c^2x^2+789c^4x^4+2650c^6x^6+1225c^8x^8)\right)\sinh^{-1}(cx)+99225b^2c^5x^5(63+90c^2x^2+35c^4x^4)\sinh^{-1}(cx)^2\right)}{31255875c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(99225*a^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - 630*a*b*Sqrt[1 + c
^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) +
 2*b^2*c*x*(662760 - 110460*c^2*x^2 + 49707*c^4*x^4 + 119250*c^6*x^6 + 4287
5*c^8*x^8) - 630*b*(-315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) + b*Sqrt[
1 + c^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x
^8))*ArcSinh[c*x] + 99225*b^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSin
h[c*x]^2))/(31255875*c^5)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^4\left(c^2d\,x^2+d\right)^2\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(342) = 684.
time = 0.31, size = 760, normalized size = 1.97

$$\frac{1}{9}b^2c^4d^2x^9\,\mathrm{arsinh}\,(cx)^2+\frac{1}{9}a^2c^4d^2x^9+\frac{2}{7}b^2c^2d^2x^7\,\mathrm{arsinh}\,(cx)^2+\frac{2}{7}a^2c^2d^2x^7+\frac{1}{5}b^2d^2x^5\,\mathrm{arsinh}\,(cx)^2+\frac{2}{2835}\left(315x^9\,\mathrm{arsinh}\,(cx)-\left(\frac{35\sqrt{c^2x^2+1}\,x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}\,x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2+1}}{c^{10}}\right)c\right)abc^4d^2-\frac{2}{893025}\left(315\left(\frac{35\sqrt{c^2x^2+1}\,x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}\,x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2+1}}{c^{10}}\right)c\,\mathrm{arsinh}\,(cx)-\frac{1225c^8x^9-1800c^6x^7+3024c^4x^5-6720c^2x^3+40320x}{c^8}\right)b^2c^4d^2+\frac{1}{5}a^2d^2x^5+\frac{4}{245}\left(35x^7\,\mathrm{arsinh}\,(cx)-\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^2d^2-\frac{4}{25725}\left(105\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\,\mathrm{arsinh}\,(cx)-\frac{75c^6x^7-126c^4x^5+280c^2x^3-1680x}{c^6}\right)b^2c^2d^2+\frac{2}{75}\left(15x^5\,\mathrm{arsinh}\,(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abd^2-\frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\,\mathrm{arsinh}\,(cx)-\frac{9c^4x^5-20c^2x^3+120x}{c^4}\right)b^2d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*c^4*d^2*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^4*d^2*x^9 + 2/7*b^2*c^2*d^2*
x^7*arcsinh(c*x)^2 + 2/7*a^2*c^2*d^2*x^7 + 1/5*b^2*d^2*x^5*arcsinh(c*x)^2 +
 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2
*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2
/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^4*d^2 - 2/893025*(315*(35*sqrt(
c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*
x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*arcs
inh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 + 403
20*x)/c^8)*b^2*c^4*d^2 + 1/5*a^2*d^2*x^5 + 4/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
```

```
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^2 - 4/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^2 + 2/75*(15*x^5*arcsinh(c*x) -
(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2
 + 1)/c^6)*c)*a*b*d^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^
2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 -
 20*c^2*x^3 + 120*x)/c^4)*b^2*d^2
```

Fricas [**A**]
time = 0.42, size = 368, normalized size = 0.95

$$\frac{42875\,(81\,a^2+2\,b^2)c^9d^2x^9+2250\,(3969\,a^2+106\,b^2)c^7d^2x^7+189\,(33075\,a^2+526\,b^2)c^5d^2x^5-220920\,b^2c^3d^2x^3+1325520\,b^2cd^2x+99225\,(35\,b^2c^9d^2x^9+90\,b^2c^7d^2x^7+63\,b^2c^5d^2x^5)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+630\left(11025\,abc^9d^2x^9+28350\,abc^7d^2x^7+19845\,abc^5d^2x^5-(1225\,b^2c^8d^2x^8+2650\,b^2c^6d^2x^6+789\,b^2c^4d^2x^4-1052\,b^2c^2d^2x^2+2104\,b^2d^2)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-630\,(1225\,abc^8d^2x^8+2650\,abc^6d^2x^6+789\,abc^4d^2x^4-1052\,abc^2d^2x^2+2104\,abd^2)\sqrt{c^2x^2+1}}{31255875\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^2*x^9 + 2250*(3969*a^2 + 106*b^2)*
c^7*d^2*x^7 + 189*(33075*a^2 + 526*b^2)*c^5*d^2*x^5 - 220920*b^2*c^3*d^2*x^
3 + 1325520*b^2*c*d^2*x + 99225*(35*b^2*c^9*d^2*x^9 + 90*b^2*c^7*d^2*x^7 +
63*b^2*c^5*d^2*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^2
*x^9 + 28350*a*b*c^7*d^2*x^7 + 19845*a*b*c^5*d^2*x^5 - (1225*b^2*c^8*d^2*x^
8 + 2650*b^2*c^6*d^2*x^6 + 789*b^2*c^4*d^2*x^4 - 1052*b^2*c^2*d^2*x^2 + 210
4*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*
c^8*d^2*x^8 + 2650*a*b*c^6*d^2*x^6 + 789*a*b*c^4*d^2*x^4 - 1052*a*b*c^2*d^2
*x^2 + 2104*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [**A**]
time = 2.15, size = 563, normalized size = 1.46

$$\begin{cases} \frac{a^2c^4d^2x^9}{9}+\frac{2a^2c^2d^2x^7}{7}+\frac{a^2d^2x^5}{5}+\frac{2abc^4d^2x^9\operatorname{asinh}(cx)}{9}-\frac{2abc^3d^2x^8\sqrt{c^2x^2+1}}{81}+\frac{4abc^2d^2x^7\operatorname{asinh}(cx)}{7}-\frac{212abcd^2x^6\sqrt{c^2x^2+1}}{3969}+\frac{2abd^2x^5\operatorname{asinh}(cx)}{5}-\frac{526abd^2x^4\sqrt{c^2x^2+1}}{33075c}+\frac{2104abd^2x^2\sqrt{c^2x^2+1}}{99225c^3}-\frac{4208abd^2\sqrt{c^2x^2+1}}{99225c^5}+\frac{b^2c^4d^2x^9\operatorname{asinh}^2(cx)}{9}+\frac{2b^2c^4d^2x^9}{729}-\frac{2b^2c^3d^2x^8\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{81}+\frac{2b^2c^2d^2x^7\operatorname{asinh}^2(cx)}{7}+\frac{212b^2c^2d^2x^7}{27783}-\frac{212b^2cd^2x^6\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3969}+\frac{b^2d^2x^5\operatorname{asinh}^2(cx)}{5}+\frac{526b^2d^2x^5}{165375}-\frac{526b^2d^2x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{33075c}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{2104b^2d^2x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{99225c^3}+\frac{4208b^2d^2x}{99225c^4}-\frac{4208b^2d^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{99225c^5} & \text{for}\: c \neq 0 \\ \frac{a^2d^2x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**9/9 + 2*a**2*c**2*d**2*x**7/7 + a**2*d**2*x**5
/5 + 2*a*b*c**4*d**2*x**9*asinh(c*x)/9 - 2*a*b*c**3*d**2*x**8*sqrt(c**2*x**
2 + 1)/81 + 4*a*b*c**2*d**2*x**7*asinh(c*x)/7 - 212*a*b*c*d**2*x**6*sqrt(c*
*2*x**2 + 1)/3969 + 2*a*b*d**2*x**5*asinh(c*x)/5 - 526*a*b*d**2*x**4*sqrt(c
**2*x**2 + 1)/(33075*c) + 2104*a*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**
3) - 4208*a*b*d**2*sqrt(c**2*x**2 + 1)/(99225*c**5) + b**2*c**4*d**2*x**9*a
sinh(c*x)**2/9 + 2*b**2*c**4*d**2*x**9/729 - 2*b**2*c**3*d**2*x**8*sqrt(c**
2*x**2 + 1)*asinh(c*x)/81 + 2*b**2*c**2*d**2*x**7*asinh(c*x)**2/7 + 212*b**
2*c**2*d**2*x**7/27783 - 212*b**2*c*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x
)/3969 + b**2*d**2*x**5*asinh(c*x)**2/5 + 526*b**2*d**2*x**5/165375 - 526*b
**2*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(33075*c) - 2104*b**2*d**2*x**
```

```
3/(297675*c**2) + 2104*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225
*c**3) + 4208*b**2*d**2*x/(99225*c**4) - 4208*b**2*d**2*sqrt(c**2*x**2 + 1)
*asinh(c*x)/(99225*c**5), Ne(c, 0)), (a**2*d**2*x**5/5, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

3.208 $\int x^3(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=296

$$-\frac{73b^2d^2x^2}{3072c^2}+\frac{73b^2d^2x^4}{9216}+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8+\frac{73bd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{1536c^3}-\frac{73bd^2x^3\sqrt{1+c^2x^2}}{23}$$

```
[Out] -73/3072*b^2*d^2*x^2/c^2+73/9216*b^2*d^2*x^4+43/3456*b^2*c^2*d^2*x^6+1/256*
b^2*c^4*d^2*x^8-1/32*b*c*d^2*x^5*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-73/30
72*d^2*(a+b*arcsinh(c*x))^2/c^4+1/24*d^2*x^4*(a+b*arcsinh(c*x))^2+1/12*d^2*
x^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/8*d^2*x^4*(c^2*x^2+1)^2*(a+b*arcsinh
(c*x))^2+73/1536*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-73/2304*b
*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-25/576*b*c*d^2*x^5*(a+b*arc
sinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.269$, Rules used = {5808, 5776, 5812, 5783, 30, 5806, 14}

$$-\frac{73d^2\left(a+b\sinh^{-1}(cx)\right)^2}{3072c^4}-\frac{1}{32}bcd^2x^5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{25}{576}bcd^2x^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{12}d^2x^4\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{73bd^2x^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2304c}+\frac{73bd^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{1536c^3}+\frac{1}{24}d^2x^4\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{256}b^2c^4d^2x^8+\frac{43b^2c^2d^2x^6}{3456}-\frac{73b^2d^2x^2}{3072c^2}+\frac{73b^2d^2x^4}{9216}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-73*b^2*d^2*x^2)/(3072*c^2) + (73*b^2*d^2*x^4)/9216 + (43*b^2*c^2*d^2*x^6)
/3456 + (b^2*c^4*d^2*x^8)/256 + (73*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(1536*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(2304*c) - (25*b*c*d^2*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/576 - (
b*c*d^2*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/32 - (73*d^2*(a + b*A
rcSinh[c*x])^2)/(3072*c^4) + (d^2*x^4*(a + b*ArcSinh[c*x])^2)/24 + (d^2*x^4
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/12 + (d^2*x^4*(1 + c^2*x^2)^2*(a + b
*ArcSinh[c*x])^2)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{8}d^2x^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{2}d\int x^3\left(d+c^2dx^2\right)(a-$$
$$=-\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{12}d^2x^4\left(1+c^2x^2\right)$$
$$=-\frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)$$
$$=\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8-\frac{73bd^2x^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{2304c}$$
$$=\frac{73b^2d^2x^4}{9216}+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8+\frac{73bd^2x\sqrt{1+c^2x^2}\left(a+\right.}{1536c^3}$$
$$=-\frac{73b^2d^2x^2}{3072c^2}+\frac{73b^2d^2x^4}{9216}+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8+\frac{73bd^2x\sqrt{}}{}$$

Mathematica [A]
time = 0.22, size = 237, normalized size = 0.80

$$\frac{d^2\left(cx\left(1152a^2c^3x^3(6+8c^2x^2+3c^4x^4)+b^2cx(-657+219c^2x^2+344c^4x^4+108c^6x^6)-6ab\sqrt{1+c^2x^2}(-219+146c^2x^2+344c^4x^4+144c^6x^6)\right)+6b\left(-bcx\sqrt{1+c^2x^2}(-219+146c^2x^2+344c^4x^4+144c^6x^6)+3a(-73+768c^4x^4+1024c^6x^6+384c^8x^8)\right)\sinh^{-1}(cx)+9b^2(-73+768c^4x^4+1024c^6x^6+384c^8x^8)\sinh^{-1}(cx)^2\right)}{27648c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(c*x*(1152*a^2*c^3*x^3*(6 + 8*c^2*x^2 + 3*c^4*x^4) + b^2*c*x*(-657 + 2
19*c^2*x^2 + 344*c^4*x^4 + 108*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(-219 + 1
46*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(
-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 3*a*(-73 + 768*c^4*x^4 +
 1024*c^6*x^6 + 384*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(-73 + 768*c^4*x^4 + 102
4*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]^2))/(27648*c^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3\left(c^2d\,x^2+d\right)^2\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(264) = 528$.
time = 0.36, size = 762, normalized size = 2.57

$$\frac{1}{8} b^{2} c^{4} d^{2} x^{8} \operatorname{arsinh}\left(c x\right)^{2} + \frac{1}{8} a^{2} c^{4} d^{2} x^{8} + \frac{1}{3} b^{2} c^{2} d^{2} x^{6} \operatorname{arsinh}\left(c x\right)^{2} + \frac{1}{3} a^{2} c^{2} d^{2} x^{6} + \frac{1}{4} b^{2} d^{2} x^{4} \operatorname{arsinh}\left(c x\right)^{2} + \frac{1}{1536} \left(384 x^{8} \operatorname{arsinh}\left(c x\right) - \left(\frac{48 \sqrt{c^{2} x^{2} + 1} x^{7}}{c^{2}} - \frac{56 \sqrt{c^{2} x^{2} + 1} x^{5}}{c^{4}} + \frac{70 \sqrt{c^{2} x^{2} + 1} x^{3}}{c^{6}} - \frac{105 \sqrt{c^{2} x^{2} + 1} x}{c^{8}} + \frac{105 \operatorname{arsinh}\left(c x\right)}{c^{9}}\right) c\right) a b c^{4} d^{2} + \frac{1}{9216} \left(\left(\frac{36 x^{8}}{c^{2}} - \frac{56 x^{6}}{c^{4}} + \frac{105 x^{4}}{c^{6}} - \frac{315 x^{2}}{c^{8}} + \frac{315 \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2}}{c^{10}}\right) c^{2} - 6 \left(\frac{48 \sqrt{c^{2} x^{2} + 1} x^{7}}{c^{2}} - \frac{56 \sqrt{c^{2} x^{2} + 1} x^{5}}{c^{4}} + \frac{70 \sqrt{c^{2} x^{2} + 1} x^{3}}{c^{6}} - \frac{105 \sqrt{c^{2} x^{2} + 1} x}{c^{8}} + \frac{105 \operatorname{arsinh}\left(c x\right)}{c^{9}}\right) c \operatorname{arsinh}\left(c x\right)\right) b^{2} c^{4} d^{2} + \frac{1}{4} a^{2} d^{2} x^{4} + \frac{1}{72} \left(48 x^{6} \operatorname{arsinh}\left(c x\right) - \left(\frac{8 \sqrt{c^{2} x^{2} + 1} x^{5}}{c^{2}} - \frac{10 \sqrt{c^{2} x^{2} + 1} x^{3}}{c^{4}} + \frac{15 \sqrt{c^{2} x^{2} + 1} x}{c^{6}} - \frac{15 \operatorname{arsinh}\left(c x\right)}{c^{7}}\right) c\right) a b c^{2} d^{2} + \frac{1}{432} \left(\left(\frac{8 x^{6}}{c^{2}} - \frac{15 x^{4}}{c^{4}} + \frac{45 x^{2}}{c^{6}} - \frac{45 \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2}}{c^{8}}\right) c^{2} - 6 \left(\frac{8 \sqrt{c^{2} x^{2} + 1} x^{5}}{c^{2}} - \frac{10 \sqrt{c^{2} x^{2} + 1} x^{3}}{c^{4}} + \frac{15 \sqrt{c^{2} x^{2} + 1} x}{c^{6}} - \frac{15 \operatorname{arsinh}\left(c x\right)}{c^{7}}\right) c \operatorname{arsinh}\left(c x\right)\right) b^{2} c^{2} d^{2} + \frac{1}{16} \left(8 x^{4} \operatorname{arsinh}\left(c x\right) - \left(\frac{2 \sqrt{c^{2} x^{2} + 1} x^{3}}{c^{2}} - \frac{3 \sqrt{c^{2} x^{2} + 1} x}{c^{4}} + \frac{3 \operatorname{arsinh}\left(c x\right)}{c^{5}}\right) c\right) a b d^{2} + \frac{1}{32} \left(\left(\frac{x^{4}}{c^{2}} - \frac{3 x^{2}}{c^{4}} + \frac{3 \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2}}{c^{6}}\right) c^{2} - 2 \left(\frac{2 \sqrt{c^{2} x^{2} + 1} x^{3}}{c^{2}} - \frac{3 \sqrt{c^{2} x^{2} + 1} x}{c^{4}} + \frac{3 \operatorname{arsinh}\left(c x\right)}{c^{5}}\right) c \operatorname{arsinh}\left(c x\right)\right) b^{2} d^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*c^4*d^2*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^4*d^2*x^8 + 1/3*b^2*c^2*d^2*
x^6*arcsinh(c*x)^2 + 1/3*a^2*c^2*d^2*x^6 + 1/4*b^2*d^2*x^4*arcsinh(c*x)^2 +
 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2
*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/
c^8 + 105*arcsinh(c*x)/c^9)*c)*a*b*c^4*d^2 + 1/9216*((36*x^8/c^2 - 56*x^6/c
^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^10)*c
^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sq
rt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^
9)*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/4*a^2*d^2*x^4 + 1/72*(48*x^6*arcsinh(c*x
) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c
^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^2*d^2 + 1/432*((8*x^6/c^2
 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6
*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*
x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^2*d^2 + 1/16*(8
*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^
4 + 3*arcsinh(c*x)/c^5)*c)*a*b*d^2 + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x
 + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(
c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*b^2*d^2
```

Fricas [**A**]
time = 0.36, size = 348, normalized size = 1.18

$$\frac{108 \left(32 a^{2} + b^{2}\right) c^{8} d^{2} x^{8} + 8 \left(1152 a^{2} + 43 b^{2}\right) c^{6} d^{2} x^{6} + 3 \left(2304 a^{2} + 73 b^{2}\right) c^{4} d^{2} x^{4} - 657 b^{2} c^{2} d^{2} x^{2} + 9 \left(384 b^{2} c^{8} d^{2} x^{8} + 1024 b^{2} c^{6} d^{2} x^{6} + 768 b^{2} c^{4} d^{2} x^{4} - 73 b^{2} d^{2}\right) \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2} + 6 \left(1152 a b c^{8} d^{2} x^{8} + 3072 a b c^{6} d^{2} x^{6} + 2304 a b c^{4} d^{2} x^{4} - 219 a b d^{2} - \left(144 b^{2} c^{7} d^{2} x^{7} + 344 b^{2} c^{5} d^{2} x^{5} + 146 b^{2} c^{3} d^{2} x^{3} - 219 b^{2} c d^{2} x\right) \sqrt{c^{2} x^{2} + 1}\right) \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right) - 6 \left(144 a b c^{7} d^{2} x^{7} + 344 a b c^{5} d^{2} x^{5} + 146 a b c^{3} d^{2} x^{3} - 219 a b c d^{2} x\right) \sqrt{c^{2} x^{2} + 1}}{27648 c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27648*(108*(32*a^2 + b^2)*c^8*d^2*x^8 + 8*(1152*a^2 + 43*b^2)*c^6*d^2*x^6
 + 3*(2304*a^2 + 73*b^2)*c^4*d^2*x^4 - 657*b^2*c^2*d^2*x^2 + 9*(384*b^2*c^8
*d^2*x^8 + 1024*b^2*c^6*d^2*x^6 + 768*b^2*c^4*d^2*x^4 - 73*b^2*d^2)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 6*(1152*a*b*c^8*d^2*x^8 + 3072*a*b*c^6*d^2*x^6 +
2304*a*b*c^4*d^2*x^4 - 219*a*b*d^2 - (144*b^2*c^7*d^2*x^7 + 344*b^2*c^5*d^2
*x^5 + 146*b^2*c^3*d^2*x^3 - 219*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) - 6*(144*a*b*c^7*d^2*x^7 + 344*a*b*c^5*d^2*x^5 + 146*a*b
*c^3*d^2*x^3 - 219*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A]
time = 2.02, size = 515, normalized size = 1.74

$$\begin{cases} \frac{a^2c^4d^2x^8}{8} + \frac{a^2c^2d^2x^6}{3} + \frac{a^2d^2x^4}{4} + \frac{abc^4d^2x^8 \operatorname{asinh}(cx)}{4} - \frac{abc^3d^2x^7\sqrt{c^2x^2+1}}{32} + \frac{2abc^2d^2x^6 \operatorname{asinh}(cx)}{3} - \frac{43abcd^2x^5\sqrt{c^2x^2+1}}{576} + \frac{abd^2x^4 \operatorname{asinh}(cx)}{2} - \frac{73abd^2x^3\sqrt{c^2x^2+1}}{2304c} + \frac{73abd^2x\sqrt{c^2x^2+1}}{1536c^3} - \frac{73abd^2 \operatorname{asinh}(cx)}{1536c^4} + \frac{b^2c^4d^2x^8 \operatorname{asinh}^2(cx)}{8} + \frac{b^2c^4d^2x^8}{256} - \frac{b^2c^3d^2x^7\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{32} + \frac{b^2c^2d^2x^6 \operatorname{asinh}^2(cx)}{3} + \frac{43b^2c^2d^2x^6}{3456} - \frac{43b^2cd^2x^5\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{576} + \frac{b^2d^2x^4 \operatorname{asinh}^2(cx)}{4} + \frac{73b^2d^2x^4}{9216} - \frac{73b^2d^2x^3\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{2304c} - \frac{73b^2d^2x^2}{3072c^2} + \frac{73b^2d^2x\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{1536c^3} - \frac{73b^2d^2 \operatorname{asinh}^2(cx)}{3072c^4} & \text{for } c \neq 0 \\ \frac{a^2d^2x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**8/8 + a**2*c**2*d**2*x**6/3 + a**2*d**2*x**4/4
 + a*b*c**4*d**2*x**8*asinh(c*x)/4 - a*b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)
/32 + 2*a*b*c**2*d**2*x**6*asinh(c*x)/3 - 43*a*b*c*d**2*x**5*sqrt(c**2*x**2
 + 1)/576 + a*b*d**2*x**4*asinh(c*x)/2 - 73*a*b*d**2*x**3*sqrt(c**2*x**2 +
1)/(2304*c) + 73*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(1536*c**3) - 73*a*b*d**2*a
sinh(c*x)/(1536*c**4) + b**2*c**4*d**2*x**8*asinh(c*x)**2/8 + b**2*c**4*d**
2*x**8/256 - b**2*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c
**2*d**2*x**6*asinh(c*x)**2/3 + 43*b**2*c**2*d**2*x**6/3456 - 43*b**2*c*d**
2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/576 + b**2*d**2*x**4*asinh(c*x)**2/4
+ 73*b**2*d**2*x**4/9216 - 73*b**2*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)
/(2304*c) - 73*b**2*d**2*x**2/(3072*c**2) + 73*b**2*d**2*x*sqrt(c**2*x**2 +
 1)*asinh(c*x)/(1536*c**3) - 73*b**2*d**2*asinh(c*x)**2/(3072*c**4), Ne(c,
0)), (a**2*d**2*x**4/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

3.209 $\int x^2(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=303

$$-\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{315c^3}-\frac{16bd^2x^2\sqrt{1+c^2x^2}}{}$$

```
[Out] -1636/11025*b^2*d^2*x/c^2+818/33075*b^2*d^2*x^3+136/6125*b^2*c^2*d^2*x^5+2/
343*b^2*c^4*d^2*x^7+8/105*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^3+2/
175*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^3-2/49*b*d^2*(c^2*x^2+1)^(
7/2)*(a+b*arcsinh(c*x))/c^3+8/105*d^2*x^3*(a+b*arcsinh(c*x))^2+4/35*d^2*x^3
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/7*d^2*x^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*
x))^2+32/315*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/315*b*d^2*x^
2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.41, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12, 380}

$$-\frac{16bd^2x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{315c}+\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{35}d^2x^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^2\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c^3}+\frac{2bd^2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{175c^3}+\frac{8bd^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{105c^3}+\frac{32bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{315c^3}+\frac{8}{105}d^2x^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{343}b^2c^4d^2x^7+\frac{136b^2c^2d^2x^5}{6125}-\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-1636*b^2*d^2*x)/(11025*c^2) + (818*b^2*d^2*x^3)/33075 + (136*b^2*c^2*d^2*
x^5)/6125 + (2*b^2*c^4*d^2*x^7)/343 + (32*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(315*c^3) - (16*b*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(315*c) + (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105*c^3)
+ (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c^3) - (2*b*d^2*(
1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c^3) + (8*d^2*x^3*(a + b*ArcSi
nh[c*x])^2)/105 + (4*d^2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (d^
2*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx &= \frac{1}{7}d^2x^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}(4d)\int x^2\left(d+c^2dx^2\right) \\
&= \frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{35c^3}-\frac{2bd^2\left(1+c^2x^2\right)^{7/2}\left(a+b\right.}{49c^3} \\
&= \frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{105c^3}+\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\right.}{175c^3} \\
&= -\frac{16bd^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{315c}+\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a\right.}{105c} \\
&= -\frac{172b^2d^2x}{3675c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd}{} \\
&= -\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32b}{}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 227, normalized size = 0.75

$$
\frac{d^2\left(11025a^2c^3x^3(35+42c^2x^2+15c^4x^4)-210ab\sqrt{1+c^2x^2}\,(-818+409c^2x^2+612c^4x^4+225c^6x^6)+2b^2cx(-85890+14315c^2x^2+12852c^4x^4+3375c^6x^6)-210b\left(-105ac^3x^3(35+42c^2x^2+15c^4x^4)+b\sqrt{1+c^2x^2}\,(-818+409c^2x^2+612c^4x^4+225c^6x^6)\right)\sinh^{-1}(cx)+11025b^2c^3x^3(35+42c^2x^2+15c^4x^4)\sinh^{-1}(cx)^2\right)}{1157625c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(11025*a^2*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - 210*a*b*Sqrt[1 + c
^2*x^2]*(-818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 2*b^2*c*x*(-8589
0 + 14315*c^2*x^2 + 12852*c^4*x^4 + 3375*c^6*x^6) - 210*b*(-105*a*c^3*x^3*(
35 + 42*c^2*x^2 + 15*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(-818 + 409*c^2*x^2 + 6
12*c^4*x^4 + 225*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c^3*x^3*(35 + 42*c^2*x^
2 + 15*c^4*x^4)*ArcSinh[c*x]^2))/(1157625*c^3)
```

<u>Maple</u> [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(c^{2} d\,x^{2}+d \right)^{2} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

<u>Maxima</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(269) = 538$.
time = 0.32, size = 619, normalized size = 2.04

$$\frac{1}{7} b^{2} c^{4} d^{2} x^{7} \operatorname{arsinh}\left(c x\right)^{2} + \frac{1}{7} a^{2} c^{4} d^{2} x^{7} + \frac{2}{5} b^{2} c^{2} d^{2} x^{5} \operatorname{arsinh}\left(c x\right)^{2} + \frac{2}{5} a^{2} c^{2} d^{2} x^{5} + \frac{2}{245} \left(35 x^{7} \operatorname{arsinh}\left(c x\right) - \left(\frac{5 \sqrt{c^{2} x^{2} + 1} x^{6}}{c^{2}} - \frac{6 \sqrt{c^{2} x^{2} + 1} x^{4}}{c^{4}} + \frac{8 \sqrt{c^{2} x^{2} + 1} x^{2}}{c^{6}} - \frac{16 \sqrt{c^{2} x^{2} + 1}}{c^{8}}\right) c\right) a b c^{4} d^{2} - \frac{2}{25725} \left(105 \left(\frac{5 \sqrt{c^{2} x^{2} + 1} x^{6}}{c^{2}} - \frac{6 \sqrt{c^{2} x^{2} + 1} x^{4}}{c^{4}} + \frac{8 \sqrt{c^{2} x^{2} + 1} x^{2}}{c^{6}} - \frac{16 \sqrt{c^{2} x^{2} + 1}}{c^{8}}\right) c \operatorname{arsinh}\left(c x\right) - \frac{75 c^{6} x^{7} - 126 c^{4} x^{5} + 280 c^{2} x^{3} - 1680 x}{c^{6}}\right) b^{2} c^{4} d^{2} + \frac{1}{3} b^{2} d^{2} x^{3} \operatorname{arsinh}\left(c x\right)^{2} + \frac{4}{75} \left(15 x^{5} \operatorname{arsinh}\left(c x\right) - \left(\frac{3 \sqrt{c^{2} x^{2} + 1} x^{4}}{c^{2}} - \frac{4 \sqrt{c^{2} x^{2} + 1} x^{2}}{c^{4}} + \frac{8 \sqrt{c^{2} x^{2} + 1}}{c^{6}}\right) c\right) a b c^{2} d^{2} - \frac{4}{1125} \left(15 \left(\frac{3 \sqrt{c^{2} x^{2} + 1} x^{4}}{c^{2}} - \frac{4 \sqrt{c^{2} x^{2} + 1} x^{2}}{c^{4}} + \frac{8 \sqrt{c^{2} x^{2} + 1}}{c^{6}}\right) c \operatorname{arsinh}\left(c x\right) - \frac{9 c^{4} x^{5} - 20 c^{2} x^{3} + 120 x}{c^{4}}\right) b^{2} c^{2} d^{2} + \frac{1}{3} a^{2} d^{2} x^{3} + \frac{2}{9} \left(3 x^{3} \operatorname{arsinh}\left(c x\right) - c \left(\frac{\sqrt{c^{2} x^{2} + 1} x^{2}}{c^{2}} - \frac{2 \sqrt{c^{2} x^{2} + 1}}{c^{4}}\right)\right) a b d^{2} - \frac{2}{27} \left(3 c \left(\frac{\sqrt{c^{2} x^{2} + 1} x^{2}}{c^{2}} - \frac{2 \sqrt{c^{2} x^{2} + 1}}{c^{4}}\right) \operatorname{arsinh}\left(c x\right) - \frac{c^{2} x^{3} - 6 x}{c^{2}}\right) b^{2} d^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^4*d^2*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^4*d^2*x^7 + 2/5*b^2*c^2*d^2*
x^5*arcsinh(c*x)^2 + 2/5*a^2*c^2*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^2 - 2/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^2 + 1/3*b^2*d^2*x^3*arcsinh(c*x)
^2 + 4/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*
x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^2 - 4/1125*(15*(3*
sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*d^2
+ 1/3*a^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^2
```

<u>Fricas</u> [A]
time = 0.35, size = 327, normalized size = 1.08

$$\frac{3375 \, {\left(49 \, a^{2} + 2 \, b^{2}\right)} c^{7} d^{2} x^{7} + 378 \, {\left(1225 \, a^{2} + 68 \, b^{2}\right)} c^{5} d^{2} x^{5} + 35 \, {\left(11025 \, a^{2} + 818 \, b^{2}\right)} c^{3} d^{2} x^{3} - 171780 \, b^{2} c d^{2} x + 11025 \, {\left(15 \, b^{2} c^{7} d^{2} x^{7} + 42 \, b^{2} c^{5} d^{2} x^{5} + 35 \, b^{2} c^{3} d^{2} x^{3}\right)} \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2} + 210 \, {\left(1575 \, a b c^{7} d^{2} x^{7} + 4410 \, a b c^{5} d^{2} x^{5} + 3675 \, a b c^{3} d^{2} x^{3} - {\left(225 \, b^{2} c^{6} d^{2} x^{6} + 612 \, b^{2} c^{4} d^{2} x^{4} + 409 \, b^{2} c^{2} d^{2} x^{2} - 818 \, b^{2} d^{2}\right)} \sqrt{c^{2} x^{2} + 1}\right)} \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right) - 210 \, {\left(225 \, a b c^{6} d^{2} x^{6} + 612 \, a b c^{4} d^{2} x^{4} + 409 \, a b c^{2} d^{2} x^{2} - 818 \, a b d^{2}\right)} \sqrt{c^{2} x^{2} + 1}}{1157625 \, c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1157625*(3375*(49*a^2 + 2*b^2)*c^7*d^2*x^7 + 378*(1225*a^2 + 68*b^2)*c^5*
d^2*x^5 + 35*(11025*a^2 + 818*b^2)*c^3*d^2*x^3 - 171780*b^2*c*d^2*x + 11025
*(15*b^2*c^7*d^2*x^7 + 42*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 210*(1575*a*b*c^7*d^2*x^7 + 4410*a*b*c^5*d^2*x^5 + 36
75*a*b*c^3*d^2*x^3 - (225*b^2*c^6*d^2*x^6 + 612*b^2*c^4*d^2*x^4 + 409*b^2*c
^2*d^2*x^2 - 818*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) -
 210*(225*a*b*c^6*d^2*x^6 + 612*a*b*c^4*d^2*x^4 + 409*a*b*c^2*d^2*x^2 - 818
*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [**A**]
time = 1.13, size = 483, normalized size = 1.59

$$\begin{cases} \frac{a^2c^4d^2x^7}{7} + \frac{2a^2c^2d^2x^5}{5} + \frac{a^2d^2x^3}{3} + \frac{2abc^4d^2x^7\operatorname{asinh}(cx)}{7} - \frac{2abc^3d^2x^6\sqrt{c^2x^2+1}}{49} + \frac{4abc^2d^2x^5\operatorname{asinh}(cx)}{5} - \frac{136abcd^2x^4\sqrt{c^2x^2+1}}{1225} + \frac{2abd^2x^3\operatorname{asinh}(cx)}{3} - \frac{818abd^2x^2\sqrt{c^2x^2+1}}{11025c} + \frac{1636abd^2\sqrt{c^2x^2+1}}{11025c^3} + \frac{b^2c^4d^2x^7\operatorname{asinh}^2(cx)}{7} + \frac{2b^2c^4d^2x^7}{343} - \frac{2b^2c^3d^2x^6\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{49} + \frac{2b^2c^2d^2x^5\operatorname{asinh}^2(cx)}{5} + \frac{136b^2c^2d^2x^5}{6125} - \frac{136b^2cd^2x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{1225} + \frac{b^2d^2x^3\operatorname{asinh}^2(cx)}{3} + \frac{818b^2d^2x^3}{33075} - \frac{818b^2d^2x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{11025c} - \frac{1636b^2d^2x}{11025c^2} + \frac{1636b^2d^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{11025c^3} & \text{for}\ c \neq 0 \\ \frac{a^2d^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**7/7 + 2*a**2*c**2*d**2*x**5/5 + a**2*d**2*x**3
/3 + 2*a*b*c**4*d**2*x**7*asinh(c*x)/7 - 2*a*b*c**3*d**2*x**6*sqrt(c**2*x**
2 + 1)/49 + 4*a*b*c**2*d**2*x**5*asinh(c*x)/5 - 136*a*b*c*d**2*x**4*sqrt(c*
*2*x**2 + 1)/1225 + 2*a*b*d**2*x**3*asinh(c*x)/3 - 818*a*b*d**2*x**2*sqrt(c
**2*x**2 + 1)/(11025*c) + 1636*a*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3) +
b**2*c**4*d**2*x**7*asinh(c*x)**2/7 + 2*b**2*c**4*d**2*x**7/343 - 2*b**2*c*
*3*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/49 + 2*b**2*c**2*d**2*x**5*asin
h(c*x)**2/5 + 136*b**2*c**2*d**2*x**5/6125 - 136*b**2*c*d**2*x**4*sqrt(c**2
*x**2 + 1)*asinh(c*x)/1225 + b**2*d**2*x**3*asinh(c*x)**2/3 + 818*b**2*d**2
*x**3/33075 - 818*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(11025*c) -
 1636*b**2*d**2*x/(11025*c**2) + 1636*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(11025*c**3), Ne(c, 0)), (a**2*d**2*x**3/3, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a+b\,\text{asinh}(c\,x))^2 \left(d\,c^2\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

3.210 $\int x(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=204

$$\frac{25}{288}b^2d^2x^2+\frac{5}{288}b^2c^2d^2x^4+\frac{b^2d^2(1+c^2x^2)^3}{108c^2}-\frac{5bd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{48c}-\frac{5bd^2x(1+c^2x^2)^{3/2}\left(a+b\right.}{72c}$$

```
[Out] 25/288*b^2*d^2*x^2+5/288*b^2*c^2*d^2*x^4+1/108*b^2*d^2*(c^2*x^2+1)^3/c^2-5/
72*b*d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-1/18*b*d^2*x*(c^2*x^2+1)^
(5/2)*(a+b*arcsinh(c*x))/c-5/96*d^2*(a+b*arcsinh(c*x))^2/c^2+1/6*d^2*(c^2*x
^2+1)^3*(a+b*arcsinh(c*x))^2/c^2-5/48*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1
)^(1/2)/c
```

Rubi [A]
time = 0.14, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.292$, Rules used = {5798, 5786, 5785, 5783, 30, 14, 267}

$$-\frac{bd^2x(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5bd^2x(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{72c}-\frac{5bd^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{48c}+\frac{d^2(c^2x^2+1)^3\left(a+b\sinh^{-1}(cx)\right)^2}{6c^2}-\frac{5d^2\left(a+b\sinh^{-1}(cx)\right)^2}{96c^2}+\frac{5}{288}b^2c^2d^2x^4+\frac{b^2d^2(c^2x^2+1)^3}{108c^2}+\frac{25}{288}b^2d^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (25*b^2*d^2*x^2)/288 + (5*b^2*c^2*d^2*x^4)/288 + (b^2*d^2*(1 + c^2*x^2)^3)/
(108*c^2) - (5*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(48*c) - (5*
b*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(72*c) - (b*d^2*x*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(18*c) - (5*d^2*(a + b*ArcSinh[c*x])^2)/
(96*c^2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(6*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{d^2(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)^2}{6c^2}-\frac{(bd^2)\int(1+c^2x^2)^{5/2}\left(a+b\right.}{3c}$$

$$=-\frac{bd^2x(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c}+\frac{d^2(1+c^2x^2)^3\left(a+b\sin\right.}{6c^2}$$

$$=\frac{b^2d^2(1+c^2x^2)^3}{108c^2}-\frac{5bd^2x(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{72c}-\frac{bd^2x}{}$$

$$=\frac{b^2d^2(1+c^2x^2)^3}{108c^2}-\frac{5bd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{48c}-\frac{5bd^2x(}{}$$

$$=\frac{25}{288}b^2d^2x^2+\frac{5}{288}b^2c^2d^2x^4+\frac{b^2d^2(1+c^2x^2)^3}{108c^2}-\frac{5bd^2x\sqrt{1+c^2x^2}}{4}$$

Mathematica [A]
time = 0.18, size = 208, normalized size = 1.02

$$\frac{d^2\left(cx\left(144a^2cx(3+3c^2x^2+c^4x^4)-6ab\sqrt{1+c^2x^2}(33+26c^2x^2+8c^4x^4)+b^2cx(99+39c^2x^2+8c^4x^4)\right)+6b\left(-bcx\sqrt{1+c^2x^2}(33+26c^2x^2+8c^4x^4)+3a(11+48c^2x^2+48c^4x^4+16c^6x^6)\right)\sinh^{-1}(cx)+9b^2(11+48c^2x^2+48c^4x^4+16c^6x^6)\sinh^{-1}(cx)^2\right)}{864c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(c*x*(144*a^2*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - 6*a*b*Sqrt[1 + c^2*x^2]*
(33 + 26*c^2*x^2 + 8*c^4*x^4) + b^2*c*x*(99 + 39*c^2*x^2 + 8*c^4*x^4)) + 6*
b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(33 + 26*c^2*x^2 + 8*c^4*x^4)) + 3*a*(11 + 48*
c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6))*ArcSinh[c*x] + 9*b^2*(11 + 48*c^2*x^2 +
 48*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x]^2))/(864*c^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(c^2d\,x^2+d\right)^2\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(182) = 364.
time = 0.33, size = 621, normalized size = 3.04

$$\frac{1}{6}b^2c^4d^2x^6\,\mathrm{arsinh}\left(cx\right)^2+\frac{1}{6}a^2c^4d^2x^6+\frac{1}{2}b^2c^2d^2x^4\,\mathrm{arsinh}\left(cx\right)^2+\frac{1}{2}a^2c^2d^2x^4+\frac{1}{144}\left(48x^6\,\mathrm{arsinh}\left(cx\right)-\left(\frac{8\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\,\mathrm{arsinh}\left(cx\right)}{c^7}\right)c\right)abc^4d^2+\frac{1}{864}\left(\left(\frac{8x^6}{c^2}-\frac{15x^4}{c^4}+\frac{45x^2}{c^6}-\frac{45\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^8}\right)c^2-6\left(\frac{8\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\,\mathrm{arsinh}\left(cx\right)}{c^7}\right)c\,\mathrm{arsinh}\left(cx\right)\right)b^2c^4d^2+\frac{1}{2}b^2d^2x^2\,\mathrm{arsinh}\left(cx\right)^2+\frac{1}{8}\left(8x^4\,\mathrm{arsinh}\left(cx\right)-\left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\,\mathrm{arsinh}\left(cx\right)}{c^5}\right)c\right)abc^2d^2+\frac{1}{16}\left(\left(\frac{x^4}{c^2}-\frac{3x^2}{c^4}+\frac{3\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2-2\left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\,\mathrm{arsinh}\left(cx\right)}{c^5}\right)c\,\mathrm{arsinh}\left(cx\right)\right)b^2c^2d^2+\frac{1}{2}a^2d^2x^2+\frac{1}{2}\left(2x^2\,\mathrm{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\mathrm{arsinh}\left(cx\right)}{c^3}\right)\right)abd^2+\frac{1}{4}\left(c^2\left(\frac{x^2}{c^2}-\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right)-2c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\mathrm{arsinh}\left(cx\right)}{c^3}\right)\mathrm{arsinh}\left(cx\right)\right)b^2d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*c^4*d^2*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^4*d^2*x^6 + 1/2*b^2*c^2*d^2*
x^4*arcsinh(c*x)^2 + 1/2*a^2*c^2*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*
sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2
+ 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^4*d^2 + 1/864*((8*x^6/c^2 - 15*x
^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqr
t(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1
)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/2*b^2*d^2*x^
2*arcsinh(c*x)^2 + 1/8*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 -
 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2*d^2 + 1/16*((x^
4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c
^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arc
sinh(c*x))*b^2*c^2*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqr
t(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^2 + 1/4*(c^2*(x^2/c^2 - log
(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c
*x)/c^3)*arcsinh(c*x))*b^2*d^2
```

Fricas [A]
time = 0.35, size = 307, normalized size = 1.50

$$\frac{8\,(18\,a^2+b^2)c^6d^2x^6+3\,(144\,a^2+13\,b^2)c^4d^2x^4+9\,(48\,a^2+11\,b^2)c^2d^2x^2+9\,(16\,b^2c^6d^2x^6+48\,b^2c^4d^2x^4+48\,b^2c^2d^2x^2+11\,b^2d^2)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(48\,abc^6d^2x^6+144\,abc^4d^2x^4+144\,abc^2d^2x^2+33\,abd^2-(8\,b^2c^5d^2x^5+26\,b^2c^3d^2x^3+33\,b^2cd^2x)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-6\,(8\,abc^5d^2x^5+26\,abc^3d^2x^3+33\,abcd^2x)\sqrt{c^2x^2+1}}{864\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/864*(8*(18*a^2 + b^2)*c^6*d^2*x^6 + 3*(144*a^2 + 13*b^2)*c^4*d^2*x^4 + 9*
(48*a^2 + 11*b^2)*c^2*d^2*x^2 + 9*(16*b^2*c^6*d^2*x^6 + 48*b^2*c^4*d^2*x^4
+ 48*b^2*c^2*d^2*x^2 + 11*b^2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(48*a
*b*c^6*d^2*x^6 + 144*a*b*c^4*d^2*x^4 + 144*a*b*c^2*d^2*x^2 + 33*a*b*d^2 - (
8*b^2*c^5*d^2*x^5 + 26*b^2*c^3*d^2*x^3 + 33*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(8*a*b*c^5*d^2*x^5 + 26*a*b*c^3*d^2*x^3 +
 33*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(196) = 392$.
time = 0.81, size = 430, normalized size = 2.11

$$\begin{cases} \frac{a^2c^4d^2x^6}{6}+\frac{a^2c^2d^2x^4}{2}+\frac{a^2d^2x^2}{2}+\frac{abc^4d^2x^6\operatorname{asinh}(cx)}{3}-\frac{abc^3d^2x^5\sqrt{c^2x^2+1}}{18}+abc^2d^2x^4\operatorname{asinh}(cx)-\frac{13abcd^2x^3\sqrt{c^2x^2+1}}{72}+abd^2x^2\operatorname{asinh}(cx)-\frac{11abd^2x\sqrt{c^2x^2+1}}{48c}+\frac{11abd^2\operatorname{asinh}(cx)}{48c^2}+\frac{b^2c^4d^2x^6\operatorname{asinh}^2(cx)}{6}+\frac{b^2c^4d^2x^6}{108}-\frac{b^2c^3d^2x^5\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{18}+\frac{b^2c^2d^2x^4\operatorname{asinh}^2(cx)}{2}+\frac{13b^2c^2d^2x^4}{288}-\frac{13b^2cd^2x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{72}+\frac{b^2d^2x^2\operatorname{asinh}^2(cx)}{2}+\frac{11b^2d^2x^2}{96}-\frac{11b^2d^2x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{48c}+\frac{11b^2d^2\operatorname{asinh}^2(cx)}{96c^2} & \text{for}\: c \neq 0 \\ \frac{a^2d^2x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**6/6 + a**2*c**2*d**2*x**4/2 + a**2*d**2*x**2/2
 + a*b*c**4*d**2*x**6*asinh(c*x)/3 - a*b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)
```

```
/18 + a*b*c**2*d**2*x**4*asinh(c*x) - 13*a*b*c*d**2*x**3*sqrt(c**2*x**2 + 1
)/72 + a*b*d**2*x**2*asinh(c*x) - 11*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(48*c)
+ 11*a*b*d**2*asinh(c*x)/(48*c**2) + b**2*c**4*d**2*x**6*asinh(c*x)**2/6 +
b**2*c**4*d**2*x**6/108 - b**2*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x
)/18 + b**2*c**2*d**2*x**4*asinh(c*x)**2/2 + 13*b**2*c**2*d**2*x**4/288 - 1
3*b**2*c*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/72 + b**2*d**2*x**2*asinh
(c*x)**2/2 + 11*b**2*d**2*x**2/96 - 11*b**2*d**2*x*sqrt(c**2*x**2 + 1)*asin
h(c*x)/(48*c) + 11*b**2*d**2*asinh(c*x)**2/(96*c**2), Ne(c, 0)), (a**2*d**2
*x**2/2, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

3.211 $\int (d+c^2dx^2)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=214

$$\frac{298}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5-\frac{16bd^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{15c}-\frac{8bd^2(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}$$

```
[Out] 298/225*b^2*d^2*x+76/675*b^2*c^2*d^2*x^3+2/125*b^2*c^4*d^2*x^5-8/45*b*d^2*(
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-2/25*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arc
sinh(c*x))/c+8/15*d^2*x*(a+b*arcsinh(c*x))^2+4/15*d^2*x*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2+1/5*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-16/15*b*d^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.18, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5786, 5772, 5798, 8, 200}

$$\frac{1}{5}d^2x(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{15}d^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^2(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}-\frac{8bd^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{45c}-\frac{16bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{15c}+\frac{8}{15}d^2x\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{125}b^2c^4d^2x^5+\frac{76}{675}b^2c^2d^2x^3+\frac{298}{225}b^2d^2x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (298*b^2*d^2*x)/225 + (76*b^2*c^2*d^2*x^3)/675 + (2*b^2*c^4*d^2*x^5)/125 -
(16*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(15*c) - (8*b*d^2*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(45*c) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a
 + b*ArcSinh[c*x]))/(25*c) + (8*d^2*x*(a + b*ArcSinh[c*x])^2)/15 + (4*d^2*x
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/15 + (d^2*x*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx &= \frac{1}{5}d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(4d)\int\left(d+c^2dx^2\right)\left(a+\right.\\ &= -\frac{2bd^2(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}+\frac{4}{15}d^2x\left(1+c^2x^2\right)\left(a+b\right.\\ &= -\frac{8bd^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{45c}-\frac{2bd^2(1+c^2x^2)^{5/2}\left(a+b\right.}{25c}\\ &= \frac{58}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5-\frac{16bd^2\sqrt{1+c^2x^2}\left(a+\right.}{15c}\\ &= \frac{298}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5-\frac{16bd^2\sqrt{1+c^2x^2}\left(a+\right.}{15c}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 191, normalized size = 0.89

$$\frac{d^2\left(225a^2cx(15+10c^2x^2+3c^4x^4)-30ab\sqrt{1+c^2x^2}\,(149+38c^2x^2+9c^4x^4)+2b^2cx(2235+190c^2x^2+27c^4x^4)-30b\left(-15acx(15+10c^2x^2+3c^4x^4)+b\sqrt{1+c^2x^2}\,(149+38c^2x^2+9c^4x^4)\right)\sinh^{-1}(cx)+225b^2cx(15+10c^2x^2+3c^4x^4)\sinh^{-1}(cx)^2\right)}{3375c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(225*a^2*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]*
(149 + 38*c^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(2235 + 190*c^2*x^2 + 27*c^4*x^4
) - 30*b*(-15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(14
9 + 38*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*b^2*c*x*(15 + 10*c^2*x^2 +
3*c^4*x^4)*ArcSinh[c*x]^2))/(3375*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(c^2 d x^2 + d\right)^2 \left(a + b \operatorname{arcsinh}\left(cx\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(190) = 380.
time = 0.31, size = 457, normalized size = 2.14

$$\frac{1}{5} b^2c^4d^2x^5 \operatorname{arsinh}(cx)^2 + \frac{1}{5}a^2c^4d^2x^5 + \frac{2}{3}b^2c^2d^2x^3\operatorname{arsinh}(cx)^2 + \frac{2}{75}\left(15x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}\,x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^4d^2 - \frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}\,x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\operatorname{arsinh}(cx) - \frac{9c^4x^5-20c^2x^3+120x}{c^4}\right)b^2c^4d^2 + \frac{2}{3}a^2c^2d^2x^3 + \frac{4}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d^2 - \frac{4}{27}\left(3c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3-6x}{c^2}\right)b^2c^2d^2 + b^2d^2x\operatorname{arsinh}(cx)^2 + 2b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^2d^2x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abd^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*c^4*d^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 + 2/3*b^2*c^2*d^2*
x^3*arcsinh(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c
^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^2
- 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c
^4)*b^2*c^4*d^2 + 2/3*a^2*c^2*d^2*x^3 + 4/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^2 - 4/27*(3*c*(sq
rt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3
- 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*arcsinh(c*x)^2 + 2*b^2*d^2*(x - sqrt(c^
2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2
 + 1))*a*b*d^2/c
```

Fricas [A]
time = 0.38, size = 278, normalized size = 1.30

$$\frac{27\left(25a^2+2b^2\right)c^5d^2x^5 + 10\left(225a^2+38b^2\right)c^3d^2x^3 + 15\left(225a^2+298b^2\right)cd^2x + 225\left(3b^2c^5d^2x^5+10b^2c^3d^2x^3+15b^2cd^2x\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 30\left(45abc^5d^2x^5+150abc^3d^2x^3+225abcd^2x-\left(9b^2c^4d^2x^4+38b^2c^2d^2x^2+149b^2d^2\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right) - 30\left(9abc^4d^2x^4+38abc^2d^2x^2+149abd^2\right)\sqrt{c^2x^2+1}}{3375c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*d^2*x^5 + 10*(225*a^2 + 38*b^2)*c^3*d^2*x^3
 + 15*(225*a^2 + 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 + 10*b^2*c^3*d^2
*x^3 + 15*b^2*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^5*d^2*
x^5 + 150*a*b*c^3*d^2*x^3 + 225*a*b*c*d^2*x - (9*b^2*c^4*d^2*x^4 + 38*b^2*c
^2*d^2*x^2 + 149*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) -
```

```
 30*(9*a*b*c^4*d^2*x^4 + 38*a*b*c^2*d^2*x^2 + 149*a*b*d^2)*sqrt(c^2*x^2 + 1
))/c
```

Sympy [A]
time = 0.56, size = 389, normalized size = 1.82

$$\begin{cases} \frac{a^2c^4d^2x^5}{5} + \frac{2a^2c^2d^2x^3}{3} + a^2d^2x + \frac{2abc^4d^2x^5\operatorname{asinh}(cx)}{5} - \frac{2abc^3d^2x^4\sqrt{c^2x^2+1}}{25} + \frac{4abc^2d^2x^3\operatorname{asinh}(cx)}{3} - \frac{76abcd^2x^2\sqrt{c^2x^2+1}}{225} + 2abd^2x\operatorname{asinh}(cx) - \frac{298abd^2\sqrt{c^2x^2+1}}{225c} + \frac{b^2c^4d^2x^5\operatorname{asinh}^2(cx)}{5} + \frac{2b^2c^4d^2x^5}{125} - \frac{2b^2c^3d^2x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{25} + \frac{2b^2c^2d^2x^3\operatorname{asinh}^2(cx)}{3} + \frac{76b^2c^2d^2x^3}{675} - \frac{76b^2cd^2x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{225} + b^2d^2x\operatorname{asinh}^2(cx) + \frac{298b^2d^2x}{225} - \frac{298b^2d^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{225c} & \text{for } c \neq 0 \\ a^2d^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**5/5 + 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
2*a*b*c**4*d**2*x**5*asinh(c*x)/5 - 2*a*b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1
)/25 + 4*a*b*c**2*d**2*x**3*asinh(c*x)/3 - 76*a*b*c*d**2*x**2*sqrt(c**2*x**
2 + 1)/225 + 2*a*b*d**2*x*asinh(c*x) - 298*a*b*d**2*sqrt(c**2*x**2 + 1)/(22
5*c) + b**2*c**4*d**2*x**5*asinh(c*x)**2/5 + 2*b**2*c**4*d**2*x**5/125 - 2*
b**2*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + 2*b**2*c**2*d**2*x*
*3*asinh(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 - 76*b**2*c*d**2*x**2*sqrt(
c**2*x**2 + 1)*asinh(c*x)/225 + b**2*d**2*x*asinh(c*x)**2 + 298*b**2*d**2*x
/225 - 298*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c), Ne(c, 0)), (a*
*2*d**2*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

3.212 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=257

$$\frac{13}{32}b^2c^2d^2x^2+\frac{1}{32}b^2c^4d^2x^4-\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))-\frac{1}{8}bcd^2x(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))-\frac{1}{3}$$

```
[Out] 13/32*b^2*c^2*d^2*x^2+1/32*b^2*c^4*d^2*x^4-1/8*b*c*d^2*x*(c^2*x^2+1)^(3/2)*
(a+b*arcsinh(c*x))-11/32*d^2*(a+b*arcsinh(c*x))^2+1/2*d^2*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^2+1/4*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/3*d^2*(a+b*arc
sinh(c*x))^3/b+d^2*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-b
*d^2*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*d^2*
polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-11/16*b*c*d^2*x*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30, 5786, 14}

$$-\frac{1}{8}bcd^2x(c^2x^2+1)^{3/2}\,(a+b\sinh^{-1}(cx))-\frac{11}{16}bcd^2x\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))+\frac{1}{4}d^2(c^2x^2+1)^2\,(a+b\sinh^{-1}(cx))^2+\frac{1}{2}d^2(c^2x^2+1)\,(a+b\sinh^{-1}(cx))^2-bd^2\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))+\frac{d^2(a+b\sinh^{-1}(cx))^3}{3b}-\frac{11}{32}d^2(a+b\sinh^{-1}(cx))^2+d^2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2+\frac{1}{32}b^2c^4d^2x^4+\frac{13}{32}b^2c^2d^2x^2-\frac{1}{2}b^2d^2\text{Li}_3\left(e^{-2\sinh^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (13*b^2*c^2*d^2*x^2)/32 + (b^2*c^4*d^2*x^4)/32 - (11*b*c*d^2*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/16 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/8 - (11*d^2*(a + b*ArcSinh[c*x])^2)/32 + (d^2*(1 + c^2*x^2)*(a +
 b*ArcSinh[c*x])^2)/2 + (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 + (d
^2*(a + b*ArcSinh[c*x])^3)/(3*b) + d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2
*ArcSinh[c*x])] - b*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])
] - (b^2*d^2*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2 \left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{4}d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{(d+c^2dx^2)\left(a+b\sinh\right.}{x}$$

$$= -\frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\right.$$

$$= -\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}\left(a\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.$$

Mathematica [A]
time = 0.25, size = 336, normalized size = 1.31

$$\frac{1}{768}d^2\left(768a^2c^2x^2+192a^2c^4x^4-624abcx\sqrt{1+c^2x^2}-96abc^3x^3\sqrt{1+c^2x^2}+1536abc^2x^2\sinh^{-1}(cx)+384abc^4x^4\sinh^{-1}(cx)+768ab\sinh^{-1}(cx)^2-256b^2\sinh^{-1}(cx)^3+624ab\tanh^{-1}\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+144b^2\cosh\left(2\sinh^{-1}(cx)\right)+288b^2\sinh^{-1}(cx)^2\cosh\left(2\sinh^{-1}(cx)\right)+3b^2\cosh\left(4\sinh^{-1}(cx)\right)+24b^2\sinh^{-1}(cx)^2\cosh\left(4\sinh^{-1}(cx)\right)+1536ab\sinh^{-1}(cx)\log\left(1-e^{-2\sinh^{-1}(cx)}\right)+768b^2\sinh^{-1}(cx)^2\log\left(1-e^{2\sinh^{-1}(cx)}\right)+768a^2\log(cx)-768ab\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+768b^2\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-384b^2\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)-288b^2\sinh^{-1}(cx)\sinh\left(2\sinh^{-1}(cx)\right)-12b^2\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^2*(768*a^2*c^2*x^2 + 192*a^2*c^4*x^4 - 624*a*b*c*x*Sqrt[1 + c^2*x^2] - 9
6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 1536*a*b*c^2*x^2*ArcSinh[c*x] + 384*a*b*c
^4*x^4*ArcSinh[c*x] + 768*a*b*ArcSinh[c*x]^2 - 256*b^2*ArcSinh[c*x]^3 + 624
*a*b*ArcTanh[(c*x)/Sqrt[1 + c^2*x^2]] + 144*b^2*Cosh[2*ArcSinh[c*x]] + 288*
b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + 3*b^2*Cosh[4*ArcSinh[c*x]] + 24*b
^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]] + 1536*a*b*ArcSinh[c*x]*Log[1 - E^(-
2*ArcSinh[c*x])] + 768*b^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 768
*a^2*Log[c*x] - 768*a*b*PolyLog[2, E^(-2*ArcSinh[c*x])] + 768*b^2*ArcSinh[c
*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - 384*b^2*PolyLog[3, E^(2*ArcSinh[c*x])]
 - 288*b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 12*b^2*ArcSinh[c*x]*Sinh[4*A
rcSinh[c*x]]))/768
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(262) = 524$.
time = 5.39, size = 586, normalized size = 2.28

method	result
derivativedivides	$2a\,d^2b\,\text{arcsinh}\,(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\,\right)+2a\,d^2b\,\text{arcsinh}\,(cx)\,c^2x^2+\frac{a\,d^2b\,\text{arcsinh}(cx)c^4x}{2}$
default	$2a\,d^2b\,\text{arcsinh}\,(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\,\right)+2a\,d^2b\,\text{arcsinh}\,(cx)\,c^2x^2+\frac{a\,d^2b\,\text{arcsinh}(cx)c^4x}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*b^2*d^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3+1/4*a^2*d^2*c^4*x^4+13/
16*a*d^2*b*arcsinh(c*x)-a*d^2*b*arcsinh(c*x)^2+2*a*d^2*b*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))+2*a*d^2*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+b^2*d^2*arcsinh(
c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2*d^2*arcsinh(c*x)*polylog(2,c*x+(c^
2*x^2+1)^(1/2))+b^2*d^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*b^2*d^
2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*a*d^2*b*arcsinh(c*x)*ln(
1+c*x+(c^2*x^2+1)^(1/2))+2*a*d^2*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))
+a^2*c^2*d^2*x^2+49/256*b^2*d^2+2*a*d^2*b*arcsinh(c*x)*c^2*x^2-2*b^2*d^2*po
lylog(3,-c*x-(c^2*x^2+1)^(1/2))+13/32*b^2*d^2*arcsinh(c*x)^2-1/3*b^2*d^2*ar
csinh(c*x)^3-2*b^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))+a^2*d^2*ln(c*x)-13/
16*b^2*d^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1/2*a*d^2*b*arcsinh(c*x)*c^4*
x^4-1/8*a*d^2*b*(c^2*x^2+1)^(1/2)*c^3*x^3-13/16*a*d^2*b*c*x*(c^2*x^2+1)^(1/
2)+b^2*d^2*arcsinh(c*x)^2*c^2*x^2+1/4*b^2*d^2*arcsinh(c*x)^2*c^4*x^4+13/32*
b^2*c^2*d^2*x^2+1/32*b^2*c^4*d^2*x^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^4*d^2*x^4 + a^2*c^2*d^2*x^2 + a^2*d^2*log(x) + integrate(b^2*c^4*
d^2*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x^3*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*b^2*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 4*a*b*c^2*d
^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
x + 2*a*b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int \frac{a^{2}}{x}\, dx + \int 2a^{2}c^{2}x\, dx + \int a^{2}c^{4}x^{3}\, dx + \int \frac{b^{2}\operatorname{asinh}^{2}{\left(cx \right)}}{x}\, dx + \int \frac{2ab\operatorname{asinh}{\left(cx \right)}}{x}\, dx + \int 2b^{2}c^{2}x\operatorname{asinh}^{2}{\left(cx \right)}\, dx + \int b^{2}c^{4}x^{3}\operatorname{asinh}^{2}{\left(cx \right)}\, dx + \int 4abc^{2}x\operatorname{asinh}{\left(cx \right)}\, dx + \int 2abc^{4}x^{3}\operatorname{asinh}{\left(cx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d**2*(Integral(a**2/x, x) + Integral(2*a**2*c**2*x, x) + Integral(a**2*c**4
*x**3, x) + Integral(b**2*asinh(c*x)**2/x, x) + Integral(2*a*b*asinh(c*x)/x
, x) + Integral(2*b**2*c**2*x*asinh(c*x)**2, x) + Integral(b**2*c**4*x**3*a
sinh(c*x)**2, x) + Integral(4*a*b*c**2*x*asinh(c*x), x) + Integral(2*a*b*c*
*4*x**3*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x, x)
```

3.213 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=229

$$\frac{32}{9}b^2c^2d^2x+\frac{2}{27}b^2c^4d^2x^3-\frac{10}{3}bcd^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))-\frac{2}{9}bcd^2(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))+\frac{8}{3}c^2d^2$$

```
[Out] 32/9*b^2*c^2*d^2*x+2/27*b^2*c^4*d^2*x^3-2/9*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))+8/3*c^2*d^2*x*(a+b*arcsinh(c*x))^2+4/3*c^2*d^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2-d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x-4*b*c*d^2*(a+
b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^2*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))+2*b^2*c*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-10/3*b*c*d^2
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5807, 5786, 5772, 5798, 8, 5808, 5806, 5816, 4267, 2317, 2438}

$$\frac{4}{3}c^2d^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{9}bcd^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{10}{3}bcd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\frac{8}{3}c^2d^2x\left(a+b\sinh^{-1}(cx)\right)^2-4bcd^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{27}b^2c^4d^2x^3+\frac{32}{9}b^2c^2d^2x-2b^2cd^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+2b^2cd^2\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (32*b^2*c^2*d^2*x)/9 + (2*b^2*c^4*d^2*x^3)/27 - (10*b*c*d^2*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/3 - (2*b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/9 + (8*c^2*d^2*x*(a + b*ArcSinh[c*x])^2)/3 + (4*c^2*d^2*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/3 - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/x - 4*b*c*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^2*
PolyLog[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^2*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
```

```
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d^2(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(4c^2d\right)\int \left(d+c^2dx^2\right)\left(a+b\right.$$

$$= \frac{2}{3}bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{4}{3}c^2d^2x\left(1+c^2x^2\right)\left(a+b\,\mathrm{si}\right.$$

$$= -\frac{2}{3}b^2c^2d^2x - \frac{2}{9}b^2c^4d^2x^3 + 2bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}b$$

$$= -\frac{16}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) +$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) -$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) -$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) -$$

Mathematica [A]

time = 0.69, size = 306, normalized size = 1.34

$$\frac{1}{54}d^2\left(-\frac{54a^2}{x}+108a^2c^2x+18a^2c^4x^3-12abc(-2+c^2x^2)\sqrt{1+c^2x^2}+36abc^4x^3\sinh^{-1}(cx)-189b^2c\sqrt{1+c^2x^2}\sinh^{-1}(cx)+216abc\left(-\sqrt{1+c^2x^2}+cx\sinh^{-1}(cx)\right)+108b^2c^2x(2+\sinh^{-1}(cx)^2)+2b^2c^2x(-12+2c^2x^2+9c^2x^2\sinh^{-1}(cx)^2)-\frac{108ab\left(\sinh^{-1}(cx)+cx\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)\right)}{x}-3b^2c\sinh^{-1}(cx)\cosh\left(3\sinh^{-1}(cx)\right)-\frac{54b^2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2cx\left(-\log\left(1-e^{-\sinh^{-1}(cx)}\right)+\log\left(1+e^{-\sinh^{-1}(cx)}\right)\right)\right)}{x}+108b^2c\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-108b^2c\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*((-54*a^2)/x + 108*a^2*c^2*x + 18*a^2*c^4*x^3 - 12*a*b*c*(-2 + c^2*x^2
)*Sqrt[1 + c^2*x^2] + 36*a*b*c^4*x^3*ArcSinh[c*x] - 189*b^2*c*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + 216*a*b*c*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x]) + 108
*b^2*c^2*x*(2 + ArcSinh[c*x]^2) + 2*b^2*c^2*x*(-12 + 2*c^2*x^2 + 9*c^2*x^2*
ArcSinh[c*x]^2) - (108*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]))
/x - 3*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] - (54*b^2*ArcSinh[c*x]*(ArcS
inh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[c*x])])
))/x + 108*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 108*b^2*c*PolyLog[2, E^(-
ArcSinh[c*x])]))/54
```

Maple [A]
time = 4.28, size = 364, normalized size = 1.59

method	result
derivativedivides	$c\left(a^2d^2\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+\frac{b^2d^2\operatorname{arcsinh}(cx)^2c^3x^3}{3}+2b^2d^2\operatorname{arcsinh}(cx)^2cx-\frac{32b^2d^2\operatorname{arcsinh}(cx)}{9}\right.$
default	$c\left(a^2d^2\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+\frac{b^2d^2\operatorname{arcsinh}(cx)^2c^3x^3}{3}+2b^2d^2\operatorname{arcsinh}(cx)^2cx-\frac{32b^2d^2\operatorname{arcsinh}(cx)}{9}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a^2*d^2*(1/3*c^3*x^3+2*c*x-1/c/x)+1/3*b^2*d^2*arcsinh(c*x)^2*c^3*x^3+2*b
^2*d^2*arcsinh(c*x)^2*c*x-32/9*b^2*d^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*b^2
*d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+32/9*b^2*d^2*c*x+2/27*b^2*d^2
*c^3*x^3+2*b^2*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*d^2*polylog(2,-c*
x-(c^2*x^2+1)^(1/2))-2/9*b^2*d^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2-2*b
^2*d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-b^2*d^2*arcsinh(c*x)^2/c/x+
2*a*d^2*b*(1/3*arcsinh(c*x)*c^3*x^3+2*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/9
*c^2*x^2*(c^2*x^2+1)^(1/2)-16/9*(c^2*x^2+1)^(1/2)-arctanh(1/(c^2*x^2+1)^(1/
2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^4*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^2 + 2*b^2*c^2*d^2*x*arcsinh(c*x)^2
+ 4*b^2*c^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 2*a^2*c^2*d^2*x +
4*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^2 - 2*(c*arcsinh(1/(c*abs(
x))) + arcsinh(c*x)/x)*a*b*d^2 - a^2*d^2/x + 1/3*(b^2*c^4*d^2*x^4 - 3*b^2*d
^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2/3*(b^2*c^7*d^2*x^6 + b^2
*c^5*d^2*x^4 - 3*b^2*c^3*d^2*x^2 - 3*b^2*c*d^2 + (b^2*c^6*d^2*x^5 - 3*b^2*c
^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2
+ (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int 2 a^{2} c^{2}\, dx + \int \frac{a^{2}}{x^{2}}\, dx + \int a^{2} c^{4} x^{2}\, dx + \int 2 b^{2} c^{2} \operatorname{asinh}^{2}{\left(c x \right)}\, dx + \int \frac{b^{2} \operatorname{asinh}^{2}{\left(c x \right)}}{x^{2}}\, dx + \int 4 a b c^{2} \operatorname{asinh}{\left(c x \right)}\, dx + \int \frac{2 a b \operatorname{asinh}{\left(c x \right)}}{x^{2}}\, dx + \int b^{2} c^{4} x^{2} \operatorname{asinh}^{2}{\left(c x \right)}\, dx + \int 2 a b c^{4} x^{2} \operatorname{asinh}{\left(c x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d**2*(Integral(2*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(a**2*c**
4*x**2, x) + Integral(2*b**2*c**2*asinh(c*x)**2, x) + Integral(b**2*asinh(c
*x)**2/x**2, x) + Integral(4*a*b*c**2*asinh(c*x), x) + Integral(2*a*b*asinh
(c*x)/x**2, x) + Integral(b**2*c**4*x**2*asinh(c*x)**2, x) + Integral(2*a*b
*c**4*x**2*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\left(d\,c^2\,x^2+d\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^2,x)
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^2, x)
```

3.214 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=272

$$\frac{1}{4}b^2c^4d^2x^2+\frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))-\frac{bcd^2(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{x}+\frac{1}{4}c^2d^2\left(a+b\sinh^{-1}\right.$$

```
[Out] 1/4*b^2*c^4*d^2*x^2-b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/x+1/4*c^2*
d^2*(a+b*arcsinh(c*x))^2+c^2*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2-1/2*d^2*(
c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x^2+2/3*c^2*d^2*(a+b*arcsinh(c*x))^3/b+2*
c^2*d^2*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^2*
ln(x)-2*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)
-b^2*c^2*d^2*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*c^3*d^2*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5807, 5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30, 14}

$$-2bc^2d^2\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcd^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^2(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}+\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)^3}{3b}+\frac{1}{4}c^2d^2\left(a+b\sinh^{-1}(cx)\right)^2+2c^2d^2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{2}bc^3d^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}b^2c^4d^2x^2-b^2c^2d^2\text{Li}_3\left(e^{-2\sinh^{-1}(cx)}\right)+b^2c^2d^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (b^2*c^4*d^2*x^2)/4 + (b*c^3*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
2 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x + (c^2*d^2*(a + b*
ArcSinh[c*x])^2)/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2 - (d^2*(1
 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (2*c^2*d^2*(a + b*ArcSinh[c
*x])^3)/(3*b) + 2*c^2*d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*ArcSinh[c*x]
)] + b^2*c^2*d^2*Log[x] - 2*b*c^2*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2
*ArcSinh[c*x])] - b^2*c^2*d^2*PolyLog[3, E^(-2*ArcSinh[c*x])]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2 \left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{d^2(1+c^2x^2)^2 \left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + (2c^2d)\int \frac{(d+c^2dx^2)\,(a+}{x}$$

$$= -\frac{bcd^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x} + c^2d^2(1+c^2x^2)\left(a+b\sin\right.$$

$$= \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2(1+c^2x^2)^{3/2}\left(a+b\,s\right.}{x}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2(1+c^2}{}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2(1+c^2}{}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2(1+c^2}{}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2(1+c^2}{}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2(1+c^2}{}$$

Mathematica [A]
time = 0.59, size = 319, normalized size = 1.17

$$\frac{1}{2}d^2\left(-\frac{a^2}{x^2}+a^2c^4x^2-\frac{2ab\left(cx\sqrt{1+c^2x^2}+\sinh^{-1}(cx)\right)}{x^2}+abc^2\left(-cx\sqrt{1+c^2x^2}+2c^2x^2\sinh^{-1}(cx)+\tanh^{-1}\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)+4a^2c^2\log(x)-\frac{b^2\left(2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)+\sinh^{-1}(cx)^2-2c^2x^2\log(cx)\right)}{x^2}+4abc^2\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)-\frac{2}{3}b^2c^2\left(2\sinh^{-1}(cx)^2\left(\sinh^{-1}(cx)-3\log\left(1-e^{2\sinh^{-1}(cx)}\right)\right)-6\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+3\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)\right)+\frac{1}{4}b^2c^2\left(\left(1+2\sinh^{-1}(cx)^2\right)\cosh\left(2\sinh^{-1}(cx)\right)-2\sinh^{-1}(cx)\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d^2*(-(a^2/x^2) + a^2*c^4*x^2 - (2*a*b*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*
x]))/x^2 + a*b*c^2*(-(c*x*Sqrt[1 + c^2*x^2]) + 2*c^2*x^2*ArcSinh[c*x] + Arc
Tanh[(c*x)/Sqrt[1 + c^2*x^2]]) + 4*a^2*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 + c^
2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 - 2*c^2*x^2*Log[c*x]))/x^2 + 4*a*b*c^2
*(ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1 - E^(-2*ArcSinh[c*x])]) - PolyLog[2,
 E^(-2*ArcSinh[c*x])]) - (2*b^2*c^2*(2*ArcSinh[c*x]^2*(ArcSinh[c*x] - 3*Log
[1 - E^(2*ArcSinh[c*x])]) - 6*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] +
 3*PolyLog[3, E^(2*ArcSinh[c*x])]))/3 + (b^2*c^2*((1 + 2*ArcSinh[c*x]^2)*Co
sh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/4))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(285) = 570$.
time = 6.55, size = 669, normalized size = 2.46

method	result
derivativedivides	$c^2\left(4a\,d^2b\,\mathrm{arcsinh}\,(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\,\right)+a\,d^2b\,\mathrm{arcsinh}\,(cx)\,c^2x^2-\frac{b^2d^2\,\mathrm{arcsinh}(cx)}{}\right.$
default	$c^2\left(4a\,d^2b\,\mathrm{arcsinh}\,(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\,\right)+a\,d^2b\,\mathrm{arcsinh}\,(cx)\,c^2x^2-\frac{b^2d^2\,\mathrm{arcsinh}(cx)}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-b^2*d^2*arcsinh(c*x)/c/x*(c^2*x^2+1)^(1/2)+1/2*a*d^2*b*arcsinh(c*x)-2
*a*d^2*b*arcsinh(c*x)^2+4*a*d^2*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+4*a*d^2
*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*b^2*d^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*
x^2+1)^(1/2))+4*b^2*d^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*b^2
*d^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+4*b^2*d^2*arcsinh(c*x)*poly
log(2,-c*x-(c^2*x^2+1)^(1/2))+4*a*d^2*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))+4*a*d^2*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/2*a^2*c^2*d^2*x^
2+1/8*b^2*d^2+a*d^2*b*arcsinh(c*x)*c^2*x^2-a*d^2*b/c/x*(c^2*x^2+1)^(1/2)-a*
d^2*b*arcsinh(c*x)/c^2/x^2-4*b^2*d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+1/4*
b^2*d^2*arcsinh(c*x)^2-2/3*b^2*d^2*arcsinh(c*x)^3-4*b^2*d^2*polylog(3,c*x+(
c^2*x^2+1)^(1/2))+2*a^2*d^2*ln(c*x)-1/2*b^2*d^2*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)*c*x-1/2*a*d^2*b*c*x*(c^2*x^2+1)^(1/2)+a*d^2*b+1/2*b^2*d^2*arcsinh(c*x)^
2*c^2*x^2-1/2*b^2*d^2*arcsinh(c*x)^2/c^2/x^2+b^2*d^2*ln(c*x+(c^2*x^2+1)^(1/
2)-1)+b^2*d^2*arcsinh(c*x)+b^2*d^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*d^2*ln
(c*x+(c^2*x^2+1)^(1/2))-1/2*a^2*d^2/c^2/x^2+1/4*b^2*c^2*d^2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^4*d^2*x^2 + 2*a^2*c^2*d^2*log(x) - a*b*d^2*(sqrt(c^2*x^2 + 1)*c/x
 + arcsinh(c*x)/x^2) - 1/2*a^2*d^2/x^2 + integrate(b^2*c^4*d^2*x*log(c*x +
sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*b^2
*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 4*a*b*c^2*d^2*log(c*x + sqrt(c^
2*x^2 + 1))/x + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int \frac{a^{2}}{x^{3}}\, dx + \int \frac{2a^{2}c^{2}}{x}\, dx + \int a^{2}c^{4}x\, dx + \int \frac{b^{2}\operatorname{asinh}^{2}{(cx)}}{x^{3}}\, dx + \int \frac{2ab\operatorname{asinh}{(cx)}}{x^{3}}\, dx + \int \frac{2b^{2}c^{2}\operatorname{asinh}^{2}{(cx)}}{x}\, dx + \int b^{2}c^{4}x\operatorname{asinh}^{2}{(cx)}\, dx + \int \frac{4abc^{2}\operatorname{asinh}{(cx)}}{x}\, dx + \int 2abc^{4}x\operatorname{asinh}{(cx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c**2/x, x) + Integral(a**2*c
**4*x, x) + Integral(b**2*asinh(c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x
)/x**3, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x, x) + Integral(b**2*c**4*
x*asinh(c*x)**2, x) + Integral(4*a*b*c**2*asinh(c*x)/x, x) + Integral(2*a*b
*c**4*x*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^3, x)
```

3.215 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=248

$$-\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))-\frac{bcd^2(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}{3x^2}+\frac{8}{3}c^4d^2x(a+$$

```
[Out] -1/3*b^2*c^2*d^2/x+2*b^2*c^4*d^2*x-1/3*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))/x^2+8/3*c^4*d^2*x*(a+b*arcsinh(c*x))^2-4/3*c^2*d^2*(c^2*x^2+1)*(a+
b*arcsinh(c*x))^2/x-1/3*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x^3-22/3*b*c
^3*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-11/3*b^2*c^3*d^2*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))+11/3*b^2*c^3*d^2*polylog(2,c*x+(c^2*x^2+1)
^(1/2))-5/3*b*c^3*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5807, 5772, 5798, 8, 5806, 5816, 4267, 2317, 2438, 14}

$$\frac{8}{3}c^4d^2x\left(a+b\sinh^{-1}(cx)\right)^2-\frac{22}{3}bc^3d^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{4c^2d^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{bcd^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{d^2(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}-\frac{5}{3}bc^3d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+2b^2c^4d^2x-\frac{11}{3}b^2c^3d^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\frac{11}{3}b^2c^3d^2\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)-\frac{b^2c^2d^2}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d^2)/x + 2*b^2*c^4*d^2*x - (5*b*c^3*d^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*x^2) + (8*c^4*d^2*x*(a + b*ArcSinh[c*x])^2)/3 - (4*c^2*d^2*(1 + c^2*x^2)*
(a + b*ArcSinh[c*x])^2)/(3*x) - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/(3*x^3) - (22*b*c^3*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 -
 (11*b^2*c^3*d^2*PolyLog[2, -E^ArcSinh[c*x]])/3 + (11*b^2*c^3*d^2*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
```

```
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^2(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(4c^2d)\int \frac{(d+c^2dx^2)\left(a+\right.}{x^2}$$

$$= -\frac{bcd^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{4c^2d^2(1+c^2x^2)\left(a+b\sinh\right.}{3x}$$

$$= \frac{11}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2(1+c^2x^2)^{3/2}\left(a+b\sin\right.}{3x^2}$$

$$= -\frac{b^2c^2d^2}{3x}-\frac{10}{3}b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd}{}$$

$$= -\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2}{}$$

$$= -\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2}{}$$

$$= -\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2}{}$$

Mathematica [A]
time = 0.57, size = 357, normalized size = 1.44

$$\frac{d^2\left(-a^2-6a^2c^2x^2-b^2c^2x^2+3a^2c^4x^4+6b^2c^4x^4-abcx\sqrt{1+c^2x^2}-6abc^3x^3\sqrt{1+c^2x^2}-2ab\sinh^{-1}(cx)-12abc^2x^2\sinh^{-1}(cx)+6abc^4x^4\sinh^{-1}(cx)-b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)-6b^2c^3x^3\sqrt{1+c^2x^2}\sinh^{-1}(cx)-b^2\sinh^{-1}(cx)^2-6b^2c^2x^2\sinh^{-1}(cx)^2+3b^2c^4x^4\sinh^{-1}(cx)^2-11abc^3x^3\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)+11b^2c^3x^3\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-11b^2c^3x^3\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+11b^2c^3x^3\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-11b^2c^3x^3\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-a^2 - 6*a^2*c^2*x^2 - b^2*c^2*x^2 + 3*a^2*c^4*x^4 + 6*b^2*c^4*x^4 -
a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh
[c*x] - 12*a*b*c^2*x^2*ArcSinh[c*x] + 6*a*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*
```

```
x] - b^2*ArcSinh[c*x]^2 - 6*b^2*c^2*x^2*ArcSinh[c*x]^2 + 3*b^2*c^4*x^4*ArcS
inh[c*x]^2 - 11*a*b*c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] + 11*b^2*c^3*x^3*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 11*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 +
E^(-ArcSinh[c*x])] + 11*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 11*b^2
*c^3*x^3*PolyLog[2, E^(-ArcSinh[c*x])]))/(3*x^3)
```

Maple [A]
time = 6.10, size = 363, normalized size = 1.46

method	result
derivativedivides	$c^3 \left(a^2d^2\left(cx - \frac{1}{3c^3x^3} - \frac{2}{cx}\right) + b^2d^2 \operatorname{arcsinh}(cx)^2 cx - 2b^2d^2 \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1} + 2b \right.$
default	$c^3 \left(a^2d^2\left(cx - \frac{1}{3c^3x^3} - \frac{2}{cx}\right) + b^2d^2 \operatorname{arcsinh}(cx)^2 cx - 2b^2d^2 \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1} + 2b \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a^2*d^2*(c*x-1/3/c^3/x^3-2/c/x)+b^2*d^2*arcsinh(c*x)^2*c*x-2*b^2*d^2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*b^2*d^2*c*x-2*b^2*d^2*arcsinh(c*x)^2/c/x-1/
3*b^2*d^2/c^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-1/3*b^2*d^2/c^3/x^3*arcsin
h(c*x)^2-1/3*b^2*d^2/c/x+11/3*b^2*d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/
2))+11/3*b^2*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-11/3*b^2*d^2*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))-11/3*b^2*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
+2*a*d^2*b*(arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-2*arcsinh(c*x)/c/x-(c
^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)-11/6*arctanh(1/(c^2*x^2+1)^(1
/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] b^2*c^4*d^2*x*arcsinh(c*x)^2 + 2*b^2*c^4*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh
(c*x)/c) + a^2*c^4*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3
*d^2 - 4*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d^2 + 1/3*((c^2
*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b
*d^2 - 2*a^2*c^2*d^2/x - 1/3*a^2*d^2/x^3 - 1/3*(6*b^2*c^2*d^2*x^2 + b^2*d^2
)*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + integrate(2/3*(6*b^2*c^5*d^2*x^4 + 7
```

```
*b^2*c^3*d^2*x^2 + b^2*c*d^2 + (6*b^2*c^4*d^2*x^3 + b^2*c^2*d^2*x)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*
sqrt(c^2*x^2 + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int a^{2}c^{4}\,dx+\int\frac{a^{2}}{x^{4}}\,dx+\int\frac{2a^{2}c^{2}}{x^{2}}\,dx+\int b^{2}c^{4}\operatorname{asinh}^{2}{(cx)}\,dx+\int\frac{b^{2}\operatorname{asinh}^{2}{(cx)}}{x^{4}}\,dx+\int 2abc^{4}\operatorname{asinh}{(cx)}\,dx+\int\frac{2ab\operatorname{asinh}{(cx)}}{x^{4}}\,dx+\int\frac{2b^{2}c^{2}\operatorname{asinh}^{2}{(cx)}}{x^{2}}\,dx+\int\frac{4abc^{2}\operatorname{asinh}{(cx)}}{x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d**2*(Integral(a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(2*a**2*c**
2/x**2, x) + Integral(b**2*c**4*asinh(c*x)**2, x) + Integral(b**2*asinh(c*x
)**2/x**4, x) + Integral(2*a*b*c**4*asinh(c*x), x) + Integral(2*a*b*asinh(c
*x)/x**4, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x**2, x) + Integral(4*a*b
*c**2*asinh(c*x)/x**2, x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^4,x)
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^4, x)
```

3.216 $\int x^4(d+c^2dx^2)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=465

$$\frac{100976b^2d^3x}{4002075c^4}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}+\frac{182b^2c^4d^3x^9}{29403}+\frac{2b^2c^6d^3x^{11}}{1331}-\frac{256bd^3\sqrt{1+c^2x^2}\,(a+}{17325c^5}$$

[Out] 100976/4002075*b^2*d^3*x/c^4-50488/12006225*b^2*d^3*x^3/c^2+12622/6670125*b^2*d^3*x^5+9410/1120581*b^2*c^2*d^3*x^7+182/29403*b^2*c^4*d^3*x^9+2/1331*b^2*c^6*d^3*x^11-16/693*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^5+4/1155*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^5-2/1617*b*d^3*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^5+8/297*b*d^3*(c^2*x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^5-2/121*b*d^3*(c^2*x^2+1)^(11/2)*(a+b*arcsinh(c*x))/c^5+16/1155*d^3*x^5*(a+b*arcsinh(c*x))^2+8/231*d^3*x^5*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+2/33*d^3*x^5*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/11*d^3*x^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2-256/17325*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+128/17325*b*d^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-32/5775*b*d^3*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c

Rubi [A]
time = 0.68, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.423$, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12, 1167}

$$\frac{1}{11}d^3x^5(c^2x^2+1)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{33}d^3x^5(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{231}d^3x^5(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{32bd^3x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{5775c}-\frac{2bd^3(c^2x^2+1)^{11/2}\left(a+b\sinh^{-1}(cx)\right)}{121c^5}+\frac{8bd^3(c^2x^2+1)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{297c^5}-\frac{2bd^3(c^2x^2+1)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{1617c^5}+\frac{4bd^3(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{1155c^5}-\frac{16bd^3(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{693c^5}-\frac{256bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{17325c^5}+\frac{128bd^3x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{17325c^3}+\frac{16d^3x^5\left(a+b\sinh^{-1}(cx)\right)^2}{1155}+\frac{2b^2c^6d^3x^{11}}{1331}+\frac{182b^2c^4d^3x^9}{29403}+\frac{100976b^2d^3x}{4002075c^4}+\frac{9410b^2c^2d^3x^7}{1120581}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}$$

Antiderivative was successfully verified.

[In] Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]

[Out] (100976*b^2*d^3*x)/(4002075*c^4) - (50488*b^2*d^3*x^3)/(12006225*c^2) + (12622*b^2*d^3*x^5)/6670125 + (9410*b^2*c^2*d^3*x^7)/1120581 + (182*b^2*c^4*d^3*x^9)/29403 + (2*b^2*c^6*d^3*x^11)/1331 - (256*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(17325*c^5) + (128*b*d^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(17325*c^3) - (32*b*d^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(5775*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(693*c^5) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(1155*c^5) - (2*b*d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(1617*c^5) + (8*b*d^3*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(297*c^5) - (2*b*d^3*(1 + c^2*x^2)^(11/2)*(a + b*ArcSinh[c*x]))/(121*c^5) + (16*d^3*x^5*(a + b*ArcSinh[c*x])^2)/1155 + (8*d^3*x^5*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/231 + (2*d^3*x^5*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/33 + (d^3*x^5*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/11

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^4\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{11}d^3x^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{11}(6d)\int x^4\left(d+c^2dx\right.$$

$$=-\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{77c^5}+\frac{4bd^3\left(1+c^2x^2\right)^{9/2}\left(a+\right.}{99c^5}$$

$$=-\frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{165c^5}+\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+\right.}{231c^5}$$

$$=-\frac{16bd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{693c^5}+\frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a\right.}{1155c}$$

$$=\frac{16b^2d^3x}{7623c^4}-\frac{8b^2d^3x^3}{22869c^2}+\frac{2b^2d^3x^5}{12705}+\frac{226b^2c^2d^3x^7}{53361}+\frac{46b^2c^4d^3x^9}{9801}+$$

$$=\frac{8368b^2d^3x}{800415c^4}-\frac{4184b^2d^3x^3}{2401245c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}+\frac{18}{}$$

$$=\frac{8368b^2d^3x}{800415c^4}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}+\frac{1}{}$$

$$=\frac{100976b^2d^3x}{4002075c^4}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}+$$

Mathematica [**A**]
time = 0.26, size = 299, normalized size = 0.64

$$\frac{d^3\left(12006225a^2c^5x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)-6930ab\sqrt{1+c^2x^2}\,(50488-25244c^2x^2+18933c^4x^4+117625c^6x^6+111475c^8x^8+33075c^{10}x^{10})+2b^2cx(174940920-29156820c^2x^2+13120569c^4x^4+58224375c^6x^6+42917875c^8x^8+10418625c^{10}x^{10})-6930b\left(-3465ac^5x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)+b\sqrt{1+c^2x^2}\,(50488-25244c^2x^2+18933c^4x^4+117625c^6x^6+111475c^8x^8+33075c^{10}x^{10})\right)\sinh^{-1}(cx)+12006225b^2c^5x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)\sinh^{-1}(cx)^2\right)}{13867189875c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(12006225*a^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)
- 6930*a*b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 11762
5*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10) + 2*b^2*c*x*(174940920 - 2915
6820*c^2*x^2 + 13120569*c^4*x^4 + 58224375*c^6*x^6 + 42917875*c^8*x^8 + 104
18625*c^10*x^10) - 6930*b*(-3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4
 + 105*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^
4 + 117625*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10))*ArcSinh[c*x] + 1200
6225*b^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*
x]^2))/(13867189875*c^5)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^4\left(c^2d\,x^2+d\right)^3\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. $2(413) = 826$.
time = 0.30, size = 1109, normalized size = 2.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*c^6*d^3*x^11*arcsinh(c*x)^2 + 1/11*a^2*c^6*d^3*x^11 + 1/3*b^2*c^4*
d^3*x^9*arcsinh(c*x)^2 + 1/3*a^2*c^4*d^3*x^9 + 3/7*b^2*c^2*d^3*x^7*arcsinh(
c*x)^2 + 3/7*a^2*c^2*d^3*x^7 + 2/7623*(693*x^11*arcsinh(c*x) - (63*sqrt(c^2
*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^
6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256
*sqrt(c^2*x^2 + 1)/c^12)*c)*a*b*c^6*d^3 - 2/26413695*(3465*(63*sqrt(c^2*x^2
 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^
6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqr
t(c^2*x^2 + 1)/c^12)*c*arcsinh(c*x) - (19845*c^10*x^11 - 26950*c^8*x^9 + 39
600*c^6*x^7 - 66528*c^4*x^5 + 147840*c^2*x^3 - 887040*x)/c^10)*b^2*c^6*d^3
+ 1/5*b^2*d^3*x^5*arcsinh(c*x)^2 + 2/945*(315*x^9*arcsinh(c*x) - (35*sqrt(c
^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x
^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*
c^4*d^3 - 2/297675*(315*(35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1
)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 1
28*sqrt(c^2*x^2 + 1)/c^10)*c*arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 +
3024*c^4*x^5 - 6720*c^2*x^3 + 40320*x)/c^8)*b^2*c^4*d^3 + 1/5*a^2*d^3*x^5 +
 6/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2
 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*
a*b*c^2*d^3 - 2/8575*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1
)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsi
nh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^
3 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x
^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d^3 - 2/1125*(15*(3*sqrt(
c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^
6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d^3
```

Fricas [A]
time = 0.43, size = 444, normalized size = 0.95

$$\frac{10418625\,(121\,a^2+2\,b^2)c^{11}d^3x^{11}+471625\,(9801\,a^2+182\,b^2)c^9d^3x^9+12375\,(480249\,a^2+9410\,b^2)c^7d^3x^7+2079\,(1334025\,a^2+12622\,b^2)c^5d^3x^5-58313640\,b^2c^3d^3x^3+349881840\,b^2cd^3x+12006225\,(105\,b^2c^{11}d^3x^{11}+385\,b^2c^9d^3x^9+495\,b^2c^7d^3x^7+231\,b^2c^5d^3x^5)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6930\left(363825\,abc^{11}d^3x^{11}+1334025\,abc^9d^3x^9+1715175\,abc^7d^3x^7+800415\,abc^5d^3x^5-(33075\,b^2c^{10}d^3x^{10}+111475\,b^2c^8d^3x^8+117625\,b^2c^6d^3x^6+18933\,b^2c^4d^3x^4-25244\,b^2c^2d^3x^2+50488\,b^2d^3)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-6930\,(33075\,abc^{10}d^3x^{10}+111475\,abc^8d^3x^8+117625\,abc^6d^3x^6+18933\,abc^4d^3x^4-25244\,abc^2d^3x^2+50488\,abd^3)\sqrt{c^2x^2+1}}{13867189875\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/13867189875*(10418625*(121*a^2 + 2*b^2)*c^11*d^3*x^11 + 471625*(9801*a^2
+ 182*b^2)*c^9*d^3*x^9 + 12375*(480249*a^2 + 9410*b^2)*c^7*d^3*x^7 + 2079*(
1334025*a^2 + 12622*b^2)*c^5*d^3*x^5 - 58313640*b^2*c^3*d^3*x^3 + 349881840
*b^2*c*d^3*x + 12006225*(105*b^2*c^11*d^3*x^11 + 385*b^2*c^9*d^3*x^9 + 495*
b^2*c^7*d^3*x^7 + 231*b^2*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 693
0*(363825*a*b*c^11*d^3*x^11 + 1334025*a*b*c^9*d^3*x^9 + 1715175*a*b*c^7*d^3
*x^7 + 800415*a*b*c^5*d^3*x^5 - (33075*b^2*c^10*d^3*x^10 + 111475*b^2*c^8*d
^3*x^8 + 117625*b^2*c^6*d^3*x^6 + 18933*b^2*c^4*d^3*x^4 - 25244*b^2*c^2*d^3
*x^2 + 50488*b^2*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 693
0*(33075*a*b*c^10*d^3*x^10 + 111475*a*b*c^8*d^3*x^8 + 117625*a*b*c^6*d^3*x^
6 + 18933*a*b*c^4*d^3*x^4 - 25244*a*b*c^2*d^3*x^2 + 50488*a*b*d^3)*sqrt(c^2
*x^2 + 1))/c^5
```

Sympy [**A**]
time = 4.36, size = 702, normalized size = 1.51

$$\begin{cases} \frac{a^2c^6d^3x^{11}}{11} + \frac{a^2c^4d^3x^9}{3} + \frac{3a^2c^2d^3x^7}{7} + \frac{a^2d^3x^5}{5} + \frac{2abc^6d^3x^{11}\operatorname{asinh}(cx)}{11} - \frac{2abc^5d^3x^{10}\sqrt{c^2x^2+1}}{121} + \frac{2abc^4d^3x^9\operatorname{asinh}(cx)}{3} - \frac{182abc^3d^3x^8\sqrt{c^2x^2+1}}{3267} + \frac{6abc^2d^3x^7\operatorname{asinh}(cx)}{7} - \frac{9410abcd^3x^6\sqrt{c^2x^2+1}}{160083} + \frac{2abd^3x^5\operatorname{asinh}(cx)}{5} - \frac{12622abd^3x^4\sqrt{c^2x^2+1}}{1334025c} + \frac{50488abd^3x^2\sqrt{c^2x^2+1}}{4002075c^3} - \frac{100976abd^3\sqrt{c^2x^2+1}}{4002075c^5} + \frac{b^2c^6d^3x^{11}\operatorname{asinh}^2(cx)}{11} + \frac{2b^2c^6d^3x^{11}}{1331} - \frac{2b^2c^5d^3x^{10}\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{121} + \frac{b^2c^4d^3x^9\operatorname{asinh}^2(cx)}{3} + \frac{182b^2c^4d^3x^9}{29403} - \frac{182b^2c^3d^3x^8\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3267} + \frac{3b^2c^2d^3x^7\operatorname{asinh}^2(cx)}{7} + \frac{9410b^2c^2d^3x^7}{1120581} - \frac{9410b^2cd^3x^6\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{160083} + \frac{b^2d^3x^5\operatorname{asinh}^2(cx)}{5} + \frac{12622b^2d^3x^5}{6670125} - \frac{12622b^2d^3x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{1334025c} - \frac{50488b^2d^3x^3}{12006225c^2} + \frac{50488b^2d^3x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{4002075c^3} + \frac{100976b^2d^3x}{4002075c^4} - \frac{100976b^2d^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{4002075c^5} & \text{for}\: c \neq 0 \\ \frac{a^2d^3x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**11/11 + a**2*c**4*d**3*x**9/3 + 3*a**2*c**2*d*
*3*x**7/7 + a**2*d**3*x**5/5 + 2*a*b*c**6*d**3*x**11*asinh(c*x)/11 - 2*a*b*
c**5*d**3*x**10*sqrt(c**2*x**2 + 1)/121 + 2*a*b*c**4*d**3*x**9*asinh(c*x)/3
 - 182*a*b*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)/3267 + 6*a*b*c**2*d**3*x**7*a
sinh(c*x)/7 - 9410*a*b*c*d**3*x**6*sqrt(c**2*x**2 + 1)/160083 + 2*a*b*d**3*
x**5*asinh(c*x)/5 - 12622*a*b*d**3*x**4*sqrt(c**2*x**2 + 1)/(1334025*c) + 5
0488*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(4002075*c**3) - 100976*a*b*d**3*sqr
t(c**2*x**2 + 1)/(4002075*c**5) + b**2*c**6*d**3*x**11*asinh(c*x)**2/11 + 2
*b**2*c**6*d**3*x**11/1331 - 2*b**2*c**5*d**3*x**10*sqrt(c**2*x**2 + 1)*asi
nh(c*x)/121 + b**2*c**4*d**3*x**9*asinh(c*x)**2/3 + 182*b**2*c**4*d**3*x**9
/29403 - 182*b**2*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)*asinh(c*x)/3267 + 3*b*
*2*c**2*d**3*x**7*asinh(c*x)**2/7 + 9410*b**2*c**2*d**3*x**7/1120581 - 9410
*b**2*c*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/160083 + b**2*d**3*x**5*as
inh(c*x)**2/5 + 12622*b**2*d**3*x**5/6670125 - 12622*b**2*d**3*x**4*sqrt(c*
*2*x**2 + 1)*asinh(c*x)/(1334025*c) - 50488*b**2*d**3*x**3/(12006225*c**2)
+ 50488*b**2*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(4002075*c**3) + 1009
76*b**2*d**3*x/(4002075*c**4) - 100976*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(4002075*c**5), Ne(c, 0)), (a**2*d**3*x**5/5, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.217 $\int x^3(d+c^2dx^2)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=376

$$-\frac{79b^2d^3x^2}{5120c^2}+\frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}+\frac{79bd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2560c^3}-79$$

```
[Out] -79/5120*b^2*d^3*x^2/c^2+79/15360*b^2*d^3*x^4+401/28800*b^2*c^2*d^3*x^6+57/
6400*b^2*c^4*d^3*x^8+1/500*b^2*c^6*d^3*x^10-1/32*b*c*d^3*x^5*(c^2*x^2+1)^(3
/2)*(a+b*arcsinh(c*x))-1/50*b*c*d^3*x^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x)
)-79/5120*d^3*(a+b*arcsinh(c*x))^2/c^4+1/40*d^3*x^4*(a+b*arcsinh(c*x))^2+1/
20*d^3*x^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+3/40*d^3*x^4*(c^2*x^2+1)^2*(a+b
*arcsinh(c*x))^2+1/10*d^3*x^4*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+79/2560*b*
d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-79/3840*b*d^3*x^3*(a+b*arcsi
nh(c*x))*(c^2*x^2+1)^(1/2)/c-31/960*b*c*d^3*x^5*(a+b*arcsinh(c*x))*(c^2*x^2
+1)^(1/2)
```

Rubi [A]

time = 1.07, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5808, 5776, 5812, 5783, 30, 5806, 14, 272, 45}

$$-\frac{79d^3\left(a+b\sinh^{-1}(cx)\right)^2}{5120c^4}-\frac{1}{50}bcd^3x^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^3x^5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{31}{960}bcd^3x^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{40}d^3x^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{20}d^3x^4\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{79bd^3x^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3840c}+\frac{79bd^3x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2560c^3}+\frac{1}{40}d^3x^4\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{500}b^2c^6d^3x^{10}+\frac{57b^2c^4d^3x^8}{6400}+\frac{401b^2c^2d^3x^6}{28800}-\frac{79b^2d^3x^2}{5120c^2}+\frac{79b^2d^3x^4}{15360}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-79*b^2*d^3*x^2)/(5120*c^2) + (79*b^2*d^3*x^4)/15360 + (401*b^2*c^2*d^3*x^
6)/28800 + (57*b^2*c^4*d^3*x^8)/6400 + (b^2*c^6*d^3*x^10)/500 + (79*b*d^3*x
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2560*c^3) - (79*b*d^3*x^3*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(3840*c) - (31*b*c*d^3*x^5*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/960 - (b*c*d^3*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/32 - (b*c*d^3*x^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/50 -
 (79*d^3*(a + b*ArcSinh[c*x])^2)/(5120*c^4) + (d^3*x^4*(a + b*ArcSinh[c*x])
^2)/40 + (d^3*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/20 + (3*d^3*x^4*(1
+ c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/40 + (d^3*x^4*(1 + c^2*x^2)^3*(a + b*A
rcSinh[c*x])^2)/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
```

```
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx &= \frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(3d)\int x^3\left(d+c^2dx^2\right. \\
&= -\frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{40}d^3x^4\left(1+c^2x^2\right. \\
&= -\frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{50}bcd^3x^5\left(1+c^2\right. \\
&= -\frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^3x^5\left(1+c^2x\right. \\
&= \frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}-\frac{79bd^3x^3\sqrt{1+c^2x}}{3} \\
&= \frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}+\frac{79bd^3}{} \\
&= -\frac{79b^2d^3x^2}{5120c^2}+\frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 285, normalized size = 0.76

$$\frac{d^3\left(cx\left(28800a^2c^3x^3(10+20c^2x^2+15c^4x^4+4c^6x^6)-30ab\sqrt{1+c^2x^2}\,(-1185+790c^2x^2+3208c^4x^4+2736c^6x^6+768c^8x^8)+b^2cx(-17775+5925c^2x^2+16040c^4x^4+10260c^6x^6+2304c^8x^8)\right)+30b\left(-bcx\sqrt{1+c^2x^2}\,(-1185+790c^2x^2+3208c^4x^4+2736c^6x^6+768c^8x^8)+15a(-79+1280c^4x^4+2560c^6x^6+1920c^8x^8+512c^{10}x^{10})\right)\sinh^{-1}(cx)+225b^2(-79+1280c^4x^4+2560c^6x^6+1920c^8x^8+512c^{10}x^{10})\sinh^{-1}(cx)^2\right)}{1152000c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(c*x*(28800*a^2*c^3*x^3*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - 3
0*a*b*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6
+ 768*c^8*x^8) + b^2*c*x*(-17775 + 5925*c^2*x^2 + 16040*c^4*x^4 + 10260*c^6
*x^6 + 2304*c^8*x^8)) + 30*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^
2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8*x^8)) + 15*a*(-79 + 1280*c^4*x^4
+ 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10))*ArcSinh[c*x] + 225*b^2*(-79
 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10)*ArcSinh[c*x]
^2))/(1152000*c^4)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3 \left(c^2 d\,x^2+d\right)^3 \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. 2(336) = 672.
time = 0.34, size = 1112, normalized size = 2.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*c^6*d^3*x^10*arcsinh(c*x)^2 + 1/10*a^2*c^6*d^3*x^10 + 3/8*b^2*c^4*
d^3*x^8*arcsinh(c*x)^2 + 3/8*a^2*c^4*d^3*x^8 + 1/2*b^2*c^2*d^3*x^6*arcsinh(
c*x)^2 + 1/2*a^2*c^2*d^3*x^6 + 1/6400*(1280*x^10*arcsinh(c*x) - (128*sqrt(c
^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)
*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(c^2*x^2 + 1)*x/c^10 - 3
15*arcsinh(c*x)/c^11)*c)*a*b*c^6*d^3 + 1/64000*((128*x^10/c^2 - 180*x^8/c^4
 + 280*x^6/c^6 - 525*x^4/c^8 + 1575*x^2/c^10 - 1575*log(c*x + sqrt(c^2*x^2
+ 1))^2/c^12)*c^2 - 10*(128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 +
1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8
+ 315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c*x)/c^11)*c*arcsinh(c*x))*b^2
*c^6*d^3 + 1/4*b^2*d^3*x^4*arcsinh(c*x)^2 + 1/512*(384*x^8*arcsinh(c*x) - (
48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x
^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*a*
b*c^4*d^3 + 1/3072*((36*x^8/c^2 - 56*x^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 +
315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^10)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/
c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqr
t(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c*arcsinh(c*x))*b^2*c^4*d^3 +
```

```
1/4*a^2*d^3*x^4 + 1/48*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2
- 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*
x)/c^7)*c)*a*b*c^2*d^3 + 1/288*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*l
og(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 1
0*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/
c^7)*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x
^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*d^
3 + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2
- 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*
x)/c^5)*c*arcsinh(c*x))*b^2*d^3
```

Fricas [**A**]
time = 0.38, size = 424, normalized size = 1.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1152000*(2304*(50*a^2 + b^2)*c^10*d^3*x^10 + 540*(800*a^2 + 19*b^2)*c^8*d
^3*x^8 + 40*(14400*a^2 + 401*b^2)*c^6*d^3*x^6 + 75*(3840*a^2 + 79*b^2)*c^4*
d^3*x^4 - 17775*b^2*c^2*d^3*x^2 + 225*(512*b^2*c^10*d^3*x^10 + 1920*b^2*c^8
*d^3*x^8 + 2560*b^2*c^6*d^3*x^6 + 1280*b^2*c^4*d^3*x^4 - 79*b^2*d^3)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 30*(7680*a*b*c^10*d^3*x^10 + 28800*a*b*c^8*d^3*x
^8 + 38400*a*b*c^6*d^3*x^6 + 19200*a*b*c^4*d^3*x^4 - 1185*a*b*d^3 - (768*b^
2*c^9*d^3*x^9 + 2736*b^2*c^7*d^3*x^7 + 3208*b^2*c^5*d^3*x^5 + 790*b^2*c^3*d
^3*x^3 - 1185*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 30*(768*a*b*c^9*d^3*x^9 + 2736*a*b*c^7*d^3*x^7 + 3208*a*b*c^5*d^3*x^5 + 7
90*a*b*c^3*d^3*x^3 - 1185*a*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [**A**]
time = 3.11, size = 654, normalized size = 1.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**10/10 + 3*a**2*c**4*d**3*x**8/8 + a**2*c**2*d*
*3*x**6/2 + a**2*d**3*x**4/4 + a*b*c**6*d**3*x**10*asinh(c*x)/5 - a*b*c**5*
d**3*x**9*sqrt(c**2*x**2 + 1)/50 + 3*a*b*c**4*d**3*x**8*asinh(c*x)/4 - 57*a
*b*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)/800 + a*b*c**2*d**3*x**6*asinh(c*x) -
 401*a*b*c*d**3*x**5*sqrt(c**2*x**2 + 1)/4800 + a*b*d**3*x**4*asinh(c*x)/2
- 79*a*b*d**3*x**3*sqrt(c**2*x**2 + 1)/(3840*c) + 79*a*b*d**3*x*sqrt(c**2*x
**2 + 1)/(2560*c**3) - 79*a*b*d**3*asinh(c*x)/(2560*c**4) + b**2*c**6*d**3*
x**10*asinh(c*x)**2/10 + b**2*c**6*d**3*x**10/500 - b**2*c**5*d**3*x**9*sqr
```

```
t(c**2*x**2 + 1)*asinh(c*x)/50 + 3*b**2*c**4*d**3*x**8*asinh(c*x)**2/8 + 57
*b**2*c**4*d**3*x**8/6400 - 57*b**2*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)*asin
h(c*x)/800 + b**2*c**2*d**3*x**6*asinh(c*x)**2/2 + 401*b**2*c**2*d**3*x**6/
28800 - 401*b**2*c*d**3*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/4800 + b**2*d**
3*x**4*asinh(c*x)**2/4 + 79*b**2*d**3*x**4/15360 - 79*b**2*d**3*x**3*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(3840*c) - 79*b**2*d**3*x**2/(5120*c**2) + 79*b**2
*d**3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2560*c**3) - 79*b**2*d**3*asinh(c*x
)**2/(5120*c**4), Ne(c, 0)), (a**2*d**3*x**4/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(d\,c^2\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.218 $\int x^2(d+c^2dx^2)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=382

$$-\frac{10516b^2d^3x}{99225c^2}+\frac{5258b^2d^3x^3}{297675}+\frac{4198b^2c^2d^3x^5}{165375}+\frac{374b^2c^4d^3x^7}{27783}+\frac{2}{729}b^2c^6d^3x^9+\frac{64bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{945c^3}$$

```
[Out] -10516/99225*b^2*d^3*x/c^2+5258/297675*b^2*d^3*x^3+4198/165375*b^2*c^2*d^3*
x^5+374/27783*b^2*c^4*d^3*x^7+2/729*b^2*c^6*d^3*x^9+16/315*b*d^3*(c^2*x^2+1
)^(3/2)*(a+b*arcsinh(c*x))/c^3+4/525*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c
*x))/c^3+2/441*b*d^3*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^3-2/81*b*d^3*(c
^2*x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^3+16/315*d^3*x^3*(a+b*arcsinh(c*x))^2+
8/105*d^3*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+2/21*d^3*x^3*(c^2*x^2+1)^2*(
a+b*arcsinh(c*x))^2+1/9*d^3*x^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+64/945*b
*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-32/945*b*d^3*x^2*(a+b*arcsinh
(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.59, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12, 380}

$$-\frac{32bd^3x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{945c}+\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{21}d^3x^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{105}d^3x^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^3\left(c^2x^2+1\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{81c^3}+\frac{2bd^3\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{441c^3}+\frac{4bd^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{525c^3}+\frac{16bd^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{315c^3}+\frac{64bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{945c^3}+\frac{16}{315}d^3x^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{729}b^2c^6d^3x^9+\frac{374b^2c^4d^3x^7}{27783}+\frac{4198b^2c^2d^3x^5}{165375}-\frac{10516b^2d^3x}{99225c^2}+\frac{5258b^2d^3x^3}{297675}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-10516*b^2*d^3*x)/(99225*c^2) + (5258*b^2*d^3*x^3)/297675 + (4198*b^2*c^2*
d^3*x^5)/165375 + (374*b^2*c^4*d^3*x^7)/27783 + (2*b^2*c^6*d^3*x^9)/729 + (
64*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c^3) - (32*b*d^3*x^2*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c) + (16*b*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/(315*c^3) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*
ArcSinh[c*x]))/(525*c^3) + (2*b*d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]
))/(441*c^3) - (2*b*d^3*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^3)
+ (16*d^3*x^3*(a + b*ArcSinh[c*x])^2)/315 + (8*d^3*x^3*(1 + c^2*x^2)*(a + b
*ArcSinh[c*x])^2)/105 + (2*d^3*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/
21 + (d^3*x^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/9
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
```

```
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx &= \frac{1}{9}d^3x^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}(2d)\int x^2\left(d+c^2dx^2\right) \\
&= \frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{63c^3}-\frac{2bd^3\left(1+c^2x^2\right)^{9/2}\left(a+b\right.}{81c^3} \\
&= \frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{105c^3}+\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\right.}{441c^3} \\
&= \frac{16bd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{315c^3}+\frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+\right.}{525c^3} \\
&= -\frac{4b^2d^3x}{567c^2}+\frac{2b^2d^3x^3}{1701}+\frac{2}{189}b^2c^2d^3x^5+\frac{38b^2c^4d^3x^7}{3969}+\frac{2}{729}b^2c^6d^3x \\
&= -\frac{3796b^2d^3x}{99225c^2}+\frac{5258b^2d^3x^3}{297675}+\frac{4198b^2c^2d^3x^5}{165375}+\frac{374b^2c^4d^3x^7}{27783}+ \\
&= -\frac{10516b^2d^3x}{99225c^2}+\frac{5258b^2d^3x^3}{297675}+\frac{4198b^2c^2d^3x^5}{165375}+\frac{374b^2c^4d^3x^7}{27783}+
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 275, normalized size = 0.72

$$\frac{d^3\left(99225a^2c^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)-630ab\sqrt{1+c^2x^2}\,(-5258+2629c^2x^2+6297c^4x^4+4675c^6x^6+1225c^8x^8)+b^2(-3312540cx+552090c^3x^3+793422c^5x^5+420750c^7x^7+85750c^9x^9)-630b\left(-315ac^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)+b\sqrt{1+c^2x^2}\,(-5258+2629c^2x^2+6297c^4x^4+4675c^6x^6+1225c^8x^8)\right)\sinh^{-1}(cx)+99225b^2c^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)\sinh^{-1}(cx)^2\right)}{31255875c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(99225*a^2*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - 63
0*a*b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6
 + 1225*c^8*x^8) + b^2*(-3312540*c*x + 552090*c^3*x^3 + 793422*c^5*x^5 + 42
0750*c^7*x^7 + 85750*c^9*x^9) - 630*b*(-315*a*c^3*x^3*(105 + 189*c^2*x^2 +
135*c^4*x^4 + 35*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 629
7*c^4*x^4 + 4675*c^6*x^6 + 1225*c^8*x^8))*ArcSinh[c*x] + 99225*b^2*c^3*x^3*
(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSinh[c*x]^2))/(31255875*c
^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(c^2d\,x^2+d\right)^3\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 922 vs. 2(340) = 680.
time = 0.34, size = 922, normalized size = 2.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*c^6*d^3*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^6*d^3*x^9 + 3/7*b^2*c^4*d^3*
x^7*arcsinh(c*x)^2 + 3/7*a^2*c^4*d^3*x^7 + 3/5*b^2*c^2*d^3*x^5*arcsinh(c*x)
^2 + 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt
(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)
*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^6*d^3 - 2/893025*(315*(35*s
qrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 +
 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*
arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 +
 40320*x)/c^8)*b^2*c^6*d^3 + 3/5*a^2*c^2*d^3*x^5 + 6/245*(35*x^7*arcsinh(c*
```

```
x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^
2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^3 - 2/8575*(105
*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^
2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 1
26*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^3 + 1/3*b^2*d^3*x^3*arcsi
nh(c*x)^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sq
rt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^3 - 2/375*(
15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*
x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^
2*d^3 + 1/3*a^2*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^3 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^
3
```

Fricas [A]
time = 0.43, size = 403, normalized size = 1.05

$$\frac{42875\,(81\,a^2+2\,b^2)c^9d^3x^9+1125\,(11907\,a^2+374\,b^2)c^7d^3x^7+189\,(99225\,a^2+4198\,b^2)c^5d^3x^5+105\,(99225\,a^2+5258\,b^2)c^3d^3x^3-3312540\,b^2cd^3x+99225\,(35\,b^2c^9d^3x^9+135\,b^2c^7d^3x^7+189\,b^2c^5d^3x^5+105\,b^2c^3d^3x^3)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+630\left(11025\,abc^9d^3x^9+42525\,abc^7d^3x^7+59535\,abc^5d^3x^5+33075\,abc^3d^3x^3-(1225\,b^2c^8d^3x^8+4675\,b^2c^6d^3x^6+6297\,b^2c^4d^3x^4+2629\,b^2c^2d^3x^2-5258\,b^2d^3)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-630\,(1225\,abc^8d^3x^8+4675\,abc^6d^3x^6+6297\,abc^4d^3x^4+2629\,abc^2d^3x^2-5258\,abd^3)\sqrt{c^2x^2+1}}{31255875\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^3*x^9 + 1125*(11907*a^2 + 374*b^2)
*c^7*d^3*x^7 + 189*(99225*a^2 + 4198*b^2)*c^5*d^3*x^5 + 105*(99225*a^2 + 52
58*b^2)*c^3*d^3*x^3 - 3312540*b^2*c*d^3*x + 99225*(35*b^2*c^9*d^3*x^9 + 135
*b^2*c^7*d^3*x^7 + 189*b^2*c^5*d^3*x^5 + 105*b^2*c^3*d^3*x^3)*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^3*x^9 + 42525*a*b*c^7*d^3*x^7 + 59
535*a*b*c^5*d^3*x^5 + 33075*a*b*c^3*d^3*x^3 - (1225*b^2*c^8*d^3*x^8 + 4675*
b^2*c^6*d^3*x^6 + 6297*b^2*c^4*d^3*x^4 + 2629*b^2*c^2*d^3*x^2 - 5258*b^2*d^
3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*c^8*d^3*
x^8 + 4675*a*b*c^6*d^3*x^6 + 6297*a*b*c^4*d^3*x^4 + 2629*a*b*c^2*d^3*x^2 -
5258*a*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 2.17, size = 626, normalized size = 1.64

$$\begin{cases} \frac{a^2c^6d^3x^9}{9}+\frac{3a^2c^4d^3x^7}{7}+\frac{3a^2c^2d^3x^5}{5}+\frac{a^2d^3x^3}{3}+\frac{2abc^6d^3x^9\operatorname{asinh}(cx)}{9}-\frac{2abc^5d^3x^8\sqrt{c^2x^2+1}}{81}+\frac{6abc^4d^3x^7\operatorname{asinh}(cx)}{7}-\frac{374abc^3d^3x^6\sqrt{c^2x^2+1}}{3969}+\frac{6abc^2d^3x^5\operatorname{asinh}(cx)}{5}-\frac{4198abcd^3x^4\sqrt{c^2x^2+1}}{33075}+\frac{2abd^3x^3\operatorname{asinh}(cx)}{3}-\frac{5258abd^3x^2\sqrt{c^2x^2+1}}{99225c}+\frac{10516abd^3\sqrt{c^2x^2+1}}{99225c^3}+\frac{b^2c^6d^3x^9\operatorname{asinh}^2(cx)}{9}+\frac{2b^2c^6d^3x^9}{729}-\frac{2b^2c^5d^3x^8\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{81}+\frac{3b^2c^4d^3x^7\operatorname{asinh}^2(cx)}{7}+\frac{374b^2c^4d^3x^7}{27783}-\frac{374b^2c^3d^3x^6\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3969}+\frac{3b^2c^2d^3x^5\operatorname{asinh}^2(cx)}{5}+\frac{4198b^2c^2d^3x^5}{165375}-\frac{4198b^2cd^3x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{33075}+\frac{b^2d^3x^3\operatorname{asinh}^2(cx)}{3}+\frac{5258b^2d^3x^3}{297675}-\frac{5258b^2d^3x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{99225c}-\frac{10516b^2d^3x}{99225c^2}+\frac{10516b^2d^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{99225c^3} & \text{for}\: c \neq 0 \\ \frac{a^2d^3x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**9/9 + 3*a**2*c**4*d**3*x**7/7 + 3*a**2*c**2*d*
*3*x**5/5 + a**2*d**3*x**3/3 + 2*a*b*c**6*d**3*x**9*asinh(c*x)/9 - 2*a*b*c*
*5*d**3*x**8*sqrt(c**2*x**2 + 1)/81 + 6*a*b*c**4*d**3*x**7*asinh(c*x)/7 - 3
74*a*b*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)/3969 + 6*a*b*c**2*d**3*x**5*asinh
(c*x)/5 - 4198*a*b*c*d**3*x**4*sqrt(c**2*x**2 + 1)/33075 + 2*a*b*d**3*x**3*
```

```
asinh(c*x)/3 - 5258*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(99225*c) + 10516*a*b
*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3) + b**2*c**6*d**3*x**9*asinh(c*x)**2/
9 + 2*b**2*c**6*d**3*x**9/729 - 2*b**2*c**5*d**3*x**8*sqrt(c**2*x**2 + 1)*a
sinh(c*x)/81 + 3*b**2*c**4*d**3*x**7*asinh(c*x)**2/7 + 374*b**2*c**4*d**3*x
**7/27783 - 374*b**2*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/3969 + 3
*b**2*c**2*d**3*x**5*asinh(c*x)**2/5 + 4198*b**2*c**2*d**3*x**5/165375 - 41
98*b**2*c*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/33075 + b**2*d**3*x**3*a
sinh(c*x)**2/3 + 5258*b**2*d**3*x**3/297675 - 5258*b**2*d**3*x**2*sqrt(c**2
*x**2 + 1)*asinh(c*x)/(99225*c) - 10516*b**2*d**3*x/(99225*c**2) + 10516*b*
*2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225*c**3), Ne(c, 0)), (a**2*d**3*
x**3/3, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(d\,c^2\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.219 $\int x(d+c^2dx^2)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=261

$$\frac{175b^2d^3x^2}{3072}+\frac{35b^2c^2d^3x^4}{3072}+\frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2}-\frac{35bd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{512c}-\frac{35bd^3x}{}$$

```
[Out] 175/3072*b^2*d^3*x^2+35/3072*b^2*c^2*d^3*x^4+7/1152*b^2*d^3*(c^2*x^2+1)^3/c
^2+1/256*b^2*d^3*(c^2*x^2+1)^4/c^2-35/768*b*d^3*x*(c^2*x^2+1)^(3/2)*(a+b*ar
csinh(c*x))/c-7/192*b*d^3*x*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c-1/32*b*d
^3*x*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c-35/1024*d^3*(a+b*arcsinh(c*x))^
2/c^2+1/8*d^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))^2/c^2-35/512*b*d^3*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.19, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5798, 5786, 5785, 5783, 30, 14, 267}

$$-\frac{bd^3x(c^2x^2+1)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{32c}-\frac{7bd^3x(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{192c}-\frac{35bd^3x(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{768c}-\frac{35bd^3x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{512c}+\frac{d^3(c^2x^2+1)^4\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2}-\frac{35d^3\left(a+b\sinh^{-1}(cx)\right)^2}{1024c^2}+\frac{35b^2c^2d^3x^4}{3072}+\frac{b^2d^3(c^2x^2+1)^4}{256c^2}+\frac{7b^2d^3(c^2x^2+1)^3}{1152c^2}+\frac{175b^2d^3x^2}{3072}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (175*b^2*d^3*x^2)/3072 + (35*b^2*c^2*d^3*x^4)/3072 + (7*b^2*d^3*(1 + c^2*x^
2)^3)/(1152*c^2) + (b^2*d^3*(1 + c^2*x^2)^4)/(256*c^2) - (35*b*d^3*x*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(512*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2)
*(a + b*ArcSinh[c*x]))/(768*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x]))/(192*c) - (b*d^3*x*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(32
*c) - (35*d^3*(a + b*ArcSinh[c*x])^2)/(1024*c^2) + (d^3*(1 + c^2*x^2)^4*(a
+ b*ArcSinh[c*x])^2)/(8*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{d^3(1+c^2x^2)^4\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2}-\frac{(bd^3)\int(1+c^2x^2)^{7/2}\left(a+b\right.}{4c}$$
$$=-\frac{bd^3x(1+c^2x^2)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{32c}+\frac{d^3(1+c^2x^2)^4\left(a+b\sin\right.}{8c^2}$$
$$=\frac{b^2d^3(1+c^2x^2)^4}{256c^2}-\frac{7bd^3x(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{192c}-\frac{bd^3x}{}$$
$$=\frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2}-\frac{35bd^3x(1+c^2x^2)^{3/2}\left(a+\right.}{768c}$$
$$=\frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2}-\frac{35bd^3x\sqrt{1+c^2x^2}\left(a+b\right.}{512c}$$
$$=\frac{175b^2d^3x^2}{3072}+\frac{35b^2c^2d^3x^4}{3072}+\frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2}$$

Mathematica [A]
time = 0.23, size = 256, normalized size = 0.98

$$\frac{d^3\left(cx\left(1152a^2cx(4+6c^2x^2+4c^4x^4+c^6x^6)+b^2cx(837+489c^2x^2+200c^4x^4+36c^6x^6)-6ab\sqrt{1+c^2x^2}\,(279+326c^2x^2+200c^4x^4+48c^6x^6)\right)+6b\left(-bcx\sqrt{1+c^2x^2}\,(279+326c^2x^2+200c^4x^4+48c^6x^6)+3a(93+512c^2x^2+768c^4x^4+512c^6x^6+128c^8x^8)\right)\sinh^{-1}(cx)+9b^2(93+512c^2x^2+768c^4x^4+512c^6x^6+128c^8x^8)\sinh^{-1}(cx)^2\right)}{9216c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(c*x*(1152*a^2*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) + b^2*c*x*(83
7 + 489*c^2*x^2 + 200*c^4*x^4 + 36*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(279
+ 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]
*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*a*(93 + 512*c^2*x^2 +
768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(93 + 512*c^
2*x^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]^2))/(9216*c^2
)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(c^2d\,x^2+d\right)^3\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(233) = 466$.
time = 0.33, size = 925, normalized size = 3.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*c^6*d^3*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^6*d^3*x^8 + 1/2*b^2*c^4*d^3*
x^6*arcsinh(c*x)^2 + 1/2*a^2*c^4*d^3*x^6 + 3/4*b^2*c^2*d^3*x^4*arcsinh(c*x)
^2 + 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt
(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1
)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*a*b*c^6*d^3 + 1/9216*((36*x^8/c^2 - 56*x
^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^1
0)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 7
0*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x
)/c^9)*c*arcsinh(c*x))*b^2*c^6*d^3 + 3/4*a^2*c^2*d^3*x^4 + 1/48*(48*x^6*arc
sinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 1
5*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^4*d^3 + 1/288*((8
*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)
*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*s
qrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^4*d^3 +
 1/2*b^2*d^3*x^2*arcsinh(c*x)^2 + 3/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^
2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2
*d^3 + 3/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c
^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh
(c*x)/c^5)*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/2*a^2*d^3*x^2 + 1/2*(2*x^2*arcsi
nh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^3 + 1/4*(c^
2*(x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x
/c^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*d^3
```

Fricas [A]
time = 0.39, size = 383, normalized size = 1.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/9216*(36*(32*a^2 + b^2)*c^8*d^3*x^8 + 8*(576*a^2 + 25*b^2)*c^6*d^3*x^6 +
3*(2304*a^2 + 163*b^2)*c^4*d^3*x^4 + 9*(512*a^2 + 93*b^2)*c^2*d^3*x^2 + 9*(
128*b^2*c^8*d^3*x^8 + 512*b^2*c^6*d^3*x^6 + 768*b^2*c^4*d^3*x^4 + 512*b^2*c
^2*d^3*x^2 + 93*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(384*a*b*c^8*d^
3*x^8 + 1536*a*b*c^6*d^3*x^6 + 2304*a*b*c^4*d^3*x^4 + 1536*a*b*c^2*d^3*x^2
```

+ 279*a*b*d^3 - (48*b^2*c^7*d^3*x^7 + 200*b^2*c^5*d^3*x^5 + 326*b^2*c^3*d^3*x^3 + 279*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(48*a*b*c^7*d^3*x^7 + 200*a*b*c^5*d^3*x^5 + 326*a*b*c^3*d^3*x^3 + 279*a*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^2

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(252) = 504$.
time = 1.65, size = 573, normalized size = 2.20

$$\begin{cases} \frac{a^2c^6d^3x^8}{8} + \frac{a^2c^4d^3x^6}{2} + \frac{3a^2c^2d^3x^4}{4} + \frac{a^2d^3x^2}{2} + \frac{abc^6d^3x^8\operatorname{asinh}(cx)}{4} - \frac{abc^5d^3x^7\sqrt{c^2x^2+1}}{32} + abc^4d^3x^6\operatorname{asinh}(cx) - \frac{25abc^3d^3x^5\sqrt{c^2x^2+1}}{192} + \frac{3abc^2d^3x^4\operatorname{asinh}(cx)}{2} - \frac{163abcd^3x^3\sqrt{c^2x^2+1}}{768} + abd^3x^2\operatorname{asinh}(cx) - \frac{93abd^3x\sqrt{c^2x^2+1}}{512c} + \frac{93abd^3\operatorname{asinh}(cx)}{512c^2} + \frac{b^2c^6d^3x^8\operatorname{asinh}^2(cx)}{8} + \frac{b^2c^6d^3x^8}{256} - \frac{b^2c^5d^3x^7\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{32} + \frac{b^2c^4d^3x^6\operatorname{asinh}^2(cx)}{2} + \frac{25b^2c^4d^3x^6}{1152} - \frac{25b^2c^3d^3x^5\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{192} + \frac{3b^2c^2d^3x^4\operatorname{asinh}^2(cx)}{4} + \frac{163b^2c^2d^3x^4}{3072} - \frac{163b^2cd^3x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{768} + \frac{b^2d^3x^2\operatorname{asinh}^2(cx)}{2} + \frac{93b^2d^3x^2}{1024} - \frac{93b^2d^3x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{512c} + \frac{93b^2d^3\operatorname{asinh}^2(cx)}{1024c^2} & \text{for}\: c \neq 0 \\ \frac{a^2d^3x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**8/8 + a**2*c**4*d**3*x**6/2 + 3*a**2*c**2*d**3
*x**4/4 + a**2*d**3*x**2/2 + a*b*c**6*d**3*x**8*asinh(c*x)/4 - a*b*c**5*d**
3*x**7*sqrt(c**2*x**2 + 1)/32 + a*b*c**4*d**3*x**6*asinh(c*x) - 25*a*b*c**3
*d**3*x**5*sqrt(c**2*x**2 + 1)/192 + 3*a*b*c**2*d**3*x**4*asinh(c*x)/2 - 16
3*a*b*c*d**3*x**3*sqrt(c**2*x**2 + 1)/768 + a*b*d**3*x**2*asinh(c*x) - 93*a
*b*d**3*x*sqrt(c**2*x**2 + 1)/(512*c) + 93*a*b*d**3*asinh(c*x)/(512*c**2) +
 b**2*c**6*d**3*x**8*asinh(c*x)**2/8 + b**2*c**6*d**3*x**8/256 - b**2*c**5*
d**3*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c**4*d**3*x**6*asinh(c*x
)**2/2 + 25*b**2*c**4*d**3*x**6/1152 - 25*b**2*c**3*d**3*x**5*sqrt(c**2*x**
2 + 1)*asinh(c*x)/192 + 3*b**2*c**2*d**3*x**4*asinh(c*x)**2/4 + 163*b**2*c*
*2*d**3*x**4/3072 - 163*b**2*c*d**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/768
 + b**2*d**3*x**2*asinh(c*x)**2/2 + 93*b**2*d**3*x**2/1024 - 93*b**2*d**3*x
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(512*c) + 93*b**2*d**3*asinh(c*x)**2/(1024*
c**2), Ne(c, 0)), (a**2*d**3*x**2/2, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(d\,c^2\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.220 $\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=291

$$\frac{4322b^2d^3x}{3675}+\frac{1514b^2c^2d^3x^3}{11025}+\frac{234b^2c^4d^3x^5}{6125}+\frac{2}{343}b^2c^6d^3x^7-\frac{32bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{35c}-\frac{16bd^3(1+c^2\cdots}{}$$

```
[Out] 4322/3675*b^2*d^3*x+1514/11025*b^2*c^2*d^3*x^3+234/6125*b^2*c^4*d^3*x^5+2/3
43*b^2*c^6*d^3*x^7-16/105*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-12/1
75*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c-2/49*b*d^3*(c^2*x^2+1)^(7/2
)*(a+b*arcsinh(c*x))/c+16/35*d^3*x*(a+b*arcsinh(c*x))^2+8/35*d^3*x*(c^2*x^2
+1)*(a+b*arcsinh(c*x))^2+6/35*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/7*
d^3*x*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2-32/35*b*d^3*(a+b*arcsinh(c*x))*(c^
2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.26, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5786, 5772, 5798, 8, 200}

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{6}{35}d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{35}d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^3\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c}-\frac{12bd^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{175c}-\frac{16bd^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{105c}-\frac{32bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{35c}+\frac{16}{35}d^3x\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{343}b^2c^6d^3x^7+\frac{234b^2c^4d^3x^5}{6125}+\frac{1514b^2c^2d^3x^3}{11025}+\frac{4322b^2d^3x}{3675}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4322*b^2*d^3*x)/3675 + (1514*b^2*c^2*d^3*x^3)/11025 + (234*b^2*c^4*d^3*x^5
)/6125 + (2*b^2*c^6*d^3*x^7)/343 - (32*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x]))/(35*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105
*c) - (12*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d^
3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c) + (16*d^3*x*(a + b*ArcSi
nh[c*x])^2)/35 + (8*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (6*d^3
*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/35 + (d^3*x*(1 + c^2*x^2)^3*(a +
 b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx &= \frac{1}{7}d^3x\left(1+c^2x^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}(6d)\int \left(d+c^2dx^2\right)^2 \left(a+\right.\\ &= -\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c}+\frac{6}{35}d^3x\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{s}\right.\\ &= -\frac{12bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{175c}-\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\right.}{49c}\\ &= \frac{2}{49}b^2d^3x+\frac{2}{49}b^2c^2d^3x^3+\frac{6}{245}b^2c^4d^3x^5+\frac{2}{343}b^2c^6d^3x^7-\frac{16bd^3(1+}{}\\ &= \frac{962b^2d^3x}{3675}+\frac{1514b^2c^2d^3x^3}{11025}+\frac{234b^2c^4d^3x^5}{6125}+\frac{2}{343}b^2c^6d^3x^7-\frac{32bd^3\sqrt{}}{}\\ &= \frac{4322b^2d^3x}{3675}+\frac{1514b^2c^2d^3x^3}{11025}+\frac{234b^2c^4d^3x^5}{6125}+\frac{2}{343}b^2c^6d^3x^7-\frac{32bd^3}{}\end{aligned}$$

Mathematica [A]

time = 0.17, size = 239, normalized size = 0.82

$$\frac{d^3\left(11025a^2cx(35+35c^2x^2+21c^4x^4+5c^6x^6)-210ab\sqrt{1+c^2x^2}\,(2161+757c^2x^2+351c^4x^4+75c^6x^6)+2b^2cx(226905+26495c^2x^2+7371c^4x^4+1125c^6x^6)-210b\left(-105acx(35+35c^2x^2+21c^4x^4+5c^6x^6)+b\sqrt{1+c^2x^2}\,(2161+757c^2x^2+351c^4x^4+75c^6x^6)\right)\sinh^{-1}(cx)+11025b^2cx(35+35c^2x^2+21c^4x^4+5c^6x^6)\sinh^{-1}(cx)^2\right)}{385875c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(11025*a^2*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - 210*a*b*Sq
rt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 2*b^2*c*x
*(226905 + 26495*c^2*x^2 + 7371*c^4*x^4 + 1125*c^6*x^6) - 210*b*(-105*a*c*x
*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(2161 + 7
57*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c*x*(35 +
35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]^2))/(385875*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(c^{2} d \,x^{2}+d \right)^{3} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(259) = 518.
time = 0.31, size = 712, normalized size = 2.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^6*d^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3*
x^5*arcsinh(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 - 2/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^6*d^3 + b^2*c^2*d^3*x^3*arcsinh(c*x)
^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*
x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 - 2/375*(15*(3*s
qrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1
)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^4*d^3 +
 a^2*c^2*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d^
3 + b^2*d^3*x*arcsinh(c*x)^2 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x
)/c) + a^2*d^3*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

Fricas [A]
time = 0.40, size = 354, normalized size = 1.22

$$\frac{1125\,(49\,a^2+2\,b^2)c^7d^3x^7+189\,(1225\,a^2+78\,b^2)c^5d^3x^5+35\,(11025\,a^2+1514\,b^2)c^3d^3x^3+105\,(3675\,a^2+4322\,b^2)cd^3x+11025\,(5\,b^2c^7d^3x^7+21\,b^2c^5d^3x^5+35\,b^2c^3d^3x^3+35\,b^2cd^3x)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+210\left(525\,abc^7d^3x^7+2205\,abc^5d^3x^5+3675\,abc^3d^3x^3+3675\,abcd^3x-(75\,b^2c^6d^3x^6+351\,b^2c^4d^3x^4+757\,b^2c^2d^3x^2+2161\,b^2d^3)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-210\,(75\,abc^6d^3x^6+351\,abc^4d^3x^4+757\,abc^2d^3x^2+2161\,abd^3)\sqrt{c^2x^2+1}}{385875\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d^3*x^7 + 189*(1225*a^2 + 78*b^2)*c^5*d
^3*x^5 + 35*(11025*a^2 + 1514*b^2)*c^3*d^3*x^3 + 105*(3675*a^2 + 4322*b^2)*
c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 + 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^3*x^
3 + 35*b^2*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d^3*x
^7 + 2205*a*b*c^5*d^3*x^5 + 3675*a*b*c^3*d^3*x^3 + 3675*a*b*c*d^3*x - (75*b
^2*c^6*d^3*x^6 + 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 + 2161*b^2*d^3)*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d^3*x^6 +
 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 + 2161*a*b*d^3)*sqrt(c^2*x^2 + 1
))/c
```

Sympy [A]
time = 1.14, size = 524, normalized size = 1.80

$$\begin{cases} \frac{a^2c^6d^3x^7}{7}+\frac{3a^2c^4d^3x^5}{5}+a^2c^2d^3x^3+a^2d^3x+\frac{2abc^6d^3x^7\operatorname{asinh}(cx)}{7}-\frac{2abc^5d^3x^6\sqrt{c^2x^2+1}}{49}+\frac{6abc^4d^3x^5\operatorname{asinh}(cx)}{5}-\frac{234abc^3d^3x^4\sqrt{c^2x^2+1}}{1225}+2abc^2d^3x^3\operatorname{asinh}(cx)-\frac{1514abcd^3x^2\sqrt{c^2x^2+1}}{3675}+2abd^3x\operatorname{asinh}(cx)-\frac{4322abd^3\sqrt{c^2x^2+1}}{3675c}+\frac{b^2c^6d^3x^7\operatorname{asinh}^2(cx)}{7}+\frac{2b^2c^6d^3x^7}{343}-\frac{2b^2c^5d^3x^6\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{49}+\frac{3b^2c^4d^3x^5\operatorname{asinh}^2(cx)}{5}+\frac{234b^2c^4d^3x^5}{6125}-\frac{234b^2c^3d^3x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{1225}+b^2c^2d^3x^3\operatorname{asinh}^2(cx)+\frac{1514b^2c^2d^3x^3}{11025}-\frac{1514b^2cd^3x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3675}+b^2d^3x\operatorname{asinh}^2(cx)+\frac{4322b^2d^3x}{3675}-\frac{4322b^2d^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3675c} & \text{for}\ c\neq 0 \\ a^2d^3x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 + a**2*c**2*d**3
*x**3 + a**2*d**3*x + 2*a*b*c**6*d**3*x**7*asinh(c*x)/7 - 2*a*b*c**5*d**3*x
**6*sqrt(c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*asinh(c*x)/5 - 234*a*b*c*
*3*d**3*x**4*sqrt(c**2*x**2 + 1)/1225 + 2*a*b*c**2*d**3*x**3*asinh(c*x) - 1
514*a*b*c*d**3*x**2*sqrt(c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*asinh(c*x) - 43
22*a*b*d**3*sqrt(c**2*x**2 + 1)/(3675*c) + b**2*c**6*d**3*x**7*asinh(c*x)**
2/7 + 2*b**2*c**6*d**3*x**7/343 - 2*b**2*c**5*d**3*x**6*sqrt(c**2*x**2 + 1)
*asinh(c*x)/49 + 3*b**2*c**4*d**3*x**5*asinh(c*x)**2/5 + 234*b**2*c**4*d**3
*x**5/6125 - 234*b**2*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/1225 +
b**2*c**2*d**3*x**3*asinh(c*x)**2 + 1514*b**2*c**2*d**3*x**3/11025 - 1514*b
**2*c*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/3675 + b**2*d**3*x*asinh(c*x
)**2 + 4322*b**2*d**3*x/3675 - 4322*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(3675*c), Ne(c, 0)), (a**2*d**3*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.221 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=337

$$\frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3(1+c^2x^2)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))-\frac{7}{36}bcd^3x(1+c^2x^2)$$

```
[Out] 71/144*b^2*c^2*d^3*x^2+7/144*b^2*c^4*d^3*x^4+1/108*b^2*d^3*(c^2*x^2+1)^3-7/
36*b*c*d^3*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-1/18*b*c*d^3*x*(c^2*x^2+1
)^(5/2)*(a+b*arcsinh(c*x))-19/48*d^3*(a+b*arcsinh(c*x))^2+1/2*d^3*(c^2*x^2+
1)*(a+b*arcsinh(c*x))^2+1/4*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/6*d^3*
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+1/3*d^3*(a+b*arcsinh(c*x))^3/b+d^3*(a+b*
arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-b*d^3*(a+b*arcsinh(c*x))*
polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*d^3*polylog(3,1/(c*x+(c^2*x^
2+1)^(1/2))^2)-19/24*b*c*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30, 5786, 14, 267}

$$-\frac{1}{18}bcd^3x(c^2x^2+1)^{5/2}(a+b\sinh^{-1}(cx))-\frac{7}{36}bcd^3x(c^2x^2+1)^{3/2}(a+b\sinh^{-1}(cx))-\frac{19}{24}bcd^3x\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))+\frac{1}{6}d^3(c^2x^2+1)^3(a+b\sinh^{-1}(cx))^2+\frac{1}{4}d^3(c^2x^2+1)^2(a+b\sinh^{-1}(cx))^2+\frac{1}{2}d^3(c^2x^2+1)(a+b\sinh^{-1}(cx))^2-bd^3\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))+\frac{d^3(a+b\sinh^{-1}(cx))^3}{3b}-\frac{19}{48}d^3(a+b\sinh^{-1}(cx))^2+d^3\log\left(1-e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2+\frac{7}{144}b^2c^4d^3x^4+\frac{71}{144}b^2c^2d^3x^2+\frac{1}{108}b^2d^3(c^2x^2+1)^3-\frac{1}{2}b^2d^3\text{Li}_3\left(e^{-2\sinh^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (71*b^2*c^2*d^3*x^2)/144 + (7*b^2*c^4*d^3*x^4)/144 + (b^2*d^3*(1 + c^2*x^2)
^3)/108 - (19*b*c*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/24 - (7*b*c
*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/36 - (b*c*d^3*x*(1 + c^2*x
^2)^(5/2)*(a + b*ArcSinh[c*x]))/18 - (19*d^3*(a + b*ArcSinh[c*x])^2)/48 + (
d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 + (d^3*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/6 + (d^3*(
a + b*ArcSinh[c*x])^3)/(3*b) + d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Arc
Sinh[c*x])] - b*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])] -
(b^2*d^3*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{6}d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh\right.}{x}$$

$$= -\frac{1}{18}bcd^3x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2\left(a+\right.$$

$$= \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{7}{36}bcd^3x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) -$$

$$= \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{7}{36}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1}$$

Mathematica [A]
time = 0.46, size = 429, normalized size = 1.27

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^3*(5184*a^2*c^2*x^2 + 2592*a^2*c^4*x^4 + 576*a^2*c^6*x^6 - 3600*a*b*c*x*
Sqrt[1 + c^2*x^2] - 1056*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 192*a*b*c^5*x^5*Sq
rt[1 + c^2*x^2] + 10368*a*b*c^2*x^2*ArcSinh[c*x] + 5184*a*b*c^4*x^4*ArcSinh
[c*x] + 1152*a*b*c^6*x^6*ArcSinh[c*x] + 3456*a*b*ArcSinh[c*x]^2 - 1152*b^2*
ArcSinh[c*x]^3 + 3600*a*b*ArcTanh[(c*x)/Sqrt[1 + c^2*x^2]] + 783*b^2*Cosh[2
*ArcSinh[c*x]] + 1566*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + 27*b^2*Cosh
[4*ArcSinh[c*x]] + 216*b^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]] + b^2*Cosh[6
*ArcSinh[c*x]] + 18*b^2*ArcSinh[c*x]^2*Cosh[6*ArcSinh[c*x]] + 6912*a*b*ArcS
inh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] + 3456*b^2*ArcSinh[c*x]^2*Log[1 - E^(
2*ArcSinh[c*x])] + 3456*a^2*Log[c*x] - 3456*a*b*PolyLog[2, E^(-2*ArcSinh[c*
x])] + 3456*b^2*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - 1728*b^2*Poly
```

```
Log[3, E^(2*ArcSinh[c*x])] - 1566*b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 1
08*b^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]] - 6*b^2*ArcSinh[c*x]*Sinh[6*ArcSin
h[c*x]]))/3456
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(334) = 668$.
time = 4.64, size = 706, normalized size = 2.09

method	result
derivativedivides	$\frac{25b^2c^2d^3x^2}{48} + \frac{11b^2c^4d^3x^4}{144} + 2d^3ab\,\mathrm{arcsinh}\,(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right) + d^3a^2\ln(cx) - \frac{11d^3}{}$
default	$\frac{25b^2c^2d^3x^2}{48} + \frac{11b^2c^4d^3x^4}{144} + 2d^3ab\,\mathrm{arcsinh}\,(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right) + d^3a^2\ln(cx) - \frac{11d^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 25/48*b^2*c^2*d^3*x^2+11/144*b^2*c^4*d^3*x^4+1/3*d^3*a*b*arcsinh(c*x)*c^6*x
^6+d^3*a^2*ln(c*x)-11/36*d^3*a*b*(c^2*x^2+1)^(1/2)*c^3*x^3-25/24*d^3*a*b*c*
x*(c^2*x^2+1)^(1/2)-1/18*b^2*d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5*x^5-11/
36*b^2*d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-25/24*b^2*d^3*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)*c*x-1/18*d^3*a*b*(c^2*x^2+1)^(1/2)*c^5*x^5+3/4*d^3*a^2*
c^4*x^4+3/2*d^3*a^2*c^2*x^2+1/6*d^3*a^2*c^6*x^6+25/24*d^3*a*b*arcsinh(c*x)+
2*d^3*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-d^3*a*b*arcsinh(c*x)^2+2*d^3*a*
b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*b^2*d^3*arcsinh(c*x)*polylog(2,c*x+(c^
2*x^2+1)^(1/2))+b^2*d^3*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*b^2*d^
3*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b^2*d^3*arcsinh(c*x)^2*ln(
1-c*x-(c^2*x^2+1)^(1/2))+1/108*b^2*d^3*c^6*x^6+3/2*d^3*a*b*arcsinh(c*x)*c^4
*x^4+3*d^3*a*b*arcsinh(c*x)*c^2*x^2+2*d^3*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^
2+1)^(1/2))+2*d^3*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+1/6*b^2*d^3*
arcsinh(c*x)^2*c^6*x^6+3/4*b^2*d^3*arcsinh(c*x)^2*c^4*x^4+3/2*b^2*d^3*arcsi
nh(c*x)^2*c^2*x^2+811/3456*b^2*d^3-2*b^2*d^3*polylog(3,-c*x-(c^2*x^2+1)^(1/
2))+25/48*b^2*d^3*arcsinh(c*x)^2-2*b^2*d^3*polylog(3,c*x+(c^2*x^2+1)^(1/2))
-1/3*b^2*d^3*arcsinh(c*x)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c^6*d^3*x^6 + 3/4*a^2*c^4*d^3*x^4 + 3/2*a^2*c^2*d^3*x^2 + a^2*d^3*l
og(x) + integrate(b^2*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^
6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x^3*log(c*x + sqrt(c
```

```
^2*x^2 + 1))^2 + 6*a*b*c^4*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2
*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b*c^2*d^3*x*log(c*x + sqrt(c^2*
x^2 + 1)) + b^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d^3*log(c*x +
sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a^2}{x}\,dx + \int 3a^2c^2x\,dx + \int 3a^2c^4x^3\,dx + \int a^2c^6x^5\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x}\,dx + \int 3b^2c^2x\operatorname{asinh}^2(cx)\,dx + \int 3b^2c^4x^3\operatorname{asinh}^2(cx)\,dx + \int b^2c^6x^5\operatorname{asinh}^2(cx)\,dx + \int 6abc^2x\operatorname{asinh}(cx)\,dx + \int 6abc^4x^3\operatorname{asinh}(cx)\,dx + \int 2abc^6x^5\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d**3*(Integral(a**2/x, x) + Integral(3*a**2*c**2*x, x) + Integral(3*a**2*c*
*4*x**3, x) + Integral(a**2*c**6*x**5, x) + Integral(b**2*asinh(c*x)**2/x,
x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(3*b**2*c**2*x*asinh(c*x)**2
, x) + Integral(3*b**2*c**4*x**3*asinh(c*x)**2, x) + Integral(b**2*c**6*x**
5*asinh(c*x)**2, x) + Integral(6*a*b*c**2*x*asinh(c*x), x) + Integral(6*a*b
*c**4*x**3*asinh(c*x), x) + Integral(2*a*b*c**6*x**5*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2\,(d\,c^2\,x^2 + d)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x,x)
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x, x)
```

3.222 $\int \frac{(d+c^2dx^2)^3 (a+b\sinh^{-1}(cx))^2}{x^2} \, dx$

Optimal. Leaf size=307

$$\frac{122}{25}b^2c^2d^3x+\frac{14}{75}b^2c^4d^3x^3+\frac{2}{125}b^2c^6d^3x^5-\frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{2}{5}bcd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.$$

```
[Out] 122/25*b^2*c^2*d^3*x+14/75*b^2*c^4*d^3*x^3+2/125*b^2*c^6*d^3*x^5-2/5*b*c*d^
3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-2/25*b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*
arcsinh(c*x))+16/5*c^2*d^3*x*(a+b*arcsinh(c*x))^2+8/5*c^2*d^3*x*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2+6/5*c^2*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-d^3*
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/x-4*b*c*d^3*(a+b*arcsinh(c*x))*arctanh(c
*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c
*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-22/5*b*c*d^3*(a+b*arcsinh(c*x))*(c^2*
x^2+1)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5807, 5786, 5772, 5798, 8, 200, 5808, 5806, 5816, 4267, 2317, 2438}

$$\frac{6}{5}c^2d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{5}c^2d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{25}bcd^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{2}{5}bcd^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{22}{5}bcd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\frac{16}{5}c^2d^3x\left(a+b\sinh^{-1}(cx)\right)^2-4bcd^3\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{125}b^2c^6d^3x^5+\frac{14}{75}b^2c^4d^3x^3+\frac{122}{25}b^2c^2d^3x-2b^2cd^3\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+2b^2cd^3\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (122*b^2*c^2*d^3*x)/25 + (14*b^2*c^4*d^3*x^3)/75 + (2*b^2*c^6*d^3*x^5)/125
- (22*b*c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c
^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x]))/25 + (16*c^2*d^3*x*(a + b*ArcSinh[c*x])^2)/5 + (8*c^2*d^3
*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/5 + (6*c^2*d^3*x*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x])^2)/5 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/x -
 4*b*c*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^3*PolyL
og[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^3*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
```

```
x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(6c^2d\right)\int \left(d+c^2dx^2\right)^2 \left(a+b\right.$$

$$= \frac{2}{5}bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{6}{5}c^2d^3x\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{s}\right.$$

$$= \frac{2}{3}bcd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{25}bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\right.$$

$$= -\frac{16}{15}b^2c^2d^3x - \frac{22}{45}b^2c^4d^3x^3 - \frac{2}{25}b^2c^6d^3x^5 + 2bcd^3\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.$$

$$= -\frac{38}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+\right.$$

Mathematica [A]
time = 0.86, size = 466, normalized size = 1.52

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^3*((-720*a^2)/x + 2160*a^2*c^2*x + 3460*b^2*c^2*x + 720*a^2*c^4*x^3 + 14
4*a^2*c^6*x^5 - (17568*a*b*c*Sqrt[1 + c^2*x^2])/5 - (2016*a*b*c^3*x^2*Sqrt[
1 + c^2*x^2])/5 - (288*a*b*c^5*x^4*Sqrt[1 + c^2*x^2])/5 - (1440*a*b*ArcSinh
[c*x])/x + 4320*a*b*c^2*x*ArcSinh[c*x] + 1440*a*b*c^4*x^3*ArcSinh[c*x] + 28
8*a*b*c^6*x^5*ArcSinh[c*x] - 3420*b^2*c*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - (7
20*b^2*ArcSinh[c*x]^2)/x + 1890*b^2*c^2*x*ArcSinh[c*x]^2 - 1440*a*b*c*ArcTa
nh[Sqrt[1 + c^2*x^2]] + 80*b^2*c^2*x*Cosh[2*ArcSinh[c*x]] + 360*b^2*c^2*x*A
rcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] - 90*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c
*x]] - (18*b^2*c*ArcSinh[c*x]*Cosh[5*ArcSinh[c*x]])/5 + 1440*b^2*c*ArcSinh[
c*x]*Log[1 - E^(-ArcSinh[c*x])] - 1440*b^2*c*ArcSinh[c*x]*Log[1 + E^(-ArcSi
nh[c*x])] + 1440*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 1440*b^2*c*PolyLog[
2, E^(-ArcSinh[c*x])] - 10*b^2*c*Sinh[3*ArcSinh[c*x]] - 45*b^2*c*ArcSinh[c*
x]^2*Sinh[3*ArcSinh[c*x]] + (18*b^2*c*Sinh[5*ArcSinh[c*x]])/25 + 9*b^2*c*Ar
cSinh[c*x]^2*Sinh[5*ArcSinh[c*x]]))/720
```

Maple [A]
time = 4.66, size = 463, normalized size = 1.51

method	result
derivativedivides	$c\left(d^3a^2\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)-2b^2d^3\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\frac{b^2d^3\,\mathrm{a}}{}\right.$
default	$c\left(d^3a^2\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)-2b^2d^3\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\frac{b^2d^3\,\mathrm{a}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(d^3*a^2*(1/5*c^5*x^5+c^3*x^3+3*c*x-1/c/x)-2*b^2*d^3*arcsinh(c*x)*ln(1+c*
x+(c^2*x^2+1)^(1/2))+1/5*b^2*d^3*arcsinh(c*x)^2*c^5*x^5+b^2*d^3*arcsinh(c*x
)^2*c^3*x^3+3*b^2*d^3*arcsinh(c*x)^2*c*x-122/25*b^2*d^3*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)-2*b^2*d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+122/25*b^2*d^3*c*x+
14/75*b^2*d^3*c^3*x^3+2/125*b^2*d^3*c^5*x^5+2*b^2*d^3*polylog(2,c*x+(c^2*x^
2+1)^(1/2))-2/25*b^2*d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4*x^4-14/25*b^2*d
^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+2*b^2*d^3*arcsinh(c*x)*ln(1-c*x-(
c^2*x^2+1)^(1/2))-b^2*d^3*arcsinh(c*x)^2/c/x+2*d^3*a*b*(1/5*arcsinh(c*x)*c^
5*x^5+arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/25*c^4*x^4
*(c^2*x^2+1)^(1/2)-7/25*c^2*x^2*(c^2*x^2+1)^(1/2)-61/25*(c^2*x^2+1)^(1/2)-a
rctanh(1/(c^2*x^2+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^6*d^3*x^5 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/
c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^6*d^3
 + a^2*c^4*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^3 + 3*b^2*c^2*d^3*x*arcsinh(c*x)^2 +
 6*b^2*c^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^2*d^3*x + 6
*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^3 - 2*(c*arcsinh(1/(c*abs(x
))) + arcsinh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/5*(b^2*c^6*d^3*x^6 + 5*b^2*c^
4*d^3*x^4 - 5*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2/5*(b^
2*c^9*d^3*x^8 + 6*b^2*c^7*d^3*x^6 + 5*b^2*c^5*d^3*x^4 - 5*b^2*c^3*d^3*x^2 -
 5*b^2*c*d^3 + (b^2*c^8*d^3*x^7 + 5*b^2*c^6*d^3*x^5 - 5*b^2*c^2*d^3*x)*sqrt
(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2*x^3 + x
)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int 3a^{2}c^{2}\,dx+\int \frac{a^{2}}{x^{2}}\,dx+\int 3a^{2}c^{4}x^{2}\,dx+\int a^{2}c^{6}x^{4}\,dx+\int 3b^{2}c^{2}\operatorname{asinh}^{2}{\left(cx\right)}\,dx+\int \frac{b^{2}\operatorname{asinh}^{2}{\left(cx\right)}}{x^{2}}\,dx+\int 6abc^{2}\operatorname{asinh}{\left(cx\right)}\,dx+\int \frac{2ab\operatorname{asinh}{\left(cx\right)}}{x^{2}}\,dx+\int 3b^{2}c^{4}x^{2}\operatorname{asinh}^{2}{\left(cx\right)}\,dx+\int b^{2}c^{6}x^{4}\operatorname{asinh}^{2}{\left(cx\right)}\,dx+\int 6abc^{4}x^{2}\operatorname{asinh}{\left(cx\right)}\,dx+\int 2abc^{6}x^{4}\operatorname{asinh}{\left(cx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*c
**4*x**2, x) + Integral(a**2*c**6*x**4, x) + Integral(3*b**2*c**2*asinh(c*x
)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*asinh
(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x) + Integral(3*b**2*c**4*x**2*
asinh(c*x)**2, x) + Integral(b**2*c**6*x**4*asinh(c*x)**2, x) + Integral(6*
a*b*c**4*x**2*asinh(c*x), x) + Integral(2*a*b*c**6*x**4*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^2, x)
```

3.223 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=354

$$\frac{21}{32}b^2c^4d^3x^2+\frac{1}{32}b^2c^6d^3x^4-\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))+\frac{7}{8}bc^3d^3x(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))$$

```
[Out] 21/32*b^2*c^4*d^3*x^2+1/32*b^2*c^6*d^3*x^4+7/8*b*c^3*d^3*x*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))-b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/x-3/32*c^
2*d^3*(a+b*arcsinh(c*x))^2+3/2*c^2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+3/4
*c^2*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-1/2*d^3*(c^2*x^2+1)^3*(a+b*arcs
inh(c*x))^2/x^2+c^2*d^3*(a+b*arcsinh(c*x))^3/b+3*c^2*d^3*(a+b*arcsinh(c*x))
^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^3*ln(x)-3*b*c^2*d^3*(a+b*arc
sinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b^2*c^2*d^3*polylog(3
,1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/16*b*c^3*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+
1)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5807, 5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30, 5786, 14, 272, 45}

$$-3bc^2d^3\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))+\frac{3}{4}c^2d^3(c^2x^2+1)^2(a+b\sinh^{-1}(cx))^2+\frac{3}{2}c^2d^3(c^2x^2+1)(a+b\sinh^{-1}(cx))^2-\frac{bcd^3(c^2x^2+1)^{5/2}(a+b\sinh^{-1}(cx))}{x}-\frac{d^3(c^2x^2+1)^3(a+b\sinh^{-1}(cx))^2}{2x^2}+\frac{c^2d^3(a+b\sinh^{-1}(cx))^3}{b}-\frac{3}{32}c^2d^3(a+b\sinh^{-1}(cx))^2+3c^2d^3\log\left(1-e^{-2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2+\frac{7}{8}bc^3d^3x(c^2x^2+1)^{3/2}(a+b\sinh^{-1}(cx))-\frac{3}{16}bc^3d^3x\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))+\frac{1}{32}b^2c^6d^3x^4+\frac{21}{32}b^2c^4d^3x^2-\frac{3}{2}b^2c^2d^3\text{Li}_3\left(e^{-2\sinh^{-1}(cx)}\right)+b^2c^2d^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (21*b^2*c^4*d^3*x^2)/32 + (b^2*c^6*d^3*x^4)/32 - (3*b*c^3*d^3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/16 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b
*ArcSinh[c*x]))/8 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x -
(3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/32 + (3*c^2*d^3*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/2 + (3*c^2*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - (d
^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (c^2*d^3*(a + b*ArcSin
h[c*x])^3)/b + 3*c^2*d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*ArcSinh[c*x])
] + b^2*c^2*d^3*Log[x] - 3*b*c^2*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*
ArcSinh[c*x])] - (3*b^2*c^2*d^3*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
```

```
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^3 (a+b\sinh^{-1}(cx))^2}{x^3}\,dx = -\frac{d^3(1+c^2x^2)^3 (a+b\sinh^{-1}(cx))^2}{2x^2} + (3c^2d)\int \frac{(d+c^2dx^2)^2 (a+b}{x}$$

$$= -\frac{bcd^3(1+c^2x^2)^{5/2} (a+b\sinh^{-1}(cx))}{x} + \frac{3}{4}c^2d^3(1+c^2x^2)^2 (a+b\,\mathrm{si}$$

$$= \frac{7}{8}bc^3d^3x(1+c^2x^2)^{3/2} (a+b\sinh^{-1}(cx)) - \frac{bcd^3(1+c^2x^2)^{5/2} (a+b}{x}$$

$$= -\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2} (a+b\sinh^{-1}(cx)) + \frac{7}{8}bc^3d^3x(1+c^2x^2)^{3/2} ($$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2} (a+b\sinh^{-1}(cx))$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2} (a+b\sinh^{-1}(cx))$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2} (a+b\sinh^{-1}(cx))$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2} (a+b\sinh^{-1}(cx))$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2} (a+b\sinh^{-1}(cx))$$

Mathematica [A]
time = 0.39, size = 512, normalized size = 1.45

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d^3*(-128*a^2 + 384*a^2*c^4*x^4 + 64*a^2*c^6*x^6 - 256*a*b*c*x*Sqrt[1 + c^
2*x^2] - 336*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 32*a*b*c^5*x^5*Sqrt[1 + c^2*x^
2] - 256*a*b*ArcSinh[c*x] + 768*a*b*c^4*x^4*ArcSinh[c*x] + 128*a*b*c^6*x^6*
ArcSinh[c*x] - 256*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 128*b^2*ArcSinh
[c*x]^2 + 768*a*b*c^2*x^2*ArcSinh[c*x]^2 - 256*b^2*c^2*x^2*ArcSinh[c*x]^3 +
 336*a*b*c^2*x^2*ArcTanh[(c*x)/Sqrt[1 + c^2*x^2]] + 80*b^2*c^2*x^2*Cosh[2*A
rcSinh[c*x]] + 160*b^2*c^2*x^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + b^2*c^
2*x^2*Cosh[4*ArcSinh[c*x]] + 8*b^2*c^2*x^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*
x]] + 1536*a*b*c^2*x^2*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] + 768*b^2*
c^2*x^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 768*a^2*c^2*x^2*Log[x]
 + 256*b^2*c^2*x^2*Log[c*x] - 768*a*b*c^2*x^2*PolyLog[2, E^(-2*ArcSinh[c*x]
)] + 768*b^2*c^2*x^2*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - 384*b^2*
c^2*x^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 160*b^2*c^2*x^2*ArcSinh[c*x]*Sinh[
2*ArcSinh[c*x]] - 4*b^2*c^2*x^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(256*x^
2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(357) = 714$.
time = 8.82, size = 788, normalized size = 2.23

method	result
derivativedivides	$c^2\left(\frac{21b^2c^2d^3x^2}{32}+\frac{b^2c^4d^3x^4}{32}+6d^3ab\operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)-\frac{d^3ab\sqrt{c^2x^2+1}}{cx}\right.$
default	$c^2\left(\frac{21b^2c^2d^3x^2}{32}+\frac{b^2c^4d^3x^4}{32}+6d^3ab\operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)-\frac{d^3ab\sqrt{c^2x^2+1}}{cx}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(21/32*b^2*c^2*d^3*x^2+1/32*b^2*c^4*d^3*x^4-d^3*a*b/c/x*(c^2*x^2+1)^(1/
2)+d^3*a*b+3*d^3*a^2*ln(c*x)-1/8*d^3*a*b*(c^2*x^2+1)^(1/2)*c^3*x^3-21/16*d^
3*a*b*c*x*(c^2*x^2+1)^(1/2)-1/8*b^2*d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*
x^3-21/16*b^2*d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1/4*d^3*a^2*c^4*x^4+3/
2*d^3*a^2*c^2*x^2+21/16*d^3*a*b*arcsinh(c*x)+6*d^3*a*b*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))-3*d^3*a*b*arcsinh(c*x)^2+6*d^3*a*b*polylog(2,c*x+(c^2*x^2+1)^
(1/2))+6*b^2*d^3*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+3*b^2*d^3*ar
csinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+6*b^2*d^3*arcsinh(c*x)*polylog(2,-
c*x-(c^2*x^2+1)^(1/2))+3*b^2*d^3*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))
+1/2*d^3*a*b*arcsinh(c*x)*c^4*x^4+3*d^3*a*b*arcsinh(c*x)*c^2*x^2+6*d^3*a*b*
arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+6*d^3*a*b*arcsinh(c*x)*ln(1+c*x+(c
^2*x^2+1)^(1/2))+1/4*b^2*d^3*arcsinh(c*x)^2*c^4*x^4+3/2*b^2*d^3*arcsinh(c*x
)^2*c^2*x^2+81/256*b^2*d^3+b^2*d^3*ln(1+c*x+(c^2*x^2+1)^(1/2))+b^2*d^3*ln(c
*x+(c^2*x^2+1)^(1/2)-1)-2*b^2*d^3*ln(c*x+(c^2*x^2+1)^(1/2))+b^2*d^3*arcsinh
```

```
(c*x)-d^3*a*b*arcsinh(c*x)/c^2/x^2-b^2*d^3*arcsinh(c*x)/c/x*(c^2*x^2+1)^(1/
2)-1/2*d^3*a^2/c^2/x^2-6*b^2*d^3*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+21/32*b^
2*d^3*arcsinh(c*x)^2-6*b^2*d^3*polylog(3,c*x+(c^2*x^2+1)^(1/2))-b^2*d^3*arc
sinh(c*x)^3-1/2*b^2*d^3*arcsinh(c*x)^2/c^2/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^6*d^3*x^4 + 3/2*a^2*c^4*d^3*x^2 + 3*a^2*c^2*d^3*log(x) - a*b*d^3*
(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a^2*d^3/x^2 + integrate(b^
2*c^6*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^6*d^3*x^3*log(c*x +
sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b
*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2*d^3*log(c*x + sqrt(c^2*
x^2 + 1))^2/x + 6*a*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d^3*log(
c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int \frac{a^{2}}{x^{3}}\, dx + \int \frac{3a^{2}c^{2}}{x}\, dx + \int 3a^{2}c^{4}x\, dx + \int a^{2}c^{6}x^{3}\, dx + \int \frac{b^{2}\operatorname{asinh}^{2}{(cx)}}{x^{3}}\, dx + \int \frac{2ab\operatorname{asinh}{(cx)}}{x^{3}}\, dx + \int \frac{3b^{2}c^{2}\operatorname{asinh}^{2}{(cx)}}{x}\, dx + \int 3b^{2}c^{4}x\operatorname{asinh}^{2}{(cx)}\, dx + \int b^{2}c^{6}x^{3}\operatorname{asinh}^{2}{(cx)}\, dx + \int \frac{6abc^{2}\operatorname{asinh}{(cx)}}{x}\, dx + \int 6abc^{4}x\operatorname{asinh}{(cx)}\, dx + \int 2abc^{6}x^{3}\operatorname{asinh}{(cx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d**3*(Integral(a**2/x**3, x) + Integral(3*a**2*c**2/x, x) + Integral(3*a**2
*c**4*x, x) + Integral(a**2*c**6*x**3, x) + Integral(b**2*asinh(c*x)**2/x**
3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(3*b**2*c**2*asinh(c*x
)**2/x, x) + Integral(3*b**2*c**4*x*asinh(c*x)**2, x) + Integral(b**2*c**6*
```

```
x**3*asinh(c*x)**2, x) + Integral(6*a*b*c**2*asinh(c*x)/x, x) + Integral(6*
a*b*c**4*x*asinh(c*x), x) + Integral(2*a*b*c**6*x**3*asinh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2\,(d\,c^2\,x^2 + d)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^3, x)
```

3.224 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=326

$$-\frac{b^2c^2d^3}{3x}+\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3-5bc^3d^3\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))+\frac{1}{9}bc^3d^3(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(c$$

```
[Out] -1/3*b^2*c^2*d^3/x+50/9*b^2*c^4*d^3*x+2/27*b^2*c^6*d^3*x^3+1/9*b*c^3*d^3*(c
^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-1/3*b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsi
nh(c*x))/x^2+16/3*c^4*d^3*x*(a+b*arcsinh(c*x))^2+8/3*c^4*d^3*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2-2*c^2*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x-1/3*d^3
*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/x^3-34/3*b*c^3*d^3*(a+b*arcsinh(c*x))*a
rctanh(c*x+(c^2*x^2+1)^(1/2))-17/3*b^2*c^3*d^3*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))+17/3*b^2*c^3*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-5*b*c^3*d^3*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5807, 5786, 5772, 5798, 8, 5808, 5806, 5816, 4267, 2317, 2438, 276}

$$\frac{16}{3}c^4d^3x(a+b\sinh^{-1}(cx))^2-\frac{34}{3}bc^3d^3\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))-\frac{2c^2d^3(c^2x^2+1)^2(a+b\sinh^{-1}(cx))^2}{x}-\frac{bcd^3(c^2x^2+1)^{5/2}(a+b\sinh^{-1}(cx))}{3x^2}-\frac{d^3(c^2x^2+1)^3(a+b\sinh^{-1}(cx))^2}{3x^3}+\frac{8}{3}c^4d^3x(c^2x^2+1)(a+b\sinh^{-1}(cx))^2+\frac{1}{9}bc^3d^3(c^2x^2+1)^{3/2}(a+b\sinh^{-1}(cx))-5bc^3d^3\sqrt{c^2x^2+1}(a+b\sinh^{-1}(cx))+\frac{2}{27}b^2c^6d^3x^3+\frac{50}{9}b^2c^4d^3x-\frac{17}{3}b^2c^3d^3\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\frac{17}{3}b^2c^3d^3\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)-\frac{b^2c^2d^3}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d^3)/x + (50*b^2*c^4*d^3*x)/9 + (2*b^2*c^6*d^3*x^3)/27 - 5*b*
c^3*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + (b*c^3*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/9 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/(3*x^2) + (16*c^4*d^3*x*(a + b*ArcSinh[c*x])^2)/3 + (8*c^4*d^3*x*(1
+ c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*c^2*d^3*(1 + c^2*x^2)^2*(a + b*Ar
cSinh[c*x])^2)/x - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(3*x^3) - (
34*b*c^3*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 - (17*b^2*c^3*
d^3*PolyLog[2, -E^ArcSinh[c*x]])/3 + (17*b^2*c^3*d^3*PolyLog[2, E^ArcSinh[c
*x]])/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
```

```
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^3 \left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^3(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3} + (2c^2d)\int \frac{(d+c^2dx^2)^2\left(a+\right.}{x^2}$$

$$= -\frac{bcd^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{2c^2d^3(1+c^2x^2)^2\left(a+b\,\mathrm{s}\right.}{x}$$

$$= \frac{17}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^3(1+c^2x^2)^{5/2}\left(a+\right.}{3x^2}$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{11}{9}b^2c^4d^3x - \frac{14}{27}b^2c^6d^3x^3 + \frac{17}{3}bc^3d^3\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{46}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sin\right.$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{50}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sin\right.$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{50}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sin\right.$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{50}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sin\right.$$

Mathematica [A]
time = 0.68, size = 461, normalized size = 1.41

$$\frac{d^3\left(-9a^2-81a^2c^2x^2-9b^2c^2x^2+81a^2c^4x^4+150b^2c^4x^4+9a^2c^6x^6+2b^2c^6x^6-9abcx\sqrt{1+c^2x^2}-150abc^3x^3\sqrt{1+c^2x^2}-6abc^5x^5\sqrt{1+c^2x^2}-18ab\sinh^{-1}(cx)-162abc^2x^2\sinh^{-1}(cx)+162abc^4x^4\sinh^{-1}(cx)+18abc^6x^6\sinh^{-1}(cx)-9b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)-150b^2c^3x^3\sqrt{1+c^2x^2}\sinh^{-1}(cx)-6b^2c^5x^5\sqrt{1+c^2x^2}\sinh^{-1}(cx)-9b^2\sinh^{-1}(cx)^2-81b^2c^2x^2\sinh^{-1}(cx)^2+81b^2c^4x^4\sinh^{-1}(cx)^2+9b^2c^6x^6\sinh^{-1}(cx)^2-153abc^3x^3\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)+153b^2c^3x^3\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-153b^2c^3x^3\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+153b^2c^3x^3\,\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-153b^2c^3x^3\,\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-9*a^2 - 81*a^2*c^2*x^2 - 9*b^2*c^2*x^2 + 81*a^2*c^4*x^4 + 150*b^2*c^
4*x^4 + 9*a^2*c^6*x^6 + 2*b^2*c^6*x^6 - 9*a*b*c*x*Sqrt[1 + c^2*x^2] - 150*a
*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 6*a*b*c^5*x^5*Sqrt[1 + c^2*x^2] - 18*a*b*Arc
Sinh[c*x] - 162*a*b*c^2*x^2*ArcSinh[c*x] + 162*a*b*c^4*x^4*ArcSinh[c*x] + 1
8*a*b*c^6*x^6*ArcSinh[c*x] - 9*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 150
*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^5*x^5*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x] - 9*b^2*ArcSinh[c*x]^2 - 81*b^2*c^2*x^2*ArcSinh[c*x]^2 + 81
*b^2*c^4*x^4*ArcSinh[c*x]^2 + 9*b^2*c^6*x^6*ArcSinh[c*x]^2 - 153*a*b*c^3*x^
3*ArcTanh[Sqrt[1 + c^2*x^2]] + 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] - 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 153
*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 153*b^2*c^3*x^3*PolyLog[2, E^
(-ArcSinh[c*x])]))/(27*x^3)
```

Maple [A]
time = 6.47, size = 468, normalized size = 1.44

method	result
derivativedivides	$c^3\left(d^3a^2\left(\frac{c^3x^3}{3}+3cx-\frac{1}{3c^3x^3}-\frac{3}{cx}\right)+\frac{17b^2d^3\,\mathrm{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{3}-\frac{17b^2d^3\,\mathrm{arcsinh}(cx}{}\right.$
default	$c^3\left(d^3a^2\left(\frac{c^3x^3}{3}+3cx-\frac{1}{3c^3x^3}-\frac{3}{cx}\right)+\frac{17b^2d^3\,\mathrm{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{3}-\frac{17b^2d^3\,\mathrm{arcsinh}(cx}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d^3*a^2*(1/3*c^3*x^3+3*c*x-1/3/c^3/x^3-3/c/x)+17/3*b^2*d^3*arcsinh(c*x
)*ln(1-c*x-(c^2*x^2+1)^(1/2))-17/3*b^2*d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1
)^(1/2))+50/9*b^2*d^3*c*x-1/3*b^2*d^3/c/x+2/27*b^2*d^3*c^3*x^3+17/3*b^2*d^3
*polylog(2,c*x+(c^2*x^2+1)^(1/2))-17/3*b^2*d^3*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))-1/3*b^2*d^3/c^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2/9*b^2*d^3*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2-3*b^2*d^3*arcsinh(c*x)^2/c/x-1/3*b^2*d^3/c
^3/x^3*arcsinh(c*x)^2+1/3*b^2*d^3*arcsinh(c*x)^2*c^3*x^3+3*b^2*d^3*arcsinh(
c*x)^2*c*x-50/9*b^2*d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*d^3*a*b*(1/3*arcsi
nh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-3*arcsinh(c*x)/
c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-25/9*(c^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x
^2+1)^(1/2)-17/6*arctanh(1/(c^2*x^2+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^6*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^6*d^3 + 3*b^2*c^4*d^3*x*arcsinh(c*x)^2
+ 6*b^2*c^4*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^4*d^3*x +
6*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3*d^3 - 6*(c*arcsinh(1/(c*ab
s(x))) + arcsinh(c*x)/x)*a*b*c^2*d^3 + 1/3*((c^2*arcsinh(1/(c*abs(x))) - sq
rt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d^3 - 3*a^2*c^2*d^3/x - 1/
3*a^2*d^3/x^3 + 1/3*(b^2*c^6*d^3*x^6 - 9*b^2*c^2*d^3*x^2 - b^2*d^3)*log(c*x
 + sqrt(c^2*x^2 + 1))^2/x^3 - integrate(2/3*(b^2*c^9*d^3*x^8 + b^2*c^7*d^3*
x^6 - 9*b^2*c^5*d^3*x^4 - 10*b^2*c^3*d^3*x^2 - b^2*c*d^3 + (b^2*c^8*d^3*x^7
 - 9*b^2*c^4*d^3*x^3 - b^2*c^2*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^4, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int 3a^{2}c^{4}\,dx+\int\frac{a^{2}}{x^{4}}\,dx+\int\frac{3a^{2}c^{2}}{x^{2}}\,dx+\int a^{2}c^{6}x^{2}\,dx+\int 3b^{2}c^{4}\operatorname{asinh}^{2}(cx)\,dx+\int\frac{b^{2}\operatorname{asinh}^{2}(cx)}{x^{4}}\,dx+\int 6abc^{4}\operatorname{asinh}(cx)\,dx+\int\frac{2ab\operatorname{asinh}(cx)}{x^{4}}\,dx+\int\frac{3b^{2}c^{2}\operatorname{asinh}^{2}(cx)}{x^{2}}\,dx+\int b^{2}c^{6}x^{2}\operatorname{asinh}^{2}(cx)\,dx+\int\frac{6abc^{2}\operatorname{asinh}(cx)}{x^{2}}\,dx+\int 2abc^{6}x^{2}\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d**3*(Integral(3*a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(3*a**2*c
**2/x**2, x) + Integral(a**2*c**6*x**2, x) + Integral(3*b**2*c**4*asinh(c*x
)**2, x) + Integral(b**2*asinh(c*x)**2/x**4, x) + Integral(6*a*b*c**4*asinh
(c*x), x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(3*b**2*c**2*asinh
(c*x)**2/x**2, x) + Integral(b**2*c**6*x**2*asinh(c*x)**2, x) + Integral(6*
a*b*c**2*asinh(c*x)/x**2, x) + Integral(2*a*b*c**6*x**2*asinh(c*x), x))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^3}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^4, x)
```

3.225 $\int \frac{x^4(a+b\sinh^{-1}(cx))^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=277

$$-\frac{22b^2x}{9c^4d}+\frac{2b^2x^3}{27c^2d}+\frac{22b\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{9c^5d}-\frac{2bx^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{9c^3d}-\frac{x(a+b\sinh^{-1}(cx)}{c^4d}$$

```
[Out] -22/9*b^2*x/c^4/d+2/27*b^2*x^3/c^2/d-x*(a+b*arcsinh(c*x))^2/c^4/d+1/3*x^3*(
a+b*arcsinh(c*x))^2/c^2/d+2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/
2))/c^5/d-2*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^
5/d+2*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d+2*I
*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d-2*I*b^2*polylog(3,I*(c*x+(
c^2*x^2+1)^(1/2)))/c^5/d+22/9*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/d-
2/9*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A]
time = 0.41, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5812, 5789, 4265, 2611, 2320, 6724, 5798, 8, 30}

$$\frac{2\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c^5d}-\frac{2ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}+\frac{22b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d}-\frac{2bx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d}+\frac{2ib^2\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{c^5d}-\frac{2ib^2\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{c^5d}-\frac{22b^2x}{9c^4d}+\frac{2b^2x^3}{27c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-22*b^2*x)/(9*c^4*d) + (2*b^2*x^3)/(27*c^2*d) + (22*b*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(9*c^5*d) - (2*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(9*c^3*d) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d) + (x^3*(a + b*ArcSinh[
c*x])^2)/(3*c^2*d) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5
*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d
) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d) + (
(2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d) - ((2*I)*b^2*PolyLog[3,
I*E^ArcSinh[c*x]])/(c^5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d} - \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2} - \frac{(2b)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{3cd}$$

$$= -\frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d} + \frac{x^3\left(a+b\sinh\right.}{3c^2d}$$

$$= \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+\right.}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+\right.}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+\right.}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+\right.}{9c^3d}$$

Mathematica [A]
time = 0.85, size = 365, normalized size = 1.32

$$\frac{-3a^2cx+a^2c^3x^3+3a^2\text{ArcTan}(cx)-\frac{2}{3}ab\left(-11\sqrt{1+c^2x^2}+c^2x^2\sqrt{1+c^2x^2}+9cx\sinh^{-1}(cx)-3c^3x^3\sinh^{-1}(cx)-9i\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+9i\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)+9i\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-9i\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\right)+3b^2\left(\frac{5}{2}\sqrt{1+c^2x^2}\sinh^{-1}(cx)-\frac{5}{4}cx\left(2+\sinh^{-1}(cx)^2\right)-\frac{1}{18}\sinh^{-1}(cx)\cosh\left(3\sinh^{-1}(cx)\right)+i\left(-\sinh^{-1}(cx)^2\left(\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-\log\left(1+ie^{-\sinh^{-1}(cx)}\right)\right)-2\sinh^{-1}(cx)\left(\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\right)-2\text{PolyLog}\left(3,-ie^{-\sinh^{-1}(cx)}\right)+2\text{PolyLog}\left(3,ie^{-\sinh^{-1}(cx)}\right)\right)+\frac{1}{108}\left(2+9\sinh^{-1}(cx)^2\right)\sinh\left(3\sinh^{-1}(cx)\right)\right)}{3c^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-3*a^2*c*x + a^2*c^3*x^3 + 3*a^2*ArcTan[c*x] - (2*a*b*(-11*Sqrt[1 + c^2*x^
2] + c^2*x^2*Sqrt[1 + c^2*x^2] + 9*c*x*ArcSinh[c*x] - 3*c^3*x^3*ArcSinh[c*x
] - (9*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (9*I)*ArcSinh[c*x]*Log[1
 + I*E^ArcSinh[c*x]] + (9*I)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9*I)*PolyLo
g[2, I*E^ArcSinh[c*x]]))/3 + 3*b^2*((5*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/2 -
```

```
(5*c*x*(2 + ArcSinh[c*x]^2))/4 - (ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]])/18 + I
*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^ArcSinh[c*x]]))
 - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[2, I/E^ArcSinh
[c*x]]) - 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] + 2*PolyLog[3, I/E^ArcSinh[c*x]
]) + ((2 + 9*ArcSinh[c*x]^2)*Sinh[3*ArcSinh[c*x]])/108))/(3*c^5*d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\,\mathrm{arcsinh}\,(cx))^2}{c^2d\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + integrate(b^2*x
^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^4*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^2*d
*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^4}{c^2x^2+1}\,dx + \int \frac{b^2x^4\,\mathrm{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^4\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**4/(c**2*x**2 + 1), x) + Integral(b**2*x**4*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{d\,c^2\,x^2+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

3.226 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=199

$$\frac{b^2x^2}{4c^2d}-\frac{bx\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2c^3d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^4d}$$

```
[Out] 1/4*b^2*x^2/c^2/d+1/4*(a+b*arcsinh(c*x))^2/c^4/d+1/2*x^2*(a+b*arcsinh(c*x))
^2/c^2/d+1/3*(a+b*arcsinh(c*x))^3/b/c^4/d-(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)/c^4/d-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^
(1/2))^2)/c^4/d+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/2*b*x
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A]

time = 0.30, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5812, 5797, 3799, 2221, 2611, 2320, 6724, 5783, 30}

$$-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^4d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d}-\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d}+\frac{b^2\text{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{b^2x^2}{4c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (b^2*x^2)/(4*c^2*d) - (b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^3*d
) + (a + b*ArcSinh[c*x])^2/(4*c^4*d) + (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*
d) + (a + b*ArcSinh[c*x])^3/(3*b*c^4*d) - ((a + b*ArcSinh[c*x])^2*Log[1 + E
^(2*ArcSinh[c*x])])/(c^4*d) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c^4*d) + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \frac{\text{Subst}\left(\int (a+\right.}{}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\,\mathrm{s}\right.}{2}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\,\mathrm{s}\right.}{2}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\,\mathrm{s}\right.}{2}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\,\mathrm{s}\right.}{2}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\,\mathrm{s}\right.}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 292, normalized size = 1.47

$$\frac{12a^2c^2x^2 - 12abcx\sqrt{1+c^2x^2} + 24abc^2x^2\sinh^{-1}(cx) + 24ab\sinh^{-1}(cx)^2 - 8b^2\sinh^{-1}(cx)^3 + 12ab\tanh^{-1}\left(\frac{cx}{\sqrt{1+c^2x^2}}\right) + 3b^2\cosh\left(2\sinh^{-1}(cx)\right) + 6b^2\sinh^{-1}(cx)^2\cosh\left(2\sinh^{-1}(cx)\right) - 24b^2\sinh^{-1}(cx)^2\log\left(1+e^{-2\sinh^{-1}(cx)}\right) - 48ab\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right) - 48ab\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right) - 12a^2\log\left(1+c^2x^2\right) + 24b^2\sinh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right) - 48ab\,\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right) - 48ab\,\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right) + 12b^2\text{PolyLog}\left(3,-e^{-2\sinh^{-1}(cx)}\right) - 6b^2\sinh^{-1}(cx)\sinh\left(2\sinh^{-1}(cx)\right)}{24c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (12*a^2*c^2*x^2 - 12*a*b*c*x*Sqrt[1 + c^2*x^2] + 24*a*b*c^2*x^2*ArcSinh[c*x
] + 24*a*b*ArcSinh[c*x]^2 - 8*b^2*ArcSinh[c*x]^3 + 12*a*b*ArcTanh[(c*x)/Sqr
t[1 + c^2*x^2]] + 3*b^2*Cosh[2*ArcSinh[c*x]] + 6*b^2*ArcSinh[c*x]^2*Cosh[2*
ArcSinh[c*x]] - 24*b^2*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] - 48*a*b
*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - 48*a*b*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] - 12*a^2*Log[1 + c^2*x^2] + 24*b^2*ArcSinh[c*x]*PolyLog[2, -E^
(-2*ArcSinh[c*x])] - 48*a*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] - 48*a*b*PolyLo
g[2, I*E^ArcSinh[c*x]] + 12*b^2*PolyLog[3, -E^(-2*ArcSinh[c*x])] - 6*b^2*Ar
cSinh[c*x]*Sinh[2*ArcSinh[c*x]])/(24*c^4*d)
```

Maple [A]
time = 3.80, size = 347, normalized size = 1.74

method	result
derivativedivides	$\frac{a^2c^2x^2}{2d}-\frac{a^2\ln\left(c^2x^2+1\right)}{2d}+\frac{b^2\operatorname{arcsinh}(cx)^3}{3d}+\frac{b^2\operatorname{arcsinh}(cx)^2c^2x^2}{2d}-\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{2d}+\frac{b^2\operatorname{arcsinh}(cx)^2}{4d}+\frac{b^2c^2x^2}{4d}+$
default	$\frac{a^2c^2x^2}{2d}-\frac{a^2\ln\left(c^2x^2+1\right)}{2d}+\frac{b^2\operatorname{arcsinh}(cx)^3}{3d}+\frac{b^2\operatorname{arcsinh}(cx)^2c^2x^2}{2d}-\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{2d}+\frac{b^2\operatorname{arcsinh}(cx)^2}{4d}+\frac{b^2c^2x^2}{4d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*a^2/d*c^2*x^2-1/2*a^2/d*ln(c^2*x^2+1)+1/3*b^2/d*arcsinh(c*x)^3+1
/2*b^2/d*arcsinh(c*x)^2*c^2*x^2-1/2*b^2/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*
x+1/4*b^2/d*arcsinh(c*x)^2+1/4*b^2/d*c^2*x^2+1/8*b^2/d-b^2/d*arcsinh(c*x)^2
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/d*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^
2+1)^(1/2))^2)+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+a*b/d*arcsin
h(c*x)^2+a*b/d*arcsinh(c*x)*c^2*x^2-1/2*a*b/d*c*x*(c^2*x^2+1)^(1/2)+1/2*a*b
/d*arcsinh(c*x)-2*a*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-a*b/d*
polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) + 1/2*(b^2*c^2*x^2 - b^2*l
og(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d) + integrate(-(b^2*c
^2*x^2 - (2*a*b*c^4 - b^2*c^4)*x^4 - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) -
 (b^2*c*x*log(c^2*x^2 + 1) + (2*a*b*c^3 - b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqr
t(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^2*d
*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^2x^2+1}\,dx + \int \frac{b^2x^3\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^3\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
[Out] (Integral(a**2*x**3/(c**2*x**2 + 1), x) + Integral(b**2*x**3*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}(c\,x)\right)}^2}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

3.227 $\int \frac{x^2 (a+b \sinh^{-1}(cx))^2}{d+c^2 dx^2} \, dx$

Optimal. Leaf size=198

$$\frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{c^3d} + \frac{x(a+b\sinh^{-1}(cx))^2}{c^2d} - \frac{2(a+b\sinh^{-1}(cx))^2 \operatorname{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d} + \cdots 2$$

```
[Out] 2*b^2*x/c^2/d+x*(a+b*arcsinh(c*x))^2/c^2/d-2*(a+b*arcsinh(c*x))^2*arctan(c*
x+(c^2*x^2+1)^(1/2))/c^3/d+2*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*
x^2+1)^(1/2)))/c^3/d-2*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^
(1/2)))/c^3/d-2*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d+2*I*b^2*p
olylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1
)^(1/2)/c^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5812, 5789, 4265, 2611, 2320, 6724, 5798, 8}

$$-\frac{2\operatorname{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{c^3d} + \frac{2ib\operatorname{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^3d} - \frac{2ib\operatorname{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^3d} + \frac{x(a+b\sinh^{-1}(cx))^2}{c^2d} - \frac{2b\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{c^3d} - \frac{2ib^2\operatorname{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{c^3d} + \frac{2ib^2\operatorname{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{c^3d} + \frac{2b^2x}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (2*b^2*x)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^3*d) +
(x*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^Arc
Sinh[c*x]])/(c^3*d) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/(c^3*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^3*d) - ((2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^3*d) + ((2*I)
*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2} - \frac{(2b)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\text{Subst}\left(\int (a+bx\right.}{}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sin\right.}{}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sin\right.}{}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sin\right.}{}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sin\right.}{}$$

Mathematica [A]
time = 0.40, size = 293, normalized size = 1.48

$$\frac{a^2x}{c^2d} - \frac{a^2\text{ArcTan}(cx)}{c^3d} + \frac{2ab\left(-\sqrt{1+c^2x^2}+cx\sinh^{-1}(cx)+i\sinh^{-1}(cx)\left(\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-\log\left(1+ie^{-\sinh^{-1}(cx)}\right)\right)+i\left(\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\right)\right)}{c^3d} + \frac{b^2\left(-2\sqrt{1+c^2x^2}\sinh^{-1}(cx)+cx\left(2+\sinh^{-1}(cx)^2\right)-i\left(-\sinh^{-1}(cx)^2\left(\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-\log\left(1+ie^{-\sinh^{-1}(cx)}\right)\right)-2\sinh^{-1}(cx)\left(\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\right)-2\left(\text{PolyLog}\left(3,-ie^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(3,ie^{-\sinh^{-1}(cx)}\right)\right)\right)\right)}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (a^2*x)/(c^2*d) - (a^2*ArcTan[c*x])/(c^3*d) + (2*a*b*(-Sqrt[1 + c^2*x^2] +
c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^
ArcSinh[c*x]]) + I*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[2, I/E^ArcSin
h[c*x]])))/(c^3*d) + (b^2*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*(2 + Arc
Sinh[c*x]^2) - I*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E
^ArcSinh[c*x]])) - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLo
g[2, I/E^ArcSinh[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSinh[c*x]] - PolyLog[3, I
/E^ArcSinh[c*x]]))))/(c^3*d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\,\text{arcsinh}\,(cx)\right)^2}{c^2d\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + integrate(b^2*x^2*log(c*x + sqrt(c^
2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2
*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^2*d
*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^2}{c^2x^2+1}\,dx + \int \frac{b^2x^2\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^2\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**2/(c**2*x**2 + 1), x) + Integral(b**2*x**2*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{d\,c^2\,x^2+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)

[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)

3.228 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=105

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}+\frac{b\left(a+b\sinh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-e^{2\sinh^{-1}}\right.}{c^2d}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^3/b/c^2/d+(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)/c^2/d+b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)/c^2/d-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5797, 3799, 2221, 2611, 2320, 6724}

$$\frac{b\mathrm{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}-\frac{b^2\mathrm{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] -1/3*(a + b*ArcSinh[c*x])^3/(b*c^2*d) + ((a + b*ArcSinh[c*x])^2*Log[1 + E^(
2*ArcSinh[c*x])])/(c^2*d) + (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSin
h[c*x])])/(c^2*d) - (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx &= \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^2d}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx,x,\sinh^{-1}(cx)\right)}{c^2d}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}-\frac{(2b)\text{Subst}}{}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}+\frac{b\left(a+b\sin\right.}{}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}+\frac{b\left(a+b\sin\right.}{}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}+\frac{b\left(a+b\sin\right.}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(105) = 210.

time = 0.18, size = 281, normalized size = 2.68

$$\frac{-6ab\sinh^{-1}(cx)^2-2b^2\sinh^{-1}(cx)^3+12ab\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+6b^2\sinh^{-1}(cx)^2\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+12ab\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+6b^2\sinh^{-1}(cx)^2\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+3a^2\log\left(1+c^2x^2\right)+12b(a+b\sinh^{-1}(cx))\,\mathrm{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+12b(a+b\sinh^{-1}(cx))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)-12b^2\mathrm{PolyLog}\left(3,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-12b^2\mathrm{PolyLog}\left(3,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)}{6c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-6*a*b*ArcSinh[c*x]^2 - 2*b^2*ArcSinh[c*x]^3 + 12*a*b*ArcSinh[c*x]*Log[1 +
 (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh
[c*x])/Sqrt[-c^2]] + 12*a*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*a^2*
Log[1 + c^2*x^2] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/
Sqrt[-c^2]] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*
x])/c] - 12*b^2*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^2*PolyLog[
3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*c^2*d)
```

Maple [A]
time = 2.09, size = 203, normalized size = 1.93

method	result
derivativedivides	$\frac{a^2\ln\left(c^2x^2+1\right)}{2d}-\frac{b^2\,\mathrm{arcsinh}(cx)^3}{3d}+\frac{b^2\,\mathrm{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{d}+\frac{b^2\,\mathrm{arcsinh}(cx)\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{c^2x^2\right.\right.}}{d}$
default	$\frac{a^2\ln\left(c^2x^2+1\right)}{2d}-\frac{b^2\,\mathrm{arcsinh}(cx)^3}{3d}+\frac{b^2\,\mathrm{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{d}+\frac{b^2\,\mathrm{arcsinh}(cx)\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{c^2x^2\right.\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*a^2/d*ln(c^2*x^2+1)-1/3*b^2/d*arcsinh(c*x)^3+b^2/d*arcsinh(c*x)^
2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2/d*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))^2)-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-a*b/d*arcsi
nh(c*x)^2+2*a*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+a*b/d*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d) + 1/2*a^2*l
og(c^2*d*x^2 + d)/(c^2*d) - integrate(-(2*a*b*c^2*x^2 - (b^2*c^2*x^2 + b^2)
```

```
*log(c^2*x^2 + 1) - (b^2*c*x*log(c^2*x^2 + 1) - 2*a*b*c*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d*x^3 + c^2*d*x + (c^3*d*x^2 + c*d)*sq
rt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a^2*x)/(c^2*d*x^2 +
 d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{c^2x^2+1}\,dx + \int \frac{b^2x\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2abx\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
[Out] (Integral(a**2*x/(c**2*x**2 + 1), x) + Integral(b**2*x*asinh(c*x)**2/(c**2*
x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

3.229 $\int \frac{(a+b\sinh^{-1}(cx))^2}{d+c^2dx^2} dx$

Optimal. Leaf size=138

$$\frac{2(a+b\sinh^{-1}(cx))^2 \operatorname{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib(a+b\sinh^{-1}(cx)) \operatorname{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{cd} + \frac{2ib(a+b\sinh$$

```
[Out] 2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-2*I*b*(a+b*arcsinh
(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b*(a+b*arcsinh(c*x))*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+
1)^(1/2)))/c/d-2*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5789, 4265, 2611, 2320, 6724}

$$\frac{2\operatorname{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{cd} - \frac{2ib\operatorname{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{cd} + \frac{2ib\operatorname{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{cd} + \frac{2ib^2\operatorname{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib^2\operatorname{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c*d) - ((2*I)*b*(a + b*A
rcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b^2*PolyLog[3, (-I)*E
^ArcSinh[c*x]])/(c*d) - ((2*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{cd}$$

$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(2ib)\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx\right.}{cd}$$

$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right.}{cd}$$

$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right.}{cd}$$

$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right.}{cd}$$

Mathematica [A]
time = 0.17, size = 274, normalized size = 1.99

$$-\frac{c\left(a^2\sqrt{-c^2}\,\text{ArcTan}(cx) - 2abc\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - b^2c\sinh^{-1}(cx)^2\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 2abc\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) + b^2c\sinh^{-1}(cx)^2\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) + 2bc\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - 2bc\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) - 2b^2c\text{PolyLog}\left(3,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 2b^2c\text{PolyLog}\left(3,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)\right)}{(-c^2)^{3/2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

```
[Out] -((c*(a^2*Sqrt[-c^2]*ArcTan[c*x] - 2*a*b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
h[c*x])/Sqrt[-c^2]] - b^2*c*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[
-c^2]] + 2*a*b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b^2*
c*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b*c*(a + b*ArcS
inh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 2*b*c*(a + b*ArcSinh[
c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b^2*c*PolyLog[3, (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] + 2*b^2*c*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c
]))/((-c^2)^(3/2)*d))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \,\mathrm{arcsinh}\left(c x\right)\right)^{2}}{c^{2} d \,x^{2}+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*arctan(c*x)/(c*d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d
*x^2 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^2 + d), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2}}{c^{2} x^{2} + 1}\, dx + \int \frac{b^{2} \operatorname{asinh}^{2}{\left(c x \right)}}{c^{2} x^{2} + 1}\, dx + \int \frac{2 a b \operatorname{asinh}{\left(c x \right)}}{c^{2} x^{2} + 1}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**2
 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2), x)
```

3.230 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x(d+c^2dx^2)}\,dx$

Optimal. Leaf size=116

$$-\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d}-\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d}+\frac{b\left(a+b\sinh^{-}\right.}{}$$

```
[Out] -2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d-b*(a+b*arcsinh
(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+b*(a+b*arcsinh(c*x))*polylog
(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^
2)/d-1/2*b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5799, 5569, 4267, 2611, 2320, 6724}

$$-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d}+\frac{b^2\text{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{b^2\text{Li}_3\left(e^{2\sinh^{-1}(cx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*(a + b*ArcSi
nh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d + (b*(a + b*ArcSinh[c*x])*PolyL
og[2, E^(2*ArcSinh[c*x])])/d + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d)
- (b^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx = \frac{\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(x)\,dx,x,\sinh^{-1}(cx)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int (a+bx)^2\text{csch}(2x)\,dx,x,\sinh^{-1}(cx)\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{(2b)\text{Subst}\left(\int (a+bx)\log\left(1-e^{2x}\right.\right.}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-e^{2\sinh^{-}}\right.}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-e^{2\sinh^{-}}\right.}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-e^{2\sinh^{-}}\right.}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 400 vs. 2(116) = 232.

time = 0.26, size = 400, normalized size = 3.45

$$-\frac{2a^3+6a^2b\sinh^{-1}(cx)+12ab^2\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+6b^3\sinh^{-1}(cx)^2\log\left(1+\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+12ab^2\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+6b^3\sinh^{-1}(cx)^2\log\left(1+\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)-6a^2b\log\left(1-e^{2\sinh^{-1}(cx)}\right)-12ab^2\sinh^{-1}(cx)\log\left(1-e^{2\sinh^{-1}(cx)}\right)-6b^3\sinh^{-1}(cx)^2\log\left(1-e^{2\sinh^{-1}(cx)}\right)+3a^2b\log\left(1+c^2x^2\right)+12b^2\left(a+b\sinh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+12b^2\left(a+b\sinh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)-6ab^2\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-6b^3\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-12b^3\mathrm{PolyLog}\left(3,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-12b^3\mathrm{PolyLog}\left(3,\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+3b^3\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

```
[Out] -1/6*(2*a^3 + 6*a^2*b*ArcSinh[c*x] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (c*E^Arc
Sinh[c*x])/Sqrt[-c^2]] + 6*b^3*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sq
rt[-c^2]] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] +
6*b^3*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6*a^2*b*Log[1
 - E^(2*ArcSinh[c*x])] - 12*a*b^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])]
- 6*b^3*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3*a^2*b*Log[1 + c^2*x^
2] + 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
+ 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6
*a*b^2*PolyLog[2, E^(2*ArcSinh[c*x])] - 6*b^3*ArcSinh[c*x]*PolyLog[2, E^(2*
ArcSinh[c*x])] - 12*b^3*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^3*
PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*b^3*PolyLog[3, E^(2*ArcSinh[c
*x])])/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(157) = 314$.
time = 3.11, size = 350, normalized size = 3.02

method	result
derivativedivides	$-\frac{a^2\ln(c^2x^2+1)}{2d}+\frac{a^2\ln(cx)}{d}-\frac{b^2\operatorname{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{d}-\frac{b^2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\left(c\right.\right.}{d}$
default	$-\frac{a^2\ln(c^2x^2+1)}{2d}+\frac{a^2\ln(cx)}{d}-\frac{b^2\operatorname{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{d}-\frac{b^2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\left(c\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/d*ln(c^2*x^2+1)+a^2/d*ln(c*x)-b^2/d*arcsinh(c*x)^2*ln(1+(c*x+(c^2*
x^2+1)^(1/2))^2)-b^2/d*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1
/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+b^2/d*arcsinh(c*x)^2*ln(1-c*
```

```
x-(c^2*x^2+1)^(1/2))+2*b^2/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-
2*b^2/d*polylog(3,c*x+(c^2*x^2+1)^(1/2))+b^2/d*arcsinh(c*x)^2*ln(1+c*x+(c^2
*x^2+1)^(1/2))+2*b^2/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*b^2
/d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+2*a*b/d*(dilog(1/(c*x+(c^2*x^2+1)^(1/2
))^2)-1/4*dilog(1/(c*x+(c^2*x^2+1)^(1/2))^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + integrate(b^2*log(c*x + sqrt(c
^2*x^2 + 1))^2/(c^2*d*x^3 + d*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*
d*x^3 + d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^3 + d*x),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^3+x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^3+x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^2x^3+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**3
 + x), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**3 + x), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2}{x\,\left(d\,c^2\,x^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)), x)
```

3.231 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=204

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx}-\frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\operatorname{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{d}-\frac{4bc\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x-2*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(
1/2))/d-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d-2*b^2*c*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))/d+2*I*b*c*(a+b*arcsinh(c*x))*polylog(2,-I*
(c*x+(c^2*x^2+1)^(1/2)))/d-2*I*b*c*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2
*x^2+1)^(1/2)))/d+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d-2*I*b^2*c*poly
log(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d+2*I*b^2*c*polylog(3,I*(c*x+(c^2*x^2+1)^
(1/2)))/d
```

Rubi [A]

time = 0.24, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5809, 5789, 4265, 2611, 2320, 6724, 5816, 4267, 2317, 2438}

$$-\frac{2c\operatorname{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d}+\frac{2ibc\operatorname{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2ibc\operatorname{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx}-\frac{4bc\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2b^2c\operatorname{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{d}+\frac{2b^2c\operatorname{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{d}-\frac{2ib^2c\operatorname{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{2ib^2c\operatorname{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x)) - (2*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/d - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d - (2*
b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d + ((2*I)*b*c*(a + b*ArcSinh[c*x])*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2,
 I*E^ArcSinh[c*x]])/d + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d - ((2*I)*b^2
*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d + ((2*I)*b^2*c*PolyLog[3, I*E^ArcSinh
[c*x]])/d
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
```

```
b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{c\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d} + \frac{(2bc)\text{Su}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\,\text{si}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\,\text{si}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\,\text{si}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\,\text{si}\right.}{}$$

Mathematica [A]
time = 0.74, size = 363, normalized size = 1.78

$$-\frac{\frac{a^2}{x}+\frac{2ab\sinh^{-1}(cx)}{x}+a^2c\text{ArcTan}(cx)+2abc\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)+\frac{1}{2}iabc\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)-\frac{1}{2}iabc\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1-ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\right)-b^2c\left(-\frac{\sinh^{-1}(cx)^2}{cx}+2\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+i\sinh^{-1}(cx)^2\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-i\sinh^{-1}(cx)^2\log\left(1+ie^{-\sinh^{-1}(cx)}\right)-2\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+2i\sinh^{-1}(cx)\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-2i\sinh^{-1}(cx)\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)-2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+2i\text{PolyLog}\left(3,-ie^{-\sinh^{-1}(cx)}\right)-2i\text{PolyLog}\left(3,ie^{-\sinh^{-1}(cx)}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a^2/x + (2*a*b*ArcSinh[c*x])/x + a^2*c*ArcTan[c*x] + 2*a*b*c*ArcTanh[Sqr
t[1 + c^2*x^2]] + (I/2)*a*b*c*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^A
rcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - (I/2)*a*b*c*(ArcSinh[c
*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh
[c*x]]) - b^2*c*(-(ArcSinh[c*x]^2/(c*x)) + 2*ArcSinh[c*x]*Log[1 - E^(-ArcSi
nh[c*x])] + I*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - I*ArcSinh[c*x]^2*L
og[1 + I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 2*Po
lyLog[2, -E^(-ArcSinh[c*x])] + (2*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh
[c*x]] - (2*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*PolyLog[2, E^(
-ArcSinh[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (2*I)*PolyLog[3,
I/E^ArcSinh[c*x]]))/d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \,\mathrm{arcsinh}\left(c x\right)\right)^{2}}{x^{2} \left(c^{2} d \,x^{2}+d \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -a^2*(c*arctan(c*x)/d + 1/(d*x)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^2*d*x^4 + d*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^4 +
d*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^4 + d*x^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^4+x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2{(cx)}}{c^2x^4+x^2}\,dx + \int \frac{2ab\operatorname{asinh}{(cx)}}{c^2x^4+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**4 + x**2), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**4 + x**2), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^2\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)), x)
```

3.232 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x^3(d+c^2dx^2)}\,dx$

Optimal. Leaf size=194

$$-\frac{bc\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{dx}-\frac{(a+b\sinh^{-1}(cx))^2}{2dx^2}+\frac{2c^2(a+b\sinh^{-1}(cx))^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d}+\frac{b^2c^2}{}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/d/x^2+2*c^2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^
2*x^2+1)^(1/2))^2)/d+b^2*c^2*ln(x)/d+b*c^2*(a+b*arcsinh(c*x))*polylog(2,-(c
*x+(c^2*x^2+1)^(1/2))^2)/d-b*c^2*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2
+1)^(1/2))^2)/d-1/2*b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b^2
*c^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d-b*c*(a+b*arcsinh(c*x))*(c^2*x^2
+1)^(1/2)/d/x
```

Rubi [A]
time = 0.28, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5809, 5799, 5569, 4267, 2611, 2320, 6724, 5800, 29}

$$\frac{bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{d}-\frac{bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{d}-\frac{bc\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{dx}+\frac{2c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{d}-\frac{(a+b\sinh^{-1}(cx))^2}{2dx^2}-\frac{b^2c^2\text{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{b^2c^2\text{Li}_3\left(e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{b^2c^2\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) - (a + b*ArcSinh[c*x]
)^2/(2*d*x^2) + (2*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/
d + (b^2*c^2*Log[x])/d + (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSi
nh[c*x])])/d - (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/
d - (b^2*c^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d) + (b^2*c^2*PolyLog[3, E
^(2*ArcSinh[c*x])])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)}\,dx &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{1+c^2x^2}}\,dx}{d} \\
&= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{c^2\text{Subst}\left(\int (a+bx)\right.}{} \\
&= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{b^2c^2\log(x)}{d} - \frac{(2c^2)}{} \\
&= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(c\right.}{} \\
&= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(c\right.}{} \\
&= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(c\right.}{} \\
&= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(c\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 419, normalized size = 2.16

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -1/2*((I/12)*b^2*c^2*Pi^3 + a^2/x^2 + (2*a*b*c*Sqrt[1 + c^2*x^2])/x + (2*a*
b*ArcSinh[c*x])/x^2 + (2*b^2*c*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/x + (b^2*Arc
Sinh[c*x]^2)/x^2 - (4*b^2*c^2*ArcSinh[c*x]^3)/3 - 2*b^2*c^2*ArcSinh[c*x]^2*
Log[1 + E^(-2*ArcSinh[c*x])] - 4*a*b*c^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c
*x]] - 4*a*b*c^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + 4*a*b*c^2*ArcSinh
[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*b^2*c^2*ArcSinh[c*x]^2*Log[1 - E^(2*A
rcSinh[c*x])] + 2*a^2*c^2*Log[x] - 2*b^2*c^2*Log[c*x] - a^2*c^2*Log[1 + c^2
*x^2] + 2*b^2*c^2*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - 4*a*b*c^2
*PolyLog[2, (-I)*E^ArcSinh[c*x]] - 4*a*b*c^2*PolyLog[2, I*E^ArcSinh[c*x]] +
```

```
2*a*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*c^2*ArcSinh[c*x]*PolyLog[
2, E^(2*ArcSinh[c*x])] + b^2*c^2*PolyLog[3, -E^(-2*ArcSinh[c*x])] - b^2*c^2
*PolyLog[3, E^(2*ArcSinh[c*x])]])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(231) = 462$.
time = 3.88, size = 670, normalized size = 3.45

method	result
derivativedivides	$c^2\left(\frac{a^2\ln(c^2x^2+1)}{2d}-\frac{b^2\,\mathrm{polylog}\left(3,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2d}-\frac{b^2\,\mathrm{arcsinh}(cx)^2}{2d\,c^2x^2}-\frac{2ab\,\mathrm{arcsinh}(cx)\ln\left(1-cx-\sqrt{\right.}}{d}\right.$
default	$c^2\left(\frac{a^2\ln(c^2x^2+1)}{2d}-\frac{b^2\,\mathrm{polylog}\left(3,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2d}-\frac{b^2\,\mathrm{arcsinh}(cx)^2}{2d\,c^2x^2}-\frac{2ab\,\mathrm{arcsinh}(cx)\ln\left(1-cx-\sqrt{\right.}}{d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a^2/d*ln(c^2*x^2+1)-b^2/d*arcsinh(c*x)/c/x*(c^2*x^2+1)^(1/2)+b^2/d
*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2/d*arcsinh(c*x)*polylog(
2,-(c*x+(c^2*x^2+1)^(1/2))^2)-2*b^2/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+
1)^(1/2))-2*a*b/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))+b^2/d*arcsinh(c*x)-a*b/d
*arcsinh(c*x)/c^2/x^2+a*b/d*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/d*a
rcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-a^2/d*ln(c*x)+a*b/d-a*b/d/c/x*(
c^2*x^2+1)^(1/2)+2*b^2/d*polylog(3,c*x+(c^2*x^2+1)^(1/2))+2*b^2/d*polylog(3
,-c*x-(c^2*x^2+1)^(1/2))-b^2/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-2
*b^2/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-b^2/d*arcsinh(c*x)^2*l
n(1+c*x+(c^2*x^2+1)^(1/2))-1/2*b^2/d*arcsinh(c*x)^2/c^2/x^2-2*a*b/d*arcsinh
(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*a^2/d/c^2/x^2-1/2*b^2*polylog(3,-(c*x
+(c^2*x^2+1)^(1/2))^2)/d+b^2/d*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2/d*ln(c*x+(
c^2*x^2+1)^(1/2))+b^2/d*ln(c*x+(c^2*x^2+1)^(1/2)-1)-2*a*b/d*polylog(2,-c*x-
(c^2*x^2+1)^(1/2))-2*a*b/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a^2 + integrate(b
^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^5 + d*x^3) + 2*a*b*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^5 + d*x^3
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^5+x^3}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^5+x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^2x^5+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**5 + x**3), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**5 + x**3), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^3\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)), x)
```

3.233 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=297

$$-\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{2c^3\left(a+b\sinh^{-1}(cx)\right)}{}$$

```
[Out] -1/3*b^2*c^2/d/x-1/3*(a+b*arcsinh(c*x))^2/d/x^3+c^2*(a+b*arcsinh(c*x))^2/d/
x+2*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d+14/3*b*c^3*(a+
b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d+7/3*b^2*c^3*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))/d-2*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x
^2+1)^(1/2)))/d+2*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(
1/2)))/d-7/3*b^2*c^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d+2*I*b^2*c^3*polylog
(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d-2*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(
1/2)))/d-1/3*b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/d/x^2
```

Rubi [A]
time = 0.46, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5809, 5789, 4265, 2611, 2320, 6724, 5816, 4267, 2317, 2438, 30}

$$\frac{2c^3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d}-\frac{2ibc^3\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{2ibc^3\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{14bc^3\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3d}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{7b^2c^3\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{3d}-\frac{7b^2c^3\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{3d}+\frac{2ib^2c^3\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{2ib^2c^3\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{b^2c^2}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -1/3*(b^2*c^2)/(d*x) - (b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*d*x^
2) - (a + b*ArcSinh[c*x])^2/(3*d*x^3) + (c^2*(a + b*ArcSinh[c*x])^2)/(d*x)
+ (2*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d + (14*b*c^3*(a +
b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d) + (7*b^2*c^3*PolyLog[2, -E^A
rcSinh[c*x]])/(3*d) - ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/d + ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[
c*x]])/d - (7*b^2*c^3*PolyLog[2, E^ArcSinh[c*x]])/(3*d) + ((2*I)*b^2*c^3*Po
lyLog[3, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*
x]])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)}\,dx=-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}-c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx+\frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{1+c^2x^2}}\,dx}{3d}$$

$$=-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx}$$

$$=-\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^2\left(a+b\sinh\right.}{dx}$$

$$=-\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^2\left(a+b\sinh\right.}{dx}$$

$$=-\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^2\left(a+b\sinh\right.}{dx}$$

$$=-\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^2\left(a+b\sinh\right.}{dx}$$

$$=-\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^2\left(a+b\sinh\right.}{dx}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 602 vs. $2(297) = 594$.
time = 7.29, size = 602, normalized size = 2.03

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -1/3*a^2/(d*x^3) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcTan[c*x])/d + (2*a*b*(-1/6
*(c*Sqrt[1 + c^2*x^2])/x^2 - ArcSinh[c*x]/(3*x^3) + (c^3*ArcTanh[Sqrt[1 + c
^2*x^2]])/6 - c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) - (I/2
)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]])/c
 + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + (I/2)*c^4*(-1/2*ArcSinh[c*x]^2/
c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, I*E^ArcSin
h[c*x]])/c)))/d + (b^2*c^3*(-4*Coth[ArcSinh[c*x]/2] + 14*ArcSinh[c*x]^2*Cot
h[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*ArcSinh[c*
x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 56*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])]
 - (24*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]^2*
Log[1 + I/E^ArcSinh[c*x]] + 56*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 56
*PolyLog[2, -E^(-ArcSinh[c*x])] - (48*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^Arc
Sinh[c*x]] + (48*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 56*PolyLog[
2, E^(-ArcSinh[c*x])] - (48*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (48*I)*Pol
yLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 - (8*ArcS
inh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSinh[c*x]/2] - 14*
ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d)
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(c x\right)\right)^{2}}{x^{4} \left(c^{2} d \,x^{2}+d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a^2 + integrate(b^2*log
(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^6 + d*x^4) + 2*a*b*log(c*x + sqrt(c^2*
x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^6 + d*x^4
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^6+x^4}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^6+x^4}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^2x^6+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d),x)
[Out] (Integral(a**2/(c**2*x**6 + x**4), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**6 + x**4), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^4\,\left(d\,c^2\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)), x)
```

3.234 $\int \frac{x^4(a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=279

$$\frac{2b^2x}{c^4d^2}+\frac{b(a+b\sinh^{-1}(cx))}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{c^5d^2}+\frac{3x(a+b\sinh^{-1}(cx))^2}{2c^4d^2}-\frac{x^3(a+b\sinh^{-1}(cx))}{2c^2d^2\,(1+c^2x^2)}$$

```
[Out] 2*b^2*x/c^4/d^2+3/2*x*(a+b*arcsinh(c*x))^2/c^4/d^2-1/2*x^3*(a+b*arcsinh(c*x
))^2/c^2/d^2/(c^2*x^2+1)-3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2
))/c^5/d^2-b^2*arctan(c*x)/c^5/d^2+3*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c
^2*x^2+1)^(1/2)))/c^5/d^2-3*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5
/d^2+3*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2+b*(a+b*arcsinh(c*
x))/c^5/d^2/(c^2*x^2+1)^(1/2)-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/
d^2
```

Rubi [A]
time = 0.37, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5810, 5812, 5789, 4265, 2611, 2320, 6724, 5798, 8, 272, 45, 5804, 396, 211}

$$-\frac{3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{c^5d^2}+\frac{3ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^5d^2}-\frac{3ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^5d^2}+\frac{3x(a+b\sinh^{-1}(cx))^2}{2c^4d^2}-\frac{x^3(a+b\sinh^{-1}(cx))^2}{2c^2d^2\,(c^2x^2+1)}-\frac{2b\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{c^5d^2}+\frac{b(a+b\sinh^{-1}(cx))}{c^5d^2\sqrt{c^2x^2+1}}-\frac{b^2\text{ArcTan}(cx)}{c^5d^2}-\frac{3ib^2\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{c^5d^2}+\frac{3ib^2\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{c^5d^2}+\frac{2b^2x}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (2*b^2*x)/(c^4*d^2) + (b*(a + b*ArcSinh[c*x]))/(c^5*d^2*Sqrt[1 + c^2*x^2])
- (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d^2) + (3*x*(a + b*ArcS
inh[c*x])^2)/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2
*x^2)) - (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) - (b^2
*ArcTan[c*x])/(c^5*d^2) + ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/(c^5*d^2) - ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcS
inh[c*x]])/(c^5*d^2) - ((3*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^2
) + ((3*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} + \frac{b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}$$

$$= -\frac{b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\text{s}\right.}{2c}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sin\right.}{2c^4}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sin\right.}{2c^4}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sin\right.}{2c^4}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sin\right.}{2c^4}$$

Mathematica [A]
time = 1.38, size = 482, normalized size = 1.73

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] ((2*a^2*x)/c^4 + (a^2*x)/(c^4 + c^6*x^2) - (3*a^2*ArcTan[c*x])/c^5 - (2*a*b
*(Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^2] - 3*c*x*ArcSinh[c*x] - 2*
c^3*x^3*ArcSinh[c*x] + (3*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)
*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] - (3*I)*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]]
- (3*I)*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*(1 + c^2*x^2)
*PolyLog[2, I*E^ArcSinh[c*x]]))/(c^5 + c^7*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt
[1 + c^2*x^2] - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + (c*x*ArcSinh[c*x]^2)/(2
+ 2*c^2*x^2) + c*x*(2 + ArcSinh[c*x]^2) + (I/2)*((4*I)*ArcTan[Tanh[ArcSinh[
```

```
c*x]/2]] + 3*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Lo
g[1 + I/E^ArcSinh[c*x]] + 6*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] -
6*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 6*PolyLog[3, (-I)/E^ArcSinh[c
*x]] - 6*PolyLog[3, I/E^ArcSinh[c*x]])))/c^5)/(2*d^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}}{\left(c^{2} d \,x^{2}+d \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2
)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*
d^2*x^2 + d^2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^
2*d^2*x^2 + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2} x^{4}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx + \int \frac{b^{2} x^{4} \operatorname{asinh}^{2}{\left(c x \right)}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx + \int \frac{2 a b x^{4} \operatorname{asinh}{\left(c x \right)}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**4*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

3.235 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=213

$$-\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^4d^2}+\frac{\left(a+b\sinh^{-1}(c\right.}{}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/c^4/d^2-1/2*x^2*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*
x^2+1)-1/3*(a+b*arcsinh(c*x))^3/b/c^4/d^2+(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)/c^4/d^2+1/2*b^2*ln(c^2*x^2+1)/c^4/d^2+b*(a+b*arcsinh(c*
x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*b^2*polylog(3,-(c*x+(
c^2*x^2+1)^(1/2))^2)/c^4/d^2-b*x*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/
2)
```

Rubi [A]
time = 0.28, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5810, 5797, 3799, 2221, 2611, 2320, 6724, 5783, 266}

$$\frac{b\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^4d^2}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{c^2x^2+1}}-\frac{b^2\text{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -((b*x*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[
c*x])^2/(2*c^4*d^2) - (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)
) - (a + b*ArcSinh[c*x])^3/(3*b*c^4*d^2) + ((a + b*ArcSinh[c*x])^2*Log[1 +
E^(2*ArcSinh[c*x])])/(c^4*d^2) + (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (b*(a
 + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^4*d^2) - (b^2*PolyLo
g[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2} + \frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2d}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x\right.}{c^4d^2}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{(a}{}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{(a}{}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{(a}{}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{(a}{}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{(a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 320, normalized size = 1.50

$$\frac{\frac{a^2}{1+c^2x^2} - \frac{ab\left(\sqrt{1+c^2x^2}-i\sinh^{-1}(cx)\right)}{i+cx} - \frac{ab\left(\sqrt{1+c^2x^2}+i\sinh^{-1}(cx)\right)}{-i+cx} - ab\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1-ie^{\sinh^{-1}(cx)}\right)\right) - ab\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right) + a^2\log\left(1+c^2x^2\right) + 4ab\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right) + 4ab\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right) + 2b^2\left(-\frac{cx\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + \frac{\sinh^{-1}(cx)^2}{2+2c^2x^2} + \frac{1}{3}\sinh^{-1}(cx)^3 + \sinh^{-1}(cx)^2\log\left(1+e^{-2\sinh^{-1}(cx)}\right) + \frac{1}{2}\log\left(1+c^2x^2\right) - \sinh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\sinh^{-1}(cx)}\right)\right)}{2c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) -
 (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - a*b*ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - a*b*ArcSinh[c*x]*(ArcSinh[c*x
] - 4*Log[1 + I*E^ArcSinh[c*x]]) + a^2*Log[1 + c^2*x^2] + 4*a*b*PolyLog[2,
(-I)*E^ArcSinh[c*x]] + 4*a*b*PolyLog[2, I*E^ArcSinh[c*x]] + 2*b^2*(-((c*x*A
rcSinh[c*x])/Sqrt[1 + c^2*x^2]) + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) + ArcSinh[
```

```
c*x]^3/3 + ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + Log[1 + c^2*x^2]/2
 - ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - PolyLog[3, -E^(-2*ArcSin
h[c*x])]/2))/(2*c^4*d^2)
```

Maple [A]
time = 7.23, size = 454, normalized size = 2.13

method	result
derivativedivides	$\frac{\frac{a^2}{2d^2(c^2x^2+1)}+\frac{a^2\ln(c^2x^2+1)}{2d^2}-\frac{b^2\operatorname{arcsinh}(cx)^3}{3d^2}-\frac{b^2\operatorname{arcsinh}(cx)cx}{d^2\sqrt{c^2x^2+1}}+\frac{b^2\operatorname{arcsinh}(cx)c^2x^2}{d^2(c^2x^2+1)}+\frac{b^2\operatorname{arcsinh}(cx)^2}{2d^2(c^2x^2+1)}+\frac{b^2\operatorname{arcsinh}(cx)}{d^2(c^2x^2+1)}+\cdots}{}$
default	$\frac{\frac{a^2}{2d^2(c^2x^2+1)}+\frac{a^2\ln(c^2x^2+1)}{2d^2}-\frac{b^2\operatorname{arcsinh}(cx)^3}{3d^2}-\frac{b^2\operatorname{arcsinh}(cx)cx}{d^2\sqrt{c^2x^2+1}}+\frac{b^2\operatorname{arcsinh}(cx)c^2x^2}{d^2(c^2x^2+1)}+\frac{b^2\operatorname{arcsinh}(cx)^2}{2d^2(c^2x^2+1)}+\frac{b^2\operatorname{arcsinh}(cx)}{d^2(c^2x^2+1)}+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*a^2/d^2/(c^2*x^2+1)+1/2*a^2/d^2*ln(c^2*x^2+1)-1/3*b^2/d^2*arcsin
h(c*x)^3-b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*c*x+b^2/d^2*arcsinh(c*x)/(c
^2*x^2+1)*c^2*x^2+1/2*b^2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)+b^2/d^2*arcsinh(c*
x)/(c^2*x^2+1)+b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-2*b^2/d^2*ln(c*x+(c^
2*x^2+1)^(1/2))+b^2/d^2*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2/
d^2*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*polylog(3,-(
c*x+(c^2*x^2+1)^(1/2))^2)/d^2-a*b/d^2*arcsinh(c*x)^2-a*b/d^2/(c^2*x^2+1)^(1
/2)*c*x+a*b/d^2/(c^2*x^2+1)*c^2*x^2+a*b/d^2*arcsinh(c*x)/(c^2*x^2+1)+a*b/d^
2/(c^2*x^2+1)+2*a*b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+a*b/d^
2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2)) + 1/2*(b^2
 + (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^
6*d^2*x^2 + c^4*d^2) - integrate(-(2*a*b*c^4*x^4 - b^2*c^2*x^2 - b^2 - (b^2
*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + (2*a*b*c^3*x^3 - b^2*c*x
 - (b^2*c^3*x^3 + b^2*c*x)*log(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1))/(c^8*d^2*x^5 + 2*c^6*d^2*x^3 + c^4*d^2*x + (c^7*d^2*x^4 +
 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x^3\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx^3\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**3*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

3.236 $\int \frac{x^2 (a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^2} \, dx$

Optimal. Leaf size=213

$$-\frac{b(a+b\sinh^{-1}(cx))}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x(a+b\sinh^{-1}(cx))^2}{2c^2d^2(1+c^2x^2)}+\frac{(a+b\sinh^{-1}(cx))^2 \operatorname{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}+\frac{b^2\operatorname{ArcTan}(cx)}{c^3d^2}-\frac{ib(a$$

```
[Out] -1/2*x*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arctan
(c*x+(c^2*x^2+1)^(1/2))/c^3/d^2+b^2*arctan(c*x)/c^3/d^2-I*b*(a+b*arcsinh(c*
x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2+I*b*(a+b*arcsinh(c*x))*po
lylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2+I*b^2*polylog(3,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c^3/d^2-I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2-b*(a
+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5810, 5789, 4265, 2611, 2320, 6724, 5798, 209}

$$\frac{\operatorname{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{c^3d^2}-\frac{ib\operatorname{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^3d^2}+\frac{ib\operatorname{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{c^3d^2}-\frac{x(a+b\sinh^{-1}(cx))^2}{2c^2d^2(c^2x^2+1)}-\frac{b(a+b\sinh^{-1}(cx))}{c^3d^2\sqrt{c^2x^2+1}}+\frac{b^2\operatorname{ArcTan}(cx)}{c^3d^2}+\frac{ib^2\operatorname{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{c^3d^2}-\frac{ib^2\operatorname{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -((b*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) - (x*(a + b*ArcSinh
[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/(c^3*d^2) + (b^2*ArcTan[c*x])/(c^3*d^2) - (I*b*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[
c*x]])/(c^3*d^2) - (I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2}+\frac{\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\text{Subst}\left(\int(a+bx)^2\text{sech}(x)\,dx,\right.}{2c^3d^2}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e\right.}{c^3d^2}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e\right.}{c^3d^2}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e\right.}{c^3d^2}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e\right.}{c^3d^2}$$

Mathematica [A]
time = 1.10, size = 385, normalized size = 1.81

$$-\frac{\frac{a^2cx}{1+c^2x^2}+\frac{2b^2\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}+\frac{b^2cx\sinh^{-1}(cx)^2}{1+c^2x^2}+\frac{ab\left(-i\sqrt{1+c^2x^2}+\sinh^{-1}(cx)\right)}{-i+cx}+\frac{ab\left(i\sqrt{1+c^2x^2}+\sinh^{-1}(cx)\right)}{i+cx}-a^2\text{ArcTan}(cx)-\frac{1}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)+\frac{1}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1-ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\right)+ib^2\left(4i\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+\sinh^{-1}(cx)^2\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)^2\log\left(1+ie^{-\sinh^{-1}(cx)}\right)+2\sinh^{-1}(cx)\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-2\sinh^{-1}(cx)\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+2\text{PolyLog}\left(3,-ie^{-\sinh^{-1}(cx)}\right)-2\text{PolyLog}\left(3,ie^{-\sinh^{-1}(cx)}\right)\right)}{2c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*((a^2*c*x)/(1 + c^2*x^2) + (2*b^2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (b
^2*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + (a*b*((-I)*Sqrt[1 + c^2*x^2] + ArcSi
nh[c*x]))/(-I + c*x) + (a*b*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x)
 - a^2*ArcTan[c*x] - (I/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^
ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + (I/2)*a*b*(ArcSinh[c*
x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[
c*x]]) + I*b^2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSinh[c*x]^2*Log[1 -
 I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 2*ArcSinh[c
*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh
[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3, I/E^ArcSinh[c*x]]
))/(c^3*d^2)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}}{\left(c^{2} d \,x^{2}+d \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + integrate(b^
2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) +
2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2} x^{2}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx + \int \frac{b^{2} x^{2} \operatorname{asinh}^{2}{\left(c x \right)}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx + \int \frac{2 a b x^{2} \operatorname{asinh}{\left(c x \right)}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**2*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

3.237 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\log\left(1+c^2x^2\right)}{2c^2d^2}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*x^2+1)-1/2*b^2*ln(c^2*x^2+1)/c^2/d^2
+b*x*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5798, 5787, 266}

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*x*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])
^2/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2}\\ &= \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\int \frac{x}{1+c^2x^2}\,dx}{d^2}\\ &= \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\log\left(1+c^2x^2\right)}{2c^2d^2}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 145, normalized size = 1.71

$$-\frac{a^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{abx}{cd^2\sqrt{1+c^2x^2}}+\frac{b\left(-a+bcx\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)}{c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\sinh^{-1}(cx)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\log\left(1+c^2x^2\right)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*a^2/(c^2*d^2*(1 + c^2*x^2)) + (a*b*x)/(c*d^2*Sqrt[1 + c^2*x^2]) + (b*(
-a + b*c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x])/(c^2*d^2*(1 + c^2*x^2)) - (b^2*
ArcSinh[c*x]^2)/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d
^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(79) = 158.
time = 3.42, size = 206, normalized size = 2.42

method	result
derivativedivides	$\frac{-\frac{a^2}{2d^2\left(c^2x^2+1\right)}+\frac{2b^2\operatorname{arcsinh}(cx)}{d^2}+\frac{b^2\operatorname{arcsinh}(cx)cx}{d^2\sqrt{c^2x^2+1}}-\frac{b^2\operatorname{arcsinh}(cx)c^2x^2}{d^2\left(c^2x^2+1\right)}-\frac{b^2\operatorname{arcsinh}(cx)^2}{2d^2\left(c^2x^2+1\right)}-\frac{b^2\operatorname{arcsinh}(cx)}{d^2\left(c^2x^2+1\right)}-\frac{b^2\ln\left(1+\left(cx+\sqrt{c\cdots}\right.\right.}{d\cdots}}{c^2}$
default	$\frac{-\frac{a^2}{2d^2\left(c^2x^2+1\right)}+\frac{2b^2\operatorname{arcsinh}(cx)}{d^2}+\frac{b^2\operatorname{arcsinh}(cx)cx}{d^2\sqrt{c^2x^2+1}}-\frac{b^2\operatorname{arcsinh}(cx)c^2x^2}{d^2\left(c^2x^2+1\right)}-\frac{b^2\operatorname{arcsinh}(cx)^2}{2d^2\left(c^2x^2+1\right)}-\frac{b^2\operatorname{arcsinh}(cx)}{d^2\left(c^2x^2+1\right)}-\frac{b^2\ln\left(1+\left(cx+\sqrt{c\cdots}\right.\right.}{d\cdots}}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a^2/d^2/(c^2*x^2+1)+2*b^2/d^2*arcsinh(c*x)+b^2/d^2*arcsinh(c*x)
/(c^2*x^2+1)^(1/2)*c*x-b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)*c^2*x^2-1/2*b^2/d^2
```

```
*arcsinh(c*x)^2/(c^2*x^2+1)-b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)-b^2/d^2*ln(1+(
c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/d^2*(-1/2/(c^2*x^2+1)*arcsinh(c*x)+1/2*c*x/
(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^2 + c^2*d^2) - 1/2*a^2/(
c^4*d^2*x^2 + c^2*d^2) + integrate(((2*a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^
2 + 1)*(2*a*b*c + b^2*c)*x + b^2)*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^2*x^5
 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2)*sqrt(c
^2*x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(79) = 158$.
time = 0.38, size = 185, normalized size = 2.18

$$\frac{2\,abc^2x^2 + 2\,\sqrt{c^2x^2+1}\,abcx - b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2 - a^2 + 2\,ab - (b^2c^2x^2+b^2)\log\left(c^2x^2+1\right) + 2\left(abc^2x^2+\sqrt{c^2x^2+1}\,b^2cx\right)\log\left(cx+\sqrt{c^2x^2+1}\right) + 2\left(abc^2x^2+ab\right)\log\left(-cx+\sqrt{c^2x^2+1}\right)}{2\left(c^4d^2x^2+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2*x^2 + 2*sqrt(c^2*x^2 + 1)*a*b*c*x - b^2*log(c*x + sqrt(c^2*x
^2 + 1))^2 - a^2 + 2*a*b - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(a*b*c^
2*x^2 + sqrt(c^2*x^2 + 1)*b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^
2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x*asinh(
c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c*
*4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

3.238 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2}-\frac{b^2\text{ArcTan}(cx)}{cd^2}-\frac{ib(a$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arctan(c*x+
(c^2*x^2+1)^(1/2))/c/d^2-b^2*arctan(c*x)/c/d^2-I*b*(a+b*arcsinh(c*x))*polyl
og(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+I*b*(a+b*arcsinh(c*x))*polylog(2,I*(
c*x+(c^2*x^2+1)^(1/2)))/c/d^2+I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c
/d^2-I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+b*(a+b*arcsinh(c*x))/
c/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5788, 5789, 4265, 2611, 2320, 6724, 5798, 209}

$$\frac{\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{cd^2}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{c^2x^2+1}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(c^2x^2+1\right)}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}-\frac{b^2\text{ArcTan}(cx)}{cd^2}+\frac{ib^2\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{cd^2}-\frac{ib^2\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]
)^2)/(2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]]
)/(c*d^2) - (b^2*ArcTan[c*x])/(c*d^2) - (I*b*(a + b*ArcSinh[c*x])*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(c*d^2) + (I*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^
ArcSinh[c*x]])/(c*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c*d^2) -
(I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{b^2\int \frac{1}{1+c^2x^2}\,dx}{d^2} + \frac{\mathrm{Subst}\left(\int (a+b\right.}{}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh}\right.}{cd^2}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh}\right.}{cd^2}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh}\right.}{cd^2}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh}\right.}{cd^2}$$

Mathematica [A]
time = 1.02, size = 403, normalized size = 1.92

$$\frac{\frac{a^2x}{1+c^2x^2}+\frac{a^2\mathrm{ArcTan}(cx)}{c}+\frac{2ab\left(\sqrt{1+c^2x^2}+cx\sinh^{-1}(cx)+i\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)+ic^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-i\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-ic^2x^2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)-i(1+c^2x^2)\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+i(1+c^2x^2)\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\right)}{c+c^3x^2}+\frac{2b^2\left(\frac{\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}+\frac{cx\sinh^{-1}(cx)^2}{2+2c^2x^2}-\frac{1}{2}i\left(-4i\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+\sinh^{-1}(cx)^2\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)^2\log\left(1+ie^{-\sinh^{-1}(cx)}\right)+2\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-2\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+2\mathrm{PolyLog}\left(3,-ie^{-\sinh^{-1}(cx)}\right)-2\mathrm{PolyLog}\left(3,ie^{-\sinh^{-1}(cx)}\right)\right)\right)}{c}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

```
[Out] ((a^2*x)/(1 + c^2*x^2) + (a^2*ArcTan[c*x])/c + (2*a*b*(Sqrt[1 + c^2*x^2] +
c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*c^2*x^2*Arc
Sinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*
x]] - I*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*(1 + c^2*x^2)*Po
lyLog[2, (-I)*E^ArcSinh[c*x]] + I*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]
]))/(c + c^3*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt[1 + c^2*x^2] + (c*x*ArcSinh[c
*x]^2)/(2 + 2*c^2*x^2) - (I/2)*((-4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSi
nh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*
x]] + 2*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyL
og[2, I/E^ArcSinh[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3,
I/E^ArcSinh[c*x]])))/c)/(2*d^2)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}{\left(c^2d\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + integrate(b^2*log(c
*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) + 2*a*b*log(c
*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^4 + 2*c
^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)
**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**
4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2, x)
```

3.239 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=193

$$-\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)}-\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2}+\frac{b^2\log\left(1+c^2x^2\right)}{2d^2}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))^2*arctanh((c*
x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*ln(c^2*x^2+1)/d^2-b*(a+b*arcsinh(c*x))*
polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b*(a+b*arcsinh(c*x))*polylog(2,(c
*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/
d^2-1/2*b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c*x*(a+b*arcsinh(c*x
))/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5811, 5799, 5569, 4267, 2611, 2320, 6724, 5787, 266}

$$-\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2x^2+1}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(c^2x^2+1\right)}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d^2}+\frac{b^2\log\left(c^2x^2+1\right)}{2d^2}+\frac{b^2\text{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{2d^2}-\frac{b^2\text{Li}_3\left(e^{2\sinh^{-1}(cx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*x*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c*
x])^2/(2*d^2*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSin
h[c*x])])/d^2 + (b^2*Log[1 + c^2*x^2])/(2*d^2) - (b*(a + b*ArcSinh[c*x])*Po
lyLog[2, -E^(2*ArcSinh[c*x])])/d^2 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(
2*ArcSinh[c*x])])/d^2 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^2) - (b^
2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx}{d}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(\ldots}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{b^2\log\left(1+c^2x^2\right)}{2d^2} + \frac{2\text{Subst}\left(\int(\ldots}{}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(\ldots\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(\ldots\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(\ldots\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(\ldots\right.}{d^2}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 1.30, size = 428, normalized size = 2.22

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] (a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) -
 (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 2*a*b*ArcSinh[c*x]
^2 + 4*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a^2*Log[c*x] - a^2*
Log[1 + c^2*x^2] + a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[
c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + a*b*(ArcSinh[c*x]*(ArcSinh[c*
x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + 2*a*b
```

*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*((I/24)*Pi^3 - (c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) - (2*ArcSinh[c*x]^3)/3 - ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + Log[1 + c^2*x^2]/2 + ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + PolyLog[3, -E^(-2*ArcSinh[c*x])]/2 - PolyLog[3, E^(2*ArcSinh[c*x])]/2))/(2*d^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(228) = 456$.
time = 3.90, size = 724, normalized size = 3.75

method	result
derivativedivides	$\frac{ab \operatorname{arcsinh}(cx)}{d^2(c^2x^2+1)} + \frac{2ab \operatorname{arcsinh}(cx) \ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{d^2} + \frac{b^2 \operatorname{arcsinh}(cx)c^2x^2}{d^2(c^2x^2+1)} - \frac{2b^2 \operatorname{polylog}\left(3,-cx-\sqrt{c^2x\right.}}{d^2}$
default	$\frac{ab \operatorname{arcsinh}(cx)}{d^2(c^2x^2+1)} + \frac{2ab \operatorname{arcsinh}(cx) \ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{d^2} + \frac{b^2 \operatorname{arcsinh}(cx)c^2x^2}{d^2(c^2x^2+1)} - \frac{2b^2 \operatorname{polylog}\left(3,-cx-\sqrt{c^2x\right.}}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*a*b/d^2*arcsinh(c*x)*l
n(1+c*x+(c^2*x^2+1)^(1/2))-b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*c*x+a*b/d
^2*arcsinh(c*x)/(c^2*x^2+1)-2*a*b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)-1/2*a^2/d^2*ln(c^2*x^2+1)+1/2*a^2/d^2/(c^2*x^2+1)+a*b/d^2/(c^2*x^2+
1)*c^2*x^2+b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)*c^2*x^2-a*b/d^2/(c^2*x^2+1)^(1/
2)*c*x-2*b^2/d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-2*b^2/d^2*polylog(3,c*x+
(c^2*x^2+1)^(1/2))+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^
2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)+b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)-b^2/d^2*a
rcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/d^2*arcsinh(c*x)*polylog(
2,-(c*x+(c^2*x^2+1)^(1/2))^2)+a*b/d^2/(c^2*x^2+1)-a*b/d^2*polylog(2,-(c*x+(
c^2*x^2+1)^(1/2))^2)+a^2/d^2*ln(c*x)+b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2
)-2*b^2/d^2*ln(c*x+(c^2*x^2+1)^(1/2))+b^2/d^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*
x^2+1)^(1/2))+2*b^2/d^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b^2/
d^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*a*b/d^2*polylog(2,c*x+(c^2
*x^2+1)^(1/2))+2*a*b/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2/d^2*arcsin
h(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + int
egrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^
2*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^
2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^5 + 2*c
^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^5+2c^2x^3+x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^5+2c^2x^3+x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)
**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**
5 + 2*c**2*x**3 + x), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^2), x)
```

3.240 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=287

$$-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)}-\frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)}-\frac{3c\left(a+b\sinh^{-1}(cx)\right)^2\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{d^2}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d^2/x/(c^2*x^2+1)-3/2*c^2*x*(a+b*arcsinh(c*x))^2/d^2/
(c^2*x^2+1)-3*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2+b^2*
c*arctan(c*x)/d^2-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d
^2-2*b^2*c*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^2+3*I*b*c*(a+b*arcsinh(c*x))
*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-3*I*b*c*(a+b*arcsinh(c*x))*polyl
og(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)/d^2-3*I*b^2*c*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+3*I*b^2*c*polylog
(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-b*c*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)^(1
/2)
```

Rubi [A]
time = 0.38, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5809, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 5811, 5816, 4267, 2317, 2438}

$$-\frac{3c\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d^2}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2x^2+1}}-\frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(c^2x^2+1\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(c^2x^2+1\right)}+\frac{3ibc\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{3ibc\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{4bc\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b^2c\text{ArcTan}(cx)}{d^2}-\frac{2b^2c\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{d^2}+\frac{2b^2c\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{d^2}-\frac{3ib^2c\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{d^2}+\frac{3ib^2c\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) - (a + b*ArcSinh[c*x]
)^2/(d^2*x*(1 + c^2*x^2)) - (3*c^2*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^
2*x^2)) - (3*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 + (b^2*c*
ArcTan[c*x])/d^2 - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d^2
 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^2 + ((3*I)*b*c*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - ((3*I)*b*c*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/d^2 + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d^
2 - ((3*I)*b^2*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d^2 + ((3*I)*b^2*c*PolyLo
g[3, I*E^ArcSinh[c*x]])/d^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
```

```
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x\left(1+c^2x^2\right)^{3/2}}\,d \ldots}{d^2}$$

$$= \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{(2bc) \ldots}{\ldots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2b^2c \ldots}{\ldots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c(\ldots}{\ldots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c(\ldots}{\ldots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c(\ldots}{\ldots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c(\ldots}{\ldots}$$

Mathematica [A]
time = 6.82, size = 549, normalized size = 1.91

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(a^2/(d^2*x)) - (a^2*c^2*x)/(2*d^2*(1 + c^2*x^2)) - (3*a^2*c*ArcTan[c*x])/
(2*d^2) + (2*a*b*c*((Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x])/(4*(-1 - I*c*x)) -
```

```
 ArcSinh[c*x]/(c*x) - (I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x])/(4*(I + c*x)) -
ArcTanh[Sqrt[1 + c^2*x^2]] + ((3*I)/4)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x
]*Log[1 + I*E^ArcSinh[c*x]] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((3*I)/4
)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyL
og[2, I*E^ArcSinh[c*x]])))/d^2 + (b^2*c*((-2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2
] - (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 4*ArcTan[Tanh[ArcSinh[c*x]/2]] - A
rcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x
])] + (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]^2
*Log[1 + I/E^ArcSinh[c*x]] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*
PolyLog[2, -E^(-ArcSinh[c*x])] + (6*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSi
nh[c*x]] - (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 4*PolyLog[2, E
^(-ArcSinh[c*x])] + (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[3
, I/E^ArcSinh[c*x]] + ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(2*d^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)^{2}}{x^{2}\left(c^{2} d\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + in
tegrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d
^2*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 +
 d^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^6 + 2*c
^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^2\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^2), x)
```

3.241 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=253

$$-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}}-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh$$

```
[Out] -c^2*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)-1/2*(a+b*arcsinh(c*x))^2/d^2/x^2/
(c^2*x^2+1)+4*c^2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d
^2+b^2*c^2*ln(x)/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2+2*b*c^2*(a+b*arcsinh(c*x
))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-2*b*c^2*(a+b*arcsinh(c*x))*pol
ylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(
1/2))^2)/d^2+b^2*c^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c*(a+b*arcs
inh(c*x))/d^2/x/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5809, 5811, 5799, 5569, 4267, 2611, 2320, 6724, 5787, 266, 277, 197, 5804, 457, 78}

$$\frac{2bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(c^2x^2+1\right)}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(c^2x^2+1\right)}+\frac{4c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d^2}-\frac{b^2c^2\text{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{d^2}+\frac{b^2c^2\text{Li}_3\left(e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{b^2c^2\log\left(c^2x^2+1\right)}{2d^2}+\frac{b^2c^2\log(x)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*x*Sqrt[1 + c^2*x^2])) - (c^2*(a + b*ArcSi
nh[c*x])^2)/(d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/(2*d^2*x^2*(1 + c^
2*x^2)) + (4*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 +
(b^2*c^2*Log[x])/d^2 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + (2*b*c^2*(a + b
*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^2 - (2*b*c^2*(a + b*ArcSi
nh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2 - (b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/d^2 + (b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/d^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
```

```
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2} \cdots$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \cdots$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\cdots}{\cdots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - \frac{b^2c^2\log\cdots}{\cdots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2(a\cdots}{\cdots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2(a\cdots}{\cdots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2(a\cdots}{\cdots}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2(a\cdots}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 594 vs. 2(253) = 506.
time = 0.63, size = 594, normalized size = 2.35

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((-2*a^2)/x^2 - (2*a*b*c)/(x*Sqrt[1 + c^2*x^2]) + a^2/(x^2 + c^2*x^4) + 4*a
^2*c^2*ArcSinh[c*x] - (4*a*b*ArcSinh[c*x])/x^2 - (2*b^2*c*ArcSinh[c*x])/(x*
Sqrt[1 + c^2*x^2]) + (2*a*b*ArcSinh[c*x])/(x^2 + c^2*x^4) - (2*b^2*ArcSinh[
c*x]^2)/x^2 + (b^2*ArcSinh[c*x]^2)/(x^2 + c^2*x^4) + 8*a*b*c^2*ArcSinh[c*x]
*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 +
(c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 8*a*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]
*E^ArcSinh[c*x])/c] + 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSin
h[c*x])/c] - 4*a^2*c^2*Log[1 - E^(2*ArcSinh[c*x])] - 8*a*b*c^2*ArcSinh[c*x]
*Log[1 - E^(2*ArcSinh[c*x])] - 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSin
h[c*x])] + 2*b^2*c^2*Log[x] + 2*a^2*c^2*Log[1 + c^2*x^2] - b^2*c^2*Log[1 +
c^2*x^2] + 8*b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[
-c^2]] + 8*b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c] - 4*a*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])] - 4*b^2*c^2*ArcSinh[c*x]*Po
lyLog[2, E^(2*ArcSinh[c*x])] - 8*b^2*c^2*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt
[-c^2]] - 8*b^2*c^2*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b^2*c^2*P
olyLog[3, E^(2*ArcSinh[c*x])])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(292) = 584$.
time = 4.39, size = 747, normalized size = 2.95

method	result
derivativedivides	$c^2\left(-\frac{ab}{d^2\sqrt{c^2x^2+1}\,cx}-\frac{2ab\,\mathrm{arcsinh}(cx)}{d^2(c^2x^2+1)}-\frac{4ab\,\mathrm{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{d^2}-\frac{b^2\,\mathrm{arcsinh}(cx)}{d^2\sqrt{c^2x^2+1}}\right.$
default	$c^2\left(-\frac{ab}{d^2\sqrt{c^2x^2+1}\,cx}-\frac{2ab\,\mathrm{arcsinh}(cx)}{d^2(c^2x^2+1)}-\frac{4ab\,\mathrm{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{d^2}-\frac{b^2\,\mathrm{arcsinh}(cx)}{d^2\sqrt{c^2x^2+1}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-a*b/d^2/(c^2*x^2+1)^(1/2)/c/x-4*a*b/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^
2+1)^(1/2))-4*a*b/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b^2/d^2*ln(1
+c*x+(c^2*x^2+1)^(1/2))+b^2/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-1)-2*a*b/d^2*arcsi
nh(c*x)/(c^2*x^2+1)+4*a*b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+
a^2/d^2*ln(c^2*x^2+1)-1/2*a^2/d^2/(c^2*x^2+1)-1/2*a^2/d^2/c^2/x^2-a*b/d^2*a
rcsinh(c*x)/(c^2*x^2+1)/c^2/x^2+4*b^2/d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))
+4*b^2/d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-b^2/d^2*arcsinh(c*x)/(c^2*x^2+1
)^(1/2)/c/x-1/2*b^2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)/c^2/x^2-b^2*polylog(3,-(
c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b^2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)+2*b^2/d^2*
arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+2*b^2/d^2*arcsinh(c*x)*polyl
```

```
og(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))^2)-2*a^2/d^2*ln(c*x)-b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-2*b^2/d^2*a
rcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-4*b^2/d^2*arcsinh(c*x)*polylog(2,
-c*x-(c^2*x^2+1)^(1/2))-2*b^2/d^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2)
)-4*a*b/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-4*a*b/d^2*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))-4*b^2/d^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c
^2*d^2*x^4 + d^2*x^2)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*
d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^
4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^7 + 2*c
^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{x^3\,{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^2), x)
```

3.242 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=401

$$-\frac{b^2c^2}{3d^2x}+\frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)}+\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x\left(1+c^2x^2\right)}+\frac{5c^4x(}{2}$$

```
[Out] -1/3*b^2*c^2/d^2/x-1/3*(a+b*arcsinh(c*x))^2/d^2/x^3/(c^2*x^2+1)+5/3*c^2*(a+
b*arcsinh(c*x))^2/d^2/x/(c^2*x^2+1)+5/2*c^4*x*(a+b*arcsinh(c*x))^2/d^2/(c^2
*x^2+1)+5*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2-b^2*c^
3*arctan(c*x)/d^2+26/3*b*c^3*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/
2))/d^2+13/3*b^2*c^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^2-5*I*b*c^3*(a+b*a
rcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+5*I*b*c^3*(a+b*arcsi
nh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-13/3*b^2*c^3*polylog(2,c*
x+(c^2*x^2+1)^(1/2))/d^2+5*I*b^2*c^3*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/
d^2-5*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+2/3*b*c^3*(a+b*arc
sinh(c*x))/d^2/(c^2*x^2+1)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))/d^2/x^2/(c^2*x^
2+1)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5809, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 5811, 5816, 4267, 2317, 2438, 331}

$$\frac{5c^3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d^2}-\frac{5ibc^3\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{5ibc^3\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{26bc^3\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3d^2}+\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x\left(c^2x^2+1\right)}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(c^2x^2+1\right)}+\frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(c^2x^2+1\right)}+\frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2x^2+1}}-\frac{b^2c^3\text{ArcTan}(cx)}{d^2}+\frac{13b^2c^3\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{3d^2}-\frac{13b^2c^3\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{3d^2}+\frac{5ib^2c^3\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{d^2}-\frac{5ib^2c^3\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{d^2}-\frac{b^2c^2}{3d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/3*(b^2*c^2)/(d^2*x) + (2*b*c^3*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[1 + c^2
*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*
ArcSinh[c*x])^2/(3*d^2*x^3*(1 + c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x])^2)/
(3*d^2*x*(1 + c^2*x^2)) + (5*c^4*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^2*
x^2)) + (5*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 - (b^2*c^
3*ArcTan[c*x])/d^2 + (26*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]]
)/(3*d^2) + (13*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^2) - ((5*I)*b*c^3
*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 + ((5*I)*b*c^3*(
a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^2 - (13*b^2*c^3*PolyLog
[2, E^ArcSinh[c*x]])/(3*d^2) + ((5*I)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh[c*x
]])/d^2 - ((5*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} - \frac{1}{3}\left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x^3(1+c^2x^2)^{3/2}}}{3d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x\left(1+c^2x^2\right)} + \left(5c^4\right)$$

$$= -\frac{b^2c^2}{3d^2x} - \frac{13bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)}$$

Mathematica [A]
time = 7.94, size = 764, normalized size = 1.91

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/3*a^2/(d^2*x^3) + (2*a^2*c^2)/(d^2*x) + (a^2*c^4*x)/(2*d^2*(1 + c^2*x^2)
) + (5*a^2*c^3*ArcTan[c*x])/(2*d^2) + (2*a*b*(-1/6*(c*Sqrt[1 + c^2*x^2])/x^
2 - (c^3*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(4*(-1 - I*c*x)) - ArcSinh[c
*x]/(3*x^3) + (c^4*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(4*(I*c + c^2*x))
+ (c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/6 - 2*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh
[Sqrt[1 + c^2*x^2]]) - ((5*I)/4)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*
x]*Log[1 + I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) +
((5*I)/4)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[
c*x]])/c + (2*PolyLog[2, I*E^ArcSinh[c*x]])/c)))/d^2 + (b^2*c^3*((24*ArcSin
h[c*x])/Sqrt[1 + c^2*x^2] + (12*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 48*ArcT
```

```
an[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 26*ArcSinh[c*x]^2*Coth[
ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*ArcSinh[c*x]
^2*Csch[ArcSinh[c*x]/2]^4)/2 - 104*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])]
- (60*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (60*I)*ArcSinh[c*x]^2*L
og[1 + I/E^ArcSinh[c*x]] + 104*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 10
4*PolyLog[2, -E^(-ArcSinh[c*x])] - (120*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^A
rcSinh[c*x]] + (120*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 104*Poly
Log[2, E^(-ArcSinh[c*x])] - (120*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (120*
I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 - (
8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSinh[c*x]/2]
 - 26*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^2)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4\left(c^2d\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
d^2*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^8 +
 2*c^2*d^2*x^6 + d^2*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8
 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^8 + 2*c
^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^8+2c^2x^6+x^4}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^8+2c^2x^6+x^4}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^4\,{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^2), x)
```

3.243 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=320

$$-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(c\right.}{8c^4d^3\left(1+c^2x\right.}$$

```
[Out] -1/12*b^2*x/c^4/d^3/(c^2*x^2+1)+1/6*b*(a+b*arcsinh(c*x))/c^5/d^3/(c^2*x^2+1
)^(3/2)-1/4*x^3*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2-3/8*x*(a+b*arcsi
nh(c*x))^2/c^4/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2
+1)^(1/2))/c^5/d^3+7/6*b^2*arctan(c*x)/c^5/d^3-3/4*I*b*(a+b*arcsinh(c*x))*p
olylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3+3/4*I*b*(a+b*arcsinh(c*x))*pol
ylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3+3/4*I*b^2*polylog(3,-I*(c*x+(c^2*
x^2+1)^(1/2)))/c^5/d^3-3/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d
^3-5/4*b*(a+b*arcsinh(c*x))/c^5/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5810, 5789, 4265, 2611, 2320, 6724, 5798, 209, 272, 45, 5804, 12, 393}

$$\frac{3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{4c^5d^3}-\frac{3ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{c^2x^2+1}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(c^2x^2+1\right)^{3/2}}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{8c^4d^3\left(c^2x^2+1\right)}+\frac{7b^2\text{ArcTan}(cx)}{6c^5d^3}+\frac{3ib^2\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{4c^5d^3}-\frac{3ib^2\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{4c^5d^3}-\frac{b^2x}{12c^4d^3\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*(b^2*x)/(c^4*d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c^5*d^3
*(1 + c^2*x^2)^(3/2)) - (5*b*(a + b*ArcSinh[c*x]))/(4*c^5*d^3*Sqrt[1 + c^2*
x^2]) - (x^3*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a
+ b*ArcSinh[c*x])^2)/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*
ArcTan[E^ArcSinh[c*x]])/(4*c^5*d^3) + (7*b^2*ArcTan[c*x])/(6*c^5*d^3) - (((
3*I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) +
 (((3*I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3)
+ (((3*I)/4)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) - (((3*I)/4)*b^
2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2cd^3}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$=\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{2c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x(a+}{8c^4}$$

$$=\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x(a}{8c}$$

$$=-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3(a+b}{4c^2d^3}$$

$$=-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3(a+b}{4c^2d^3}$$

$$=-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3(a+b}{4c^2d^3}$$

$$=-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3(a+b}{4c^2d^3}$$

Mathematica [A]
time = 2.14, size = 552, normalized size = 1.72

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((6*a^2*c*x)/(1 + c^2*x^2)^2 - (15*a^2*c*x)/(1 + c^2*x^2) + (15*a*b*(Sqrt[1
 + c^2*x^2] - I*ArcSinh[c*x]))/(-1 + I*c*x) + (15*a*b*(Sqrt[1 + c^2*x^2] +
I*ArcSinh[c*x]))/(-1 - I*c*x) - (I*a*b*((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*
ArcSinh[c*x]))/(-I + c*x)^2 + (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*Arc
Sinh[c*x]))/(I + c*x)^2 + 9*a^2*ArcTan[c*x] + ((9*I)/2)*a*b*(ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c
*x]]) - ((9*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c
*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((-2*c*x)/(1 + c^2*x^2) + (4*
ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (30*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (
6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 - (15*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^
2) + 56*ArcTan[Tanh[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^Arc
Sinh[c*x]] + (9*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSin
```

```
h[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E
^ArcSinh[c*x]] - (18*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3,
 I/E^ArcSinh[c*x]]))/(24*c^5*d^3)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\left(a+b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}}{\left(c^{2} d \,x^{2}+d \right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arc
tan(c*x)/(c^5*d^3)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6
*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^4*log(c*x + sqrt(
c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^6*d
^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2} x^{4}}{c^{6} x^{6} + 3 c^{4} x^{4} + 3 c^{2} x^{2} + 1}\, dx + \int \frac{b^{2} x^{4} \operatorname{asinh}^{2}{\left(c x \right)}}{c^{6} x^{6} + 3 c^{4} x^{4} + 3 c^{2} x^{2} + 1}\, dx + \int \frac{2 a b x^{4} \operatorname{asinh}{\left(c x \right)}}{c^{6} x^{6} + 3 c^{4} x^{4} + 3 c^{2} x^{2} + 1}\, dx}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**4*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

3.244 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=167

$$-\frac{b^2}{12c^4d^3\,(1+c^2x^2)}+\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\,(1+c^2x^2)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3}+\frac{x^4\left(a+b\sinh^{-1}(c\right.}{4d^3\,(1+c^2x^2)}$$

```
[Out] -1/12*b^2/c^4/d^3/(c^2*x^2+1)+1/6*b*x^3*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1
)^(3/2)-1/4*(a+b*arcsinh(c*x))^2/c^4/d^3+1/4*x^4*(a+b*arcsinh(c*x))^2/d^3/(
c^2*x^2+1)^2-1/3*b^2*ln(c^2*x^2+1)/c^4/d^3+1/2*b*x*(a+b*arcsinh(c*x))/c^3/d
^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5800, 5810, 5783, 266, 272, 45}

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\,(c^2x^2+1)^2}+\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\,(c^2x^2+1)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{c^2x^2+1}}-\frac{b^2}{12c^4d^3\,(c^2x^2+1)}-\frac{b^2\log\left(c^2x^2+1\right)}{3c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*b^2/(c^4*d^3*(1 + c^2*x^2)) + (b*x^3*(a + b*ArcSinh[c*x]))/(6*c*d^3*(
1 + c^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(2*c^3*d^3*Sqrt[1 + c^2*x^
2]) - (a + b*ArcSinh[c*x])^2/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x])^2)/(4*
d^3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/(3*c^4*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3}$$

$$= \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{b^2\int\frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{6d^3} - \frac{b\int\frac{x^2(a+}{(1-}}{}$$

$$= \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}} + \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{b^2\mathrm{S}}{}$$

$$= \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3} + \frac{x^4(a}{4c}$$

$$= -\frac{b^2}{12c^4d^3\left(1+c^2x^2\right)} + \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}} - \frac{(a+b}{}$$

Mathematica [A]
time = 0.20, size = 186, normalized size = 1.11

$$-\frac{3a^2+b^2+6a^2c^2x^2+b^2c^2x^2-6abcx\sqrt{1+c^2x^2}-8abc^3x^3\sqrt{1+c^2x^2}+2b\left(-bcx\sqrt{1+c^2x^2}\,(3+4c^2x^2)+a(3+6c^2x^2)\right)\sinh^{-1}(cx)+3b^2(1+2c^2x^2)\sinh^{-1}(cx)^2+4(b+bc^2x^2)^2\log\left(1+c^2x^2\right)}{12c^4d^3\left(1+c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*(3*a^2 + b^2 + 6*a^2*c^2*x^2 + b^2*c^2*x^2 - 6*a*b*c*x*Sqrt[1 + c^2*x
^2] - 8*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(3 +
 4*c^2*x^2)) + a*(3 + 6*c^2*x^2))*ArcSinh[c*x] + 3*b^2*(1 + 2*c^2*x^2)*ArcS
inh[c*x]^2 + 4*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(c^4*d^3*(1 + c^2*x^2)^2
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(151) = 302$.
time = 6.33, size = 481, normalized size = 2.88

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{1}{4\left(c^2x^2+1\right)^2}-\frac{1}{2\left(c^2x^2+1\right)}\right)}{d^3}+\frac{4b^2\operatorname{arcsinh}(cx)}{3d^3}+\frac{2b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c^3x^3}{3d^3\left(c^4x^4+2c^2x^2+1\right)}-\frac{2b^2\operatorname{arcsinh}(cx)c^4x^4}{3d^3\left(c^4x^4+2c^2x^2+1\right)}-\frac{b^2\operatorname{arcsinh}(c}{2d^3\left(c^4x^4+2c\right.}}{}$
default	$\frac{\frac{a^2\left(\frac{1}{4\left(c^2x^2+1\right)^2}-\frac{1}{2\left(c^2x^2+1\right)}\right)}{d^3}+\frac{4b^2\operatorname{arcsinh}(cx)}{3d^3}+\frac{2b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c^3x^3}{3d^3\left(c^4x^4+2c^2x^2+1\right)}-\frac{2b^2\operatorname{arcsinh}(cx)c^4x^4}{3d^3\left(c^4x^4+2c^2x^2+1\right)}-\frac{b^2\operatorname{arcsinh}(c}{2d^3\left(c^4x^4+2c\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a^2/d^3*(1/4/(c^2*x^2+1)^2-1/2/(c^2*x^2+1))+4/3*b^2/d^3*arcsinh(c*x)
+2/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-2
/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c^4*x^4-1/2*b^2/d^3/(c^4*x^4+
2*c^2*x^2+1)*arcsinh(c*x)^2*c^2*x^2+1/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)*c*x-4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x
)*c^2*x^2-1/4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2-1/12*b^2/d^3/(c^
4*x^4+2*c^2*x^2+1)*c^2*x^2-2/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)-1
/12*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)-2/3*b^2/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)+2*a*b/d^3*(1/4*arcsinh(c*x)/(c^2*x^2+1)^2-1/2/(c^2*x^2+1)*arcsinh(c*x)-1
/12/(c^2*x^2+1)^(3/2)*c*x+1/3*c*x/(c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c^2*x^2 + 1)*a^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 1/4*(2*b
^2*c^2*x^2 + b^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^8*d^3*x^4 + 2*c^6*d^3*x
^2 + c^4*d^3) + integrate(1/2*(3*b^2*c^2*x^2 + 2*(2*a*b*c^4 + b^2*c^4)*x^4
+ b^2 + (b^2*c*x + 2*(2*a*b*c^3 + b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1))/(c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^
3*x + (c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2
+ 1)), x)
```

<u>**Fricas**</u> [**A**]
time = 0.39, size = 283, normalized size = 1.69

$$\frac{8\,abc^4x^4-(6\,a^2-16\,ab+b^2)c^2x^2-3\,(2\,b^2c^2x^2+b^2)\log\left(cx+\sqrt{c^2x^2+1}\right)^2-3\,a^2+8\,ab-b^2-4\,(b^2c^4x^4+2\,b^2c^2x^2+b^2)\log\left(c^2x^2+1\right)+2\left(3\,abc^4x^4+(4\,b^2c^3x^3+3\,b^2cx)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+6\,(abc^4x^4+2\,abc^2x^2+ab)\log\left(-cx+\sqrt{c^2x^2+1}\right)+2\,(4\,abc^3x^3+3\,abcx)\sqrt{c^2x^2+1}}{12\,(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(8*a*b*c^4*x^4 - (6*a^2 - 16*a*b + b^2)*c^2*x^2 - 3*(2*b^2*c^2*x^2 + b
^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 3*a^2 + 8*a*b - b^2 - 4*(b^2*c^4*x^4 +
 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + (4*b^2*c^3*x^3
+ 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x
^4 + 2*a*b*c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(4*a*b*c^3*x^3
+ 3*a*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

<u>**Sympy**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{b^2x^3\operatorname{asinh}^2{(cx)}}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{2abx^3\operatorname{asinh}{(cx)}}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**3*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

<u>**Giac**</u> [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

3.245 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=318

$$\frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2d^3\left(1+c^2x^2\right)}+$$

```
[Out] 1/12*b^2*x/c^2/d^3/(c^2*x^2+1)-1/6*b*(a+b*arcsinh(c*x))/c^3/d^3/(c^2*x^2+1)
^(3/2)-1/4*x*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2+1/8*x*(a+b*arcsinh(
c*x))^2/c^2/d^3/(c^2*x^2+1)+1/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)
^(1/2))/c^3/d^3-1/6*b^2*arctan(c*x)/c^3/d^3-1/4*I*b*(a+b*arcsinh(c*x))*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b*(a+b*arcsinh(c*x))*polylo
g(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b^2*polylog(3,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c^3/d^3-1/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+
1/4*b*(a+b*arcsinh(c*x))/c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5810, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 205}

$$\frac{\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{4c^3d^3}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2d^3\left(c^2x^2+1\right)}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{c^2x^2+1}}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(c^2x^2+1\right)^{3/2}}-\frac{b^2\text{ArcTan}(cx)}{6c^3d^3}+\frac{ib^2\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{4c^3d^3}-\frac{ib^2\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{4c^3d^3}+\frac{b^2x}{12c^2d^3\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b^2*x)/(12*c^2*d^3*(1 + c^2*x^2)) - (b*(a + b*ArcSinh[c*x]))/(6*c^3*d^3*(1
 + c^2*x^2)^(3/2)) + (b*(a + b*ArcSinh[c*x]))/(4*c^3*d^3*Sqrt[1 + c^2*x^2])
 - (x*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSi
nh[c*x])^2)/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^Ar
cSinh[c*x]])/(4*c^3*d^3) - (b^2*ArcTan[c*x])/(6*c^3*d^3) - ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b^2*PolyLog[
3, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) - ((I/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x]
])/(c^3*d^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{b\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2cd^3} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2d^3\left(1+c^2x^2\right)} + \cdots$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)} - \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)} - \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)} - \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)} - \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)} - \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}} - \cdots$$

<u>Mathematica</u> [**A**]
time = 1.71, size = 550, normalized size = 1.73

$$\frac{-\frac{6a^2cx}{(1+c^2x^2)^2}+\frac{3a^2cx}{1+c^2x^2}+\frac{ab\left((2+icx)\sqrt{1+c^2x^2}+3i\sinh^{-1}(cx)\right)}{(-i+cx)^2}+\frac{3ab\left(-i\sqrt{1+c^2x^2}+\sinh^{-1}(cx)\right)}{-i+cx}+\frac{3ab\left(i\sqrt{1+c^2x^2}+\sinh^{-1}(cx)\right)}{i+cx}-\frac{iab\left((2i+cx)\sqrt{1+c^2x^2}+3\sinh^{-1}(cx)\right)}{(i+cx)^2}+3a^2\mathrm{ArcTan}(cx)+\frac{3}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)-\frac{3}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1-ie^{\sinh^{-1}(cx)}\right)\right)-4\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\right)+b^2\left(\frac{2cx}{1+c^2x^2}-\frac{4\sinh^{-1}(cx)}{(1+c^2x^2)^{3/2}}+\frac{6\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}-\frac{6cx\sinh^{-1}(cx)^2}{(1+c^2x^2)^2}+\frac{3cx\sinh^{-1}(cx)^2}{1+c^2x^2}-8\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)-3i\sinh^{-1}(cx)^2\log\left(1-ie^{-\sinh^{-1}(cx)}\right)+3i\sinh^{-1}(cx)^2\log\left(1+ie^{-\sinh^{-1}(cx)}\right)-6i\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+6i\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)-6i\mathrm{PolyLog}\left(3,-ie^{-\sinh^{-1}(cx)}\right)+6i\mathrm{PolyLog}\left(3,ie^{-\sinh^{-1}(cx)}\right)\right)}{24c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((-6*a^2*c*x)/(1 + c^2*x^2)^2 + (3*a^2*c*x)/(1 + c^2*x^2) + (a*b*((2 + I*c*
x)*Sqrt[1 + c^2*x^2] + (3*I)*ArcSinh[c*x]))/(-I + c*x)^2 + (3*a*b*((-I)*Sqr
t[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (3*a*b*(I*Sqrt[1 + c^2*x^2] +
ArcSinh[c*x]))/(I + c*x) - (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSin
h[c*x]))/(I + c*x)^2 + 3*a^2*ArcTan[c*x] + ((3*I)/2)*a*b*(ArcSinh[c*x]*(Arc
Sinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]
]) - ((3*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]
]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((2*c*x)/(1 + c^2*x^2) - (4*ArcS
inh[c*x])/(1 + c^2*x^2)^(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x
*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (3*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 8
*ArcTan[Tanh[ArcSinh[c*x]/2]] - (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*
x]] + (3*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (6*I)*ArcSinh[c*x]*P
```

```
olyLog[2, (-I)/E^ArcSinh[c*x]] + (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[
c*x]] - (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (6*I)*PolyLog[3, I/E^ArcSin
h[c*x]]))/(24*c^3*d^3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}}{\left(c^{2} d \,x^{2}+d \right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x
)/(c^3*d^3)) + integrate(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^
6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2
 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^6*d
^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2} x^{2}}{c^{6} x^{6} + 3 c^{4} x^{4} + 3 c^{2} x^{2} + 1}\, dx + \int \frac{b^{2} x^{2} \operatorname{asinh}^{2}{\left(c x \right)}}{c^{6} x^{6} + 3 c^{4} x^{4} + 3 c^{2} x^{2} + 1}\, dx + \int \frac{2 a b x^{2} \operatorname{asinh}{\left(c x \right)}}{c^{6} x^{6} + 3 c^{4} x^{4} + 3 c^{2} x^{2} + 1}\, dx}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

3.246 $\int \frac{x(a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=145

$$\frac{b^2}{12c^2d^3(1+c^2x^2)}+\frac{bx(a+b\sinh^{-1}(cx))}{6cd^3(1+c^2x^2)^{3/2}}+\frac{bx(a+b\sinh^{-1}(cx))}{3cd^3\sqrt{1+c^2x^2}}-\frac{(a+b\sinh^{-1}(cx))^2}{4c^2d^3(1+c^2x^2)^2}-\frac{b^2\log(1+c^2x^2)}{6c^2d^3}$$

```
[Out] 1/12*b^2/c^2/d^3/(c^2*x^2+1)+1/6*b*x*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(
3/2)-1/4*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2-1/6*b^2*ln(c^2*x^2+1)/c
^2/d^3+1/3*b*x*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5798, 5788, 5787, 266, 267}

$$\frac{bx(a+b\sinh^{-1}(cx))}{3cd^3\sqrt{c^2x^2+1}}+\frac{bx(a+b\sinh^{-1}(cx))}{6cd^3(c^2x^2+1)^{3/2}}-\frac{(a+b\sinh^{-1}(cx))^2}{4c^2d^3(c^2x^2+1)^2}+\frac{b^2}{12c^2d^3(c^2x^2+1)}-\frac{b^2\log(c^2x^2+1)}{6c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] b^2/(12*c^2*d^3*(1 + c^2*x^2)) + (b*x*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 + c
^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^3*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])^2/(4*c^2*d^3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/
(6*c^2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
```

```
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2cd^3}\\ &= \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{b^2\int\frac{x}{\left(1+c^2x^2\right)^2}\,dx}{6d^3}+\frac{b\int\frac{a+b\sinh^{-1}}{(1+c^2x^2}}{3cd^3}\\ &= \frac{b^2}{12c^2d^3\left(1+c^2x^2\right)}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{1+c^2x^2}}-\frac{(a+b\,\mathrm{si}}{4c^2d^3\,(}\\ &= \frac{b^2}{12c^2d^3\left(1+c^2x^2\right)}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{1+c^2x^2}}-\frac{(a+b\,\mathrm{si}}{4c^2d^3\,(}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 152, normalized size = 1.05

$$\frac{-3a^2+b^2+b^2c^2x^2+6abcx\sqrt{1+c^2x^2}+4abc^3x^3\sqrt{1+c^2x^2}+2b\left(-3a+bcx\sqrt{1+c^2x^2}\,(3+2c^2x^2)\right)\sinh^{-1}(cx)-3b^2\sinh^{-1}(cx)^2-2(b+bc^2x^2)^2\log\left(1+c^2x^2\right)}{12d^3\left(c+c^3x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

[Out] (-3*a^2 + b^2 + b^2*c^2*x^2 + 6*a*b*c*x*Sqrt[1 + c^2*x^2] + 4*a*b*c^3*x^3*S
qrt[1 + c^2*x^2] + 2*b*(-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2))*Arc
Sinh[c*x] - 3*b^2*ArcSinh[c*x]^2 - 2*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(1
2*d^3*(c + c^3*x^2)^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(131) = 262$.
time = 4.23, size = 410, normalized size = 2.83

method	result
derivativedivides	$-\frac{a^2}{4d^3(c^2x^2+1)^2}+\frac{2b^2\operatorname{arcsinh}(cx)}{3d^3}+\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c^3x^3}{3d^3(c^4x^4+2c^2x^2+1)}-\frac{b^2\operatorname{arcsinh}(cx)c^4x^4}{3d^3(c^4x^4+2c^2x^2+1)}+\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c}{2d^3(c^4x^4+2c^2x^2+1)}$
default	$-\frac{a^2}{4d^3(c^2x^2+1)^2}+\frac{2b^2\operatorname{arcsinh}(cx)}{3d^3}+\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c^3x^3}{3d^3(c^4x^4+2c^2x^2+1)}-\frac{b^2\operatorname{arcsinh}(cx)c^4x^4}{3d^3(c^4x^4+2c^2x^2+1)}+\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c}{2d^3(c^4x^4+2c^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/c^2*(-1/4*a^2/d^3/(c^2*x^2+1)^2+2/3*b^2/d^3*arcsinh(c*x)+1/3*b^2/d^3/(c^4
*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-1/3*b^2/d^3/(c^4*x
^4+2*c^2*x^2+1)*arcsinh(c*x)*c^4*x^4+1/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)*c*x-2/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*
x)*c^2*x^2-1/4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2+1/12*b^2/d^3/(c
^4*x^4+2*c^2*x^2+1)*c^2*x^2-1/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+
1/12*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)-1/3*b^2/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)+2*a*b/d^3*(-1/4*arcsinh(c*x)/(c^2*x^2+1)^2+1/12/(c^2*x^2+1)^(3/2)*c*x+1
/6*c*x/(c^2*x^2+1)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")

[Out] -1/4*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*
d^3) - 1/4*a^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + integrate(1/2*((4*
a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^2 + 1)*(4*a*b*c + b^2*c)*x + b^2)*log(c
*x + sqrt(c^2*x^2 + 1))/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3 + c^2*
d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^2*x^2
+ 1)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(131) = 262$.
time = 0.46, size = 273, normalized size = 1.88

$$\frac{4\,abc^4x^4 + (8\,ab + b^2)c^2x^2 - 3\,b^2\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 - 3\,a^2 + 4\,ab + b^2 - 2\,(b^2c^4x^4 + 2\,b^2c^2x^2 + b^2)\log\left(c^2x^2 + 1\right) + 2\left(3\,abc^4x^4 + 6\,abc^2x^2 + (2\,b^2c^3x^3 + 3\,b^2cx)\sqrt{c^2x^2 + 1}\right)\log\left(cx + \sqrt{c^2x^2 + 1}\right) + 6\,(abc^4x^4 + 2\,abc^2x^2 + ab)\log\left(-cx + \sqrt{c^2x^2 + 1}\right) + 2\,(2\,abc^3x^3 + 3\,abcx)\sqrt{c^2x^2 + 1}}{12\,(c^6d^3x^4 + 2\,c^4d^3x^2 + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*b*c^4*x^4 + (8*a*b + b^2)*c^2*x^2 - 3*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 - 3*a^2 + 4*a*b + b^2 - 2*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c
^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + (2*b^2*c^3*x^3 + 3*b^2*c*x
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x^4 + 2*a*b*
c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(2*a*b*c^3*x^3 + 3*a*b*c*x
)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x\,\mathrm{asinh}^2\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx\,\mathrm{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b**2*x*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Int
egral(2*a*b*x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d
**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{asinh}(c\,x)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

3.247 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=309

$$-\frac{b^2x}{12d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{8d^3\left(1+c^2x^2\right)}$$

```
[Out] -1/12*b^2*x/d^3/(c^2*x^2+1)+1/6*b*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(3/2
)+1/4*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+3/8*x*(a+b*arcsinh(c*x))^2/d
^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d^3
-5/6*b^2*arctan(c*x)/c/d^3-3/4*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^
2*x^2+1)^(1/2)))/c/d^3+3/4*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c/d^3+3/4*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3-3/4
*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/4*b*(a+b*arcsinh(c*x))/
c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 205}

$$\frac{3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{4cd^3}+\frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{c^2x^2+1}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{8d^3\left(c^2x^2+1\right)}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{3ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}-\frac{5b^2\text{ArcTan}(cx)}{6cd^3}-\frac{b^2x}{12d^3\left(c^2x^2+1\right)}+\frac{3ib^2\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{4cd^3}-\frac{3ib^2\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{4cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*(b^2*x)/(d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 +
c^2*x^2)^(3/2)) + (3*b*(a + b*ArcSinh[c*x]))/(4*c*d^3*Sqrt[1 + c^2*x^2]) +
(x*(a + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*
x])^2)/(8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c
*x]])/(4*c*d^3) - (5*b^2*ArcTan[c*x])/(6*c*d^3) - (((3*I)/4)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b*(a + b*Arc
Sinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b^2*PolyLog[3
, (-I)*E^ArcSinh[c*x]])/(c*d^3) - (((3*I)/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x
]])/(c*d^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3} + \frac{3\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx}{4d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{8d^3\left(1+c^2x^2\right)} - \frac{b^2\int \frac{\;}{(1+}}{6}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh\right.}{4d^3\left(1+c\right.}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh\right.}{4d^3\left(1+c\right.}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh\right.}{4d^3\left(1+c\right.}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh\right.}{4d^3\left(1+c\right.}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh\right.}{4d^3\left(1+c\right.}$$

Mathematica [A]
time = 1.64, size = 546, normalized size = 1.77

$$\frac{\frac{6a^2x}{\left(1+c^2x^2\right)^2} + \frac{9a^2x}{1+c^2x^2} + \frac{9a^2\text{ArcTan}(cx)}{c} + \frac{ab\left(\frac{9\left(-i\sqrt{1+c^2x^2}+\sinh^{-1}(cx)\right)}{-i+cx} + \frac{9\left(i\sqrt{1+c^2x^2}+\sinh^{-1}(cx)\right)}{i+cx} - \frac{i\left((-2i+cx)\sqrt{1+c^2x^2}+3\sinh^{-1}(cx)\right)}{(-i+cx)^2} + \frac{i\left((2i+cx)\sqrt{1+c^2x^2}+3\sinh^{-1}(cx)\right)}{(i+cx)^2} + \frac{9}{2}i\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right) - \frac{9}{2}i\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1-ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\right)\right)}{c} + \frac{b^2\left(-\frac{2cx}{1+c^2x^2} + \frac{4\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}} + \frac{18\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + \frac{6cx\sinh^{-1}(cx)^2}{\left(1+c^2x^2\right)^2} + \frac{9cx\sinh^{-1}(cx)^2}{1+c^2x^2} - 40\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right) - 9i\sinh^{-1}(cx)^2\log\left(1-ie^{-\sinh^{-1}(cx)}\right) + 9i\sinh^{-1}(cx)^2\log\left(1+ie^{-\sinh^{-1}(cx)}\right) - 18i\sinh^{-1}(cx)\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right) + 18i\sinh^{-1}(cx)\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right) - 18i\text{PolyLog}\left(3,-ie^{-\sinh^{-1}(cx)}\right) + 18i\text{PolyLog}\left(3,ie^{-\sinh^{-1}(cx)}\right)\right)}{c}}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((6*a^2*x)/(1 + c^2*x^2)^2 + (9*a^2*x)/(1 + c^2*x^2) + (9*a^2*ArcTan[c*x])/
c + (a*b*((9*((-I)*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (9*(I*Sq
rt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x) - (I*((-2*I + c*x)*Sqrt[1 + c^2*
x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 + (I*((2*I + c*x)*Sqrt[1 + c^2*x^2] +
3*ArcSinh[c*x]))/(I + c*x)^2 + ((9*I)/2)*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Lo
g[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((9*I)/2)*(
ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*
E^ArcSinh[c*x]])))/c + (b^2*((-2*c*x)/(1 + c^2*x^2) + (4*ArcSinh[c*x])/(1 +
 c^2*x^2)^(3/2) + (18*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (6*c*x*ArcSinh[c*x]
^2)/(1 + c^2*x^2)^2 + (9*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 40*ArcTan[Tanh
[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (9*I)*
ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSinh[c*x]*PolyLog[2, (
-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - (1
8*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3, I/E^ArcSinh[c*x]])
)/c)/(24*d^3)
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{(c^2 d\, x^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c
*x)/(c*d^3)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^6 +
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(
c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^6 + 3*c
^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b
**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(2*a*b*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^3, x)
```

3.248 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=275

$$-\frac{b^2}{12d^3\left(1+c^2x^2\right)}-\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}}-\frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}+\frac{\left(a+b\sinh^{-1}(cx\right.}{2d^3\left(1+c^2x^2\right)}$$

```
[Out] -1/12*b^2/d^3/(c^2*x^2+1)-1/6*b*c*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2
)+1/4*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+1/2*(a+b*arcsinh(c*x))^2/d^3/(
c^2*x^2+1)-2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+2/
3*b^2*ln(c^2*x^2+1)/d^3-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1
/2))^2)/d^3+b*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+1
/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-1/2*b^2*polylog(3,(c*x+(c^
2*x^2+1)^(1/2))^2)/d^3-4/3*b*c*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5811, 5799, 5569, 4267, 2611, 2320, 6724, 5787, 266, 5788, 267}

$$-\frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{c^2x^2+1}}-\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(c^2x^2+1\right)^{3/2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3\left(c^2x^2+1\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d^3}-\frac{b^2}{12d^3\left(c^2x^2+1\right)}+\frac{2b^2\log\left(c^2x^2+1\right)}{3d^3}+\frac{b^2\text{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{b^2\text{Li}_3\left(e^{2\sinh^{-1}(cx)}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/12*b^2/(d^3*(1 + c^2*x^2)) - (b*c*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^
2*x^2)^(3/2)) - (4*b*c*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) +
(a + b*ArcSinh[c*x])^2/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])^2/(2*
d^3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])
/d^3 + (2*b^2*Log[1 + c^2*x^2])/(3*d^3) - (b*(a + b*ArcSinh[c*x])*PolyLog[2
, -E^(2*ArcSinh[c*x])])/d^3 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSi
nh[c*x])])/d^3 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^3) - (b^2*PolyL
og[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
```

```
1] && NeQ[p, -3/2]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx}{d}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3\left(1+c^2x^2\right)} - \frac{(bc)\int}{}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\,\mathrm{s}}{4d^3\,(1}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\,\mathrm{s}}{4d^3\,(1}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\,\mathrm{s}}{4d^3\,(1}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\,\mathrm{s}}{4d^3\,(1}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\,\mathrm{s}}{4d^3\,(1}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\,\mathrm{s}}{4d^3\,(1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.62, size = 560, normalized size = 2.04

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((6*a^2)/(1 + c^2*x^2)^2 + (12*a^2)/(1 + c^2*x^2) + 24*a^2*Log[c*x] - 12*a^
2*Log[1 + c^2*x^2] + a*b*((-15*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c
*x) - (15*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 24*ArcSinh[c*x
]^2 - ((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(-I + c*x)^2 - ((2*
I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(I + c*x)^2 + 48*ArcSinh[c*x]*
Log[1 - E^(2*ArcSinh[c*x])] + 12*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*
E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + 12*(ArcSinh[c*x]*(A
rcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]
) + 24*PolyLog[2, E^(2*ArcSinh[c*x])]) + b^2*(I*Pi^3 - 2/(1 + c^2*x^2) - (4
*c*x*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (32*c*x*ArcSinh[c*x])/Sqrt[1 + c^2
```

```
*x^2] + (6*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (12*ArcSinh[c*x]^2)/(1 + c^2*x
^2) - 16*ArcSinh[c*x]^3 - 24*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] +
24*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 16*Log[1 + c^2*x^2] + 24*Ar
cSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 24*ArcSinh[c*x]*PolyLog[2, E^
(2*ArcSinh[c*x])] + 12*PolyLog[3, -E^(-2*ArcSinh[c*x])] - 12*PolyLog[3, E^(
2*ArcSinh[c*x])]))/(24*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(300) = 600$.
time = 5.57, size = 1129, normalized size = 4.11

method	result	size
derivativedivides	Expression too large to display	1129
default	Expression too large to display	1129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c^2*x^2+8/3*a*b/d^3/(c^4*x^4+2*c
^2*x^2+1)*c^2*x^2+1/4*a^2/d^3/(c^2*x^2+1)^2+4/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+
1)*c^4*x^4+a^2/d^3*ln(c*x)+4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c
^4*x^4+1/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2*c^2*x^2+8/3*b^2/d^3
/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c^2*x^2-4/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+
1)*(c^2*x^2+1)^(1/2)*c^3*x^3-1/12*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*c^2*x^2-2*b
^2/d^3*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-2*b^2/d^3*polylog(3,c*x+(c^2*x^2+1
)^(1/2))-3/2*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c*x*(c^2*x^2+1)^(1/2)+4/3*b^2/d^
3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/12*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)-1/2*a^
2/d^3*ln(c^2*x^2+1)+1/2*a^2/d^3/(c^2*x^2+1)-8/3*b^2/d^3*ln(c*x+(c^2*x^2+1)^
(1/2))-4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3
*x^3-3/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1
/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*a*b/d^3/(c^4*x^4+2*c^2
*x^2+1)*arcsinh(c*x)-2*a*b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
+2*a*b/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*a*b/d^3*arcsinh(c*x)*
ln(1-c*x-(c^2*x^2+1)^(1/2))+3/4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^
2+4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+b^2/d^3*arcsinh(c*x)^2*ln(
1+c*x+(c^2*x^2+1)^(1/2))+2*b^2/d^3*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))+b^2/d^3*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2/d^3*arcsinh
(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+4/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)-a*
b/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/d^3*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))+2*a*b/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-b^2/d^3*arcsinh(c*x
)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/d^3*arcsinh(c*x)*polylog(2,-(c*x+(c
^2*x^2+1)^(1/2))^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^
2 + 1)/d^3 + 4*log(x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x) + 2*a*b*log(c*x + sqrt
(c^2*x^2 + 1))/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^7 + 3*c
^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b
**2*asinh(c*x)**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integra
l(2*a*b*asinh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\, x))^2}{x\, (d\, c^2\, x^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^3), x)
```

3.249 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=389

$$\frac{b^2c^2x}{12d^3(1+c^2x^2)}-\frac{bc(a+b\sinh^{-1}(cx))}{6d^3(1+c^2x^2)^{3/2}}-\frac{7bc(a+b\sinh^{-1}(cx))}{4d^3\sqrt{1+c^2x^2}}-\frac{(a+b\sinh^{-1}(cx))^2}{d^3x(1+c^2x^2)^2}-\frac{5c^2x(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2}$$

```
[Out] 1/12*b^2*c^2*x/d^3/(c^2*x^2+1)-1/6*b*c*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(
3/2)-(a+b*arcsinh(c*x))^2/d^3/x/(c^2*x^2+1)^2-5/4*c^2*x*(a+b*arcsinh(c*x))^
2/d^3/(c^2*x^2+1)^2-15/8*c^2*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)-15/4*c*
(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3+11/6*b^2*c*arctan(c*
x)/d^3-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d^3-2*b^2*c*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^3+15/4*I*b*c*(a+b*arcsinh(c*x))*polylog
(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-15/4*I*b*c*(a+b*arcsinh(c*x))*polylog(2,
I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d^3
-15/4*I*b^2*c*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+15/4*I*b^2*c*polylo
g(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-7/4*b*c*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+
1)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5809, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 205, 5811, 5816, 4267, 2317, 2438}

$$-\frac{15c\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{4d^3}-\frac{7bc(a+b\sinh^{-1}(cx))}{4d^3\sqrt{c^2x^2+1}}-\frac{bc(a+b\sinh^{-1}(cx))}{6d^3(c^2x^2+1)^{3/2}}-\frac{15c^2x(a+b\sinh^{-1}(cx))^2}{8d^3(c^2x^2+1)}-\frac{5c^2x(a+b\sinh^{-1}(cx))^2}{4d^3(c^2x^2+1)^2}-\frac{(a+b\sinh^{-1}(cx))^2}{d^3x(c^2x^2+1)^2}+\frac{15ibc\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{4d^3}-\frac{15ibc\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{4d^3}-\frac{4bc\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{d^3}+\frac{11b^2c\text{ArcTan}(cx)}{6d^3}+\frac{b^2c^2x}{12d^3(c^2x^2+1)}-\frac{2b^2c\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{d^3}+\frac{2b^2c\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{d^3}-\frac{15ib^2c\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{4d^3}+\frac{15ib^2c\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] (b^2*c^2*x)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(6*d^3*(1 +
 c^2*x^2)^(3/2)) - (7*b*c*(a + b*ArcSinh[c*x]))/(4*d^3*Sqrt[1 + c^2*x^2]) -
 (a + b*ArcSinh[c*x])^2/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c
*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*
(1 + c^2*x^2)) - (15*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^
3) + (11*b^2*c*ArcTan[c*x])/(6*d^3) - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E
^ArcSinh[c*x]])/d^3 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^3 + (((15*I)/
4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 - (((15*I)
/4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 + (2*b^2*c*P
olyLog[2, E^ArcSinh[c*x]])/d^3 - (((15*I)/4)*b^2*c*PolyLog[3, (-I)*E^ArcSin
h[c*x]])/d^3 + (((15*I)/4)*b^2*c*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x(1+c^2x^2)^{5/2}}\,d}{d^3}$$

$$= \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3\left(1+c^2x^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{(2bc}{}$$

$$= -\frac{b^2c^2x}{3d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-}\right.}{d^3x\left(1+c\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-}\right.}{d^3x\left(1+c\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-}\right.}{d^3x\left(1+c\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-}\right.}{d^3x\left(1+c\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-}\right.}{d^3x\left(1+c\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-}\right.}{d^3x\left(1+c\right.}$$

<u>Mathematica</u> [**A**]
time = 7.09, size = 716, normalized size = 1.84

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(a^2/(d^3*x)) - (a^2*c^2*x)/(4*d^3*(1 + c^2*x^2)^2) - (7*a^2*c^2*x)/(8*d^3
*(1 + c^2*x^2)) - (15*a^2*c*ArcTan[c*x])/(8*d^3) + (2*a*b*c*((7*(Sqrt[1 + c
^2*x^2] + I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(c*x) - (7*(I*S
qrt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I + c*x)) + ((I/48)*((-2*I + c*x)*Sq
rt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 - ((I/48)*((2*I + c*x)*Sqrt
[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 - ArcTanh[Sqrt[1 + c^2*x^2]] +
 ((15*I)/16)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]
] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((15*I)/16)*(-1/2*ArcSinh[c*x]^2 +
 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyLog[2, I*E^ArcSinh[c*x]])
```

```
))/d^3 + (b^2*c*((2*c*x)/(1 + c^2*x^2) - (4*ArcSinh[c*x])/(1 + c^2*x^2)^(3/
2) - (42*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x*ArcSinh[c*x]^2)/(1 + c^2*
x^2)^2 - (21*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 88*ArcTan[Tanh[ArcSinh[c*x
]/2]] - 12*ArcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 48*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] + (45*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (45*I)
*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - 48*ArcSinh[c*x]*Log[1 + E^(-Arc
Sinh[c*x])] + 48*PolyLog[2, -E^(-ArcSinh[c*x])] + (90*I)*ArcSinh[c*x]*PolyL
og[2, (-I)/E^ArcSinh[c*x]] - (90*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x
]] - 48*PolyLog[2, E^(-ArcSinh[c*x])] + (90*I)*PolyLog[3, (-I)/E^ArcSinh[c*
x]] - (90*I)*PolyLog[3, I/E^ArcSinh[c*x]] + 12*ArcSinh[c*x]^2*Tanh[ArcSinh[
c*x]/2]))/(24*d^3)
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(c x\right)\right)^{2}}{x^{2} \left(c^{2} d \,x^{2}+d\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*
x) + 15*c*arctan(c*x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2) + 2*a*b*log(c*x + sq
rt(c^2*x^2 + 1))/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^8 + 3*c
^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2{(cx)}}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{2ab\operatorname{asinh}{(cx)}}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integra
l(b**2*asinh(c*x)**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + I
ntegral(2*a*b*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x)
)/d**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^2\,{\left(d\,c^2\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^3), x)
```

3.250 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=381

$$\frac{b^2c^2}{12d^3\left(1+c^2x^2\right)}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}}-\frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}}+\frac{4bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}}-\frac{3c^2\left(a+b\sinh\right.}{4d^3\left(1+c^2\right.}$$

```
[Out] 1/12*b^2*c^2/d^3/(c^2*x^2+1)-b*c*(a+b*arcsinh(c*x))/d^3/x/(c^2*x^2+1)^(3/2)
-5/6*b*c^3*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2)-3/4*c^2*(a+b*arcsinh(
c*x))^2/d^3/(c^2*x^2+1)^2-1/2*(a+b*arcsinh(c*x))^2/d^3/x^2/(c^2*x^2+1)^2-3/
2*c^2*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)+6*c^2*(a+b*arcsinh(c*x))^2*arcta
nh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+b^2*c^2*ln(x)/d^3-7/6*b^2*c^2*ln(c^2*x^2+
1)/d^3+3*b*c^2*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3
-3*b*c^2*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-3/2*b^
2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*b^2*c^2*polylog(3,(c*x+
(c^2*x^2+1)^(1/2))^2)/d^3+4/3*b*c^3*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1
/2)
```

Rubi [A]
time = 0.55, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 19, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$, Rules used = {5809, 5811, 5799, 5569, 4267, 2611, 2320, 6724, 5787, 266, 5788, 267, 277, 198, 197, 5804, 12, 1265, 907}

$$\frac{3bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3\left(c^2x^2+1\right)}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(c^2x^2+1\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{6c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d^3}+\frac{4bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{c^2x^2+1}}-\frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(c^2x^2+1\right)^{3/2}}-\frac{3b^2c^2\text{Li}_3\left(-e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{3b^2c^2\text{Li}_3\left(e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{b^2c^2}{12d^3\left(c^2x^2+1\right)}-\frac{7b^2c^2\log\left(c^2x^2+1\right)}{6d^3}+\frac{b^2c^2\log(x)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] (b^2*c^2)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(d^3*x*(1 + c
^2*x^2)^(3/2)) - (5*b*c^3*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3/2
)) + (4*b*c^3*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a
 + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])^2/(2*d
^3*x^2*(1 + c^2*x^2)^2) - (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^3*(1 + c^2*x^
2)) + (6*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (b^2
*c^2*Log[x])/d^3 - (7*b^2*c^2*Log[1 + c^2*x^2])/(6*d^3) + (3*b*c^2*(a + b*A
rcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^3 - (3*b*c^2*(a + b*ArcSinh
[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^3 - (3*b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/(2*d^3) + (3*b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
```

```
1] && NeQ[p, -3/2]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3x^2\left(1+c^2x^2\right)^2} - \left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2(1+c^2x^2)^{5/2}}\,d}{d^3}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{4bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\left(1+c^2x^2\right)^{3/2}} - \frac{8bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{8bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}}$$

$$= \frac{b^2c^2}{4d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x(a+}{3d^3\sqrt{}}$$

$$= \frac{b^2c^2}{4d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x(a+}{3d^3\sqrt{}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x(a}{3d^3}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x(a}{3d^3}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x(a}{3d^3}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x(a}{3d^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.33, size = 688, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/4*((2*a^2)/x^2 + (a^2*c^2)/(1 + c^2*x^2)^2 + (4*a^2*c^2)/(1 + c^2*x^2) +
 12*a^2*c^2*Log[x] - 6*a^2*c^2*Log[1 + c^2*x^2] - (a*b*((27*c^2*(Sqrt[1 + c
^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) + (27*c^2*(Sqrt[1 + c^2*x^2] + I*ArcSi
nh[c*x]))/(-I + c*x) - (24*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + (c
^2*((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 + (c^2*(
(2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 - 36*c^2*(ArcS
```

```
inh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E
^ArcSinh[c*x]]) - 36*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSin
h[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + 72*c^2*(ArcSinh[c*x]*(ArcSinh[
c*x] - 2*Log[1 - E^(2*ArcSinh[c*x])]) - PolyLog[2, E^(2*ArcSinh[c*x])]))))/6
 - 4*b^2*c^2*((-1/8*I)*Pi^3 + (12 + 12*c^2*x^2)^(-1) + (c*x*ArcSinh[c*x])/(
6*(1 + c^2*x^2)^(3/2)) + (7*c*x*ArcSinh[c*x])/(3*Sqrt[1 + c^2*x^2]) - (Sqrt
[1 + c^2*x^2]*ArcSinh[c*x])/(c*x) - ArcSinh[c*x]^2/(2*c^2*x^2) - ArcSinh[c*
x]^2/(4*(1 + c^2*x^2)^2) - ArcSinh[c*x]^2/(1 + c^2*x^2) + 2*ArcSinh[c*x]^3
+ 3*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] - 3*ArcSinh[c*x]^2*Log[1 -
E^(2*ArcSinh[c*x])] + Log[c*x] - (7*Log[1 + c^2*x^2])/6 - 3*ArcSinh[c*x]*Po
lyLog[2, -E^(-2*ArcSinh[c*x])] - 3*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x
])] - (3*PolyLog[3, -E^(-2*ArcSinh[c*x])])/2 + (3*PolyLog[3, E^(2*ArcSinh[c
*x])])/2))/d^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1364 vs. $2(402) = 804$.

time = 6.08, size = 1365, normalized size = 3.58

method	result	size
derivativedivides	Expression too large to display	1365
default	Expression too large to display	1365

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c^2*x^2-8/3*a*b/d^3/(c^4
*x^4+2*c^2*x^2+1)*c^2*x^2-1/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)/c^2/x^2*arcsinh
(c*x)^2-1/4*a^2/d^3/(c^2*x^2+1)^2-4/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c^4*x^4
-3*a^2/d^3*ln(c*x)-4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c^4*x^4-3
/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2*c^2*x^2-8/3*b^2/d^3/(c^4*x^
4+2*c^2*x^2+1)*arcsinh(c*x)*c^2*x^2+4/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*(c^2*
x^2+1)^(1/2)*c^3*x^3+1/12*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*c^2*x^2+6*b^2/d^3*p
olylog(3,-c*x-(c^2*x^2+1)^(1/2))+6*b^2/d^3*polylog(3,c*x+(c^2*x^2+1)^(1/2))
-a*b/d^3/(c^4*x^4+2*c^2*x^2+1)/c/x*(c^2*x^2+1)^(1/2)-a*b/d^3/(c^4*x^4+2*c^2
*x^2+1)/c^2/x^2*arcsinh(c*x)-b^2/d^3/(c^4*x^4+2*c^2*x^2+1)/c/x*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)+1/2*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c*x*(c^2*x^2+1)^(1/2)+
b^2/d^3*ln(1+c*x+(c^2*x^2+1)^(1/2))+b^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-1)-1/2
*a^2/d^3/c^2/x^2-7/3*b^2/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/12*b^2/d^3/(
c^4*x^4+2*c^2*x^2+1)+3/2*a^2/d^3*ln(c^2*x^2+1)-a^2/d^3/(c^2*x^2+1)+8/3*b^2/
d^3*ln(c*x+(c^2*x^2+1)^(1/2))+4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)*c^3*x^3+1/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*
(c^2*x^2+1)^(1/2)*c*x-3/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-9/2
*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+6*a*b/d^3*arcsinh(c*x)*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)-6*a*b/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-
6*a*b/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-9/4*b^2/d^3/(c^4*x^4+2*c
```

```
^2*x^2+1)*arcsinh(c*x)^2-4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)-3*b
^2/d^3*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-6*b^2/d^3*arcsinh(c*x)*po
lylog(2,-c*x-(c^2*x^2+1)^(1/2))-3*b^2/d^3*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+
1)^(1/2))-6*b^2/d^3*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-4/3*a*b/d
^3/(c^4*x^4+2*c^2*x^2+1)+3*a*b/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-6*
a*b/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-6*a*b/d^3*polylog(2,c*x+(c^2*x^2+
1)^(1/2))+3*b^2/d^3*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+3*b^2/d^
3*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^
2) - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + integrate(b^2*log(c*
x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3
*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3
*c^2*d^3*x^5 + d^3*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^9 + 3*c
^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + Integra
l(b**2*asinh(c*x)**2/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + I
```

```
ntegral(2*a*b*asinh(c*x)/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x)
)/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{x^3\,\left(d\,c^2\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^3), x)
```

3.251 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=529

$$-\frac{b^2c^2}{2d^3x}+\frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)}+\frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)}-\frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}+\frac{29bc^3\left(a+b\sinh^{-1}(cx)\right)}{12d^3\sqrt{1+c^2x^2}}$$

```
[Out] -1/2*b^2*c^2/d^3/x+1/6*b^2*c^2/d^3/x/(c^2*x^2+1)+1/12*b^2*c^4*x/d^3/(c^2*x^
2+1)-1/6*b*c^3*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2)-1/3*b*c*(a+b*arcsin
h(c*x))/d^3/x^2/(c^2*x^2+1)^(3/2)-1/3*(a+b*arcsinh(c*x))^2/d^3/x^3/(c^2*x^2
+1)^2+7/3*c^2*(a+b*arcsinh(c*x))^2/d^3/x/(c^2*x^2+1)^2+35/12*c^4*x*(a+b*arc
sinh(c*x))^2/d^3/(c^2*x^2+1)^2+35/8*c^4*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2
+1)+35/4*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3-17/6*b^
2*c^3*arctan(c*x)/d^3+38/3*b*c^3*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)
^(1/2))/d^3+19/3*b^2*c^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^3+35/4*I*b*c^3
*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-35/4*I*b*c^3*(
a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-19/3*b^2*c^3*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))/d^3-35/4*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2
+1)^(1/2)))/d^3+35/4*I*b^2*c^3*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+29
/12*b*c^3*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 17, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {5809, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 205, 5811, 5816, 4267, 2317, 2438, 296, 331}

$$\frac{35c^3\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3}-\frac{35ibc^3\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{35ibc^3\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{38bc^3\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3d^3}+\frac{7c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x\left(c^2x^2+1\right)^2}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(c^2x^2+1\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{35c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{8d^3\left(c^2x^2+1\right)}+\frac{35c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{12d^3\left(c^2x^2+1\right)^2}+\frac{29bc^3\left(a+b\sinh^{-1}(cx)\right)}{12d^3\sqrt{c^2x^2+1}}-\frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(c^2x^2+1\right)^{3/2}}-\frac{17b^2c^3\text{ArcTan}(cx)}{6d^3}+\frac{19b^2c^3\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{3d^3}-\frac{19b^2c^3\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{3d^3}+\frac{35ib^2c^3\text{Li}_3\left(-ie^{\sinh^{-1}(cx)}\right)}{4d^3}-\frac{35ib^2c^3\text{Li}_3\left(ie^{\sinh^{-1}(cx)}\right)}{4d^3}+\frac{b^2c^2}{6d^3x\left(c^2x^2+1\right)}-\frac{b^2c^2}{2d^3x}+\frac{b^2c^4x}{12d^3\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/2*(b^2*c^2)/(d^3*x) + (b^2*c^2)/(6*d^3*x*(1 + c^2*x^2)) + (b^2*c^4*x)/(1
2*d^3*(1 + c^2*x^2)) - (b*c^3*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3
/2)) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^3*x^2*(1 + c^2*x^2)^(3/2)) + (29*b*c
^3*(a + b*ArcSinh[c*x]))/(12*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^
2/(3*d^3*x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x])^2)/(3*d^3*x*(1
+ c^2*x^2)^2) + (35*c^4*x*(a + b*ArcSinh[c*x])^2)/(12*d^3*(1 + c^2*x^2)^2)
+ (35*c^4*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*
ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (17*b^2*c^3*ArcTan[c*x])/
(6*d^3) + (38*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d^3) +
 (19*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^3) - (((35*I)/4)*b*c^3*(a +
b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)/4)*b*c^3*(a
```

```
 + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 - (19*b^2*c^3*PolyLog[
2, E^ArcSinh[c*x]])/(3*d^3) + (((35*I)/4)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh
[c*x]])/d^3 - (((35*I)/4)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x^3\left(1+c^2x^2\right)^2} - \frac{1}{3}\left(7c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^3}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(c}{x^3(1+c^2x^2)^{5/}}}{3d^3}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x\left(1+c^2x^2\right)^2} + \frac{1}{3}(3$$

$$= \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} - \frac{19bc^3\left(a+b\sinh^{-1}(cx)\right)}{9d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{(a+b\,\mathrm{si}}{3d^3x^3\,(}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{19b^2c^4x}{18d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc(a+}{3d^3x^2}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc(a+}{3d^3x^2}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc(a+}{3d^3x^2}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc(a+}{3d^3x^2}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc(a+}{3d^3x^2}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc(a+}{3d^3x^2}$$

Mathematica [A]
time = 8.80, size = 937, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/3*a^2/(d^3*x^3) + (3*a^2*c^2)/(d^3*x) + (a^2*c^4*x)/(4*d^3*(1 + c^2*x^2)
^2) + (11*a^2*c^4*x)/(8*d^3*(1 + c^2*x^2)) + (35*a^2*c^3*ArcTan[c*x])/(8*d^
3) + (2*a*b*(-1/6*(c*Sqrt[1 + c^2*x^2])/x^2 + ((I/48)*c^3*((2*I - c*x)*Sqrt
[1 + c^2*x^2] - 3*ArcSinh[c*x]))/(-I + c*x)^2 - (11*c^3*(Sqrt[1 + c^2*x^2]
+ I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(3*x^3) + (11*c^4*(I*Sq
rt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I*c + c^2*x)) + ((I/48)*c^3*((2*I + c
```

```
*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 + (c^3*ArcTanh[Sqrt[1
+ c^2*x^2]])/6 - 3*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) -
 ((35*I)/16)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSi
nh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + ((35*I)/16)*c^4*(-1/
2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*Poly
Log[2, I*E^ArcSinh[c*x]])/c)))/d^3 + (b^2*c^3*((-2*c*x)/(1 + c^2*x^2) + (4*
ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (66*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (
6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (33*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^
2) - 136*ArcTan[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 38*ArcSinh
[c*x]^2*Coth[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x
*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 152*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] - (105*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (105*I)*A
rcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 152*ArcSinh[c*x]*Log[1 + E^(-ArcS
inh[c*x])] - 152*PolyLog[2, -E^(-ArcSinh[c*x])] - (210*I)*ArcSinh[c*x]*Poly
Log[2, (-I)/E^ArcSinh[c*x]] + (210*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c
*x]] + 152*PolyLog[2, E^(-ArcSinh[c*x])] - (210*I)*PolyLog[3, (-I)/E^ArcSin
h[c*x]] + (210*I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSin
h[c*x]/2]^2 - (8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[
ArcSinh[c*x]/2] - 38*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)^{2}}{x^{4} \left(c^{2} d\,x^{2}+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2
 - 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + integrate(b^2*log(c*x + sq
rt(c^2*x^2 + 1))^2/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4)
 + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2
*d^3*x^6 + d^3*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^10 + 3*
c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integr
al(b**2*asinh(c*x)**2/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) +
 Integral(2*a*b*asinh(c*x)/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4),
 x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^4\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^3), x)
```

3.252 $\int (\pi + c^2\pi x^2)^{5/2} \left(a + b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=300

$$\frac{245b^2\pi^{5/2}x\sqrt{1+c^2x^2}}{1152}+\frac{65b^2\pi^{5/2}x(1+c^2x^2)^{3/2}}{1728}+\frac{1}{108}b^2\pi^{5/2}x\left(1+c^2x^2\right)^{5/2}-\frac{115b^2\pi^{5/2}\sinh^{-1}(cx)}{1152c}-\frac{5}{16}bc\pi^{5/2}x$$

```
[Out] 65/1728*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(3/2)+1/108*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(
5/2)-115/1152*b^2*Pi^(5/2)*arcsinh(c*x)/c-5/16*b*c*Pi^(5/2)*x^2*(a+b*arcsin
h(c*x))-5/48*b*Pi^(5/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c-1/18*b*Pi^(5/2)*
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c+5/24*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arc
sinh(c*x))^2+1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2+5/48*Pi^(5/2)
*(a+b*arcsinh(c*x))^3/b/c+245/1152*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(1/2)+5/16*Pi
^2*x*(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$-\frac{\pi^{5/2}b(c^2x^2+1)^3\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5\pi^{5/2}b(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{48c}+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5}{16}\pi^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{5}{16}\pi^{5/2}bcx^2\left(a+b\sinh^{-1}(cx)\right)+\frac{5\pi^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc}+\frac{1}{108}\pi^{5/2}b^2x\left(c^2x^2+1\right)^{5/2}+\frac{65\pi^{5/2}b^2x(c^2x^2+1)^{3/2}}{1728}+\frac{245\pi^{5/2}b^2x\sqrt{c^2x^2+1}}{1152}-\frac{115\pi^{5/2}b^2\sinh^{-1}(cx)}{1152c}$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (245*b^2*Pi^(5/2)*x*Sqrt[1 + c^2*x^2])/1152 + (65*b^2*Pi^(5/2)*x*(1 + c^2*x
^2)^(3/2))/1728 + (b^2*Pi^(5/2)*x*(1 + c^2*x^2)^(5/2))/108 - (115*b^2*Pi^(5
/2)*ArcSinh[c*x])/(1152*c) - (5*b*c*Pi^(5/2)*x^2*(a + b*ArcSinh[c*x]))/16 -
 (5*b*Pi^(5/2)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(48*c) - (b*Pi^(5/2)*(
1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(18*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x])^2)/16 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcS
inh[c*x])^2)/24 + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5
*Pi^(5/2)*(a + b*ArcSinh[c*x])^3)/(48*b*c)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(\pi+c^2\pi x^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{6}x\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{6}(5\pi)\int \left(\pi+c^2\pi x^2\right)^{3/2}$$

$$= -\frac{b\pi^2\left(1+c^2x^2\right)^{5/2}\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{18c}+\frac{5}{24}\pi x\left(\pi+c\right.$$

$$= \frac{1}{108}b^2\pi^2x\left(1+c^2x^2\right)^2\sqrt{\pi+c^2\pi x^2}-\frac{5b\pi^2\left(1+c^2x^2\right)^{3/2}\sqrt{\pi+c^2\pi}}{48c}$$

$$= \frac{65b^2\pi^2x(1+c^2x^2)\sqrt{\pi+c^2\pi x^2}}{1728}+\frac{1}{108}b^2\pi^2x\left(1+c^2x^2\right)^2\sqrt{\pi+c^2}$$

$$= \frac{245b^2\pi^2x\sqrt{\pi+c^2\pi x^2}}{1152}+\frac{65b^2\pi^2x(1+c^2x^2)\sqrt{\pi+c^2\pi x^2}}{1728}+\frac{1}{108}$$

$$= \frac{245b^2\pi^2x\sqrt{\pi+c^2\pi x^2}}{1152}+\frac{65b^2\pi^2x(1+c^2x^2)\sqrt{\pi+c^2\pi x^2}}{1728}+\frac{1}{108}$$

Mathematica [A]
time = 0.62, size = 284, normalized size = 0.95

$$\frac{\pi^{5/2}\left(9504a^2cx\sqrt{1+c^2x^2}+7488a^2c^3x^3\sqrt{1+c^2x^2}+2304a^2c^5x^5\sqrt{1+c^2x^2}+1440b^2\sinh^{-1}(cx)^3-3240ab\cosh\left(2\sinh^{-1}(cx)\right)-324ab\cosh\left(4\sinh^{-1}(cx)\right)-24ab\cosh\left(6\sinh^{-1}(cx)\right)+1620b^2\sinh\left(2\sinh^{-1}(cx)\right)+81b^2\sinh\left(4\sinh^{-1}(cx)\right)+4b^2\sinh\left(6\sinh^{-1}(cx)\right)+72b\sinh^{-1}(cx)^2\left(60a+45b\sinh\left(2\sinh^{-1}(cx)\right)+9b\sinh\left(4\sinh^{-1}(cx)\right)+b\sinh\left(6\sinh^{-1}(cx)\right)\right)+12\sinh^{-1}(cx)\left(360a^2-270b^2\cosh\left(2\sinh^{-1}(cx)\right)-27b^2\cosh\left(4\sinh^{-1}(cx)\right)-2b^2\cosh\left(6\sinh^{-1}(cx)\right)+540ab\sinh\left(2\sinh^{-1}(cx)\right)+108ab\sinh\left(4\sinh^{-1}(cx)\right)+12ab\sinh\left(6\sinh^{-1}(cx)\right)\right)\right)}{13824c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Pi^(5/2)*(9504*a^2*c*x*Sqrt[1 + c^2*x^2] + 7488*a^2*c^3*x^3*Sqrt[1 + c^2*x
^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2] + 1440*b^2*ArcSinh[c*x]^3 - 3240*a
*b*Cosh[2*ArcSinh[c*x]] - 324*a*b*Cosh[4*ArcSinh[c*x]] - 24*a*b*Cosh[6*ArcS
inh[c*x]] + 1620*b^2*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sinh[4*ArcSinh[c*x]] + 4
*b^2*Sinh[6*ArcSinh[c*x]] + 72*b*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh
[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]) + 12*ArcSinh[c*
x]*(360*a^2 - 270*b^2*Cosh[2*ArcSinh[c*x]] - 27*b^2*Cosh[4*ArcSinh[c*x]] -
2*b^2*Cosh[6*ArcSinh[c*x]] + 540*a*b*Sinh[2*ArcSinh[c*x]] + 108*a*b*Sinh[4*
ArcSinh[c*x]] + 12*a*b*Sinh[6*ArcSinh[c*x]])))/(13824*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\pi\,c^2x^2+\pi\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a^2*c^4*x^4 + 2*pi^2*a^2*c^2*x^2 + pi^
2*a^2 + (pi^2*b^2*c^4*x^4 + 2*pi^2*b^2*c^2*x^2 + pi^2*b^2)*arcsinh(c*x)^2 +
 2*(pi^2*a*b*c^4*x^4 + 2*pi^2*a*b*c^2*x^2 + pi^2*a*b)*arcsinh(c*x)), x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(287) = 574.
time = 36.58, size = 583, normalized size = 1.94

$$\begin{cases} \frac{\pi^{\frac{5}{2}} a^2 c^4 x^5 \sqrt{c^2x^2+1}}{6} + \frac{13\pi^{\frac{5}{2}} a^2 c^2 x^3 \sqrt{c^2x^2+1}}{24} + \frac{11\pi^{\frac{5}{2}} a^2 x \sqrt{c^2x^2+1}}{16} + \frac{5\pi^{\frac{5}{2}} a^2 \operatorname{asinh}(cx)}{16c} - \frac{\pi^{\frac{5}{2}} abc^5x^6}{18} + \frac{\pi^{\frac{5}{2}} abc^4x^5 \sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{3} - \frac{13\pi^{\frac{5}{2}} abc^3x^4}{48} + \frac{13\pi^{\frac{5}{2}} abc^2x^3\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{12} - \frac{11\pi^{\frac{5}{2}} abcx^2}{16} + \frac{11\pi^{\frac{5}{2}} abx\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{8} + \frac{5\pi^{\frac{5}{2}} ab \operatorname{asinh}^2(cx)}{16c} - \frac{\pi^{\frac{5}{2}} b^2c^5x^6 \operatorname{asinh}(cx)}{18} + \frac{\pi^{\frac{5}{2}} b^2c^4x^5\sqrt{c^2x^2+1} \operatorname{asinh}^2(cx)}{6} + \frac{\pi^{\frac{5}{2}} b^2c^4x^5\sqrt{c^2x^2+1}}{108} - \frac{13\pi^{\frac{5}{2}} b^2c^3x^4 \operatorname{asinh}(cx)}{48} + \frac{13\pi^{\frac{5}{2}} b^2c^2x^3\sqrt{c^2x^2+1} \operatorname{asinh}^2(cx)}{24} + \frac{97\pi^{\frac{5}{2}} b^2c^2x^3\sqrt{c^2x^2+1}}{1728} - \frac{11\pi^{\frac{5}{2}} b^2cx^2 \operatorname{asinh}(cx)}{16} + \frac{11\pi^{\frac{5}{2}} b^2x\sqrt{c^2x^2+1} \operatorname{asinh}^2(cx)}{16} + \frac{299\pi^{\frac{5}{2}} b^2x\sqrt{c^2x^2+1}}{1152} + \frac{5\pi^{\frac{5}{2}} b^2 \operatorname{asinh}^3(cx)}{48c} - \frac{299\pi^{\frac{5}{2}} b^2 \operatorname{asinh}(cx)}{1152c} & \text{for}\: c \neq 0 \\ \pi^{\frac{5}{2}} a^2 x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((pi**(5/2)*a**2*c**4*x**5*sqrt(c**2*x**2 + 1)/6 + 13*pi**(5/2)*a*
*2*c**2*x**3*sqrt(c**2*x**2 + 1)/24 + 11*pi**(5/2)*a**2*x*sqrt(c**2*x**2 +
1)/16 + 5*pi**(5/2)*a**2*asinh(c*x)/(16*c) - pi**(5/2)*a*b*c**5*x**6/18 + p
i**(5/2)*a*b*c**4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/3 - 13*pi**(5/2)*a*b*
c**3*x**4/48 + 13*pi**(5/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/12
 - 11*pi**(5/2)*a*b*c*x**2/16 + 11*pi**(5/2)*a*b*x*sqrt(c**2*x**2 + 1)*asin
h(c*x)/8 + 5*pi**(5/2)*a*b*asinh(c*x)**2/(16*c) - pi**(5/2)*b**2*c**5*x**6*
asinh(c*x)/18 + pi**(5/2)*b**2*c**4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/
6 + pi**(5/2)*b**2*c**4*x**5*sqrt(c**2*x**2 + 1)/108 - 13*pi**(5/2)*b**2*c*
```

```
*3*x**4*asinh(c*x)/48 + 13*pi**(5/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)*asi
nh(c*x)**2/24 + 97*pi**(5/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/1728 - 11*p
i**(5/2)*b**2*c*x**2*asinh(c*x)/16 + 11*pi**(5/2)*b**2*x*sqrt(c**2*x**2 + 1
)*asinh(c*x)**2/16 + 299*pi**(5/2)*b**2*x*sqrt(c**2*x**2 + 1)/1152 + 5*pi**
(5/2)*b**2*asinh(c*x)**3/(48*c) - 299*pi**(5/2)*b**2*asinh(c*x)/(1152*c), N
e(c, 0)), (pi**(5/2)*a**2*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.253 $\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=210

$$\frac{15}{64}b^2\pi^{3/2}x\sqrt{1+c^2x^2}+\frac{1}{32}b^2\pi^{3/2}x\left(1+c^2x^2\right)^{3/2}-\frac{9b^2\pi^{3/2}\sinh^{-1}(cx)}{64c}-\frac{3}{8}bc\pi^{3/2}x^2\left(a+b\sinh^{-1}(cx)\right)-\frac{b\pi^{3/2}(1}{}$$

```
[Out] 1/32*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(3/2)-9/64*b^2*Pi^(3/2)*arcsinh(c*x)/c-3/8*
b*c*Pi^(3/2)*x^2*(a+b*arcsinh(c*x))-1/8*b*Pi^(3/2)*(c^2*x^2+1)^2*(a+b*arcsi
nh(c*x))/c+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*Pi^(3/2)*(a
+b*arcsinh(c*x))^3/b/c+15/64*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(1/2)+3/8*Pi*x*(a+b
*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{\pi^{3/2}b\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3}{8}\pi^{3/2}bcx^2\left(a+b\sinh^{-1}(cx)\right)+\frac{\pi^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc}+\frac{1}{32}\pi^{3/2}b^2x\left(c^2x^2+1\right)^{3/2}+\frac{15}{64}\pi^{3/2}b^2x\sqrt{c^2x^2+1}-\frac{9\pi^{3/2}b^2\sinh^{-1}(cx)}{64c}$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (15*b^2*Pi^(3/2)*x*Sqrt[1 + c^2*x^2])/64 + (b^2*Pi^(3/2)*x*(1 + c^2*x^2)^(3
/2))/32 - (9*b^2*Pi^(3/2)*ArcSinh[c*x])/(64*c) - (3*b*c*Pi^(3/2)*x^2*(a + b
*ArcSinh[c*x]))/8 - (b*Pi^(3/2)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(8*c)
 + (3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (x*(Pi + c^2*P
i*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (Pi^(3/2)*(a + b*ArcSinh[c*x])^3)/
(8*b*c)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{4}x\left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)^2 + \frac{1}{4}(3\pi)\int \sqrt{\pi + c^2\pi x^2}$$

$$= -\frac{b\pi(1 + c^2x^2)^{3/2}\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{8c} + \frac{3}{8}\pi x\sqrt{\pi +}$$

$$= \frac{1}{32}b^2\pi x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2} - \frac{3bc\pi x^2\sqrt{\pi + c^2\pi x^2}\left(a + b\sin\right.}{8\sqrt{1 + c^2x^2}}$$

$$= \frac{15}{64}b^2\pi x\sqrt{\pi + c^2\pi x^2} + \frac{1}{32}b^2\pi x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2} - \frac{3bc\pi}{}$$

$$= \frac{15}{64}b^2\pi x\sqrt{\pi + c^2\pi x^2} + \frac{1}{32}b^2\pi x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2} - \frac{9b^2\pi}{}$$

Mathematica [A]
time = 0.39, size = 202, normalized size = 0.96

$$\frac{\pi^{3/2}\left(160a^2cx\sqrt{1+c^2x^2} + 64a^2c^3x^3\sqrt{1+c^2x^2} + 32b^2\sinh^{-1}(cx)^3 - 64ab\cosh\left(2\sinh^{-1}(cx)\right) - 4ab\cosh\left(4\sinh^{-1}(cx)\right) + 32b^2\sinh\left(2\sinh^{-1}(cx)\right) + b^2\sinh\left(4\sinh^{-1}(cx)\right) + 8b\sinh^{-1}(cx)^2\left(12a + 8b\sinh\left(2\sinh^{-1}(cx)\right) + b\sinh\left(4\sinh^{-1}(cx)\right)\right) + 4\sinh^{-1}(cx)\left(-16b^2\cosh\left(2\sinh^{-1}(cx)\right) - b^2\cosh\left(4\sinh^{-1}(cx)\right) + 4a\left(6a + 8b\sinh\left(2\sinh^{-1}(cx)\right) + b\sinh\left(4\sinh^{-1}(cx)\right)\right)\right)\right)}{256c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Pi^(3/2)*(160*a^2*c*x*Sqrt[1 + c^2*x^2] + 64*a^2*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 32*b^2*ArcSinh[c*x]^3 - 64*a*b*Cosh[2*ArcSinh[c*x]] - 4*a*b*Cosh[4*ArcSi
nh[c*x]] + 32*b^2*Sinh[2*ArcSinh[c*x]] + b^2*Sinh[4*ArcSinh[c*x]] + 8*b*Arc
Sinh[c*x]^2*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) + 4*
ArcSinh[c*x]*(-16*b^2*Cosh[2*ArcSinh[c*x]] - b^2*Cosh[4*ArcSinh[c*x]] + 4*a
*(6*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))))/(256*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\pi\,c^2x^2 + \pi\right)^{\frac{3}{2}} \left(a + b\,\mathrm{arcsinh}\left(cx\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a^2*c^2*x^2 + pi*a^2 + (pi*b^2*c^2*x^2 +
 pi*b^2)*arcsinh(c*x)^2 + 2*(pi*a*b*c^2*x^2 + pi*a*b)*arcsinh(c*x)), x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(197) = 394$.
time = 3.70, size = 405, normalized size = 1.93

$$\begin{cases} \frac{\pi^{\frac{3}{2}} a^2 c^2 x^3 \sqrt{c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}} a^2 x \sqrt{c^2x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}} a^2 \operatorname{asinh}(cx)}{8c} - \frac{\pi^{\frac{3}{2}} abc^3x^4}{8} + \frac{\pi^{\frac{3}{2}} abc^2x^3 \sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{2} - \frac{5\pi^{\frac{3}{2}} abcx^2}{8} + \frac{5\pi^{\frac{3}{2}} abx \sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{4} + \frac{3\pi^{\frac{3}{2}} ab \operatorname{asinh}^2(cx)}{8c} - \frac{\pi^{\frac{3}{2}} b^2c^3x^4 \operatorname{asinh}(cx)}{8} + \frac{\pi^{\frac{3}{2}} b^2c^2x^3 \sqrt{c^2x^2+1} \operatorname{asinh}^2(cx)}{4} + \frac{\pi^{\frac{3}{2}} b^2c^2x^3 \sqrt{c^2x^2+1}}{32} - \frac{5\pi^{\frac{3}{2}} b^2cx^2 \operatorname{asinh}(cx)}{8} + \frac{5\pi^{\frac{3}{2}} b^2x \sqrt{c^2x^2+1} \operatorname{asinh}^2(cx)}{8} + \frac{17\pi^{\frac{3}{2}} b^2x \sqrt{c^2x^2+1}}{64} + \frac{\pi^{\frac{3}{2}} b^2 \operatorname{asinh}^3(cx)}{8c} - \frac{17\pi^{\frac{3}{2}} b^2 \operatorname{asinh}(cx)}{64c} & \text{for } c \neq 0 \\ \pi^{\frac{3}{2}} a^2 x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((pi**(3/2)*a**2*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a**
2*x*sqrt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a**2*asinh(c*x)/(8*c) - pi**(3/2)*a
*b*c**3*x**4/8 + pi**(3/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/2 -
 5*pi**(3/2)*a*b*c*x**2/8 + 5*pi**(3/2)*a*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x
)/4 + 3*pi**(3/2)*a*b*asinh(c*x)**2/(8*c) - pi**(3/2)*b**2*c**3*x**4*asinh(
c*x)/8 + pi**(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/4 + pi*
*(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/32 - 5*pi**(3/2)*b**2*c*x**2*asin
h(c*x)/8 + 5*pi**(3/2)*b**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/8 + 17*pi**
(3/2)*b**2*x*sqrt(c**2*x**2 + 1)/64 + pi**(3/2)*b**2*asinh(c*x)**3/(8*c) -
17*pi**(3/2)*b**2*asinh(c*x)/(64*c), Ne(c, 0)), (pi**(3/2)*a**2*x, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))^2\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.254 $\int \sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=122

$$\frac{1}{4}b^2\sqrt{\pi}\, x\sqrt{1+c^2x^2} - \frac{b^2\sqrt{\pi}\, \sinh^{-1}(cx)}{4c} - \frac{1}{2}bc\sqrt{\pi}\, x^2\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{2}x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)^2$$

```
[Out] -1/4*b^2*arcsinh(c*x)*Pi^(1/2)/c-1/2*b*c*x^2*(a+b*arcsinh(c*x))*Pi^(1/2)+1/
6*(a+b*arcsinh(c*x))^3*Pi^(1/2)/b/c+1/4*b^2*x*Pi^(1/2)*(c^2*x^2+1)^(1/2)+1/
2*x*(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5785, 5783, 5776, 327, 221}

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{1}{2}\sqrt{\pi}\, bcx^2\left(a+b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc} + \frac{1}{4}\sqrt{\pi}\, b^2x\sqrt{c^2x^2+1} - \frac{\sqrt{\pi}\, b^2\sinh^{-1}(cx)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*Sqrt[Pi]*x*Sqrt[1 + c^2*x^2])/4 - (b^2*Sqrt[Pi]*ArcSinh[c*x])/(4*c) -
(b*c*Sqrt[Pi]*x^2*(a + b*ArcSinh[c*x]))/2 + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b
*ArcSinh[c*x])^2)/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^3)/(6*b*c)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{\pi + c^2\pi x^2}\int \frac{(a+b\sinh^{-1}(cx}{\sqrt{1+c^2x^2}}}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcx^2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}\left(a + b\sin\right.$$

$$= \frac{1}{4}b^2x\sqrt{\pi + c^2\pi x^2} - \frac{bcx^2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{\cdot}$$

$$= \frac{1}{4}b^2x\sqrt{\pi + c^2\pi x^2} - \frac{b^2\sqrt{\pi + c^2\pi x^2}\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}} - \frac{bcx^2\sqrt{\pi + c^2\pi x}}{2\sqrt{}}$$

Mathematica [A]
time = 0.21, size = 124, normalized size = 1.02

$$\frac{\sqrt{\pi}\left(4b^2\sinh^{-1}(cx)^3 + 6b\sinh^{-1}(cx)^2\left(2a + b\sinh\left(2\sinh^{-1}(cx)\right)\right) + 3\left(4a^2cx\sqrt{1+c^2x^2} - 2ab\cosh\left(2\sinh^{-1}(cx)\right) + b^2\sinh\left(2\sinh^{-1}(cx)\right)\right) + 6\sinh^{-1}(cx)\left(-b^2\cosh\left(2\sinh^{-1}(cx)\right) + 2a\left(a + b\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)\right)}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[Pi]*(4*b^2*ArcSinh[c*x]^3 + 6*b*ArcSinh[c*x]^2*(2*a + b*Sinh[2*ArcSin
h[c*x]]) + 3*(4*a^2*c*x*Sqrt[1 + c^2*x^2] - 2*a*b*Cosh[2*ArcSinh[c*x]] + b^
2*Sinh[2*ArcSinh[c*x]]) + 6*ArcSinh[c*x]*(-(b^2*Cosh[2*ArcSinh[c*x]]) + 2*a
*(a + b*Sinh[2*ArcSinh[c*x]]))))/(24*c)
```

Maple [A]
time = 1.21, size = 179, normalized size = 1.47

method	result
default	$\frac{a^{2}x\sqrt{\pi\,c^{2}x^{2}+\pi}}{2}+\frac{a^{2}\pi\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{2\sqrt{\pi\,c^{2}}}+\frac{b^{2}\sqrt{\pi}\,\left(6\,\mathrm{arcsinh}(cx)^{2}\sqrt{c^{2}x^{2}+1}\,xc-6\,\mathrm{arcsinh}(cx)c^{2}x^{2}\right.}{12c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a^2*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c
^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/12*b^2*Pi^(1/2)*(6*arcsinh(c*x)^2*(c^2*x
^2+1)^(1/2)*x*c-6*arcsinh(c*x)*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+2*arcsinh(c*
x)^3-3*arcsinh(c*x))/c+1/2*a*b*Pi^(1/2)*(2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c
*x-c^2*x^2+arcsinh(c*x)^2-1)/c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int a^{2}\sqrt{c^{2}x^{2}+1}\,dx+\int b^{2}\sqrt{c^{2}x^{2}+1}\,\mathrm{asinh}^{2}\left(cx\right)dx+\int 2ab\sqrt{c^{2}x^{2}+1}\,\mathrm{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(1/2)*(a+b*asinh(c*x))**2,x)
```

[Out] sqrt(pi)*(Integral(a**2*sqrt(c**2*x**2 + 1), x) + Integral(b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)**2, x) + Integral(2*a*b*sqrt(c**2*x**2 + 1)*asinh(c*x), x))

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{\Pi\,c^2\,x^2+\Pi}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2),x)

[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2), x)

3.255 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{\pi}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3/b/c/Pi^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {5783}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3\sqrt{\pi}\,bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{\pi}}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(21) = 42$.
time = 0.81, size = 72, normalized size = 2.88

method	result	size
default	$\frac{a^2 \ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b^2 \operatorname{arcsinh}(cx)^3}{3c\sqrt{\pi}}+\frac{ab \operatorname{arcsinh}(cx)^2}{c\sqrt{\pi}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/3*b^
2/c/Pi^(1/2)*arcsinh(c*x)^3+a*b*arcsinh(c*x)^2/c/Pi^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.
time = 0.29, size = 47, normalized size = 1.88

$$\frac{b^2 \operatorname{arsinh}(cx)^3}{3\sqrt{\pi}\,c}+\frac{ab \operatorname{arsinh}(cx)^2}{\sqrt{\pi}\,c}+\frac{a^2 \operatorname{arsinh}(cx)}{\sqrt{\pi}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*arcsinh(c*x)^3/(sqrt(pi)*c) + a*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a^2
*arcsinh(c*x)/(sqrt(pi)*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(pi + pi*c^2*x
^2), x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(19) = 38$.

time = 1.62, size = 88, normalized size = 3.52

$$\begin{cases} a^2 \left(\begin{cases} \frac{\sqrt{-\frac{1}{c^2}} \operatorname{asin}\left(x\sqrt{-c^2}\right)}{\sqrt{\pi}} & \text{for } \pi c^2 < 0 \\ \frac{\sqrt{\frac{1}{c^2}} \operatorname{asinh}\left(x\sqrt{c^2}\right)}{\sqrt{\pi}} & \text{for } \pi c^2 > 0 \end{cases} \right) & \text{for } b = 0 \\ \frac{a^2 x}{\sqrt{\pi}} & \text{for } c = 0 \\ \frac{(a+b\operatorname{asinh}(cx))^3}{3\sqrt{\pi}\, bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((a**2*Piecewise((sqrt(-1/c**2)*asin(x*sqrt(-c**2))/sqrt(pi), pi*c
**2 < 0), (sqrt(c**(-2))*asinh(x*sqrt(c**2))/sqrt(pi), pi*c**2 > 0)), Eq(b,
 0)), (a**2*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**3/(3*sqrt(pi)*b*c),
 True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.256 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{(a+b\sinh^{-1}(cx))^2}{c\pi^{3/2}}+\frac{x(a+b\sinh^{-1}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2b(a+b\sinh^{-1}(cx))\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c\pi^{3/2}}-\frac{b^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c\pi^{3/2}}$$

```
[Out] (a+b*arcsinh(c*x))^2/c/Pi^(3/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)/c/Pi^(3/2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(3/2)
+x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {5787, 5797, 3799, 2221, 2317, 2438}

$$\frac{x(a+b\sinh^{-1}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{(a+b\sinh^{-1}(cx))^2}{\pi^{3/2}c}-\frac{2b\log\left(e^{2\sinh^{-1}(cx)}+1\right)(a+b\sinh^{-1}(cx))}{\pi^{3/2}c}-\frac{b^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{\pi^{3/2}c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(c*Pi^(3/2)) + (x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[P
i + c^2*Pi*x^2]) - (2*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(
c*Pi^(3/2)) - (b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*Pi^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}$$

$$=\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\sinh^{-1}(\right.}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$=\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}}-\frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subs}}{c\pi}$$

$$=\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\,\text{si}\right.}{c\pi}$$

$$=\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\,\text{si}\right.}{c\pi}$$

$$=\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\,\text{si}\right.}{c\pi}$$

Mathematica [A]
time = 0.24, size = 153, normalized size = 1.47

$$\frac{-b^2\left(-cx+\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2+2b\sinh^{-1}(cx)\left(acx-b\sqrt{1+c^2x^2}\log\left(1+e^{-2\sinh^{-1}(cx)}\right)\right)+a\left(acx-b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)\right)+b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)}{c\pi^{3/2}\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*c
*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqrt[
1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*Ar
cSinh[c*x])])/(c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(114) = 228$.
time = 2.39, size = 306, normalized size = 2.94

method	result
default	$\frac{a^{2} x}{\pi \sqrt{\pi c^{2} x^{2}+\pi}}-\frac{b^{2} \operatorname{arcsinh}(c x)^{2} c x^{2}}{\pi^{\frac{3}{2}}\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \operatorname{arcsinh}(c x)^{2} x}{\pi^{\frac{3}{2}} \sqrt{c^{2} x^{2}+1}}-\frac{b^{2} \operatorname{arcsinh}(c x)^{2}}{\pi^{\frac{3}{2}} c\left(c^{2} x^{2}+1\right)}+\frac{2 b^{2} \operatorname{arcsinh}(c x)^{2}}{c \pi^{\frac{3}{2}}}-\frac{2 b^{2} \operatorname{arcsinh}(c x) \ln (}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)-b^2/Pi^(3/2)*arcsinh(c*x)^2*c/(c^2*x^2+1)*x^
2+b^2/Pi^(3/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)*x-b^2/Pi^(3/2)*arcsinh(c*x)
^2/c/(c^2*x^2+1)+2*b^2/c/Pi^(3/2)*arcsinh(c*x)^2-2*b^2/c/Pi^(3/2)*arcsinh(c
*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^
2)/c/Pi^(3/2)+4*a*b/c/Pi^(3/2)*arcsinh(c*x)-2*a*b/Pi^(3/2)*arcsinh(c*x)*c/(
c^2*x^2+1)*x^2+2*a*b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*x-2*a*b/Pi^(3/
2)*arcsinh(c*x)/c/(c^2*x^2+1)-2*a*b/c/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(pi + pi*c^2*x^2)^(3/2), x) +
2*a*b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a^2*x/(pi*sqrt(pi + pi*c^
2*x^2)) - a*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2)/(pi^2*c^4*x^4 + 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) +
Integral(b**2*asinh(c*x)**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2
 + 1)), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqr
t(c**2*x**2 + 1)), x))/pi**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))^2}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.257 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^{5/2}\left(1+c^2x^2\right)}+\frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{3c\pi^{5/2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx\right.}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

```
[Out] 1/3*b*(a+b*arcsinh(c*x))/c/Pi^(5/2)/(c^2*x^2+1)+2/3*(a+b*arcsinh(c*x))^2/c/
Pi^(5/2)+1/3*x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-4/3*b*(a+b*arc
sinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-2/3*b^2*polylog(2,-(c
*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-1/3*b^2*x/Pi^(5/2)/(c^2*x^2+1)^(1/2)+2/
3*x*(a+b*arcsinh(c*x))^2/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197}

$$\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}c\left(c^2x^2+1\right)}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^{5/2}c}-\frac{4b\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}c}-\frac{b^2x}{3\pi^{5/2}\sqrt{c^2x^2+1}}-\frac{2b^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{3\pi^{5/2}c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/3*(b^2*x)/(Pi^(5/2)*Sqrt[1 + c^2*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*P
i^(5/2)*(1 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])^2)/(3*c*Pi^(5/2)) + (x*(a
+ b*ArcSinh[c*x])^2)/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c
*x])^2)/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (4*b*(a + b*ArcSinh[c*x])*Log[1 +
E^(2*ArcSinh[c*x])])/(3*c*Pi^(5/2)) - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])
])/(3*c*Pi^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx &= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{3\pi}-\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{x\left(a+b\sinh^{-1}\right.}{\left(1+c^2x^2\right)}}{3\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+ \\
&= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+ \\
&= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+ \\
&= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+ \\
&= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+
\end{aligned}
$$

Mathematica [A]
time = 0.41, size = 293, normalized size = 1.44

$$
\frac{3a^2cx-b^2cx+2a^2c^3x^3-b^2c^3x^3+ab\sqrt{1+c^2x^2}-b^2\left(-3cx-2c^3x^3+2\sqrt{1+c^2x^2}+2c^2x^2\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2-b\sinh^{-1}(cx)\left(-6acx-4ac^3x^3-b\sqrt{1+c^2x^2}+4b(1+c^2x^2)^{3/2}\log\left(1+e^{-2\sinh^{-1}(cx)}\right)\right)-2ab\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)-2abc^2x^2\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)+2b^2(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)}{3c\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 - b^2*c^3*x^3 + a*b*Sqrt[1 + c^2*x^2]
- b^2*(-3*c*x - 2*c^3*x^3 + 2*Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^
2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-6*a*c*x - 4*a*c^3*x^3 - b*Sqrt[1 + c^
2*x^2] + 4*b*(1 + c^2*x^2)^(3/2)*Log[1 + E^(-2*ArcSinh[c*x])]) - 2*a*b*Sqrt
[1 + c^2*x^2]*Log[1 + c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^
2*x^2] + 2*b^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])])/(3*c*P
i^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. $2(192) = 384$.
time = 6.10, size = 1729, normalized size = 8.48

method	result	size
default	Expression too large to display	1729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2+2/3*b^2/Pi^(
5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8-4*a*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(
c^2*x^2+1)^2*arcsinh(c*x)*x^6+17/3*b^2/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+
1)^(3/2)*arcsinh(c*x)^2*x^3-3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*arcs
inh(c*x)*x^2+8*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^
4-4/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(c*x)*x^6-4*b^2/Pi^
(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(c*x)*x^4+2*b^2/Pi^(5/2)*c^4/(3*
c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x^5+14/3*b^2/Pi^(5/2)*c/(3*c^2*
x^2+4)/(c^2*x^2+1)^2*x^2-b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2-2/3*b
^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6-b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4
)/(c^2*x^2+1)^(3/2)*x^5+4*b^2/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcs
inh(c*x)^2*x-5/3*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4+4/3*b^2/Pi^
(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)+10/3*b^2/Pi^(5/2)*c^5/(3*c
^2*x^2+4)/(c^2*x^2+1)^2*x^6-7/3*b^2/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^
(3/2)*x^3+6*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^4-22/3*b^2/Pi^(5
/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2*x^2+16/3*b^2/Pi^(5/2)*c/(3
*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2-2*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)
/(c^2*x^2+1)^2*arcsinh(c*x)^2*x^6+16/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*
x^2+1)^2*arcsinh(c*x)*x^6+4/3*b^2/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*
arcsinh(c*x)*x^8-20/3*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(
c*x)^2*x^4-4/3*b^2/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*x+8/3*a*b/c/Pi^
(5/2)*arcsinh(c*x)+34/3*a*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*ar
csinh(c*x)*x^3-44/3*a*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)
*x^2-4/3*a*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-40/3*a*b/Pi^(5/2)*c
^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4+4*a*b/Pi^(5/2)*c^4/(3*c^2*x
^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5+a^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2
)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))+4/3*b^2/c/Pi^(5/2)*arcsinh(c*x)^2+4/3*a
*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8-4/3*a*b/Pi^(5/2)*c^5/(3*c^2
*x^2+4)/(c^2*x^2+1)*x^6+16/3*a*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x
^6-4*a*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4+8*a*b/Pi^(5/2)*c^3/(3*c
^2*x^2+4)/(c^2*x^2+1)^2*x^4-3*a*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2+
8*a*b/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x+16/3*a*b/Pi^(
5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2-16/3*a*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c
^2*x^2+1)^2*arcsinh(c*x)+4/3*a*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2+4/3
```

```
*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2-4/3*b^2/c/Pi^(5/2)*arcsinh(c*x)
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-2/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))
^2)/c/Pi^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/
2)*c^2)) + 2/3*a*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi
*c^2*x^2)))*arcsinh(c*x) + 1/3*a^2*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(p
i^2*sqrt(pi + pi*c^2*x^2))) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/
(pi + pi*c^2*x^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2)/(pi^3*c^6*x^6 + 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2}}{c^{4}x^{4}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\,dx+\int \frac{b^{2}\operatorname{asinh}^{2}(cx)}{c^{4}x^{4}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{c^{4}x^{4}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2
+ 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b**2*asinh(c*x)**2/(c**4*x**4*sq
rt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)),
 x) + Integral(2*a*b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**
2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**(5/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.258 $\int x^3 \sqrt{d+c^2 d x^2} \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=358

$$-\frac{52b^2\sqrt{d+c^2dx^2}}{225c^4}+\frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{26b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{675c^4}+\frac{2b^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^4}+\frac{4b^2x\sqrt{\cdot}}{1}$$

```
[Out] -52/225*b^2*(c^2*d*x^2+d)^(1/2)/c^4-26/675*b^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1
/2)/c^4+2/125*b^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^4-2/15*(a+b*arcsinh(c
*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/15*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/c^2+1/5*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+4/15*a*b*x*(c^2*
d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)+4/15*b^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^
(1/2)/c^3/(c^2*x^2+1)^(1/2)-2/45*b*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)/c/(c^2*x^2+1)^(1/2)-2/25*b*c*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5806, 5812, 5798, 5772, 267, 5776, 272, 45}

$$\frac{x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^2}-\frac{2bcx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{45c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^4}+\frac{4abx\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}+\frac{2b^2(c^2x^2+1)^2\sqrt{c^2dx^2+d}}{125c^4}-\frac{26b^2(c^2x^2+1)\sqrt{c^2dx^2+d}}{675c^4}-\frac{52b^2\sqrt{c^2dx^2+d}}{225c^4}+\frac{4b^2x\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{15c^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-52*b^2*Sqrt[d + c^2*d*x^2])/(225*c^4) + (4*a*b*x*Sqrt[d + c^2*d*x^2])/(15
*c^3*Sqrt[1 + c^2*x^2]) - (26*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/(675*c
^4) + (2*b^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c^4) + (4*b^2*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(15*c^3*Sqrt[1 + c^2*x^2]) - (2*b*x^3*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*c*Sqrt[1 + c^2*x^2]) - (2*b*c*x^5*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) - (2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4) + (x^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
```

```
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{5}x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d+c^2dx^2}\int\frac{x^3(a+b\sinh}{\sqrt{1+c}}}{5\sqrt{1+c^2x^2}}$$

$$= -\frac{2bcx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{1+c^2x^2}}+\frac{x^2\sqrt{d+c^2dx^2}\,(a+b}{15c^2}$$

$$= -\frac{2bx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45c\sqrt{1+c^2x^2}}-\frac{2bcx^5\sqrt{d+c^2dx^2}\,(a+}{25\sqrt{1+c^2x}}$$

$$= \frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{2bx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45c\sqrt{1+c^2x^2}}-\frac{2bc}{}$$

$$= \frac{2b^2\sqrt{d+c^2dx^2}}{25c^4}+\frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{4b^2(1+c^2x^2)\sqrt{d+c^2d}}{75c^4}$$

$$= -\frac{52b^2\sqrt{d+c^2dx^2}}{225c^4}+\frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{26b^2(1+c^2x^2)\sqrt{d+}}{675c^4}$$

Mathematica [A]
time = 0.17, size = 222, normalized size = 0.62

$$\frac{\sqrt{d+c^2dx^2}\left(225(-2+3c^2x^2)(a+ac^2x^2)^2-30abcx\sqrt{1+c^2x^2}(-30+5c^2x^2+9c^4x^4)+2b^2(-428-439c^2x^2+16c^4x^4+27c^6x^6)-30b\left(-15a(1+c^2x^2)^2(-2+3c^2x^2)+bcx\sqrt{1+c^2x^2}(-30+5c^2x^2+9c^4x^4)\right)\sinh^{-1}(cx)+225(-2+3c^2x^2)(b+bc^2x^2)^2\sinh^{-1}(cx)^2\right)}{3375c^4(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(225*(-2 + 3*c^2*x^2)*(a + a*c^2*x^2)^2 - 30*a*b*c*x*S
qrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(-428 - 439*c^2*x^2
+ 16*c^4*x^4 + 27*c^6*x^6) - 30*b*(-15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) +
 b*c*x*Sqrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*
(-2 + 3*c^2*x^2)*(b + b*c^2*x^2)^2*ArcSinh[c*x]^2))/(3375*c^4*(1 + c^2*x^2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(310) = 620$.
time = 2.33, size = 1162, normalized size = 3.25

method	result	size
default	Expression too large to display	1162

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b^2*
(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c
^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(
25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*
c*x+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2
+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)
+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c
*x+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+1
))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/
2)*c*x+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)+1/4000*(d*(c^
2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2
*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*(25*arcsinh(c*x
)^2+10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*
(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^
3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))/c^4/(c^2*x^
2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c
^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*
(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)/c^
4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*
(1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(
c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(
c*x))/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1
)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x
^2+1)^(1/2)*c*x+1)*(1+5*arcsinh(c*x))/c^4/(c^2*x^2+1))
```

Maxima [A]
time = 0.35, size = 302, normalized size = 0.84

$$\frac{1}{15}\,b^2\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}(cx)^2+\frac{2}{15}\,ab\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}(cx)+\frac{1}{15}\,a^2\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)+\frac{2}{3375}\,b^2\left(\frac{27\,\sqrt{c^2x^2+1}\,c^2\sqrt{d}\,x^4-11\,\sqrt{c^2x^2+1}\,\sqrt{d}\,x^2-\frac{428\,\sqrt{c^2x^2+1}\,\sqrt{d}}{c^2}}{c^2}-\frac{15\left(9\,c^4\sqrt{d}\,x^5+5\,c^2\sqrt{d}\,x^3-30\,\sqrt{d}\,x\right)\operatorname{arsinh}(cx)}{c^3}\right)-\frac{2\left(9\,c^4\sqrt{d}\,x^5+5\,c^2\sqrt{d}\,x^3-30\,\sqrt{d}\,x\right)ab}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*b^2*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^
4*d))*arcsinh(c*x)^2 + 2/15*a*b*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c
^2*d*x^2 + d)^(3/2)/(c^4*d))*arcsinh(c*x) + 1/15*a^2*(3*(c^2*d*x^2 + d)^(3/
```

```
2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 2/3375*b^2*((27*sqrt(c^
2*x^2 + 1)*c^2*sqrt(d)*x^4 - 11*sqrt(c^2*x^2 + 1)*sqrt(d)*x^2 - 428*sqrt(c^
2*x^2 + 1)*sqrt(d)/c^2)/c^2 - 15*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x^3 - 3
0*sqrt(d)*x)*arcsinh(c*x)/c^3) - 2/225*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x
^3 - 30*sqrt(d)*x)*a*b/c^3
```

Fricas [A]
time = 0.39, size = 316, normalized size = 0.88

$$\frac{225\left(3b^{2}c^{6}x^{6}+4b^{2}c^{4}x^{4}-b^{2}c^{2}x^{2}-2b^{2}\right)\sqrt{c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}+30\left(45abc^{6}x^{6}+60abc^{4}x^{4}-15abc^{2}x^{2}-30ab-\left(9b^{2}c^{5}x^{5}+5b^{2}c^{3}x^{3}-30b^{2}cx\right)\sqrt{c^{2}x^{2}+1}\right)\sqrt{c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(27\left(25a^{2}+2b^{2}\right)c^{6}x^{6}+4\left(225a^{2}+8b^{2}\right)c^{4}x^{4}-\left(225a^{2}+878b^{2}\right)c^{2}x^{2}-450a^{2}-856b^{2}-30\left(9abc^{5}x^{5}+5abc^{3}x^{3}-30abcx\right)\sqrt{c^{2}x^{2}+1}\right)\sqrt{c^{2}dx^{2}+d}}{3375\left(c^{6}x^{2}+c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 + 4*b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 + 60*a*b*c^4
*x^4 - 15*a*b*c^2*x^2 - 30*a*b - (9*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 30*b^2*c*
x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (2
7*(25*a^2 + 2*b^2)*c^6*x^6 + 4*(225*a^2 + 8*b^2)*c^4*x^4 - (225*a^2 + 878*b
^2)*c^2*x^2 - 450*a^2 - 856*b^2 - 30*(9*a*b*c^5*x^5 + 5*a*b*c^3*x^3 - 30*a*
b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \operatorname{asinh}(c x)\right)^2 \sqrt{d c^2 x^2 + d}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.259 $\int x^2 \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=291

$$\frac{b^2x\sqrt{d+c^2dx^2}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{64c^3\sqrt{1+c^2x^2}}-\frac{bx^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{1+c^2x^2}}-\frac{bc}{}$$

```
[Out] 1/64*b^2*x*(c^2*d*x^2+d)^(1/2)/c^2+1/32*b^2*x^3*(c^2*d*x^2+d)^(1/2)+1/8*x*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arcsinh(c*x))^2*(c
^2*d*x^2+d)^(1/2)-1/64*b^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)
^(1/2)-1/8*b*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)
-1/8*b*c*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/24*
(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5806, 5812, 5783, 5776, 327, 221}

$$-\frac{bx^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2x^2+1}}+\frac{x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2}-\frac{bcx^4\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^3}{24bc^3\sqrt{c^2x^2+1}}+\frac{b^2x\sqrt{c^2dx^2+d}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{c^2dx^2+d}-\frac{b^2\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{64c^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/(64*c^2) + (b^2*x^3*Sqrt[d + c^2*d*x^2])/32 - (
b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c^3*Sqrt[1 + c^2*x^2]) - (b*x^2*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/4 - (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(24
*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{4}x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{d+c^2dx^2}\int\frac{x^2(a+b\sinh^{-1}}{\sqrt{1+c}}}{4\sqrt{1+c^2x^2}}$$

$$= -\frac{bcx^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{1+c^2x^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sin\right.}{8c^2}$$

$$= \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{bx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{1+c^2x^2}} - \frac{bcx}{}$$

$$= \frac{b^2x\sqrt{d+c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{bx^2\sqrt{d+c^2dx^2}\left(a+\right.}{8c\sqrt{1+c^2x}}$$

$$= \frac{b^2x\sqrt{d+c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{b^2\sqrt{d+c^2dx^2}\sinh^{-1}}{64c^3\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.89, size = 207, normalized size = 0.71

$$-\frac{-96a^2cx(1+2c^2x^2)\sqrt{d+c^2dx^2}+96a^2\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{12ab\sqrt{d+c^2dx^2}\left(8\sinh^{-1}(cx)^2+\cosh(4\sinh^{-1}(cx))-4\sinh^{-1}(cx)\sinh(4\sinh^{-1}(cx))\right)}{\sqrt{1+c^2x^2}}+\frac{b^2\sqrt{d+c^2dx^2}\left(32\sinh^{-1}(cx)^3+12\sinh^{-1}(cx)\cosh(4\sinh^{-1}(cx))-3(1+8\sinh^{-1}(cx)^2)\sinh(4\sinh^{-1}(cx))\right)}{\sqrt{1+c^2x^2}}}{768c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/768*(-96*a^2*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 96*a^2*Sqrt[d]*Lo
g[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (12*a*b*Sqrt[d + c^2*d*x^2]*(8*Arc
Sinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/
Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(32*ArcSinh[c*x]^3 + 12*ArcSin
h[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*ArcSinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]
))/Sqrt[1 + c^2*x^2])/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(251) = 502$.
time = 2.70, size = 618, normalized size = 2.12

method	result
default	$\frac{a^2x(c^2d\,x^2+d)^{\frac{3}{2}}}{4c^2d} - \frac{a^2x\sqrt{c^2d\,x^2+d}}{8c^2} - \frac{a^2d\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^2\sqrt{c^2d}} + b^2\left(-\frac{\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{24\sqrt{c^2x^2+1}\,c^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a^2/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a
^2/c^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(-
1/24*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3+1/512*(d*(c
^2*x^2+1))^(1/2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*x^
2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*
x)+1)/c^3/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^
(1/2)*x^4*c^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2
))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)/c^3/(c^2*x^2+1))+2*a*b*(-1/16*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/256*(d*(c^2*x^2+1))^
(1/2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+
1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^3/(c^2*x^2+1)+1/256
*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3-8*
c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))/c^3/(
c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\sqrt{d\,c^2\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.260 $\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=180

$$\frac{4b^2\sqrt{d+c^2dx^2}}{9c^2}+\frac{2b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{27c^2}-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\left(a+b\right.}{9\sqrt{1+c^2x^2}}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/c^2/d+4/9*b^2*(c^2*d*x^2+d)^(1
/2)/c^2+2/27*b^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2-2/3*b*x*(a+b*arcsinh(c
*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/9*b*c*x^3*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {5798, 5784, 455, 45}

$$-\frac{2bx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{2bcx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}+\frac{2b^2(c^2x^2+1)\sqrt{c^2dx^2+d}}{27c^2}+\frac{4b^2\sqrt{c^2dx^2+d}}{9c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4*b^2*Sqrt[d + c^2*d*x^2])/(9*c^2) + (2*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x
^2])/(27*c^2) - (2*b*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c*Sqrt[
1 + c^2*x^2]) - (2*b*c*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqr
t[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
```

```
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{\left(2b\sqrt{d+c^2dx^2}\right)\int(1+c^2}{3c\sqrt{1+}}$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\,(a+b}{9\sqrt{1+c^2x^2}}$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\,(a+b}{9\sqrt{1+c^2x^2}}$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\,(a+b}{9\sqrt{1+c^2x^2}}$$

$$=\frac{4b^2\sqrt{d+c^2dx^2}}{9c^2}+\frac{2b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{27c^2}-\frac{2bx\sqrt{d+c^2dx^2}}{3c\sqrt{1}}$$

Mathematica [A]
time = 0.15, size = 166, normalized size = 0.92

$$\frac{\sqrt{d+c^2dx^2}\left(-6abcx\sqrt{1+c^2x^2}\,(3+c^2x^2)+9(a+ac^2x^2)^2+2b^2(7+8c^2x^2+c^4x^4)+6b\left(3a(1+c^2x^2)^2-bcx\sqrt{1+c^2x^2}\,(3+c^2x^2)\right)\sinh^{-1}(cx)+9(b+bc^2x^2)^2\sinh^{-1}(cx)^2\right)}{27c^2\,(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-6*a*b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2) + 9*(a + a
*c^2*x^2)^2 + 2*b^2*(7 + 8*c^2*x^2 + c^4*x^4) + 6*b*(3*a*(1 + c^2*x^2)^2 -
b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2))*ArcSinh[c*x] + 9*(b + b*c^2*x^2)^2*A
rcSinh[c*x]^2))/(27*c^2*(1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(156) = 312$.
time = 0.98, size = 657, normalized size = 3.65

method	result
default	$\frac{a^2(c^2d\,x^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{d\,(c^2x^2+1)}\,\left(4c^4x^4+4\sqrt{c^2x^2+1}\,x^3c^3+5c^2x^2+3\sqrt{c^2x^2+1}\,cx+1\right)\left(9\,\mathrm{arcsinh}(cx)^2-6\,\mathrm{arc}\right.}{216c^2(c^2x^2+1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2/c^2/d*(c^2*d*x^2+d)^(3/2)+b^2*(1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x
^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcs
inh(c*x)^2-6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2
*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/(c^2*x^
2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c
*x)^2+2*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*
x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arc
sinh(c*x)^2+6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*
c*x+1)*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^
2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))/c^2/(c^2*x^2
+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2
*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))/c^2/(c^2*x^2+1))
```

Maxima [A]
time = 0.29, size = 183, normalized size = 1.02

$$\frac{2}{27}\,b^2\left(\frac{\sqrt{c^2x^2+1}\,d^{\frac{3}{2}}x^2+\frac{7\sqrt{c^2x^2+1}\,d^{\frac{3}{2}}}{c^2}}{d}-\frac{3\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)\mathrm{arsinh}\,(cx)}{cd}\right)+\frac{(c^2dx^2+d)^{\frac{3}{2}}b^2\,\mathrm{arsinh}\,(cx)^2}{3\,c^2d}+\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}ab\,\mathrm{arsinh}\,(cx)}{3\,c^2d}-\frac{2\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)ab}{9\,cd}+\frac{(c^2dx^2+d)^{\frac{3}{2}}a^2}{3\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*b^2*((sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 + 7*sqrt(c^2*x^2 + 1)*d^(3/2)/c^2)
/d - 3*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*arcsinh(c*x)/(c*d)) + 1/3*(c^2*d*x^2
 + d)^(3/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*arcs
inh(c*x)/(c^2*d) - 2/9*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*a*b/(c*d) + 1/3*(c^2
*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

Fricas [A]
time = 0.42, size = 249, normalized size = 1.38

$$\frac{9\,(b^2c^4x^4+2\,b^2c^2x^2+b^2)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3\,abc^4x^4+6\,abc^2x^2+3\,ab-(b^2c^3x^3+3\,b^2cx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left((9\,a^2+2\,b^2)c^4x^4+2\,(9\,a^2+8\,b^2)c^2x^2+9\,a^2+14\,b^2-6\,(abc^3x^3+3\,abcx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{27\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + 3*a*b - (b^2*c^3*x
^3 + 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + 2*(9*a^2 + 8*b^2)*c^2*x^2 + 9*a^2 + 1
4*b^2 - 6*(a*b*c^3*x^3 + 3*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))
/(c^4*x^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.261 $\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=184

$$\frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}}-\frac{bcx^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] 1/4*b^2*x*(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2
)-1/4*b^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^2*
(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*(a+b*arcsinh(c
*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5785, 5783, 5776, 327, 221}

$$\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}b^2x\sqrt{c^2dx^2+d}-\frac{b^2\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{4c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/4 - (b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(4*c
*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d+c^2dx^2}\int\frac{(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}}{2\sqrt{1+c^2x^2}}$$

$$=-\frac{bcx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sin\right.$$

$$=\frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{bcx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d}$$

$$=\frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}}-\frac{bcx^2\sqrt{d+c^2dx^2}}{2\sqrt{1}}$$

Mathematica [A]
time = 0.74, size = 200, normalized size = 1.09

$$\frac{1}{24}\left(12a^2x\sqrt{d+c^2dx^2}+\frac{12a^2\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{c}+\frac{b^2\sqrt{d+c^2dx^2}\left(4\sinh^{-1}(cx)^3-6\sinh^{-1}(cx)\cosh\left(2\sinh^{-1}(cx)\right)+\left(3+6\sinh^{-1}(cx)^2\right)\sinh\left(2\sinh^{-1}(cx)\right)\right)}{c\sqrt{1+c^2x^2}}+\frac{6ab\sqrt{d+c^2dx^2}\left(-\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{c\sqrt{1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (12*a^2*x*Sqrt[d + c^2*d*x^2] + (12*a^2*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d
+ c^2*d*x^2]])/c + (b^2*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c
```

```
*x]*Cosh[2*ArcSinh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(c
*Sqrt[1 + c^2*x^2]) + (6*a*b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2
*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2])
)/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(158) = 316$.
time = 1.67, size = 480, normalized size = 2.61

method	result
default	$\frac{a^2 x\sqrt{c^2 d\,x^2+d}}{2}+\frac{a^2 d\ln\left(\frac{x\,c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{2\sqrt{c^2 d}}+b^2\left(\frac{\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d\,(c^2x^2+1)}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x*(c^2*d*x^2+d)^(1/2)+1/2*a^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c
*arcsinh(c*x)^3+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)
^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)/c/(c^2*
x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*
c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)/c/(c^2*x^2+1))+2
*a*b*(1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2+1/16*(d*
(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)
^(1/2))*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3
*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x)
)/c/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.262 $\int \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=338

$$2b^2\sqrt{d+c^2dx^2}-\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2b^2cx\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2\sqrt{d}}{}$$

```
[Out] 2*b^2*(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-2*a*b*c*
x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*x*arcsinh(c*x)*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1
/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b*(a+b*arc
sinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x
^2+1)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5806, 5816, 4267, 2611, 2320, 6724, 5772, 267}

$$-\frac{2b\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2b\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{2abcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}+\frac{2b^2\sqrt{c^2dx^2+d}\,\text{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{c^2dx^2+d}\,\text{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+2b^2\sqrt{c^2dx^2+d}-\frac{2b^2cx\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] 2*b^2*Sqrt[d + c^2*d*x^2] - (2*a*b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^
2] - (2*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] +
 (2*b*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] - (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx &= \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d+c^2dx^2}\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d+c^2}}{} \\
&= -\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2} \\
&= 2b^2\sqrt{d+c^2dx^2}-\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(c}{\sqrt{1+c^2x^2}} \\
&= 2b^2\sqrt{d+c^2dx^2}-\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(c}{\sqrt{1+c^2x^2}} \\
&= 2b^2\sqrt{d+c^2dx^2}-\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(c}{\sqrt{1+c^2x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.79, size = 352, normalized size = 1.04

$$a^2\sqrt{d+c^2dx^2}+a^2\sqrt{d}\log(cx)-a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{2ab\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\sinh^{-1}(cx)+\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{1+c^2x^2}}+\frac{b^2\sqrt{d+c^2dx^2}\left(2\sqrt{1+c^2x^2}-2cx\sinh^{-1}(cx)+\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2+\sinh^{-1}(cx)^2\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)^2\log\left(1+e^{-\sinh^{-1}(cx)}\right)+2\sinh^{-1}(cx)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-2\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+2\text{PolyLog}\left(3,-e^{-\sinh^{-1}(cx)}\right)-2\text{PolyLog}\left(3,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] a^2*Sqrt[d + c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] + (2*a*b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*
Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^
(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 +
 c^2*x^2] - 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh
[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x
])] + 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyLo
g[2, E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3, E
^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(353) = 706$.
time = 2.00, size = 823, normalized size = 2.43

method	result
default	$-\sqrt{d}\ \ln\left(\frac{2d+2\sqrt{d}\ \sqrt{c^2 d\,x^2+d}}{x}\right) a^2 + a^2\sqrt{c^2 d\,x^2+d} + \frac{b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)^2 x^2 c^2}{c^2x^2+1} - \frac{2b^2 \sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a^2+a^2*(c^2*d*x^2+d)^(1
/2)+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)^2*x^2*c^2-2*b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c+2*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)*x^2*c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x
)^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2
*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/
2))+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^
2*x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)*arcsinh(c*x)*x^2*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*x*c+2*
a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*a*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*a*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1
)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d))*a^2 + integrate(sqrt
(c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*sqrt(c^2*d*x^2 + d
)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x, x)
```

3.263 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}+\frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{1+c^2x^2}}+$$

```
[Out] -(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x+c*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*c*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2
)/b/(c^2*x^2+1)^(1/2)+2*b*c*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2
))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*polylog(2,1/(c*x+(c^2*x^2
+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5805, 5775, 3797, 2221, 2317, 2438, 5783}

$$\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}+\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\frac{2bc\sqrt{c^2dx^2+d}\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{b^2c\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x) + (c*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + (c*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2]) + (2*b*c*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] - (b^2*c*Sqr
t[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int \frac{a+b\sinh^{-}}{x}}{\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{3b\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.77, size = 232, normalized size = 1.11

$$-\frac{a^2\sqrt{d+c^2dx^2}}{x}+\frac{ab\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\sinh^{-1}(cx)+cx\sinh^{-1}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}}+a^2c\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{b^2c\sqrt{d+c^2dx^2}\left(\sinh^{-1}(cx)\left(\left(3-\frac{3\sqrt{1+c^2x^2}}{cx}\right)\sinh^{-1}(cx)+\sinh^{-1}(cx)^2+6\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)}{3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[d + c^2*d*x^2])/x) + (a*b*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x
^2]) + a^2*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b^2*c*Sqrt
[d + c^2*d*x^2]*(ArcSinh[c*x]*((3 - (3*Sqrt[1 + c^2*x^2])/(c*x))*ArcSinh[c*
x] + ArcSinh[c*x]^2 + 6*Log[1 - E^(-2*ArcSinh[c*x])]) - 3*PolyLog[2, E^(-2*
ArcSinh[c*x])]))/(3*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(207) = 414$.
time = 2.48, size = 625, normalized size = 2.99

method	result
default	$-\frac{a^2(c^2d\,x^2+d)^{\frac{3}{2}}}{dx}+a^2c^2x\sqrt{c^2d\,x^2+d}+\frac{a^2c^2d\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{d\,(c^2x^2+1)}\ \text{arcsinh}(cx}{3\sqrt{c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(3/2)+a^2*c^2*x*(c^2*d*x^2+d)^(1/2)+a^2*c^2*d*ln(x*c
^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsi
nh(c*x)^2*x/(c^2*x^2+1)*c^2-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x
^2+1)^(1/2)*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/x/(c^2*x^2+1)+2*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*l
n(1-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
polylog(2,c*x+(c^2*x^2+1)^(1/2))*c+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)^2*c-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*c-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*x/(c^2*x^2+1)*c^2-2*a*b*(d*
(c^2*x^2+1))^(1/2)*arcsinh(c*x)/x/(c^2*x^2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] (c*sqrt(d)*arcsinh(c*x) - sqrt(c^2*d*x^2 + d)/x)*a^2 + integrate(sqrt(c^2*d
*x^2 + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^2 + 2*sqrt(c^2*d*x^2 + d)*a*
b*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d\,c^2\,x^2+d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

3.264 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=358

$$-\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}-\frac{c^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\tan}{\sqrt{1+c^2x^2}}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2-b*c*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-c^2*(a+b*arcsinh(c*x))^2*arctanh(c*x+
(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*arctanh((c
^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c^2*(a+b*arcsinh(c
*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)+b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)+b^2*c^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5805, 5776, 272, 65, 214, 5816, 4267, 2611, 2320, 6724}

$$-\frac{bc^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}-\frac{c^2\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}+\frac{b^2c^2\sqrt{c^2dx^2+d}\,\text{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{b^2c^2\sqrt{c^2dx^2+d}\,\text{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{b^2c^2\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^
2*c^2*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (
b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*Sqrt[d + c^2*d*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
```

```
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{a+b\sinh^{-1}(c}{x^2}}{\sqrt{1+c^2x^2}}$$

$$=-\frac{bc\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx\right.}{2x^2}$$

$$=-\frac{bc\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx\right.}{2x^2}$$

$$=-\frac{bc\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx\right.}{2x^2}$$

$$=-\frac{bc\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx\right.}{2x^2}$$

$$=-\frac{bc\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx\right.}{2x^2}$$

Mathematica [A]
time = 3.39, size = 446, normalized size = 1.25

$$\frac{1}{8}\left(-\frac{4a^2\sqrt{d+c^2dx^2}}{x^2}+4a^2c^2\sqrt{d}\,\log(x)-4a^2c^2\sqrt{d}\,\log\left(d+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)+\frac{2abc^2\sqrt{d+c^2dx^2}\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)+4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{\sqrt{1+c^2x^2}}+\frac{b^2c^2\sqrt{d+c^2dx^2}\left(-4\sinh^{-1}(cx)\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)^2\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)^2\log\left(1-e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)^2\log\left(1+e^{-\sinh^{-1}(cx)}\right)+8\log\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+8\sinh^{-1}(cx)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-8\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+8\text{PolyLog}\left(3,-e^{-\sinh^{-1}(cx)}\right)-8\text{PolyLog}\left(3,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)^2\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{\sqrt{1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] ((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 + 4*a^2*c^2*Sqrt[d]*Log[x] - 4*a^2*c^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*Sqrt[d + c^2*d*x^2]
*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh
[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])
] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - Ar
cSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x
^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] -
ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSin
h[c*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh
[c*x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*
PolyLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLo
g[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh
[c*x]*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 869 vs. $2(371) = 742$.
time = 3.90, size = 870, normalized size = 2.43

method	result
default	$-\frac{a^2 (c^2 d x^2+d)^{\frac{3}{2}}}{2 d x^2}-\frac{a^2 \sqrt{d}\, \ln\left(\frac{2d+2\sqrt{d}\, \sqrt{c^2 d x^2+d}}{x}\right) c^2}{2}+\frac{a^2 \sqrt{c^2 d x^2+d}\, c^2}{2}-\frac{b^2 \operatorname{arcsinh}(cx)^2 \sqrt{d\,(c^2x^2+}}{2(c^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/d/x^2*(c^2*d*x^2+d)^(3/2)-1/2*a^2*d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d
*x^2+d)^(1/2))/x)*c^2+1/2*a^2*(c^2*d*x^2+d)^(1/2)*c^2-1/2*b^2*arcsinh(c*x)^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*c^2-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1
/2)/x/(c^2*x^2+1)^(1/2)*c-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/x^2/
(c^2*x^2+1)-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*
ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2+b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*
c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(
3,c*x+(c^2*x^2+1)^(1/2))*c^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
arctanh(c*x+(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*ar
csinh(c*x)*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/x/(c^2*x^2+1)^(1/2)*c-a*b*arcsinh(
c*x)*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
```

```
^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2+a*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^
2-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(c^2*sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d)*c^2 + (c^2*d*
x^2 + d)^(3/2)/(d*x^2))*a^2 + integrate(sqrt(c^2*d*x^2 + d)*b^2*log(c*x + s
qrt(c^2*x^2 + 1))^2/x^3 + 2*sqrt(c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c^2*x^2
+ 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,(c^2x^2+1)}\,(a+b\,\mathrm{asinh}\,(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^3, x)
```

3.265 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=294

$$-\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}}-\frac{bc\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}+\frac{c^3\sqrt{d+c^2d}}{3}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/d/x^3-1/3*b^2*c^2*(c^2*d*x^2+
d)^(1/2)/x+1/3*b^2*c^3*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1
/3*c^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2/3*b*c^3
*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-1/3*b^2*c^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
*(c^2*d*x^2+d)^(1/2)/x^2
```

Rubi [A]
time = 0.22, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5800, 5802, 283, 221, 5775, 3797, 2221, 2317, 2438}

$$-\frac{bc\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2x^2+1}}+\frac{2bc^3\sqrt{c^2dx^2+d}\,\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}-\frac{b^2c^2\sqrt{c^2dx^2+d}}{3x}-\frac{b^2c^3\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}+\frac{b^2c^3\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*Sqrt[d + c^2*d*x^2])/x + (b^2*c^3*Sqrt[d + c^2*d*x^2]*ArcSinh
[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(3*x^2) + (c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)^2)/(3*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(3*d*x^3) + (2*b*c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-
2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (b^2*c^3*Sqrt[d + c^2*d*x^2]*Poly
Log[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

nomialQ[a, b, c, n, m, p, x]

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)}{\ldots}}{3\sqrt{1+c^2x^2}\ldots}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{(d+c^2dx^2)^{3/2}\left(a\ldots\right.}{3d\ldots}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} - \frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\ldots}{\ldots}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\ldots}{\ldots}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\ldots}{\ldots}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\ldots}{\ldots}$$

Mathematica [A]
time = 0.54, size = 240, normalized size = 0.82

$$-\frac{\sqrt{d+c^2dx^2}\left(abcx+a^2\sqrt{1+c^2x^2}+a^2c^2x^2\sqrt{1+c^2x^2}+b^2c^2x^2\sqrt{1+c^2x^2}+b^2\left(-c^3x^3+\sqrt{1+c^2x^2}+c^2x^2\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2-b\sinh^{-1}(cx)\left(-bcx-2a(1+c^2x^2)^{3/2}+2bc^3x^3\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-2abc^3x^3\log(cx)+b^2c^3x^3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)}{3x^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(Sqrt[d + c^2*d*x^2]*(a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + a^2*c^2*x^2*Sq
rt[1 + c^2*x^2] + b^2*c^2*x^2*Sqrt[1 + c^2*x^2] + b^2*(-(c^3*x^3) + Sqrt[1
+ c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-(
b*c*x) - 2*a*(1 + c^2*x^2)^(3/2) + 2*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]
```

```
) - 2*a*b*c^3*x^3*Log[c*x] + b^2*c^3*x^3*PolyLog[2, E^(-2*ArcSinh[c*x])]))/
(x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2556 vs. $2(274) = 548$.

time = 4.07, size = 2557, normalized size = 8.70

method	result	size
default	Expression too large to display	2557

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*c^2-1/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh
(c*x)^2+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c
*x-(c^2*x^2+1)^(1/2))*c^3+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)
*x^2/(c^2*x^2+1)^(1/2)*c^5-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1
)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3+b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7+1/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3*arcsinh(c*x)*c^6-2*a*b*(d*(c^2*x^2+1))
^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-10/3*a*b*(d
*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-
20/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsin
h(c*x)*c^4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1
)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(
3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/3*a*b*(d*(c^2*x^
2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/3*a*b*(d*(c^2*x^2+1
))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-2/3*a*b*(d*(c^2*x^
2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+1/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x*arcsinh(c*x)*c^4+2/3*b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2)
)*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)
*c^8-5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*
c^6-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^4
-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcs
inh(c*x)*c^6-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*
x^2+1)*arcsinh(c*x)^2*c^4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^
2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
```

```
4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(
3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)^2*c^8-1/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-3*b^2
*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)
^2*c^6+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)^2*c^5-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^
2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*a^2/d/x^3*(c^2*d*x^2+d)^(3/2)+2*a*
b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsi
nh(c*x)*c^7-6*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^
2+1)*arcsinh(c*x)*c^6+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x
^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+3*c^2*x^2+1)*x^3*c^6-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)^2*c^3+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,
c*x+(c^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3-4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*c^3+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*
x^2+1)*x^3*c^6+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x*c^4-
a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3+2/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1
)*c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima
")
```

```
[Out] -1/3*((-1)^(2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(2*c^2*d + 2*d/x^2) - c^2*d^(
3/2)*log(x^2 + 1/c^2) + sqrt(c^4*d*x^4 + 2*c^2*d*x^2 + d)*d/x^2)*a*b*c/d -
1/3*b^2*((c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x
^2 + 1))^2/x^3 - 3*integrate(2/3*((c^2*x^2 + 1)*c^2*sqrt(d)*x + (c^3*sqrt(d
)*x^2 + c*sqrt(d))*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c*x^4 +
 sqrt(c^2*x^2 + 1)*x^3), x)) - 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*arcsinh(c*x)/(
d*x^3) - 1/3*(c^2*d*x^2 + d)^(3/2)*a^2/(d*x^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,(c^2x^2+1)}\,(a+b\,\mathrm{asinh}\,(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^4, x)
```

3.266 $\int x^3(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=482

$$-\frac{304b^2d\sqrt{d+c^2dx^2}}{3675c^4}+\frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{152b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{11025c^4}-\frac{38b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{6125c^4}+$$

```
[Out] 1/7*x^4*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-304/3675*b^2*d*(c^2*d*x^2+
d)^(1/2)/c^4-152/11025*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^4-38/6125*b^
2*d*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^4+2/343*b^2*d*(c^2*x^2+1)^3*(c^2*d*
x^2+d)^(1/2)/c^4-2/35*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/35*d
*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+3/35*d*x^4*(a+b*arcsinh(c
*x))^2*(c^2*d*x^2+d)^(1/2)+4/35*a*b*d*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)
^(1/2)+4/35*b^2*d*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-
2/105*b*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-16
/175*b*c*d*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/4
9*b*c^3*d*x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5808, 5806, 5812, 5798, 5772, 267, 5776, 272, 45, 14, 5803, 12, 457, 78}

$$\frac{dx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{35c^2}-\frac{16bcdx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{175\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{35}dx^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bdx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{105c\sqrt{c^2x^2+1}}-\frac{2d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{35c^4}+\frac{4abdx\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}-\frac{2bc^3dx^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{49\sqrt{c^2x^2+1}}+\frac{2b^2d\left(c^2x^2+1\right)^3\sqrt{c^2dx^2+d}}{343c^4}-\frac{38b^2d\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}}{6125c^4}-\frac{304b^2d\sqrt{c^2dx^2+d}}{3675c^4}-\frac{152b^2d\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}}{11025c^4}+\frac{4b^2dx\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{35c^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-304*b^2*d*Sqrt[d + c^2*d*x^2])/(3675*c^4) + (4*a*b*d*x*Sqrt[d + c^2*d*x^2
])/(35*c^3*Sqrt[1 + c^2*x^2]) - (152*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2
])/(11025*c^4) - (38*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(6125*c^4)
+ (2*b^2*d*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^4) + (4*b^2*d*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(35*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(105*c*Sqrt[1 + c^2*x^2]) - (16*b*c*d*
x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(175*Sqrt[1 + c^2*x^2]) - (2*
b*c^3*d*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(49*Sqrt[1 + c^2*x^2]
) - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^4) + (d*x^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^2) + (3*d*x^4*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])^2)/35 + (x^4*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{7}x^4\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}(3d)\int x^3\sqrt{d+c^2d}$$

$$=-\frac{2bcdx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{35\sqrt{1+c^2x^2}}-\frac{2bc^3dx^7\sqrt{d+c^2dx}}{49\sqrt{1}}$$

$$=-\frac{16bcdx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{175\sqrt{1+c^2x^2}}-\frac{2bc^3dx^7\sqrt{d+c^2d}}{49\sqrt{}}$$

$$=-\frac{2bdx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{105c\sqrt{1+c^2x^2}}-\frac{16bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1}}$$

$$=\frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{2bdx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{105c\sqrt{1+c^2x^2}}-$$

$$=\frac{62b^2d\sqrt{d+c^2dx^2}}{1225c^4}+\frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{74b^2d(1+c^2x^2)\sqrt{}}{3675c^4}$$

$$=-\frac{304b^2d\sqrt{d+c^2dx^2}}{3675c^4}+\frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{152b^2d(1+c^2x}{110}$$

Mathematica [A]
time = 0.25, size = 251, normalized size = 0.52

$$\frac{d\sqrt{d+c^2dx^2}\left(11025a^2(1+c^2x^2)^3(-2+5c^2x^2)-210abcx\sqrt{1+c^2x^2}(-210+35c^2x^2+168c^4x^4+75c^6x^6)+2b^2(-18692-20371c^2x^2+499c^4x^4+3303c^6x^6+1125c^8x^8)-210b\left(-105a(1+c^2x^2)^3(-2+5c^2x^2)+bcx\sqrt{1+c^2x^2}(-210+35c^2x^2+168c^4x^4+75c^6x^6)\right)\sinh^{-1}(cx)+11025b^2(1+c^2x^2)^3(-2+5c^2x^2)\sinh^{-1}(cx)^2\right)}{385875c^4(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(11025*a^2*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - 210*a*
b*c*x*Sqrt[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 2*
```

```
b^2*(-18692 - 20371*c^2*x^2 + 499*c^4*x^4 + 3303*c^6*x^6 + 1125*c^8*x^8) -
210*b*(-105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(-
210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*(1 +
 c^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^4*(1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1765 vs. $2(420) = 840$.
time = 2.20, size = 1766, normalized size = 3.66

method	result	size
default	Expression too large to display	1766

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b^2*
(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144
*x^6*c^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3
*c^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*c*x+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*
x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x
^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c
^2*x^2+1)^(1/2)*c*x+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*d/c^4/(c^2*x^2
+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c
^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d/c^4
/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*
(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d/
c^4/(c^2*x^2+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)
*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2+6*arcsinh(c
*x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*
x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(
c^2*x^2+1)^(1/2)*c*x+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d/c^4/(c^2*x^
2+1)+1/43904*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8-64*(c^2*x^2+1)^(1/2)*x^7*c^7
+144*x^6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4-56*(c^2*x^2+1)^(1/2)
*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x+1)*(49*arcsinh(c*x)^2+14*arcsin
h(c*x)+2)*d/c^4/(c^2*x^2+1))+2*a*b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^
8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+10
4*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*c*x+1
)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*x^
6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+
13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x
^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)*d/c
^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1
```

```
)*(1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4
-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcs
inh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^
2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5
*(c^2*x^2+1)^(1/2)*c*x+1)*(1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(c
^2*x^2+1))^(1/2)*(64*x^8*c^8-64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6-112*(
c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4-56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^
2-7*(c^2*x^2+1)^(1/2)*c*x+1)*(1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1))
```

Maxima [A]
time = 0.34, size = 346, normalized size = 0.72

$$\frac{1}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)b^2\operatorname{arsinh}(cx)^2+\frac{2}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)ab\operatorname{arsinh}(cx)+\frac{1}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)a^2+\frac{2}{385875}b^2\left(\frac{1125\sqrt{c^2x^2+1}\,c^4d^{\frac{3}{2}}x^6+2178\sqrt{c^2x^2+1}\,c^2d^{\frac{3}{2}}x^4-1679\sqrt{c^2x^2+1}\,d^{\frac{3}{2}}x^2-\frac{18692\sqrt{c^2x^2+1}\,d^{\frac{3}{2}}}{c^2}}{c^2}-\frac{105\left(75\,c^6d^{\frac{3}{2}}x^7+168\,c^4d^{\frac{3}{2}}x^5+35\,c^2d^{\frac{3}{2}}x^3-210\,d^{\frac{3}{2}}x\right)\operatorname{arsinh}(cx)}{c^3}\right)-\frac{2\left(75\,c^6d^{\frac{3}{2}}x^7+168\,c^4d^{\frac{3}{2}}x^5+35\,c^2d^{\frac{3}{2}}x^3-210\,d^{\frac{3}{2}}x\right)ab}{3675\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d)
)*b^2*arcsinh(c*x)^2 + 2/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d
*x^2 + d)^(5/2)/(c^4*d))*a*b*arcsinh(c*x) + 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x
^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 + 2/385875*b^2*((1125*sqr
t(c^2*x^2 + 1)*c^4*d^(3/2)*x^6 + 2178*sqrt(c^2*x^2 + 1)*c^2*d^(3/2)*x^4 - 1
679*sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 - 18692*sqrt(c^2*x^2 + 1)*d^(3/2)/c^2)/c^
2 - 105*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x^3 - 21
0*d^(3/2)*x)*arcsinh(c*x)/c^3) - 2/3675*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/
2)*x^5 + 35*c^2*d^(3/2)*x^3 - 210*d^(3/2)*x)*a*b/c^3
```

Fricas [A]
time = 0.48, size = 402, normalized size = 0.83

$$\frac{11025\,(5\,b^2c^8dx^8+13\,b^2c^6dx^6+9\,b^2c^4dx^4-b^2c^2dx^2-2\,b^2d)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+210\left(525\,abc^8dx^8+1365\,abc^6dx^6+945\,abc^4dx^4-105\,abc^2dx^2-210\,abd-(75\,b^2c^7dx^7+168\,b^2c^5dx^5+35\,b^2c^3dx^3-210\,b^2cdx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(1125\,(49\,a^2+2\,b^2)c^8dx^8+9\,(15925\,a^2+734\,b^2)c^6dx^6+(99225\,a^2+998\,b^2)c^4dx^4-(11025\,a^2+40742\,b^2)c^2dx^2-2\,(11025\,a^2+18692\,b^2)d-210\,(75\,abc^7dx^7+168\,abc^5dx^5+35\,abc^3dx^3-210\,abcdx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{385875\,(c^6x^2+c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] 1/385875*(11025*(5*b^2*c^8*d*x^8 + 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 - b^2
*c^2*d*x^2 - 2*b^2*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 +
210*(525*a*b*c^8*d*x^8 + 1365*a*b*c^6*d*x^6 + 945*a*b*c^4*d*x^4 - 105*a*b*c
^2*d*x^2 - 210*a*b*d - (75*b^2*c^7*d*x^7 + 168*b^2*c^5*d*x^5 + 35*b^2*c^3*d
*x^3 - 210*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 + 1)) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 + 9*(15925*a^2 + 734*b^2)
*c^6*d*x^6 + (99225*a^2 + 998*b^2)*c^4*d*x^4 - (11025*a^2 + 40742*b^2)*c^2*
d*x^2 - 2*(11025*a^2 + 18692*b^2)*d - 210*(75*a*b*c^7*d*x^7 + 168*a*b*c^5*d
```

```
*x^5 + 35*a*b*c^3*d*x^3 - 210*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2
+ d))/(c^6*x^2 + c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(d \left(c^2 x^2 + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{asinh}\left(c x\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.267 $\int x^2(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=405

$$-\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}+\frac{7b^2d\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{1152c^3\sqrt{1+c^2x^2}}-\frac{bdx^2\sqrt{\ }}{}$$

```
[Out] 1/6*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-7/1152*b^2*d*x*(c^2*d*x^2+
d)^(1/2)/c^2+43/1728*b^2*d*x^3*(c^2*d*x^2+d)^(1/2)+1/108*b^2*c^2*d*x^5*(c^2
*d*x^2+d)^(1/2)+1/16*d*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+1/8*d
*x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+7/1152*b^2*d*arcsinh(c*x)*(c^
2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-1/16*b*d*x^2*(a+b*arcsinh(c*x))*(c^2
*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/48*b*c*d*x^4*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/18*b*c^3*d*x^6*(a+b*arcsinh(c*x))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/48*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(
1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5808, 5806, 5812, 5783, 5776, 327, 221, 14, 5803, 12, 470}

$$-\frac{bdx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16c\sqrt{c^2x^2+1}}+\frac{dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16c^2}-\frac{7bcdx^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{48\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{8}dx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc^3\sqrt{c^2x^2+1}}-\frac{bc^3dx^6\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{18\sqrt{c^2x^2+1}}-\frac{7b^2dx\sqrt{c^2dx^2+d}}{1152c^2}+\frac{1}{108}b^2c^2dx^5\sqrt{c^2dx^2+d}+\frac{43b^2dx^3\sqrt{c^2dx^2+d}}{1728}+\frac{7b^2d\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{1152c^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-7*b^2*d*x*Sqrt[d + c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d + c^2*d*
x^2])/1728 + (b^2*c^2*d*x^5*Sqrt[d + c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d + c^
2*d*x^2]*ArcSinh[c*x])/(1152*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^2*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(48*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(18*Sqrt[1 + c^2*x^2]) + (d*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2)/8 + (x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(48*b*c^3*Sqrt[1
 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{2}d\int x^2\sqrt{d+c^2dx^2}\,($$

$$= -\frac{bcdx^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{12\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}\,(}{18\sqrt{1+}}$$

$$= -\frac{7bcdx^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{48\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}}{18\sqrt{1+}}$$

$$= \frac{1}{64}b^2dx^3\sqrt{d+c^2dx^2}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}-\frac{bdx^2\sqrt{d+c}}{1}$$

$$= \frac{b^2dx\sqrt{d+c^2dx^2}}{128c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c}$$

$$= -\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d}$$

$$= -\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d}$$

Mathematica [A]
time = 0.84, size = 508, normalized size = 1.25

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (864*a^2*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 4032*a^2*c^3*d*x^3*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*d*x^5*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2] - 288*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 216*a
*b*d*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 108*a*b*d*Sqrt[d + c^2*d*x^
2]*Cosh[4*ArcSinh[c*x]] - 24*a*b*d*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]]
 - 864*a^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2
]] - 108*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 27*b^2*d*Sqrt[d +
 c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSi
nh[c*x]] + 12*b*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(18*b*Cosh[2*ArcSinh[c*x
]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 36*a*Sinh[2*ArcS
inh[c*x]] + 36*a*Sinh[4*ArcSinh[c*x]] + 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-12*a - 3*b*Sinh[2*ArcSinh[c*x]] + 3*b
```

```
*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c^3*Sqrt[1 + c^2*x^
2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(351) = 702$.
time = 2.70, size = 1552, normalized size = 3.83

method	result	size
default	Expression too large to display	1552

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a^2/c^2*x*(c^2*d*x^2+d)^(3/2)-1/16
*a^2/c^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a^2/c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+
(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(-1/48*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/c^3*arcsinh(c*x)^3*d+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32
*(c^2*x^2+1)^(1/2)*x^6*c^6+64*x^5*c^5+48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x
^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2
-6*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^
5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+
(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)-1/
256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2
*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)-1/256*
(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2
+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)+1/1024*(d*
(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3-8*c^2*
x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arcsinh(
c*x)+1)*d/c^3/(c^2*x^2+1)+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7-32*(c^2*
x^2+1)^(1/2)*x^6*c^6+64*x^5*c^5-48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^3-18*
c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2+6*arc
sinh(c*x)+1)*d/c^3/(c^2*x^2+1))+2*a*b*(-1/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/c^3*arcsinh(c*x)^2*d+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32*
(c^2*x^2+1)^(1/2)*x^6*c^6+64*x^5*c^5+48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^
3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))
*d/c^3/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/
2)*x^4*c^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*
(-1+4*arcsinh(c*x))*d/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^
3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*
d/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2
+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/5
12*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3-
8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d/c
^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7-32*(c^2*x^2+1)^(1/2
)*x^6*c^6+64*x^5*c^5-48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^3-18*c^2*x^2*(c^
```

```
2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1
))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^4 + a^2*d*x^2 + (b^2*c^2*d*x^4 + b^2*d*x^2)*arcsinh(c
*x)^2 + 2*(a*b*c^2*d*x^4 + a*b*d*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b \operatorname{asinh}(c x))^2 \left(d c^2 x^2 + d\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.268 $\int x(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=267

$$\frac{16b^2d\sqrt{d+c^2dx^2}}{75c^2}+\frac{8b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{225c^2}+\frac{2b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^2}-\frac{2bdx\sqrt{d+c^2dx^2}\,(a+b\sinh}{5c\sqrt{1+c^2x^2}}$$

```
[Out] 1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/c^2/d+16/75*b^2*d*(c^2*d*x^2+d
)^(1/2)/c^2+8/225*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2+2/125*b^2*d*(c^
2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c/(c^2*x^2+1)^(1/2)-4/15*b*c*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5798, 200, 5784, 12, 1261, 712}

$$-\frac{2bdx\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{5c\sqrt{c^2x^2+1}}+\frac{(c^2dx^2+d)^{5/2}\,(a+b\sinh^{-1}(cx))^2}{5c^2d}-\frac{4bcdx^3\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{15\sqrt{c^2x^2+1}}-\frac{2bc^3dx^5\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{25\sqrt{c^2x^2+1}}+\frac{2b^2d(c^2x^2+1)^2\sqrt{c^2dx^2+d}}{125c^2}+\frac{16b^2d\sqrt{c^2dx^2+d}}{75c^2}+\frac{8b^2d(c^2x^2+1)\sqrt{c^2dx^2+d}}{225c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (16*b^2*d*Sqrt[d + c^2*d*x^2])/(75*c^2) + (8*b^2*d*(1 + c^2*x^2)*Sqrt[d + c
^2*d*x^2])/(225*c^2) + (2*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c
^2) - (2*b*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c*Sqrt[1 + c^2*
x^2]) - (4*b*c*d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 +
 c^2*x^2]) - (2*b*c^3*d*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*S
qrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(5*c^2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d} - \frac{\left(2bd\sqrt{d+c^2dx^2}\right)\int(1+}{5c\sqrt{1}}$$

$$= -\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}\,(a}{15\sqrt{1+c^2}}$$

$$= -\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}\,(a}{15\sqrt{1+c^2}}$$

$$= -\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}\,(a}{15\sqrt{1+c^2}}$$

$$= -\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}\,(a}{15\sqrt{1+c^2}}$$

$$= \frac{16b^2d\sqrt{d+c^2dx^2}}{75c^2} + \frac{8b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{225c^2} + \frac{2b^2d(1+c^2}{}$$

Mathematica [A]
time = 0.22, size = 198, normalized size = 0.74

$$\frac{d\sqrt{d+c^2dx^2}\left(225a^2(1+c^2x^2)^3-30abcx\sqrt{1+c^2x^2}\,(15+10c^2x^2+3c^4x^4)+2b^2(149+187c^2x^2+47c^4x^4+9c^6x^6)+30b\left(15a(1+c^2x^2)^3-bcx\sqrt{1+c^2x^2}\,(15+10c^2x^2+3c^4x^4)\right)\sinh^{-1}(cx)+225b^2(1+c^2x^2)^3\sinh^{-1}(cx)^2\right)}{1125c^2\,(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(225*a^2*(1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt[1 + c^2*x
^2]*(15 + 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(149 + 187*c^2*x^2 + 47*c^4*x^4 +
 9*c^6*x^6) + 30*b*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15 + 10
*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] + 225*b^2*(1 + c^2*x^2)^3*ArcSinh[c*x]^
2))/(1125*c^2*(1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. 2(233) = 466.
time = 0.95, size = 1149, normalized size = 4.30

method	result	size
default	Expression too large to display	1149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2/c^2/d*(c^2*d*x^2+d)^(5/2)+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*x^6
*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+1
3*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*
d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2
)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2-6*arcsinh(
c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(
1/2)*c*x+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2+2*arcsinh
(c*x)+2)*d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^
2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2+6
*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6
-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^
2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d/c^
2/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1
)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x
^2+1)^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1
))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/
2)*c*x+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)*d/c^2/(c^2*x^2+1)+1/16*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))*d/c
^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^
3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))*d/c^2/(c^2*x^
2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+2
8*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1
)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

Maxima [A]
time = 0.33, size = 230, normalized size = 0.86

$$\frac{(c^2dx^2+d)^{\frac{5}{2}}b^2\operatorname{arsinh}(cx)^2}{5\,c^2d}+\frac{2}{1125}\,b^2\left(\frac{9\,\sqrt{c^2x^2+1}\,c^2d^{\frac{5}{2}}x^4+38\,\sqrt{c^2x^2+1}\,d^{\frac{5}{2}}x^2+\frac{149\,\sqrt{c^2x^2+1}\,d^{\frac{5}{2}}}{c^2}}{d}-\frac{15\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)\operatorname{arsinh}(cx)}{cd}\right)+\frac{2\,(c^2dx^2+d)^{\frac{5}{2}}ab\operatorname{arsinh}(cx)}{5\,c^2d}+\frac{(c^2dx^2+d)^{\frac{5}{2}}a^2}{5\,c^2d}-\frac{2\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)ab}{75\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*d*x^2 + d)^(5/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/1125*b^2*((9*sqrt(
c^2*x^2 + 1)*c^2*d^(5/2)*x^4 + 38*sqrt(c^2*x^2 + 1)*d^(5/2)*x^2 + 149*sqrt(
c^2*x^2 + 1)*d^(5/2)/c^2)/d - 15*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 +
15*d^(5/2)*x)*arcsinh(c*x)/(c*d)) + 2/5*(c^2*d*x^2 + d)^(5/2)*a*b*arcsinh(c
*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2)*a^2/(c^2*d) - 2/75*(3*c^4*d^(5/2)*x
^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*a*b/(c*d)
```

Fricas [A]
time = 0.37, size = 332, normalized size = 1.24

$$\frac{225\,(b^2c^6dx^6+3\,b^2c^4dx^4+3\,b^2c^2dx^2+b^2d)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(15\,abc^6dx^6+45\,abc^4dx^4+45\,abc^2dx^2+15\,abd-(3\,b^2c^5dx^5+10\,b^2c^3dx^3+15\,b^2cdx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(9\,(25\,a^2+2\,b^2)c^6dx^6+(675\,a^2+94\,b^2)c^4dx^4+(675\,a^2+374\,b^2)c^2dx^2+(225\,a^2+298\,b^2)d-30\,(3\,abc^5dx^5+10\,abc^3dx^3+15\,abcdx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{1125\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*(b^2*c^6*d*x^6 + 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 + b^2*d)*sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(15*a*b*c^6*d*x^6 + 45
*a*b*c^4*d*x^4 + 45*a*b*c^2*d*x^2 + 15*a*b*d - (3*b^2*c^5*d*x^5 + 10*b^2*c^
3*d*x^3 + 15*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 + 1)) + (9*(25*a^2 + 2*b^2)*c^6*d*x^6 + (675*a^2 + 94*b^2)*c^4*d
*x^4 + (675*a^2 + 374*b^2)*c^2*d*x^2 + (225*a^2 + 298*b^2)*d - 30*(3*a*b*c^
5*d*x^5 + 10*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^
2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.269 $\int \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=294

$$\frac{15}{64}b^2dx\sqrt{d+c^2dx^2}+\frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{9b^2d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{64c\sqrt{1+c^2x^2}}-\frac{3bcdx^2\sqrt{d+c^2dx^2}\left(a\right.}{8\sqrt{1+c^2}}$$

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+15/64*b^2*d*x*(c^2*d*x^2+d)^
(1/2)+1/32*b^2*d*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)-1/8*b*d*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(a+b*arcsinh(c*x))^2*(c^
2*d*x^2+d)^(1/2)-9/64*b^2*d*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^
(1/2)-3/8*b*c*d*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)+1/8*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\frac{d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bd\left(c^2x^2+1\right)^{3/2}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3bcdx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{15}{64}b^2dx\sqrt{c^2dx^2+d}+\frac{1}{32}b^2dx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}-\frac{9b^2d\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{64c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (15*b^2*d*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*d*x*(1 + c^2*x^2)*Sqrt[d + c^2*d
*x^2])/32 - (9*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c*Sqrt[1 + c^2*x
^2]) - (3*b*c*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c
^2*x^2]) - (b*d*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(8*c) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (x*(d + c
^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (d*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(8*b*c*Sqrt[1 + c^2*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{4}(3d) \int \sqrt{d+c^2dx^2} \, (a$$

$$= -\frac{bd\left(1+c^2x^2\right)^{3/2} \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)}{8c} + \frac{3}{8}dx\sqrt{d+c^2}$$

$$= \frac{1}{32}b^2dx\left(1+c^2x^2\right) \sqrt{d+c^2dx^2} - \frac{3bcdx^2\sqrt{d+c^2dx^2} \left(a+b\sinh\right.}{8\sqrt{1+c^2x^2}}$$

$$= \frac{15}{64}b^2dx\sqrt{d+c^2dx^2} + \frac{1}{32}b^2dx\left(1+c^2x^2\right) \sqrt{d+c^2dx^2} - \frac{3bcdx^2}{}$$

$$= \frac{15}{64}b^2dx\sqrt{d+c^2dx^2} + \frac{1}{32}b^2dx\left(1+c^2x^2\right) \sqrt{d+c^2dx^2} - \frac{9b^2d\sqrt{}}{}$$

Mathematica [A]
time = 1.36, size = 329, normalized size = 1.12

$$\frac{96a^2cdx\sqrt{1+c^2x^2}\left(5+2c^2x^2\right)\sqrt{d+c^2dx^2}+288a^2d^{3/2}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+32b^2d\sqrt{d+c^2dx^2}\left(4\sinh^{-1}(cx)^3-6\sinh^{-1}(cx)\cosh\left(2\sinh^{-1}(cx)\right)+\left(3+6\sinh^{-1}(cx)^2\right)\sinh\left(2\sinh^{-1}(cx)\right)\right)-192abd\sqrt{d+c^2dx^2}\left(\cosh\left(2\sinh^{-1}(cx)\right)-2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)-12abd\sqrt{d+c^2dx^2}\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)-b^2d\sqrt{d+c^2dx^2}\left(32\sinh^{-1}(cx)^3+12\sinh^{-1}(cx)\cosh\left(4\sinh^{-1}(cx)\right)-3\left(1+8\sinh^{-1}(cx)^2\right)\sinh\left(4\sinh^{-1}(cx)\right)\right)}{768c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (96*a^2*c*d*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 288*a
^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 32*
b^2*d*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcSinh
[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 192*a*b*d*Sqrt[d +
c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*Ar
cSinh[c*x]])) - 12*a*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*Arc
Sinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - b^2*d*Sqrt[d + c^2*d*x^
2]*(32*ArcSinh[c*x]^3 + 12*ArcSinh[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*Arc
Sinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]))/(768*c*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(254) = 508$.
time = 1.54, size = 959, normalized size = 3.26

method	result

default	$\frac{x(c^2dx^2+d)^{\frac{3}{2}}a^2}{4}+\frac{3a^2dx\sqrt{c^2dx^2+d}}{8}+\frac{3a^2d^2\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{8\sqrt{c^2x^2+1}\,c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/8*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^3*d+1/512*(d*(c^2*x^2+1))^(
1/2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1
)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d/c/(c
^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)
+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d/c/(c^2*x^2+
1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-
(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d/c/(c^2*x^2+1)+1/51
2*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3-8
*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arc
sinh(c*x)+1)*d/c/(c^2*x^2+1))+2*a*b*(3/16*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/c*arcsinh(c*x)^2*d+1/256*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*(c^2*x^2
+1)^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^
(1/2))*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^
3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*
x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^
2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/25
6*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3-8
*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d/c/
(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.270 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=498

$$\frac{22}{9}b^2d\sqrt{d+c^2dx^2}-\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{2b^2cdx\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}-\frac{2b}{\;}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+22/9*b^2*d*(c^2*d*x^2+d)^(1/2)
+2/27*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+d*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)-2*a*b*c*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*d*x*a
rcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/3*b*c*d*x*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*b*c^3*d*x^3*(a+b*arcsinh(c*
x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*d*(a+b*arcsinh(c*x))^2*arctanh(
c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*d*(a+b*arc
sinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)+2*b*d*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b^2*d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*d*polylog(3,c*x+(c^2*x^2+1)^(1/2))*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5808, 5806, 5816, 4267, 2611, 2320, 6724, 5772, 267, 5784, 455, 45}

$$-\frac{2bd\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{2abcdx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}-\frac{2bcdx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2d\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}-\frac{2bc^3dx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}+\frac{2b^2d\sqrt{c^2dx^2+d}\,\text{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{2b^2d\sqrt{c^2dx^2+d}\,\text{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{22}{9}b^2d\sqrt{c^2dx^2+d}+\frac{2}{27}b^2d\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}-\frac{2b^2cdx\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (22*b^2*d*Sqrt[d + c^2*d*x^2])/9 - (2*a*b*c*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[1
 + c^2*x^2] + (2*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/27 - (2*b^2*c*d*x
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (2*b*c*d*x*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d*x^3*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*
x])^2)/3 - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSin
h[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*
b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(2*b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{si}\right.}{x}$$

$$= -\frac{2bcdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2dx^2}\,(}{9\sqrt{1+c}}$$

$$= -\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2bcdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} -$$

$$= -\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cdx\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} - \frac{2bcdx}{}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{}$$

Mathematica [A]
time = 1.60, size = 520, normalized size = 1.04

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (a^2*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 - (2*a*b*d*Sqrt[d + c^2*d*x^2]*
(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2
]) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] + (2*a*b*d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/
Sqrt[1 + c^2*x^2] + (b^2*d*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 + c^2*x^2] - 2*c*x
*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh[c*x]^2*(Log[1 -
E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]) + 2*ArcSinh[c*x]*(PolyLog[
2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]) + 2*(PolyLog[3, -E^
(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])])))/Sqrt[1 + c^2*x^2] + (b^
```

```
2*d*Sqrt[d + c^2*d*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9
*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcS
inh[c*x]])))/(108*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(491) = 982$.
time = 2.51, size = 1053, normalized size = 2.11

method	result
default	$a^2\sqrt{c^2 d\,x^2+d}\;d+\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\;\mathrm{polylog}\left(3,-cx-\sqrt{c^2x^2+1}\right)d}{\sqrt{c^2x^2+1}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\;d\,\mathrm{arcsinh}(cx)^2x^4c^4}{3c^2x^2+3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(c^2*d*x^2+d)^(1/2)*d-2/9*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)
*arcsinh(c*x)*x^3*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*x*c+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^
4*c^4+5/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^2*c^2+2*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2
))*d-a^2*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*d^(3/2)-2*b^2*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2)
)*d+2/27*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*x^4*c^4+70/27*b^2*(d*(c^2*
x^2+1))^(1/2)*d/(c^2*x^2+1)*x^2*c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*d-b^2*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*d+2*b^2*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(
1/2))*d+10/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-2
*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2
))*d+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2+2/3*a*b*(d*
(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4+68/27*b^2*(d*(c^2*x^2
+1))^(1/2)*d/(c^2*x^2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d-8/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2
*x^2+1)^(1/2)*x*c-2/9*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*x^3*c^3
-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x
^2+1)^(1/2))*d+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*d+8/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*
x)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))*d+1/3*(c^2*d*x^2+d)^(3/2)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2) - 3*sqrt(c^2*
d*x^2 + d)*d)*a^2 + integrate((c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c^2*
x^2 + 1))^2/x + 2*(c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x,
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x, x)
```

3.271 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=398

$$\frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{5b^2cd\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{2\sqrt{1+c^2x^2}}+bcd\sqrt{1+c^2x^2}$$

```
[Out] -(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x+1/4*b^2*c^2*d*x*(c^2*d*x^2+d)^(
1/2)+3/2*c^2*d*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*arcsi
nh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/2*b*c^3*d*x^2*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+c*d*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*c*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^
(1/2)/b/(c^2*x^2+1)^(1/2)+2*b*c*d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1
)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*d*polylog(2,1/(c*x+
(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*c*d*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {5807, 5785, 5783, 5776, 327, 221, 5801, 5775, 3797, 2221, 2317, 2438, 201}

$$\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2+\frac{cd\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^3}{2b\sqrt{c^2x^2+1}}+\frac{cd\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}+bcd\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\frac{2bcd\sqrt{c^2dx^2+d}\,\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{3bc^3dx^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}-\frac{b^2cd\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{1}{4}b^2c^2dx\sqrt{c^2dx^2+d}-\frac{5b^2cd\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (b^2*c^2*d*x*Sqrt[d + c^2*d*x^2])/4 - (5*b^2*c*d*Sqrt[d + c^2*d*x^2]*ArcSin
h[c*x])/(4*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*
d*x^2]*(a + b*ArcSinh[c*x]) + (3*c^2*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/2 + (c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x + (c*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^
2*x^2] - (b^2*c*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt
[1 + c^2*x^2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
```

```
b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(3c^2d\right)\int \sqrt{d+c^2dx^2}\,(a$$

$$= bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{3}{2}c^2dx\sqrt{d+c^2d$$

$$= -\frac{1}{2}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{3bc^3dx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{b^2cd\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{2\sqrt{1+c^2x^2}} - \frac{3bc^3dx^2\sqrt{}}{}$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{5b^2cd\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}} - \frac{3bc^3dx^2}{}$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{5b^2cd\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}} - \frac{3bc^3dx^2}{}$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{5b^2cd\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}} - \frac{3bc^3dx^2}{}$$

Mathematica [**A**]
time = 2.06, size = 369, normalized size = 0.93

$$\frac{12a^2d(-2+c^2x^2)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+24abd\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\sinh^{-1}(cx)+cx\sinh^{-1}(cx)^2+2cx\log(cx)\right)+36a^2cd^{3/2}x\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-8b^2d\sqrt{d+c^2dx^2}\left(\sinh^{-1}(cx)\left(3\sqrt{1+c^2x^2}\sinh^{-1}(cx)-cx\sinh^{-1}(cx)\left(3+\sinh^{-1}(cx)\right)-6cx\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)+3cx\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)+b^2cdx\sqrt{d+c^2dx^2}\left(4\sinh^{-1}(cx)^3-6\sinh^{-1}(cx)\cosh\left(2\sinh^{-1}(cx)\right)+\left(3+6\sinh^{-1}(cx)^2\right)\sinh\left(2\sinh^{-1}(cx)\right)\right)-6abcdx\sqrt{d+c^2dx^2}\left(\cosh\left(2\sinh^{-1}(cx)\right)-2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{24x\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (12*a^2*d*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 24*a*b*d*S
qrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2
+ 2*c*x*Log[c*x]) + 36*a^2*c*d^(3/2)*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] - 8*b^2*d*Sqrt[d + c^2*d*x^2]*(ArcSinh[c*x]*(3*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]*(3 + ArcSinh[c*x]) - 6*c*x*Log
[1 - E^(-2*ArcSinh[c*x])]) + 3*c*x*PolyLog[2, E^(-2*ArcSinh[c*x])]) + b^2*c
*d*x*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcSinh[
c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 6*a*b*c*d*x*Sqrt[d +
 c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*A
rcSinh[c*x]])))/(24*x*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(372) = 744$.
time = 2.45, size = 954, normalized size = 2.40

method	result
default	$-\frac{ab\sqrt{d\left(c^{2}x^{2}+1\right)}\, c^{2}d\,\mathrm{arcsinh}(cx)x}{c^{2}x^{2}+1}+\frac{ab\sqrt{d\left(c^{2}x^{2}+1\right)}\, c^{4}d\,\mathrm{arcsinh}(cx)x^{3}}{c^{2}x^{2}+1}+\frac{b^{2}\sqrt{d\left(c^{2}x^{2}+1\right)}\, c^{4}d\,\mathrm{arcsinh}(cx)^{2}x}{2c^{2}x^{2}+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*b*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)*x+3/2*a*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c*d-1/2*a*b*(d*(c^2*x^2+1))
^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2
+1)^(1/2)*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*d/x/(c^2*x^
2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2
))^2-1)*c*d+3/2*a^2*c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)+3/2*a^2*c^2*d*x*(c^2*d*x^2+d)^(1/2)-a^2/d/x*(c^2*d*x^2+d)^(5/2
)+a^2*c^2*x*(c^2*d*x^2+d)^(3/2)+a*b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)
*arcsinh(c*x)*x^3-1/4*a*b*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1/2)+2*b^2
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^
(1/2))*c*d+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-
c*x-(c^2*x^2+1)^(1/2))*c*d-1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*x^2+1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcs
inh(c*x)^2*x^3-1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)
^2*x-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2*d/x/(c^2*x^2+1)+1/4*b^2*(d*(c
^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/
(c^2*x^2+1)*x+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^
3*c*d-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c*d+2*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c*
d-1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+2*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c*d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

3.272 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=541

$$2b^2c^2d\sqrt{d+c^2dx^2}-\frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{3b^2c^3dx\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}-\frac{bcd\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(c\right.}{x\sqrt{1+c^2x^2}}$$

```
[Out] -1/2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2+2*b^2*c^2*d*(c^2*d*x^2+d)
^(1/2)+3/2*c^2*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-3*a*b*c^3*d*x*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*b^2*c^3*d*x*arcsinh(c*x)*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)-b*c*d*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x/(c^
2*x^2+1)^(1/2)+b*c^3*d*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)-3*c^2*d*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*d*arctanh((c^2*x^2+1)^(1/2))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*b*c^2*d*(a+b*arcsinh(c*x))*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3*b*c^2*d*(a+b*arcs
inh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)+3*b^2*c^2*d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-3*b^2*c^2*d*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {5807, 5806, 5816, 4267, 2611, 2320, 6724, 5772, 267, 14, 5803, 457, 81, 65, 214}

$$-\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{3}{2}c^2d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcd\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}-\frac{3c^2d\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}-\frac{3abc^3dx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}+\frac{bc^3dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{3b^2c^2d\sqrt{c^2dx^2+d}\,\text{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{3b^2c^2d\sqrt{c^2dx^2+d}\,\text{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+2b^2c^2d\sqrt{c^2dx^2+d}-\frac{b^2c^2d\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}}-\frac{3b^2c^3dx\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] 2*b^2*c^2*d*Sqrt[d + c^2*d*x^2] - (3*a*b*c^3*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[
1 + c^2*x^2] - (3*b^2*c^3*d*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^
2*x^2] - (b*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x
^2]) + (b*c^3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^
2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 - ((d + c^2*d*x
^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*
d*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (3*b*
c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Po
lyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (3*b^2*c^2*d*Sqrt[d + c^2*d*x
```

```
^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (3*b^2*c^2*d*Sqrt[d +
c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))^2}{x^3}\,dx = -\frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))^2}{2x^2} + \frac{1}{2}(3c^2d)\int \frac{\sqrt{d+c^2dx^2}\,(a}{}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}\,(a+b\,\mathrm{s}}{\sqrt{1+c^2x^2}}$$

$$= -\frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc}{}$$

$$= -b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx}}{\sqrt{1+c^2}}$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x}}$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x}}$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x}}$$

Mathematica [A]
time = 7.26, size = 771, normalized size = 1.43

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (a^2*c^2*d - (a^2*d)/(2*x^2))*Sqrt[d*(1 + c^2*x^2)] + (3*a^2*c^2*d^(3/2)*Lo
g[x])/2 - (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/2 + (2
*a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/
Sqrt[1 + c^2*x^2] + b^2*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(2 - (2*c*x*ArcSinh[c*x
])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[c*x]^2*(Log[1 - E^(-ArcSin
h[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]
*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 +
 c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x
])]))/Sqrt[1 + c^2*x^2]) + (a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[ArcSin
```

```
h[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2,
 -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[
ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2*x^2]) + (b^2*c
^2*d*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - ArcSinh[
c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])]
 - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh[c*x]/2]
] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*PolyLog[
2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLog[3, E^(
-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Ta
nh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(536) = 1072$.
time = 3.69, size = 1131, normalized size = 2.09

method	result	size
default	Expression too large to display	1131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/d/x^2*(c^2*d*x^2+d)^(5/2)-3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d-3/2*a^2*c^2*ln((2*d+2*
d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*d^(3/2)+3/2*a^2*c^2*(c^2*d*x^2+d)^(1/2)*d+1
/2*a^2*c^2*(c^2*d*x^2+d)^(3/2)-3/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d-3*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*
d-2*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x-b^2*ar
csinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d/x/(c^2*x^2+1)^(1/2)*c+3/2*b^2*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^
2*d+3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*
x+(c^2*x^2+1)^(1/2))*c^2*d+b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcs
inh(c*x)^2*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)+2*a*b*(d*(c^2*
x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)*x^2+3*a*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d-2*a*b*(d
*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x+a*b*(d*(c^2*x^2+1))^(1/2)*c^2
*d/(c^2*x^2+1)*arcsinh(c*x)-a*b*(d*(c^2*x^2+1))^(1/2)*d/x/(c^2*x^2+1)^(1/2)
*c-a*b*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)+3*a*b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d-3*a*b
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*
c^2*d+3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2
+1)^(1/2))*c^2*d-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+
(c^2*x^2+1)^(1/2))*c^2*d-3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*poly
log(3,c*x+(c^2*x^2+1)^(1/2))*c^2*d-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(
```

```
1/2)*d/x^2/(c^2*x^2+1)+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*x^2+1/
2*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(3*c^2*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2)*c^2 - 3*s
qrt(c^2*d*x^2 + d)*c^2*d + (c^2*d*x^2 + d)^(5/2)/(d*x^2))*a^2 + integrate((
c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + 2*(c^2*d*x^2
+ d)^(3/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^3, x)
```

3.273 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=378

$$-\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3d\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}}-\frac{bcd\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{c^2d\sqrt{d}\cdots}{}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3-1/3*b^2*c^2*d*(c^2*d*x^2+
d)^(1/2)/x-c^2*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x+1/3*b^2*c^3*d*a
rcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+4/3*c^3*d*(a+b*arcsinh(c*
x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*c^3*d*(a+b*arcsinh(c*x))^3*
(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+8/3*b*c^3*d*(a+b*arcsinh(c*x))*ln(1
-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-4/3*b^2
*c^3*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-1/3*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
/x^2
```

Rubi [A]
time = 0.45, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5807, 5805, 5775, 3797, 2221, 2317, 2438, 5783, 5802, 283, 221}

$$-\frac{c^2d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{x}-\frac{bcd\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}+\frac{c^3d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}+\frac{4c^3d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2x^2+1}}+\frac{8bc^3d\sqrt{c^2dx^2+d}\,\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}-\frac{b^2c^2d\sqrt{c^2dx^2+d}}{3x}-\frac{4b^2c^3d\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}+\frac{b^2c^3d\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d*Sqrt[d + c^2*d*x^2])/x + (b^2*c^3*d*Sqrt[d + c^2*d*x^2]*Arc
Sinh[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) - (c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^2)/x + (4*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*Sq
rt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
 (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2])
 + (8*b*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSin
h[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (4*b^2*c^3*d*Sqrt[d + c^2*d*x^2]*PolyLog[
2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3} + \left(c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\,\left(a\right.}{x}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{c^2d\sqrt{d+c^2d}}{}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} - \frac{bcd\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}\right.}{3x^2}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+}}{}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+}}{}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+}}{}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+}}{}$$

Mathematica [**A**]
time = 0.95, size = 458, normalized size = 1.21

$$\frac{-abcdx\sqrt{d+c^2dx^2} - a^2d\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2} - 4a^2c^2dx^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2} - b^2c^2dx^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2} + bd\sqrt{d+c^2dx^2}\left(3ac^3x^3 - b\left(-4c^3x^3+\sqrt{1+c^2x^2}+4c^2x^2\sqrt{1+c^2x^2}\right)\right)\sinh^{-1}(cx)^2 + b^2c^3dx^3\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)^3 + bd\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)\left(-bcx-2a\sqrt{1+c^2x^2}\,(1+4c^2x^2)+8bc^3x^3\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right) + 8abc^3dx^3\sqrt{d+c^2dx^2}\,\log(cx) + 3a^2c^3d^{3/2}x^3\sqrt{1+c^2x^2}\,\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right) - 4b^2c^3dx^3\sqrt{d+c^2dx^2}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{3x^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (-(a*b*c*d*x*Sqrt[d + c^2*d*x^2]) - a^2*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2] - 4*a^2*c^2*d*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - b^2*c^2*d*x^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + b*d*Sqrt[d + c^2*d*x^2]*(3*a*c^3*
x^3 - b*(-4*c^3*x^3 + Sqrt[1 + c^2*x^2] + 4*c^2*x^2*Sqrt[1 + c^2*x^2]))*Arc
Sinh[c*x]^2 + b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + b*d*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2
) + 8*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]) + 8*a*b*c^3*d*x^3*Sqrt[d + c^
2*d*x^2]*Log[c*x] + 3*a^2*c^3*d^(3/2)*x^3*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqr
t[d]*Sqrt[d + c^2*d*x^2]] - 4*b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*PolyLog[2,
E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2795 vs. $2(350) = 700$.
time = 4.06, size = 2796, normalized size = 7.40

method	result	size
default	Expression too large to display	2796

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c^3*d+8/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
*c^3*d-1/3*a^2/d/x^3*(c^2*d*x^2+d)^(5/2)+2/3*a^2*c^4*x*(c^2*d*x^2+d)^(3/2)+
a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d-16/3*a*b*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3*d+16/3*a*b*(d*(c^2*
x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3*c^6+4/3*a*b*(d*(c^2*x^2+1))^(1
/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x*c^4-3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*
x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3*d-64*a*b*(d*(c^2*x^2+1))^(1
/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8+64*a*b*(d*(
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*c^7-104*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*
x^2+1)*arcsinh(c*x)*c^6+24*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^
2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-146/3*a*b*(d*(c^2*x^2+1))^(1/2)
*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-28/3*a*b*(d*(c^2
*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-16
/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8
-20/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*
c^6-8*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^
(1/2)*c^5+8/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*c^3-4/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^
2*x^2+1)*x/(c^2*x^2+1)*c^4-1/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^
2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-32*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4
+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)^2*c^8-52*b^2*(d*(c^2*x^2+1))^(1/
2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^2*c^6+12*b^2*(d*
(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh
(c*x)^2*c^5-8*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2
*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+
9*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c+32*b^2*(d*(c^2*x^2+1))^(1
/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7-73/
3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*arcsin
h(c*x)^2*c^4-2/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x^3/(
c^2*x^2+1)*arcsinh(c*x)-2/3*a^2*c^2/d/x*(c^2*d*x^2+d)^(5/2)+a^2*c^4*d*x*(c^
2*d*x^2+d)^(1/2)+a^2*c^4*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/
(c^2*d)^(1/2)-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c
^2*x^2+1)*arcsinh(c*x)*c^4-14/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c
^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2-16/3*b^2*(d*(c^2*x^2+1))^(1/2)*d
```

```
/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-20/3*b^2*(d*(c^2
*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6+
4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*
arcsinh(c*x)^2*c^3-3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3*d-20/3*b^2*(d*(c^2*x^2+
1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-29/3*b^2*(d*(c^2*x
^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6+4/3*b^2*(d*(c^2
*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x*arcsinh(c*x)*c^4+16/3*b^2*(d*(c
^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3*arcsinh(c*x)*c^6+8*b^2*(d*(
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7+3*b^
2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^
5+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^
2*x^2+1)^(1/2))*c^3*d-10/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^
2+1)*x/(c^2*x^2+1)*c^4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^
2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^
2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4
*x^4+9*c^2*x^2+1)*x^3*c^6+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2
*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)^2*c^3*d+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*pol
ylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3*d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^4, x)
```

3.274 $\int x^3(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=625

$$-\frac{160b^2d^2\sqrt{d+c^2dx^2}}{3969c^4}+\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{80b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{11907c^4}-\frac{4b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1323c^4}$$

```
[Out] 5/63*d*x^4*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/9*x^4*(c^2*d*x^2+d)^(
5/2)*(a+b*arcsinh(c*x))^2-160/3969*b^2*d^2*(c^2*d*x^2+d)^(1/2)/c^4-80/11907
*b^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^4-4/1323*b^2*d^2*(c^2*x^2+1)^2*(
c^2*d*x^2+d)^(1/2)/c^4-50/27783*b^2*d^2*(c^2*x^2+1)^3*(c^2*d*x^2+d)^(1/2)/c
^4+2/729*b^2*d^2*(c^2*x^2+1)^4*(c^2*d*x^2+d)^(1/2)/c^4-2/63*d^2*(a+b*arcsin
h(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/63*d^2*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*
x^2+d)^(1/2)/c^2+1/21*d^2*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+4/63
*a*b*d^2*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)+4/63*b^2*d^2*x*arcsinh
(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-2/189*b*d^2*x^3*(a+b*arcsin
h(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/21*b*c*d^2*x^5*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-38/441*b*c^3*d^2*x^7*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/81*b*c^5*d^2*x^9*(a+b*a
rcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5808, 5806, 5812, 5798, 5772, 267, 5776, 272, 45, 14, 5803, 12, 457, 78, 276, 1265, 911, 1167}

$$\frac{d^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{63c^2}-\frac{2bcd^2x^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{21\sqrt{c^2x^2+1}}+\frac{1}{21}d^2x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{189c\sqrt{c^2x^2+1}}+\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5}{63}dx^4\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bc^5d^2x^9\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{81\sqrt{c^2x^2+1}}-\frac{2d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{63c^4}+\frac{4abd^2x\sqrt{c^2dx^2+d}}{63c^3\sqrt{c^2x^2+1}}-\frac{38bc^3d^2x^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{441\sqrt{c^2x^2+1}}+\frac{2b^2d^2\left(c^2x^2+1\right)^4\sqrt{c^2dx^2+d}}{729c^4}-\frac{50b^2d^2\left(c^2x^2+1\right)^3\sqrt{c^2dx^2+d}}{27783c^4}-\frac{160b^2d^2\sqrt{c^2dx^2+d}}{3969c^4}-\frac{4b^2d^2\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}}{1323c^4}-\frac{80b^2d^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}}{11907c^4}+\frac{4b^2d^2x\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{63c^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-160*b^2*d^2*Sqrt[d + c^2*d*x^2])/(3969*c^4) + (4*a*b*d^2*x*Sqrt[d + c^2*d
*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (80*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*
d*x^2])/(11907*c^4) - (4*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(1323
*c^4) - (50*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(27783*c^4) + (2*b
^2*d^2*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2])/(729*c^4) + (4*b^2*d^2*x*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x])/(63*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d^2*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(189*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(21*Sqrt[1 + c^2*x^2]) - (38
*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(441*Sqrt[1 + c^2*
x^2]) - (2*b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(81*Sqrt
[1 + c^2*x^2]) - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^4
) + (d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^2) + (d^2*x^
```

```
4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/21 + (5*d*x^4*(d + c^2*d*x^2)
^(3/2)*(a + b*ArcSinh[c*x])^2)/63 + (x^4*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])^2)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{9}(5d)\int x^3\left(d+c^2dx\right.$$

$$=-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45\sqrt{1+c^2x^2}}-\frac{4bc^3d^2x^7\sqrt{d+c^2d}}{63\sqrt{}}$$

$$=-\frac{8bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{105\sqrt{1+c^2x^2}}-\frac{38bc^3d^2x^7\sqrt{d+c}}{441}$$

$$=-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{21\sqrt{1+c^2x^2}}-\frac{38bc^3d^2x^7\sqrt{d+c}}{441}$$

$$=-\frac{2bd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{189c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^5\sqrt{d+c^2dx}}{21\sqrt{1}}$$

$$=\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{2bd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{189c\sqrt{1+c^2x^2}}$$

$$=\frac{134b^2d^2\sqrt{d+c^2dx^2}}{3969c^4}+\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{122b^2d^2(1+c^2}{119}$$

$$=-\frac{160b^2d^2\sqrt{d+c^2dx^2}}{3969c^4}+\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{80b^2d^2(1+c}{11}$$

Mathematica [A]
time = 0.29, size = 277, normalized size = 0.44

$$\frac{d^2\sqrt{d+c^2dx^2}\left(3969a^2(1+c^2x^2)^4(-2+7c^2x^2)-126abcx\sqrt{1+c^2x^2}\,(-126+21c^2x^2+189c^4x^4+171c^6x^6+49c^8x^8)+2b^2(-6140-7039c^2x^2+106c^4x^4+2152c^6x^6+1490c^8x^8+343c^{10}x^{10})-126b\left(-63a(1+c^2x^2)^4(-2+7c^2x^2)+bcx\sqrt{1+c^2x^2}\,(-126+21c^2x^2+189c^4x^4+171c^6x^6+49c^8x^8)\right)\sinh^{-1}(cx)+3969b^2(1+c^2x^2)^4(-2+7c^2x^2)\sinh^{-1}(cx)^2\right)}{250047c^4(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(3969*a^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - 126*a
*b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 4
9*c^8*x^8) + 2*b^2*(-6140 - 7039*c^2*x^2 + 106*c^4*x^4 + 2152*c^6*x^6 + 149
0*c^8*x^8 + 343*c^10*x^10) - 126*b*(-63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)
+ b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 +
49*c^8*x^8))*ArcSinh[c*x] + 3969*b^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSi
nh[c*x]^2))/(250047*c^4*(1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2013 vs. $2(549) = 1098$.

time = 2.22, size = 2014, normalized size = 3.22

method	result	size
default	Expression too large to display	2014

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b^2*
(1/373248*(d*(c^2*x^2+1))^(1/2)*(256*x^10*c^10+256*(c^2*x^2+1)^(1/2)*x^9*c^
9+704*x^8*c^8+576*(c^2*x^2+1)^(1/2)*x^7*c^7+688*x^6*c^6+432*(c^2*x^2+1)^(1/
2)*x^5*c^5+280*c^4*x^4+120*(c^2*x^2+1)^(1/2)*x^3*c^3+41*c^2*x^2+9*(c^2*x^2+
1)^(1/2)*c*x+1)*(81*arcsinh(c*x)^2-18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3
/175616*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*
x^6*c^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*
c^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*c*x+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x
)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2
+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2-6*
arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c
^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1
)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*
x)^2+2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*
c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(9
*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3/175616*(d*(c^2*x^2+
1))^(1/2)*(64*x^8*c^8-64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6-112*(c^2*x^2
+1)^(1/2)*x^5*c^5+104*c^4*x^4-56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^
2*x^2+1)^(1/2)*c*x+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^
2+1)+1/373248*(d*(c^2*x^2+1))^(1/2)*(256*x^10*c^10-256*(c^2*x^2+1)^(1/2)*x^
9*c^9+704*x^8*c^8-576*(c^2*x^2+1)^(1/2)*x^7*c^7+688*x^6*c^6-432*(c^2*x^2+1)
^(1/2)*x^5*c^5+280*c^4*x^4-120*(c^2*x^2+1)^(1/2)*x^3*c^3+41*c^2*x^2-9*(c^2*
x^2+1)^(1/2)*c*x+1)*(81*arcsinh(c*x)^2+18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+
1))+2*a*b*(1/41472*(d*(c^2*x^2+1))^(1/2)*(256*x^10*c^10+256*(c^2*x^2+1)^(1/
2)*x^9*c^9+704*x^8*c^8+576*(c^2*x^2+1)^(1/2)*x^7*c^7+688*x^6*c^6+432*(c^2*x
^2+1)^(1/2)*x^5*c^5+280*c^4*x^4+120*(c^2*x^2+1)^(1/2)*x^3*c^3+41*c^2*x^2+9*
(c^2*x^2+1)^(1/2)*c*x+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d
*(c^2*x^2+1))^(1/2)*(64*x^8*c^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6+11
2*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2
*x^2+7*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/5
76*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3
*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)*d^2/c
^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1
```

```
)*(1+arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x
^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*ar
csinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8-6
4*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c
^4*x^4-56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x+1)*(
1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*x^
10*c^10-256*(c^2*x^2+1)^(1/2)*x^9*c^9+704*x^8*c^8-576*(c^2*x^2+1)^(1/2)*x^7
*c^7+688*x^6*c^6-432*(c^2*x^2+1)^(1/2)*x^5*c^5+280*c^4*x^4-120*(c^2*x^2+1)^
(1/2)*x^3*c^3+41*c^2*x^2-9*(c^2*x^2+1)^(1/2)*c*x+1)*(1+9*arcsinh(c*x))*d^2/
c^4/(c^2*x^2+1))
```

Maxima [**A**]
time = 0.30, size = 390, normalized size = 0.62

$$\frac{1}{63}\left(\frac{7\left(c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)b^2\operatorname{arsinh}(cx)^2+\frac{2}{63}\left(\frac{7\left(c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)ab\operatorname{arsinh}(cx)+\frac{1}{63}\left(\frac{7\left(c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)a^2+\frac{2}{250047}b^2\left(\frac{343\sqrt{c^2x^2+1}c^6d^{\frac{5}{2}}x^8+1147\sqrt{c^2x^2+1}c^4d^{\frac{5}{2}}x^6+1005\sqrt{c^2x^2+1}c^2d^{\frac{5}{2}}x^4-899\sqrt{c^2x^2+1}d^{\frac{5}{2}}x^2-\frac{6140\sqrt{c^2x^2+1}d^{\frac{5}{2}}}{c^2}}{c^2}-\frac{63\left(49c^8d^{\frac{5}{2}}x^9+171c^6d^{\frac{5}{2}}x^7+189c^4d^{\frac{5}{2}}x^5+21c^2d^{\frac{5}{2}}x^3-126d^{\frac{5}{2}}x\right)\operatorname{arsinh}(cx)}{c^3}\right)-\frac{2\left(49c^8d^{\frac{5}{2}}x^9+171c^6d^{\frac{5}{2}}x^7+189c^4d^{\frac{5}{2}}x^5+21c^2d^{\frac{5}{2}}x^3-126d^{\frac{5}{2}}x\right)ab}{3969c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d)
)*b^2*arcsinh(c*x)^2 + 2/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d
*x^2 + d)^(7/2)/(c^4*d))*a*b*arcsinh(c*x) + 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x
^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 + 2/250047*b^2*((343*sqrt
(c^2*x^2 + 1)*c^6*d^(5/2)*x^8 + 1147*sqrt(c^2*x^2 + 1)*c^4*d^(5/2)*x^6 + 10
05*sqrt(c^2*x^2 + 1)*c^2*d^(5/2)*x^4 - 899*sqrt(c^2*x^2 + 1)*d^(5/2)*x^2 -
6140*sqrt(c^2*x^2 + 1)*d^(5/2)/c^2)/c^2 - 63*(49*c^8*d^(5/2)*x^9 + 171*c^6*
d^(5/2)*x^7 + 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*arc
sinh(c*x)/c^3) - 2/3969*(49*c^8*d^(5/2)*x^9 + 171*c^6*d^(5/2)*x^7 + 189*c^4
*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*a*b/c^3
```

Fricas [**A**]
time = 0.40, size = 525, normalized size = 0.84

$$\frac{3969\left(7b^2c^{10}d^2x^{10}+26b^2c^8d^2x^8+34b^2c^6d^2x^6+16b^2c^4d^2x^4-b^2c^2d^2x^2-2b^2d^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+126\left(441abc^{10}d^2x^{10}+1638abc^8d^2x^8+2142abc^6d^2x^6+1008abc^4d^2x^4-63abc^2d^2x^2-126abd^2-\left(49b^2c^9d^2x^9+171b^2c^7d^2x^7+189b^2c^5d^2x^5+21b^2c^3d^2x^3-126b^2cd^2x\right)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(343\left(81a^2+2b^2\right)c^{10}d^2x^{10}+2\left(51597a^2+1490b^2\right)c^8d^2x^8+2\left(67473a^2+2152b^2\right)c^6d^2x^6+4\left(15876a^2+53b^2\right)c^4d^2x^4-\left(3969a^2+14078b^2\right)c^2d^2x^2-2\left(3969a^2+6140b^2\right)d^2-126\left(49abc^9d^2x^9+171abc^7d^2x^7+189abc^5d^2x^5+21abc^3d^2x^3-126abcd^2x\right)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{250047\left(c^6x^2+c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] 1/250047*(3969*(7*b^2*c^10*d^2*x^10 + 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x
^6 + 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 - 2*b^2*d^2)*sqrt(c^2*d*x^2 + d)*
log(c*x + sqrt(c^2*x^2 + 1))^2 + 126*(441*a*b*c^10*d^2*x^10 + 1638*a*b*c^8*
d^2*x^8 + 2142*a*b*c^6*d^2*x^6 + 1008*a*b*c^4*d^2*x^4 - 63*a*b*c^2*d^2*x^2
- 126*a*b*d^2 - (49*b^2*c^9*d^2*x^9 + 171*b^2*c^7*d^2*x^7 + 189*b^2*c^5*d^2
*x^5 + 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*
```

```
x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (343*(81*a^2 + 2*b^2)*c^10*d^2*x^10
 + 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 + 2152*b^2)*c^6*d^2*
x^6 + 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a^2 + 14078*b^2)*c^2*d^2*x
^2 - 2*(3969*a^2 + 6140*b^2)*d^2 - 126*(49*a*b*c^9*d^2*x^9 + 171*a*b*c^7*d^
2*x^7 + 189*a*b*c^5*d^2*x^5 + 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x)*sqrt(c^
2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.275 $\int x^2(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=536

$$-\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2}+\frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296}+\frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824}+\frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2}+\frac{35}{}$$

```
[Out] 5/48*d*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*x^3*(c^2*d*x^2+d)^(
5/2)*(a+b*arcsinh(c*x))^2-359/36864*b^2*d^2*x*(c^2*d*x^2+d)^(1/2)/c^2+1079/
55296*b^2*d^2*x^3*(c^2*d*x^2+d)^(1/2)+209/13824*b^2*c^2*d^2*x^5*(c^2*d*x^2+
d)^(1/2)+1/256*b^2*c^4*d^2*x^7*(c^2*d*x^2+d)^(1/2)+5/128*d^2*x*(a+b*arcsinh
(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)+359/36864*b^2*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2
+1)^(1/2)-5/128*b*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2
+1)^(1/2)-59/384*b*c*d^2*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-17/144*b*c^3*d^2*x^6*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)-1/32*b*c^5*d^2*x^8*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-5/384*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^
2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.69, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5808, 5806, 5812, 5783, 5776, 327, 221, 14, 5803, 12, 470, 272, 45, 1281}

$$-\frac{5bd^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{128c\sqrt{c^2x^2+1}}+\frac{5d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{128c^2}-\frac{59bcd^2x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{384\sqrt{c^2x^2+1}}+\frac{5}{64}d^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5}{48}dx^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bc^5d^2x^8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{32\sqrt{c^2x^2+1}}-\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{384bc^3\sqrt{c^2x^2+1}}-\frac{17bc^3d^2x^6\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{144\sqrt{c^2x^2+1}}-\frac{359b^2d^2x\sqrt{c^2dx^2+d}}{36864c^2}+\frac{209b^2c^2d^2x^5\sqrt{c^2dx^2+d}}{13824}+\frac{1079b^2d^2x^3\sqrt{c^2dx^2+d}}{55296}+\frac{1}{256}b^2c^4d^2x^7\sqrt{c^2dx^2+d}+\frac{359b^2d^2\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{36864c^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-359*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/(36864*c^2) + (1079*b^2*d^2*x^3*Sqrt[d
 + c^2*d*x^2])/55296 + (209*b^2*c^2*d^2*x^5*Sqrt[d + c^2*d*x^2])/13824 + (b
^2*c^4*d^2*x^7*Sqrt[d + c^2*d*x^2])/256 + (359*b^2*d^2*Sqrt[d + c^2*d*x^2]*
ArcSinh[c*x])/(36864*c^3*Sqrt[1 + c^2*x^2]) - (5*b*d^2*x^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(128*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(384*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*x^6*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(144*Sqrt[1 + c^2*x^2]) - (
b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(32*Sqrt[1 + c^2*x^
2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(128*c^2) + (5*d
^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/64 + (5*d*x^3*(d + c^2*d
*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/48 + (x^3*(d + c^2*d*x^2)^(5/2)*(a + b*
ArcSinh[c*x])^2)/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(38
4*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
```

```
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
```

```
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{8}(5d)\int x^2\left(d+c^2dx^2\right.$$

$$=-\frac{bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{16\sqrt{1+c^2x^2}}-\frac{bc^3d^2x^6\sqrt{d+c^2dx^2}}{12\sqrt{1+}}$$

$$=-\frac{11bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{96\sqrt{1+c^2x^2}}-\frac{17bc^3d^2x^6\sqrt{d+c}}{144}$$

$$=\frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2}-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{384\sqrt{1+c^2x^2}}$$

$$=\frac{5}{512}b^2d^2x^3\sqrt{d+c^2dx^2}+\frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824}+\frac{1}{256}b^2c^4d$$

$$=\frac{5b^2d^2x\sqrt{d+c^2dx^2}}{1024c^2}+\frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296}+\frac{209b^2c^2d^2x^5}{138}$$

$$=-\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2}+\frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296}+\frac{209b^2c^2d}{}$$

$$=-\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2}+\frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296}+\frac{209b^2c^2d}{}$$

Mathematica [A]
time = 1.41, size = 619, normalized size = 1.15

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(34560*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 271872*a^2*c^3*
x^3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 313344*a^2*c^5*x^5*Sqrt[1 + c^2
*x^2]*Sqrt[d + c^2*d*x^2] + 110592*a^2*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c
^2*d*x^2] - 11520*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 13824*a*b*Sqrt[d
 + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 3456*a*b*Sqrt[d + c^2*d*x^2]*Cosh[4*Ar
cSinh[c*x]] - 1536*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 216*a*b*S
qrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]] - 34560*a^2*Sqrt[d]*Sqrt[1 + c^2*x^
2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6912*b^2*Sqrt[d + c^2*d*x^2]*
Sinh[2*ArcSinh[c*x]] + 864*b^2*Sqrt[d + c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 2
56*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] + 27*b^2*Sqrt[d + c^2*d*x^2
]*Sinh[8*ArcSinh[c*x]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(576*b*Cosh[
2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*x]] - 64*b*Cosh[6*ArcSinh[c*x]] -
9*b*Cosh[8*ArcSinh[c*x]] - 1152*a*Sinh[2*ArcSinh[c*x]] + 576*a*Sinh[4*ArcSi
nh[c*x]] + 384*a*Sinh[6*ArcSinh[c*x]] + 72*a*Sinh[8*ArcSinh[c*x]]) + 288*b*
Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-120*a - 48*b*Sinh[2*ArcSinh[c*x]] + 24
*b*Sinh[4*ArcSinh[c*x]] + 16*b*Sinh[6*ArcSinh[c*x]] + 3*b*Sinh[8*ArcSinh[c*
x]])))/(884736*c^3*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(468) = 936$.
time = 2.77, size = 2280, normalized size = 4.25

method	result	size
default	Expression too large to display	2280

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a^2/c^2*x*(c^2*d*x^2+d)^(5/2)-5/19
2*a^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a^2/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-5
/128*a^2/c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2
)+b^2*(-5/384*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3*d^
2+1/65536*(d*(c^2*x^2+1))^(1/2)*(128*x^9*c^9+128*(c^2*x^2+1)^(1/2)*x^8*c^8+
320*x^7*c^7+256*(c^2*x^2+1)^(1/2)*x^6*c^6+272*x^5*c^5+160*(c^2*x^2+1)^(1/2)
*x^4*c^4+88*c^3*x^3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*(
32*arcsinh(c*x)^2-8*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/6912*(d*(c^2*x^2+
1))^(1/2)*(32*x^7*c^7+32*(c^2*x^2+1)^(1/2)*x^6*c^6+64*x^5*c^5+48*(c^2*x^2+1
)^(1/2)*x^4*c^4+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(
1/2))*(18*arcsinh(c*x)^2-6*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/2048*(d*(c
^2*x^2+1))^(1/2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*x^
2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*
x)+1)*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*
```

```
(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)
+1)*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c
^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1
)*d^2/c^3/(c^2*x^2+1)+1/2048*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)
^(1/2)*x^4*c^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/
2))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/6912*(d*(c^2*
x^2+1))^(1/2)*(32*x^7*c^7-32*(c^2*x^2+1)^(1/2)*x^6*c^6+64*x^5*c^5-48*(c^2*x
^2+1)^(1/2)*x^4*c^4+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+
1)^(1/2))*(18*arcsinh(c*x)^2+6*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/65536*
(d*(c^2*x^2+1))^(1/2)*(128*x^9*c^9-128*(c^2*x^2+1)^(1/2)*x^8*c^8+320*x^7*c^
7-256*(c^2*x^2+1)^(1/2)*x^6*c^6+272*x^5*c^5-160*(c^2*x^2+1)^(1/2)*x^4*c^4+8
8*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1)^(1/2))*(32*arcsinh
(c*x)^2+8*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1))+2*a*b*(-5/256*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d^2+1/16384*(d*(c^2*x^2+1))^(1
/2)*(128*x^9*c^9+128*(c^2*x^2+1)^(1/2)*x^8*c^8+320*x^7*c^7+256*(c^2*x^2+1)^
(1/2)*x^6*c^6+272*x^5*c^5+160*(c^2*x^2+1)^(1/2)*x^4*c^4+88*c^3*x^3+32*c^2*x
^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*(-1+8*arcsinh(c*x))*d^2/c^3/(
c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32*(c^2*x^2+1)^(1/2)*x^
6*c^6+64*x^5*c^5+48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^3+18*c^2*x^2*(c^2*x^
2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)
+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3
*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x
))*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^
2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d^2/c^3/(c^2*x^
2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c
*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1/1024*(d*(c^2
*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3-8*c^2*x^2*
(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d^2/c^3/(c^2*
x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7-32*(c^2*x^2+1)^(1/2)*x^6*c^
6+64*x^5*c^5-48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)
^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1/16
384*(d*(c^2*x^2+1))^(1/2)*(128*x^9*c^9-128*(c^2*x^2+1)^(1/2)*x^8*c^8+320*x^
7*c^7-256*(c^2*x^2+1)^(1/2)*x^6*c^6+272*x^5*c^5-160*(c^2*x^2+1)^(1/2)*x^4*c
^4+88*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1)^(1/2))*(1+8*ar
csinh(c*x))*d^2/c^3/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^6 + 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2*
x^6 + 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^6
+ 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.276 $\int x(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=366

$$\frac{32b^2d^2\sqrt{d+c^2dx^2}}{245c^2}+\frac{16b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{735c^2}+\frac{12b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1225c^2}+\frac{2b^2d^2(1+c^2x^2)^3\sqrt{d+}}{343c^2}$$

```
[Out] 1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))^2/c^2/d+32/245*b^2*d^2*(c^2*d*x^
2+d)^(1/2)/c^2+16/735*b^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2+12/1225*b
^2*d^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^2+2/343*b^2*d^2*(c^2*x^2+1)^3*(c
^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/
(c^2*x^2+1)^(1/2)-2/7*b*c*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-2/49*b*c^5*d^2*x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5798, 200, 5784, 12, 1813, 1864}

$$-\frac{2bd^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{c^2x^2+1}}-\frac{2bcd^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{7\sqrt{c^2x^2+1}}+\frac{(c^2dx^2+d)^{7/2}\left(a+b\sinh^{-1}(cx)\right)^2}{7c^2d}-\frac{2bc^5d^2x^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{49\sqrt{c^2x^2+1}}-\frac{6bc^3d^2x^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{35\sqrt{c^2x^2+1}}+\frac{2b^2d^2(c^2x^2+1)^3\sqrt{c^2dx^2+d}}{343c^2}+\frac{32b^2d^2\sqrt{c^2dx^2+d}}{245c^2}+\frac{12b^2d^2(c^2x^2+1)^2\sqrt{c^2dx^2+d}}{1225c^2}+\frac{16b^2d^2(c^2x^2+1)\sqrt{c^2dx^2+d}}{735c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (32*b^2*d^2*Sqrt[d + c^2*d*x^2])/(245*c^2) + (16*b^2*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/(735*c^2) + (12*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2
])/(1225*c^2) + (2*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^2) -
 (2*b*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c*Sqrt[1 + c^2*x^2
]) - (2*b*c*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*Sqrt[1 + c
^2*x^2]) - (6*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(35*S
qrt[1 + c^2*x^2]) - (2*b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
]))/(49*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x])^2)
/(7*c^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left(d+c^2dx^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)^2}{7c^2d}-\frac{\left(2bd^2\sqrt{d+c^2dx^2}\right)\int(1+}{7c\sqrt{1}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\,(}{7\sqrt{1+c}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\,(}{7\sqrt{1+c}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\,(}{7\sqrt{1+c}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\,(}{7\sqrt{1+c}}$$

$$=\frac{32b^2d^2\sqrt{d+c^2dx^2}}{245c^2}+\frac{16b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{735c^2}+\frac{12b^2d^2(1}{}$$

Mathematica [A]
time = 0.25, size = 224, normalized size = 0.61

$$\frac{d^2\sqrt{d+c^2dx^2}\left(3675a^2(1+c^2x^2)^4-210abcx\sqrt{1+c^2x^2}\,(35+35c^2x^2+21c^4x^4+5c^6x^6)+2b^2(2161+2918c^2x^2+1108c^4x^4+426c^6x^6+75c^8x^8)+210b\left(35a(1+c^2x^2)^4-bcx\sqrt{1+c^2x^2}\,(35+35c^2x^2+21c^4x^4+5c^6x^6)\right)\sinh^{-1}(cx)+3675b^2(1+c^2x^2)^4\sinh^{-1}(cx)^2\right)}{25725c^2\,(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(3675*a^2*(1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[1 + c
^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(2161 + 2918*c^2
*x^2 + 1108*c^4*x^4 + 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*(1 + c^2*x^2)
^4 - b*c*x*Sqrt[1 + c^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6))*Ar
cSinh[c*x] + 3675*b^2*(1 + c^2*x^2)^4*ArcSinh[c*x]^2))/(25725*c^2*(1 + c^2*
x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1772 vs. $2(322)=644$.
time = 0.95, size = 1773, normalized size = 4.84

method	result	size
default	Expression too large to display	1773

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^2/c^2/d*(c^2*d*x^2+d)^(7/2)+b^2*(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*x^
8*c^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6+112*(c^2*x^2+1)^(1/2)*x^5*c^
5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*c
*x+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3200*(d*(
c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c
^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(25*arcsinh(c
*x)^2-10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2*x^2+1))^(1/2)*(4
*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(
9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1)
)^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)
*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)
*c*x+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+
1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3
200*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x
^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*(25*a
rcsinh(c*x)^2+10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/43904*(d*(c^2*x^2+1)
)^(1/2)*(64*x^8*c^8-64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6-112*(c^2*x^2+1
)^(1/2)*x^5*c^5+104*c^4*x^4-56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^2*
x^2+1)^(1/2)*c*x+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+
1))+2*a*b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*x^8*c^8+64*(c^2*x^2+1)^(1/2)*x^
7*c^7+144*x^6*c^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^
(1/2)*x^3*c^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+7*arcsinh(c*x))*d^2
/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/
2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)
^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))
^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)
*c*x+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)
*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)*d^2/c^2/(c^2*x^2+1)+5/1
28*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))
*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(
1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))*d^2/c^
2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*
x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1
/2)*c*x+1)*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1
/2)*(64*x^8*c^8-64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6-112*(c^2*x^2+1)^(1
/2)*x^5*c^5+104*c^4*x^4-56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+
1)^(1/2)*c*x+1)*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1))
```

Maxima [A]
time = 0.29, size = 274, normalized size = 0.75

$$\frac{(c^2dx^2+d)^{\frac{7}{2}}b^2\operatorname{arsinh}(cx)^2}{7\,c^2d}+\frac{2\,(c^2dx^2+d)^{\frac{7}{2}}ab\operatorname{arsinh}(cx)}{7\,c^2d}+\frac{2}{25725}\,b^2\left(\frac{75\,\sqrt{c^2x^2+1}\,c^4d^{\frac{7}{2}}x^6+351\,\sqrt{c^2x^2+1}\,c^2d^{\frac{7}{2}}x^4+757\,\sqrt{c^2x^2+1}\,d^{\frac{7}{2}}x^2+\frac{2161\,\sqrt{c^2x^2+1}\,d^{\frac{7}{2}}}{c^2}}{d}-\frac{105\left(5\,c^6d^{\frac{7}{2}}x^7+21\,c^4d^{\frac{7}{2}}x^5+35\,c^2d^{\frac{7}{2}}x^3+35\,d^{\frac{7}{2}}x\right)\operatorname{arsinh}(cx)}{cd}\right)+\frac{(c^2dx^2+d)^{\frac{7}{2}}a^2}{7\,c^2d}-\frac{2\left(5\,c^6d^{\frac{7}{2}}x^7+21\,c^4d^{\frac{7}{2}}x^5+35\,c^2d^{\frac{7}{2}}x^3+35\,d^{\frac{7}{2}}x\right)ab}{245\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*(c^2*d*x^2 + d)^(7/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/7*(c^2*d*x^2 + d)^
(7/2)*a*b*arcsinh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 + 1)*c^4*d^(
7/2)*x^6 + 351*sqrt(c^2*x^2 + 1)*c^2*d^(7/2)*x^4 + 757*sqrt(c^2*x^2 + 1)*d^
(7/2)*x^2 + 2161*sqrt(c^2*x^2 + 1)*d^(7/2)/c^2)/d - 105*(5*c^6*d^(7/2)*x^7
+ 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*arcsinh(c*x)/(c*d
)) + 1/7*(c^2*d*x^2 + d)^(7/2)*a^2/(c^2*d) - 2/245*(5*c^6*d^(7/2)*x^7 + 21*
c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*a*b/(c*d)
```

Fricas [A]
time = 0.38, size = 446, normalized size = 1.22

$$\frac{3675\,(b^2c^8d^2x^8+4\,b^2c^6d^2x^6+6\,b^2c^4d^2x^4+4\,b^2c^2d^2x^2+b^2d^2)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+210\left(35\,abc^8d^2x^8+140\,abc^6d^2x^6+210\,abc^4d^2x^4+140\,abc^2d^2x^2+35\,abd^2-(5\,b^2c^7d^2x^7+21\,b^2c^5d^2x^5+35\,b^2c^3d^2x^3+35\,b^2cd^2x)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(75\,(49\,a^2+2\,b^2)c^8d^2x^8+12\,(1225\,a^2+71\,b^2)c^6d^2x^6+2\,(11025\,a^2+1108\,b^2)c^4d^2x^4+4\,(3675\,a^2+1459\,b^2)c^2d^2x^2+(3675\,a^2+4322\,b^2)d^2-210\,(5\,abc^7d^2x^7+21\,abc^5d^2x^5+35\,abc^3d^2x^3+35\,abcd^2x)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{25725\,(c^4x^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/25725*(3675*(b^2*c^8*d^2*x^8 + 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 + 4*
b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))
^2 + 210*(35*a*b*c^8*d^2*x^8 + 140*a*b*c^6*d^2*x^6 + 210*a*b*c^4*d^2*x^4 +
140*a*b*c^2*d^2*x^2 + 35*a*b*d^2 - (5*b^2*c^7*d^2*x^7 + 21*b^2*c^5*d^2*x^5
+ 35*b^2*c^3*d^2*x^3 + 35*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 + 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 + 12*(12
25*a^2 + 71*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4 + 4*(36
75*a^2 + 1459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2 - 210*(5*a*b*c^7
*d^2*x^7 + 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 + 35*a*b*c*d^2*x)*sqrt(c
^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.277 $\int \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=420

$$\frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152}+\frac{65b^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}-\frac{115b^2d^2\sqrt{d+c^2}}{1152c\sqrt{1}}$$

```
[Out] 5/24*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/6*x*(c^2*d*x^2+d)^(5/2)
*(a+b*arcsinh(c*x))^2+245/1152*b^2*d^2*x*(c^2*d*x^2+d)^(1/2)+65/1728*b^2*d^
2*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+1/108*b^2*d^2*x*(c^2*x^2+1)^2*(c^2*d*x^
2+d)^(1/2)-5/48*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1
/2)/c-1/18*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c
+5/16*d^2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-115/1152*b^2*d^2*arcsi
nh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/16*b*c*d^2*x^2*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/48*d^2*(a+b*arcsinh(c*x))^
3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\sqrt{c^2x^2+1}}+\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bd^2(c^2x^2+1)^{5/2}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5bd^2(c^2x^2+1)^{3/2}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{48c}-\frac{5bcd^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5}{24}dx\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{108}b^2d^2x\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}+\frac{245b^2d^2x\sqrt{c^2dx^2+d}}{1152}+\frac{65b^2d^2x(c^2x^2+1)\sqrt{c^2dx^2+d}}{1728}-\frac{115b^2d^2\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{1152c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (245*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/1728 + (b^2*d^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/108
 - (115*b^2*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(1152*c*Sqrt[1 + c^2*x^2]
) - (5*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*Sqrt[1 + c
^2*x^2]) - (5*b*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(48*c) - (b*d^2*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(18*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/16
+ (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/24 + (x*(d + c^2*d*x
^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^3)/(48*b*c*Sqrt[1 + c^2*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2\, dx = \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{6}(5d)\int \left(d+c^2dx^2\right)^{3/2} (\ldots$$

$$= -\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{18c} + \frac{5}{24}dx\left(d+c^2\ldots\right.$$

$$= \frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2} - \frac{5bd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx\ldots}}{48c}$$

$$= \frac{65b^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}}{1728} + \frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2d\ldots}$$

$$= \frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152} + \frac{65b^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}}{1728} + \frac{1}{108}b^2\ldots$$

$$= \frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152} + \frac{65b^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}}{1728} + \frac{1}{108}b^2\ldots$$

Mathematica [A]
time = 1.04, size = 499, normalized size = 1.19

$$\frac{d^2\left(9504a^2cx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+7488a^2c^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+2304a^2c^5x^5\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+1440b^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)^3-3240ab\sqrt{d+c^2dx^2}\cosh\left(2\sinh^{-1}(cx)\right)-324ab\sqrt{d+c^2dx^2}\cosh\left(4\sinh^{-1}(cx)\right)-24ab\sqrt{d+c^2dx^2}\cosh\left(6\sinh^{-1}(cx)\right)+4320a^2\sqrt{d}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+1620b^2\sqrt{d+c^2dx^2}\sinh\left(2\sinh^{-1}(cx)\right)+81b^2\sqrt{d+c^2dx^2}\sinh\left(4\sinh^{-1}(cx)\right)+4b^2\sqrt{d+c^2dx^2}\sinh\left(6\sinh^{-1}(cx)\right)-12b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)\left(270b\cosh\left(2\sinh^{-1}(cx)\right)+27b\cosh\left(4\sinh^{-1}(cx)\right)+2b\cosh\left(6\sinh^{-1}(cx)\right)-540a\sinh\left(2\sinh^{-1}(cx)\right)-108a\sinh\left(4\sinh^{-1}(cx)\right)-12a\sinh\left(6\sinh^{-1}(cx)\right)\right)+72b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)^2\left(60a+45b\sinh\left(2\sinh^{-1}(cx)\right)+9b\sinh\left(4\sinh^{-1}(cx)\right)+b\sinh\left(6\sinh^{-1}(cx)\right)\right)\right)}{13824c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(9504*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 7488*a^2*c^3*x^3
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2] + 1440*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 3240*a
*b*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 324*a*b*Sqrt[d + c^2*d*x^2]*C
osh[4*ArcSinh[c*x]] - 24*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 432
0*a^2*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] +
1620*b^2*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sqrt[d + c^2*d*x
^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] -
```

```
 12*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] + 27*b*C
osh[4*ArcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSinh[c*x]]
 - 108*a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*Sqrt[d +
c^2*d*x^2]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*Ar
cSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]])))/(13824*c*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. $2(366) = 732$.
time = 1.61, size = 1568, normalized size = 3.73

method	result	size
default	Expression too large to display	1568

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(c^2*d*x^2+d)^(5/2)*a^2+5/24*a^2*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a^2*d^2
*x*(c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+b^2*(5/48*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*ar
csinh(c*x)^3*d^2+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32*(c^2*x^2+1)^(1
/2)*x^6*c^6+64*x^5*c^5+48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^3+18*c^2*x^2*(
c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2-6*arcsinh(c*x)
+1)*d^2/c/(c^2*x^2+1)+3/1024*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*(c^2*x^2+1)
^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/
2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+15/256*(d*(c^2*x^
2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))
*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1
))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2
*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+3/1024*(d*(c^2*x^2+1))^
(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+
1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)*d^2/c
/(c^2*x^2+1)+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7-32*(c^2*x^2+1)^(1/2)*
x^6*c^6+64*x^5*c^5-48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^3-18*c^2*x^2*(c^2*
x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2+6*arcsinh(c*x)+1)*
d^2/c/(c^2*x^2+1))+2*a*b*(5/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*ar
csinh(c*x)^2*d^2+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32*(c^2*x^2+1)^(1
/2)*x^6*c^6+64*x^5*c^5+48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^3+18*c^2*x^2*(
c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2/c/(c^2*x^
2+1)+3/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*
c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(
c*x))*d^2/c/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(
c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d^2/c/(c^2*x^
2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*
c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+3/512*(d*(c^2*x
^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3-8*c^2*x^2*(c
```

```
^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d^2/c/(c^2*x^2+
1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7-32*(c^2*x^2+1)^(1/2)*x^6*c^6+64
*x^5*c^5-48*(c^2*x^2+1)^(1/2)*x^4*c^4+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/
2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.278 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=635

$$\frac{598}{225}b^2d^2\sqrt{d+c^2dx^2}-\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{74}{675}b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}+\frac{2}{125}b^2d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2d$$

```
[Out] 1/3*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/5*(c^2*d*x^2+d)^(5/2)*(a+b
*arcsinh(c*x))^2+598/225*b^2*d^2*(c^2*d*x^2+d)^(1/2)+74/675*b^2*d^2*(c^2*x^
2+1)*(c^2*d*x^2+d)^(1/2)+2/125*b^2*d^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)+d^
2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-2*a*b*c*d^2*x*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-2*b^2*c*d^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-16/15*b*c*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)-22/45*b*c^3*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-2/25*b*c^5*d^2*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-2*d^2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*d^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b*d^2*(a+b*arcsin
h(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)+2*b^2*d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-2*b^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 16, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5808, 5806, 5816, 4267, 2611, 2320, 6724, 5772, 267, 5784, 455, 45, 200, 12, 1261, 712}

$$-\frac{2bd^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2x^2+1}}+\frac{2bd^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2x^2+1}}-\frac{2abcd^2x\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}-\frac{16bcd^2x\sqrt{c^2dx^2+d}(a+b\sinh^{-1}(cx))}{15\sqrt{c^2x^2+1}}+d^2\sqrt{c^2dx^2+d}(a+b\sinh^{-1}(cx))^2-\frac{2d^2\sqrt{c^2dx^2+d}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{1}{5}(c^2dx^2+d)^{5/2}(a+b\sinh^{-1}(cx))^2+\frac{1}{3}d(c^2dx^2+d)^{3/2}(a+b\sinh^{-1}(cx))^2-\frac{2bc^5d^2x^5\sqrt{c^2dx^2+d}(a+b\sinh^{-1}(cx))}{25\sqrt{c^2x^2+1}}-\frac{22bc^3d^2x^3\sqrt{c^2dx^2+d}(a+b\sinh^{-1}(cx))}{45\sqrt{c^2x^2+1}}+\frac{2b^2d^2\sqrt{c^2dx^2+d}\,\text{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{2b^2d^2\sqrt{c^2dx^2+d}\,\text{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{598}{225}b^2d^2\sqrt{c^2dx^2+d}+\frac{2}{125}b^2d^2(c^2x^2+1)^2\sqrt{c^2dx^2+d}+\frac{74}{675}b^2d^2(c^2x^2+1)\sqrt{c^2dx^2+d}-\frac{2b^2cd^2x\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (598*b^2*d^2*Sqrt[d + c^2*d*x^2])/225 - (2*a*b*c*d^2*x*Sqrt[d + c^2*d*x^2])
/Sqrt[1 + c^2*x^2] + (74*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/675 + (
2*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/125 - (2*b^2*c*d^2*x*Sqrt[d
+ c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (16*b*c*d^2*x*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 + c^2*x^2]) - (22*b*c^3*d^2*x^3*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*Sqrt[1 + c^2*x^2]) - (2*b*c^5*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) + d^2
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + (d*(d + c^2*d*x^2)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/3 + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/5 -
(2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/
```

```
Sqrt[1 + c^2*x^2] - (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyL
og[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b^2*d
^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*
b^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
```

```
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\,\mathrm{s}\right.}{x}$$

$$= -\frac{2bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5\sqrt{1+c^2x^2}} - \frac{4bc^3d^2x^3\sqrt{d+c^2dx^2}\,(}{15\sqrt{1+c}}$$

$$= -\frac{16bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}} - \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+}}$$

$$= -\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}}$$

$$= -\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cd^2x\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} - \frac{16bc}{}$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right.$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right.$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right.$$

Mathematica [A]
time = 2.94, size = 710, normalized size = 1.12

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^2*(3600*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(23 + 11*c^2*x^2 + 3*c
^4*x^4) - 24000*a*b*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^
(3/2)*ArcSinh[c*x]) - 480*a*b*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 + 9
*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) -
 b^2*Sqrt[d + c^2*d*x^2]*(480*c*x*(-30 + 5*c^2*x^2 + 9*c^4*x^4)*ArcSinh[c*x
] + 6750*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 125*(2 + 9*ArcSinh[c*x]^2
)*Cosh[3*ArcSinh[c*x]] - 27*(2 + 25*ArcSinh[c*x]^2)*Cosh[5*ArcSinh[c*x]]) +
 54000*a^2*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*x] - 54000*a^2*Sqrt[d]*Sqrt[1 +
```

```
c^2*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 108000*a*b*Sqrt[d + c^2*d*x
^2]*(c*x - Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - ArcSinh[c*x]*Log[1 - E^(-ArcSin
h[c*x])] + ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - PolyLog[2, -E^(-ArcSin
h[c*x])] + PolyLog[2, E^(-ArcSinh[c*x])]) + 54000*b^2*Sqrt[d + c^2*d*x^2]*(
2*Sqrt[1 + c^2*x^2] - 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2
 + ArcSinh[c*x]^2*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])])
 + 2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[
c*x])]) + 2*(PolyLog[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])]
)) + 1000*b^2*Sqrt[d + c^2*d*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2
) + (2 + 9*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + S
inh[3*ArcSinh[c*x]]))))/(54000*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1320 vs. $2(614) = 1228$.
time = 1.93, size = 1321, normalized size = 2.08

method	result	size
default	Expression too large to display	1321

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -46/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*x*c-2/25*a*b*(d*(c^2
*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*x^5*c^5-22/45*a*b*(d*(c^2*x^2+1))^(1/2
)*d^2/(c^2*x^2+1)^(1/2)*x^3*c^3+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-2*a*b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2-2*a*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+46
/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)+2*a*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2+68/15*a*
b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+1/5*b^2*(d*(c^
2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^6*c^6+14/15*b^2*(d*(c^2*x^
2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^4*c^4+34/15*b^2*(d*(c^2*x^2+1)
)^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^2*c^2-2/25*b^2*(d*(c^2*x^2+1))^(1/
2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^5*c^5-22/45*b^2*(d*(c^2*x^2+1))^(1/
2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3*c^3-46/15*b^2*(d*(c^2*x^2+1))^(1/
2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*d^2-a^2*d^(5/2)*ln((2*d+2*d^(
1/2)*(c^2*d*x^2+d)^(1/2))/x)+a^2*(c^2*d*x^2+d)^(1/2)*d^2+1/3*a^2*d*(c^2*d*x
^2+d)^(3/2)+2/125*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*x^6*c^6+532/337
5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*x^4*c^4+9872/3375*b^2*(d*(c^2*x
^2+1))^(1/2)*d^2/(c^2*x^2+1)*x^2*c^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2+b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-b
^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2
```

```
+1)^(1/2))*d^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+2/5*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2
*x^2+1)*arcsinh(c*x)*x^6*c^6+28/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1
)*arcsinh(c*x)*x^4*c^4+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylo
g(3,-c*x-(c^2*x^2+1)^(1/2))*d^2+23/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^
2+1)*arcsinh(c*x)^2+9394/3375*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)+1/5
*(c^2*d*x^2+d)^(5/2)*a^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2) - 5*(c^2*
d*x^2 + d)^(3/2)*d - 15*sqrt(c^2*d*x^2 + d)*d^2)*a^2 + integrate((c^2*d*x^2
 + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*(c^2*d*x^2 + d)^(5/2)*
a*b*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x, x)
```

3.279 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=530

$$\frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2}+\frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}-\frac{89b^2cd^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{64\sqrt{1+c^2x^2}}-\frac{15bc^3d^2x^2\sqrt{d-}}{8}$$

[Out] 5/4*c^2*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x+31/64*b^2*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+1/32*b^2*c^2*d^2*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)-1/8*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+15/8*c^2*d^2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-89/64*b^2*c*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-15/8*b*c^3*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+c*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/8*c*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+2*b*c*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*c*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)

Rubi [A]
time = 0.44, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {5807, 5786, 5785, 5783, 5776, 327, 221, 5798, 201, 5801, 5775, 3797, 2221, 2317, 2438}

$$\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5cd^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{8b\sqrt{c^2x^2+1}}+\frac{cd^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}-\frac{1}{8}bcd^2\left(c^2x^2+1\right)^{3/2}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+bcd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{2bcd^2\sqrt{c^2dx^2+d}\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{5}{4}c^2dx\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}-\frac{15bc^3d^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{b^2cd^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{31}{64}b^2c^2d^2x\sqrt{c^2dx^2+d}+\frac{1}{32}b^2c^2d^2x\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}-\frac{89b^2cd^2\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{64\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]

[Out] (31*b^2*c^2*d^2*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*c^2*d^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*Sqrt[1 + c^2*x^2]) - (15*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + b*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]) - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (15*c^2*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x + (5*c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(8*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] - (b^2*c*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2]

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
```

```
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} + (5c^2d)\int (d+c^2dx^2)^{3/2}\,(a$$

$$= \frac{1}{2}bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{5}{4}c^2dx(d+c$$

$$= -\frac{1}{8}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}$$

$$= -\frac{11}{16}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{1}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{11}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{89}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{89}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{89}{}$$

Mathematica [A]
time = 1.53, size = 550, normalized size = 1.04

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(-256*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 288*a^2*c^2*x^2*Sqrt
[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 64*a^2*c^4*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 160*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 128*a*b*c*
x*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 4*a*b*c*x*Sqrt[d + c^2*d*x^2]*
Cosh[4*ArcSinh[c*x]] + 512*a*b*c*x*Sqrt[d + c^2*d*x^2]*Log[c*x] + 480*a^2*c
*Sqrt[d]*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 256
*b^2*c*x*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])] + 64*b^2*c*x*S
qrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + b^2*c*x*Sqrt[d + c^2*d*x^2]*Sinh[
4*ArcSinh[c*x]] - 4*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(128*a*Sqrt[1 + c^2*
```

```
x^2] + 32*b*c*x*Cosh[2*ArcSinh[c*x]] + b*c*x*Cosh[4*ArcSinh[c*x]] - 128*b*c
*x*Log[1 - E^(-2*ArcSinh[c*x])] - 64*a*c*x*Sinh[2*ArcSinh[c*x]] - 4*a*c*x*S
inh[4*ArcSinh[c*x]]) + 8*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(60*a*c*x + 3
2*b*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]] + b*c*x*Si
nh[4*ArcSinh[c*x]])))/(256*x*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(490) = 980$.
time = 2.45, size = 1223, normalized size = 2.31

method	result	size
default	Expression too large to display	1223

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 11/4*a*b*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3+15/8*a^
2*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-a^2/d/x*(c^2*d*x^2+d)^(7/2)+5/4*a^2*c^2*d*x
*(c^2*d*x^2+d)^(3/2)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcs
inh(c*x)^2*x^5+11/8*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c
*x)^2*x^3+1/8*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*
x+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*
x^2+1)^(1/2))*c*d^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c
*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c*d^2-1/8*b^2*(d*(c^2*x^2+1))^(1/2)*c^5*d^2
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-9/8*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-33/64*a*b*(d*(c^2*x^2+1))^(1/2)*c*d^2/(c^
2*x^2+1)^(1/2)+1/4*a*b*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh(c*
x)*x+15/8*a^2*c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)
^(1/2)+1/2*a*b*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-2
*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*d^2/x/(c^2*x^2+1)-2*a*b*(d*(c^2*x^2
+1))^(1/2)*c*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/8*a*b*(d*(c^2*x^2+1))^(1/
2)*c^5*d^2/(c^2*x^2+1)^(1/2)*x^4-9/8*a*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2
*x^2+1)^(1/2)*x^2+15/8*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)^2*c*d^2+a^2*c^2*x*(c^2*d*x^2+d)^(5/2)-b^2*(d*(c^2*x^2+1))^(1/2)*arcsin
h(c*x)^2*d^2/x/(c^2*x^2+1)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))*c*d^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c*d^2-b^2*(d*(c^2*x^2+1))^(1/2)*c
*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-33/64*b^2*(d*(c^2*x^2+1))^(1/2)*c*d^2
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/32*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2
*x^2+1)*x^5+35/64*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*x^3+33/64*b
^2*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*x+5/8*b^2*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c*d^2+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c*d^2
```

Maxima [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\operatorname{asinh}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^2,x)
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

3.280 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=687

$$\frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2}-\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}-\frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\,\sinh^{-}}{\sqrt{1+c^2x^2}}$$

```
[Out] 5/6*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-1/2*(c^2*d*x^2+d)^(5/2)*
(a+b*arcsinh(c*x))^2/x^2+40/9*b^2*c^2*d^2*(c^2*d*x^2+d)^(1/2)+2/27*b^2*c^2*
d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+5/2*c^2*d^2*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)-5*a*b*c^3*d^2*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*b^2*c
^3*d^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c*d^2*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)+1/3*b*c^3*d^2*x*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*b*c^5*d^2*x^3*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*c^2*d^2*(a+b*arcsinh(c*
x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-
b^2*c^2*d^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)-5*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5*b^2*c^2*d^2*poly
log(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*b^2*c
^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/
2)
```

Rubi [A]
time = 0.63, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 20, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5807, 5808, 5806, 5816, 4267, 2611, 2320, 6724, 5772, 267, 5784, 455, 45, 276, 5803, 12, 1265, 911, 1167, 214}

$$-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2x^2+1}}+\frac{5}{2}c^2d^2\sqrt{c^2dx^2+d}(a+b\sinh^{-1}(cx))^2-\frac{bcd^2\sqrt{c^2dx^2+d}(a+b\sinh^{-1}(cx))}{x\sqrt{c^2x^2+1}}-\frac{5c^2d^2\sqrt{c^2dx^2+d}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{5}{6}c^2d(c^2dx^2+d)^{3/2}(a+b\sinh^{-1}(cx))^2-\frac{(c^2dx^2+d)^{5/2}(a+b\sinh^{-1}(cx))^2}{2x^2}-\frac{2bc^5d^2x^3\sqrt{c^2dx^2+d}(a+b\sinh^{-1}(cx))}{9\sqrt{c^2x^2+1}}-\frac{5abc^3d^2x\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}+\frac{bc^3d^2x\sqrt{c^2dx^2+d}(a+b\sinh^{-1}(cx))}{3\sqrt{c^2x^2+1}}+\frac{5b^2c^2d^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{5b^2c^2d^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{40}{9}b^2c^2d^2\sqrt{c^2dx^2+d}+\frac{2}{27}b^2c^2d^2(c^2x^2+1)\sqrt{c^2dx^2+d}-\frac{b^2c^2d^2\sqrt{c^2dx^2+d}\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}}-\frac{5b^2c^3d^2x\sqrt{c^2dx^2+d}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (40*b^2*c^2*d^2*Sqrt[d + c^2*d*x^2])/9 - (5*a*b*c^3*d^2*x*Sqrt[d + c^2*d*x^
2])/Sqrt[1 + c^2*x^2] + (2*b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/2
7 - (5*b^2*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] -
(b*c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2]) +
(b*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 +
c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (5*c
^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/6 - ((d + c^2*d*x^2)^(5/
2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*
```

```
ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*d^2*S
qrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (5*b*c^2
*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*P
olyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (5*b^2*c^2*d^2*Sqrt[d + c^2*
d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b^2*c^2*d^2*Sqrt
[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
```

```
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^2}{x^3}\,dx = -\frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^2}{2x^2}+\frac{1}{2}(5c^2d)\int \frac{(d+c^2dx^2)^{3/2}}{}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x\sqrt{1+c^2x^2}}+\frac{2bc^3d^2x\sqrt{d+c^2dx^2}\,(a+}{\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x\sqrt{1+c^2x^2}}+\frac{bc^3d^2x\sqrt{d+c^2dx^2}\,(a+}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{bcd^2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{x\sqrt{1+c^2x^2}}+$$

$$= -\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}-$$

$$= \frac{55}{9}b^2c^2d^2\sqrt{d+c^2dx^2}-\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{5}{27}b^2c^2d^2(1+c^2$$

$$= \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2}-\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2c^2d^2(1+c^2$$

$$= \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2}-\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2c^2d^2(1+c^2$$

Mathematica [A]
time = 7.35, size = 990, normalized size = 1.44

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*((7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^4*d^2
*x^2)/3) + 2*a*b*c^2*d^2*(-1/9*(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/Sq
rt[1 + c^2*x^2] + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3) + (
5*a^2*c^2*d^(5/2)*Log[x])/2 - (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d*(1
+ c^2*x^2)]])/2 + (4*a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c
^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*
x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2,
 E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + 2*b^2*c^2*d^2*Sqrt[d*(1 + c^2*x^2
)]*(2 - (2*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[
c*x]^2*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 +
c^2*x^2] + (2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(
-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] -
PolyLog[3, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]) + (b^2*c^2*d^2*Sqrt[d*(1
 + c^2*x^2)]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9*ArcSinh[c*
x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcSinh[c*x]]))
)/(108*Sqrt[1 + c^2*x^2]) + (a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[Arc
Sinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 -
 E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog
[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Se
ch[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2*x^2]) + (b^
2*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Arc
Sinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c
*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh[c*
x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*Pol
yLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLog[3
, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*
x]*Tanh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1403 vs. $2(664) = 1328$.
time = 3.79, size = 1404, normalized size = 2.04

method	result	size
default	Expression too large to display	1404

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -5/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c
^2*x^2+1)^(1/2))*c^2*d^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*
arcsinh(c*x)^2*x^4+8/3*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsin
h(c*x)^2*x^2-2/9*b^2*(d*(c^2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*arcsin
h(c*x)*x^3-14/3*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)*arcsinh
(c*x)*x-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d^2/x/(c^2*x^2+1)^(1/2)*c-5*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))*c^2*d^2+5/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2+5*b^2*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d^2+1/
2*a^2*c^2*(c^2*d*x^2+d)^(5/2)+122/27*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2
*x^2+1)+2/3*a*b*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4+
16/3*a*b*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-5*a*b*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1
/2))*c^2*d^2+5*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(
1-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2-5/2*a^2*c^2*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2
*d*x^2+d)^(1/2))/x)+5/2*a^2*c^2*(c^2*d*x^2+d)^(1/2)*d^2-1/2*a^2/d/x^2*(c^2*
d*x^2+d)^(7/2)+5/6*a^2*c^2*d*(c^2*d*x^2+d)^(3/2)+5*a*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d^2-2/9*a*b*(d*(c^
2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*x^3-14/3*a*b*(d*(c^2*x^2+1))^(1/2
)*c^3*d^2/(c^2*x^2+1)^(1/2)*x+11/3*a*b*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x
^2+1)*arcsinh(c*x)-a*b*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d^2/x^2/(c^2*x^2+
1)-5*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))*c^2*d^2-a*b*(d*(c^2*x^2+1))^(1/2)*d^2/x/(c^2*x^2+1)^(1/2)*c+5*b^2*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^
2*d^2+2/27*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*x^4+124/27*b^2*(d*
(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*x^2+11/6*b^2*(d*(c^2*x^2+1))^(1/2)*c
^2*d^2/(c^2*x^2+1)*arcsinh(c*x)^2-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1
/2)*d^2/x^2/(c^2*x^2+1)-5*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polyl
og(3,c*x+(c^2*x^2+1)^(1/2))*c^2*d^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*arctanh(c*x+(c^2*x^2+1)^(1/2))*c^2*d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(15*c^2*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2)*c^2 -
5*(c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(c^2*d*x^2 + d)*c^2*d^2 + 3*(c^2*d*x
^2 + d)^(7/2)/(d*x^2))*a^2 + integrate((c^2*d*x^2 + d)^(5/2)*b^2*log(c*x +
sqrt(c^2*x^2 + 1))^2/x^3 + 2*(c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c^2*x
^2 + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^3, x)
```

3.281 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=561

$$\frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x}-\frac{23b^2c^3d^2\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{12\sqrt{1+c^2x^2}}-\frac{5bc^5d^2x^2\sqrt{d+c^2d}}{2\sqrt{1}}$$

```
[Out] -5/3*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x-1/3*(c^2*d*x^2+d)^(5/
2)*(a+b*arcsinh(c*x))^2/x^3+7/12*b^2*c^4*d^2*x*(c^2*d*x^2+d)^(1/2)-1/3*b^2*
c^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/x-1/3*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2+5/2*c^4*d^2*x*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)-23/12*b^2*c^3*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)-5/2*b*c^5*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)+7/3*c^3*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)+5/6*c^3*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1
)^(1/2)+14/3*b*c^3*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-7/3*b^2*c^3*d^2*polylog(2,1/(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {5807, 5785, 5783, 5776, 327, 221, 5801, 5775, 3797, 2221, 2317, 2438, 201, 5802, 283}

$$-\frac{bcd^2(c^2x^2+1)^{3/2}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{5c^2d(c^2dx^2+d)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{(c^2dx^2+d)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}-\frac{5bc^5d^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5c^3d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{6b\sqrt{c^2x^2+1}}+\frac{7c^3d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2x^2+1}}+\frac{7}{3}bc^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{14bc^3d^2\sqrt{c^2dx^2+d}\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}-\frac{b^2c^2d^2(c^2x^2+1)\sqrt{c^2dx^2+d}}{3x}+\frac{7}{12}b^2c^4d^2x\sqrt{c^2dx^2+d}-\frac{7b^2c^3d^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}-\frac{23b^2c^3d^2\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{12\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d + c^2*d*x^2])/12 - (b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])/(3*x) - (23*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(
12*Sqrt[1 + c^2*x^2]) - (5*b*c^5*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (7*b*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/2 + (7*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^2)/(3*Sqrt[1 + c^2*x^2]) - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
(5*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*Sqrt[1 + c^2*x^
2]) + (14*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*
ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (7*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*
PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/}}{}$$

$$= -\frac{bcd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{5c^2d(d+c}{}$$

$$= -\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}+\frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}$$

$$= -\frac{2}{3}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{5bc^5}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{2b^2c^3}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{23b^2c}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{23b^2c}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{23b^2c}{}$$

Mathematica [A]
time = 1.61, size = 616, normalized size = 1.10

$$\frac{d^2\left(-8abcx\sqrt{d+c^2dx^2}-8a^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}-56a^2c^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}-8b^2c^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+12a^2c^4x^4\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+20b^2c^3x^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)^3-6abc^3x^3\sqrt{d+c^2dx^2}\cosh\left(2\sinh^{-1}(cx)\right)+112abc^3x^3\sqrt{d+c^2dx^2}\log(cx)+60a^2c^3\sqrt{d}x^3\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-56b^2c^3x^3\sqrt{d+c^2dx^2}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+3b^2c^3x^3\sqrt{d+c^2dx^2}\sinh\left(2\sinh^{-1}(cx)\right)-2b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)\left(4bcx+8a\sqrt{1+c^2x^2}+56ac^2x^2\sqrt{1+c^2x^2}+3bc^3x^3\cosh\left(2\sinh^{-1}(cx)\right)-56bc^3x^3\log\left(1-e^{-2\sinh^{-1}(cx)}\right)-6ac^3x^3\sinh\left(2\sinh^{-1}(cx)\right)\right)+2b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)^2\left(30ac^3x^3-4b\left(-7c^3x^3+\sqrt{1+c^2x^2}+7c^2x^2\sqrt{1+c^2x^2}\right)+3bc^3x^3\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{24x^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-8*a*b*c*x*Sqrt[d + c^2*d*x^2] - 8*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2] - 56*a^2*c^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 8*b^2*c^2*
x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 12*a^2*c^4*x^4*Sqrt[1 + c^2*x^2
]*Sqrt[d + c^2*d*x^2] + 20*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 -
 6*a*b*c^3*x^3*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] + 112*a*b*c^3*x^3*S
qrt[d + c^2*d*x^2]*Log[c*x] + 60*a^2*c^3*Sqrt[d]*x^3*Sqrt[1 + c^2*x^2]*Log[
c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 56*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*P
olyLog[2, E^(-2*ArcSinh[c*x])] + 3*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*Sinh[2*A
rcSinh[c*x]] - 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(4*b*c*x + 8*a*Sqrt[1 +
 c^2*x^2] + 56*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 3*b*c^3*x^3*Cosh[2*ArcSinh[c*x
]] - 56*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])] - 6*a*c^3*x^3*Sinh[2*ArcSinh
[c*x]]) + 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(30*a*c^3*x^3 - 4*b*(-7*c^
3*x^3 + Sqrt[1 + c^2*x^2] + 7*c^2*x^2*Sqrt[1 + c^2*x^2]) + 3*b*c^3*x^3*Sinh
[2*ArcSinh[c*x]])))/(24*x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3310 vs. 2(511) = 1022.
time = 4.11, size = 3311, normalized size = 5.90

method	result	size
default	Expression too large to display	3311

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/d/x^3*(c^2*d*x^2+d)^(7/2)+4/3*a^2*c^4*x*(c^2*d*x^2+d)^(5/2)+70*a*b
*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*
arcsinh(c*x)*c^5-46/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+
1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2+294*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4
*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7-294*a*b*(d*(c^2*x
^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8
-406*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1
)*arcsinh(c*x)*c^6-380/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x
^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-56/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63
*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-71/3*b^2*(d*(c^2*x^2+1))^(1/2)*d
^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-16/3*b^2*(d*(c^2*x^2+1))^(
1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/3*b^2*(d*(c^2*x^2+1)
```

)^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2
+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2+14/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^
2+1)^(1/2))*d^2*c^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsi
nh(c*x)^2*x^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x
)^2*x+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*
x-(c^2*x^2+1)^(1/2))*d^2*c^3-5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15
*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/2*b^2*(d*(c^2*x^2+1))^(1/2
)*d^2*c^5/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+7/3*b^2*(d*(c^2*x^2+1))^(1/2)*
d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3+5*b^2*(d
*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5
+21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)
^(1/2)*c^7+49/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3
*arcsinh(c*x)*c^6+7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+
1)*x*arcsinh(c*x)*c^4-7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*
x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-147*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63
*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)^2*c^8-203*b^2*(d*(c^2*x
^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^2*c
^6-56/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^
2+1)*arcsinh(c*x)*c^6-2/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*
x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)-1/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c
^4*x^4+15*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-21*a*b*(d*(c^2*x^2+1))^(1/2)*d
^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5-49/3*a*b*(d*(c^2*x^2
+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-1/4*a*b*(d*(c^
2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)+5/3*a^2*c^4*d*x*(c^2*d*x^2+d)^(3/
2)+5/2*a^2*c^4*d^2*x*(c^2*d*x^2+d)^(1/2)+5/2*a^2*c^4*d^3*ln(x*c^2*d/(c^2*d)
^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+14/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^
2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+a*b*(d*(c^2*
x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^3+a*b*(d*(c^2*x^2+1))^(1/2
)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x-49/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63
*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-1/2*a*b*(d*(c^2*x^2
+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^2+7/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/
(63*c^4*x^4+15*c^2*x^2+1)*x*c^4-28/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*d^2*c^3+49/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+1
5*c^2*x^2+1)*x^3*c^6-5*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2
+1)/(c^2*x^2+1)^(1/2)*c^3+5/2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)^2*d^2*c^3+14/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((
c*x+(c^2*x^2+1)^(1/2))^2-1)*d^2*c^3-4/3*a^2*c^2/d/x*(c^2*d*x^2+d)^(7/2)-56/
3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*c
^6-7/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1
)*c^4-190/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*
x^2+1)*arcsinh(c*x)^2*c^4-23/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15
*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2+35*b^2*(d*(c^2*x^2+1))^(1/2)*d
^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5+147*b
^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2

```
)*arcsinh(c*x)^2*c^7-21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^
2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d
^2/(63*c^4*x^4+15*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c-7/3*b^2*(
d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*c^6+1/3*b^2*(d*(c^2*
x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-1/4*b^2*(
d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/4*b^2*(d*(c^2
*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*x^3+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2*c^3+14/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2*c^3-14/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arc...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{5}{2}} (a+b\operatorname{asinh}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^4, x)
```

3.282 $\int \frac{x^4 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=153

$$-\frac{15x\sqrt{1+a^2x^2}}{64a^4}+\frac{x^3\sqrt{1+a^2x^2}}{32a^2}+\frac{15\sinh^{-1}(ax)}{64a^5}+\frac{3x^2\sinh^{-1}(ax)}{8a^3}-\frac{x^4\sinh^{-1}(ax)}{8a}-\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax}{8a^4}$$

```
[Out] 15/64*arcsinh(a*x)/a^5+3/8*x^2*arcsinh(a*x)/a^3-1/8*x^4*arcsinh(a*x)/a+1/8*
arcsinh(a*x)^3/a^5-15/64*x*(a^2*x^2+1)^(1/2)/a^4+1/32*x^3*(a^2*x^2+1)^(1/2)
/a^2-3/8*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)^2*(a^2
*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5812, 5783, 5776, 327, 221}

$$\frac{\sinh^{-1}(ax)^3}{8a^5}+\frac{15\sinh^{-1}(ax)}{64a^5}+\frac{3x^2\sinh^{-1}(ax)}{8a^3}+\frac{x^3\sqrt{a^2x^2+1}}{32a^2}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{4a^2}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{8a^4}-\frac{x^4\sinh^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-15*x*Sqrt[1 + a^2*x^2])/(64*a^4) + (x^3*Sqrt[1 + a^2*x^2])/(32*a^2) + (15
*ArcSinh[a*x])/(64*a^5) + (3*x^2*ArcSinh[a*x])/(8*a^3) - (x^4*ArcSinh[a*x])
/(8*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(8*a^4) + (x^3*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x]^2)/(4*a^2) + ArcSinh[a*x]^3/(8*a^5)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\sinh^{-1}(ax)\,dx}{2a}$$

$$= -\frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4a^2} + \frac{1}{8}\int$$

$$= \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{8a^4} +$$

$$= -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(ax)}{8a} - 3x\sqrt{1+a}$$

$$= -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{15\sinh^{-1}(ax)}{64a^5} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(}{8a}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 0.64

$$\frac{ax\sqrt{1+a^2x^2}\,(-15+2a^2x^2)+(15+24a^2x^2-8a^4x^4)\sinh^{-1}(ax)+8ax\sqrt{1+a^2x^2}\,(-3+2a^2x^2)\sinh^{-1}(ax)^2+8\sinh^{-1}(ax)^3}{64a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]

[Out] (a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2) + (15 + 24*a^2*x^2 - 8*a^4*x^4)*ArcSinh[a*x] + 8*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2 + 8*ArcSinh[a*x]^3)/(64*a^5)

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)

[Out] int(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)

Fricas [A]
time = 0.38, size = 131, normalized size = 0.86

$$\frac{8\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2+8\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3-(8\,a^4x^4-24\,a^2x^2-15)\log\left(ax+\sqrt{a^2x^2+1}\right)+(2\,a^3x^3-15\,ax)\sqrt{a^2x^2+1}}{64\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/64*(8*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 8*log(a*x + sqrt(a^2*x^2 + 1))^3 - (8*a^4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1)) + (2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1))/a^5

Sympy [A]
time = 0.86, size = 146, normalized size = 0.95

$$\begin{cases} -\frac{x^4 \operatorname{asinh}(ax)}{8a}+\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}}{32a^2}+\frac{3x^2\operatorname{asinh}(ax)}{8a^3}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{8a^4}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}+\frac{\operatorname{asinh}^3(ax)}{8a^5}+\frac{15\operatorname{asinh}(ax)}{64a^5} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)

```
[Out] Piecewise((-x**4*asinh(a*x)/(8*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/
(4*a**2) + x**3*sqrt(a**2*x**2 + 1)/(32*a**2) + 3*x**2*asinh(a*x)/(8*a**3)
- 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(8*a**4) - 15*x*sqrt(a**2*x**2 + 1)
/(64*a**4) + asinh(a*x)**3/(8*a**5) + 15*asinh(a*x)/(64*a**5), Ne(a, 0)), (
0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^4*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.283 $\int \frac{x^3 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=122

$$-\frac{14\sqrt{1+a^2x^2}}{9a^4}+\frac{2(1+a^2x^2)^{3/2}}{27a^4}+\frac{4x\sinh^{-1}(ax)}{3a^3}-\frac{2x^3\sinh^{-1}(ax)}{9a}-\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}}{3a}$$

```
[Out] 2/27*(a^2*x^2+1)^(3/2)/a^4+4/3*x*arcsinh(a*x)/a^3-2/9*x^3*arcsinh(a*x)/a-14
/9*(a^2*x^2+1)^(1/2)/a^4-2/3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*a
rcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5812, 5798, 5772, 267, 5776, 272, 45}

$$\frac{4x\sinh^{-1}(ax)}{3a^3}+\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{3a^2}+\frac{2(a^2x^2+1)^{3/2}}{27a^4}-\frac{14\sqrt{a^2x^2+1}}{9a^4}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{3a^4}-\frac{2x^3\sinh^{-1}(ax)}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-14*Sqrt[1 + a^2*x^2])/(9*a^4) + (2*(1 + a^2*x^2)^(3/2))/(27*a^4) + (4*x*A
rcSinh[a*x])/(3*a^3) - (2*x^3*ArcSinh[a*x])/(9*a) - (2*Sqrt[1 + a^2*x^2]*Ar
cSinh[a*x]^2)/(3*a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} dx = \frac{x^2\sqrt{1+a^2x^2} \sinh^{-1}(ax)^2}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} dx}{3a^2} - \frac{2\int x^2 \sinh^{-1}(ax)\, dx}{3a}$$
$$= -\frac{2x^3 \sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2} \sinh^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2} \sinh^{-1}(ax)^2}{3a^2} + \frac{2}{9}\int \frac{\ }{\sqrt{\ }}$$
$$= \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3 \sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2} \sinh^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2} \sinh^{-1}}{3a^2}$$
$$= -\frac{4\sqrt{1+a^2x^2}}{3a^4} + \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3 \sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2} \sinh^{-1}(ax)^2}{3a^4} + \frac{x^2}{\ }$$
$$= -\frac{14\sqrt{1+a^2x^2}}{9a^4} + \frac{2(1+a^2x^2)^{3/2}}{27a^4} + \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3 \sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}}{3a}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.65

$$\frac{2(-20+a^2x^2)\sqrt{1+a^2x^2} - 6ax(-6+a^2x^2)\sinh^{-1}(ax) + 9(-2+a^2x^2)\sqrt{1+a^2x^2} \sinh^{-1}(ax)^2}{27a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
[Out] (2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] - 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x] +
9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(27*a^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
[Out] int(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.28, size = 101, normalized size = 0.83

$$\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^2 + \frac{2\left(\sqrt{a^2x^2+1}\,x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)}{27\,a^2} - \frac{2\,(a^2x^3-6\,x)\operatorname{arsinh}(ax)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^2 +
2/27*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)/a^2 - 2/9*(a^2*x^3
- 6*x)*arcsinh(a*x)/a^3
```

Fricas [A]
time = 0.40, size = 98, normalized size = 0.80

$$\frac{9\sqrt{a^2x^2+1}\,(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^2-6\,(a^3x^3-6\,ax)\log\left(ax+\sqrt{a^2x^2+1}\right)+2\sqrt{a^2x^2+1}\,(a^2x^2-20)}{27\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*
(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1)*(a^2*x
^2 - 20))/a^4
```

Sympy [A]
time = 0.57, size = 121, normalized size = 0.99

$$\begin{cases} -\frac{2x^3\operatorname{asinh}(ax)}{9a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}}{27a^2}+\frac{4x\operatorname{asinh}(ax)}{3a^3}-\frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^4}-\frac{40\sqrt{a^2x^2+1}}{27a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-2*x**3*asinh(a*x)/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**
2/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)/(27*a**2) + 4*x*asinh(a*x)/(3*a**3)
 - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**4) - 40*sqrt(a**2*x**2 + 1)/(2
7*a**4), Ne(a, 0)), (0, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^3*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.284 $\int \frac{x^2 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sqrt{1+a^2x^2}}{4a^2}-\frac{\sinh^{-1}(ax)}{4a^3}-\frac{x^2\sinh^{-1}(ax)}{2a}+\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}$$

```
[Out] -1/4*arcsinh(a*x)/a^3-1/2*x^2*arcsinh(a*x)/a-1/6*arcsinh(a*x)^3/a^3+1/4*x*(
a^2*x^2+1)^(1/2)/a^2+1/2*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5812, 5783, 5776, 327, 221}

$$-\frac{\sinh^{-1}(ax)^3}{6a^3}-\frac{\sinh^{-1}(ax)}{4a^3}+\frac{x\sqrt{a^2x^2+1}}{4a^2}+\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2a^2}-\frac{x^2\sinh^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x*Sqrt[1 + a^2*x^2])/(4*a^2) - ArcSinh[a*x]/(4*a^3) - (x^2*ArcSinh[a*x])/(
2*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*a^2) - ArcSinh[a*x]^3/(6*a^3
)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx &= \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{2a^2}-\frac{\int x\sinh^{-1}(ax)\,dx}{a}\\ &= -\frac{x^2\sinh^{-1}(ax)}{2a}+\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}+\frac{1}{2}\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{x\sqrt{1+a^2x^2}}{4a^2}-\frac{x^2\sinh^{-1}(ax)}{2a}+\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}-\frac{\int \frac{1}{\sqrt{1+}}}{4}\\ &= \frac{x\sqrt{1+a^2x^2}}{4a^2}-\frac{\sinh^{-1}(ax)}{4a^3}-\frac{x^2\sinh^{-1}(ax)}{2a}+\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(a}{6a^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 0.83

$$\frac{3ax\sqrt{1+a^2x^2}-3(1+2a^2x^2)\sinh^{-1}(ax)+6ax\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2-2\sinh^{-1}(ax)^3}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a*x*Sqrt[1 + a^2*x^2] - 3*(1 + 2*a^2*x^2)*ArcSinh[a*x] + 6*a*x*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3)/(12*a^3)
```

Maple [A]
time = 2.47, size = 69, normalized size = 0.79

method	result	size
default	$-\frac{-6\,\mathrm{arcsinh}(ax)^{2}\sqrt{a^{2}x^{2}+1}\,ax+6x^{2}\,\mathrm{arcsinh}(ax)a^{2}+2\,\mathrm{arcsinh}(ax)^{3}-3\sqrt{a^{2}x^{2}+1}\,ax+3\,\mathrm{arcsinh}(ax)}{12a^{3}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-6*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+6*x^2*arcsinh(a*x)*a^2+2*arc
sinh(a*x)^3-3*(a^2*x^2+1)^(1/2)*a*x+3*arcsinh(a*x))/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Fricas [A]
time = 0.35, size = 102, normalized size = 1.17

$$\frac{6\sqrt{a^{2}x^{2}+1}\,ax\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}-2\,\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}+3\sqrt{a^{2}x^{2}+1}\,ax-3\,(2\,a^{2}x^{2}+1)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)}{12\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*log(a*x +
sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 + 1)))/a^3
```

Sympy [A]
time = 0.33, size = 78, normalized size = 0.90

$$\begin{cases} -\frac{x^{2}\,\mathrm{asinh}\,(ax)}{2a}+\frac{x\sqrt{a^{2}x^{2}+1}\,\mathrm{asinh}^{2}\,(ax)}{2a^{2}}+\frac{x\sqrt{a^{2}x^{2}+1}}{4a^{2}}-\frac{\mathrm{asinh}^{3}\,(ax)}{6a^{3}}-\frac{\mathrm{asinh}\,(ax)}{4a^{3}} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2*asinh(a*x)/(2*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(2*
a**2) + x*sqrt(a**2*x**2 + 1)/(4*a**2) - asinh(a*x)**3/(6*a**3) - asinh(a*x
)/(4*a**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.285 $\int \frac{x \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{1+a^2x^2}}{a^2}-\frac{2x\sinh^{-1}(ax)}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a^2}$$

```
[Out] -2*x*arcsinh(a*x)/a+2*(a^2*x^2+1)^(1/2)/a^2+arcsinh(a*x)^2*(a^2*x^2+1)^(1/2
)/a^2
```

Rubi [A]
time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5798, 5772, 267}

$$\frac{2\sqrt{a^2x^2+1}}{a^2}+\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{a^2}-\frac{2x\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2])/a^2 - (2*x*ArcSinh[a*x])/a + (Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^2)/a^2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a^2} - \frac{2\int \sinh^{-1}(ax)\,dx}{a}$$
$$= -\frac{2x\sinh^{-1}(ax)}{a} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a^2} + 2\int \frac{x}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{2\sqrt{1+a^2x^2}}{a^2} - \frac{2x\sinh^{-1}(ax)}{a} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a^2}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.92

$$\frac{2\sqrt{1+a^2x^2} - 2ax\sinh^{-1}(ax) + \sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2] - 2*a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
2)/a^2
```

Maple [A]
time = 2.29, size = 64, normalized size = 1.23

method	result	size
default	$\frac{x^2 \operatorname{arcsinh}(ax)^2 a^2+\operatorname{arcsinh}(ax)^2-2\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+2a^2x^2+2}{a^2\sqrt{a^2x^2+1}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(x^2*arcsinh(a*x)^2*a^2+arcsinh(a*x)^2-2*arcsinh(a*
x)*(a^2*x^2+1)^(1/2)*a*x+2*a^2*x^2+2)
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.92

$$\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)^2}{a^2} - \frac{2\left(ax\operatorname{arsinh}(ax) - \sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a^2 - 2*(a*x*arcsinh(a*x) - sqrt(a^2*x^2 +
 1))/a^2
```

Fricas [A]
time = 0.42, size = 70, normalized size = 1.35

$$-\frac{2\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\,\sqrt{a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(2*a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2
*x^2 + 1))^2 - 2*sqrt(a^2*x^2 + 1))/a^2
```

Sympy [A]
time = 0.26, size = 49, normalized size = 0.94

$$\begin{cases} -\frac{2x\operatorname{asinh}(ax)}{a}+\frac{\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{a^2}+\frac{2\sqrt{a^2x^2+1}}{a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-2*x*asinh(a*x)/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**2/a**2 + 2*s
qrt(a**2*x**2 + 1)/a**2, Ne(a, 0)), (0, True))
```

Giac [A]
time = 0.41, size = 74, normalized size = 1.42

$$\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^2}-\frac{2\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt(
a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.286 $\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

[Out] 1/3*arcsinh(a*x)^3/a

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5783}

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]

[Out] ArcSinh[a*x]^3/(3*a)

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx = \frac{\sinh^{-1}(ax)^3}{3a}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]

[Out] ArcSinh[a*x]^3/(3*a)

Maple [A]
time = 0.25, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^3}{3a}$	12
default	$\frac{\operatorname{arcsinh}(ax)^3}{3a}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsinh(a*x)^3/a
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.85

$$\frac{\operatorname{arsinh}(ax)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*arcsinh(a*x)^3/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 0.36, size = 23, normalized size = 1.77

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/a
```

Sympy [A]
time = 0.20, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^3(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Mupad [B]
time = 0.09, size = 11, normalized size = 0.85

$$\frac{\operatorname{asinh}(a\,x)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(a^2*x^2 + 1)^(1/2),x)
[Out] asinh(a*x)^3/(3*a)
```

3.287 $\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=68

$$-2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)$$

[Out] -2*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))

Rubi [A]
time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5816, 4267, 2611, 2320, 6724}

$$-2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)+2\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)-2\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]

[Out] -2*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 2*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] + 2*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] + 2*PolyLog[3, -E^ArcSinh[a*x]] - 2*PolyLog[3, E^ArcSinh[a*x]]

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
```

```
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + 2$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2$$

Mathematica [A]
time = 0.08, size = 100, normalized size = 1.47

$$\sinh^{-1}(ax)^2\log\left(1-e^{-\sinh^{-1}(ax)}\right) - \sinh^{-1}(ax)^2\log\left(1+e^{-\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right) + 2\text{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right) - 2\text{PolyLog}\left(3,e^{-\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] - ArcSinh[a*x]^2*Log[1 + E^(-ArcS
inh[a*x])] + 2*ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*ArcSinh[a*x]
*PolyLog[2, E^(-ArcSinh[a*x])] + 2*PolyLog[3, -E^(-ArcSinh[a*x])] - 2*PolyL
og[3, E^(-ArcSinh[a*x])]
```

Maple [A]
time = 2.60, size = 144, normalized size = 2.12

method	result
default	$\operatorname{arcsinh}(ax)^2 \ln\left(1-ax-\sqrt{a^2x^2+1}\right)+2\operatorname{arcsinh}(ax)\operatorname{polylog}\left(2, ax+\sqrt{a^2x^2+1}\right)-2\,\mathrm{polylo}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^
2*x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))-arcsinh(a*x)^2*ln(1+a*x+
(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*polyl
og(3,-a*x-(a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^2}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(x*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^2/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.288 $\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=66

$$-a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x}+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)+a\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)$$

```
[Out] -a*arcsinh(a*x)^2+2*a*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+a*polylo
g(2,(a*x+(a^2*x^2+1)^(1/2))^2)-arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5800, 5775, 3797, 2221, 2317, 2438}

$$-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{x}+a\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-a\sinh^{-1}(ax)^2+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -(a*ArcSinh[a*x]^2) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x + 2*a*ArcSinh[a*
x]*Log[1 - E^(2*ArcSinh[a*x])] + a*PolyLog[2, E^(2*ArcSinh[a*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x} + (2a)\int \frac{\sinh^{-1}(ax)}{x}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x} + (2a)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax)\right) \\
&= -a\sinh^{-1}(ax)^2 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x} - (4a)\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right. \\
&= -a\sinh^{-1}(ax)^2 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x} + 2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) - (2 \\
&= -a\sinh^{-1}(ax)^2 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x} + 2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) - a\text{S} \\
&= -a\sinh^{-1}(ax)^2 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x} + 2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) + a\text{I}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 65, normalized size = 0.98

$$
a\left(\sinh^{-1}(ax)\left(\sinh^{-1}(ax) - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{ax} + 2\log\left(1-e^{-2\sinh^{-1}(ax)}\right)\right) - \text{PolyLog}\left(2, e^{-2\sinh^{-1}(ax)}\right)\right)
$$

Antiderivative was successfully verified.

[In] Integrate[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]

[Out] a*(ArcSinh[a*x]*(ArcSinh[a*x] - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(a*x) + 2*
Log[1 - E^(-2*ArcSinh[a*x])]) - PolyLog[2, E^(-2*ArcSinh[a*x])])

Maple [A]
time = 4.24, size = 132, normalized size = 2.00

method	result
default	$\frac{\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)^2}{x}-2a\operatorname{arcsinh}(ax)^2+2a\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+2a$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] (a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^2-2*a*arcsinh(a*x)^2+2*a*arcsinh(a*x
)*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(2,a*x+(a^2*x^2+1)^(1/2))+2*a*arcs
inh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*a*polylog(2,-a*x-(a^2*x^2+1)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2
+ sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(sqrt(a^2*x^2 +
1)*a*x^2 + (a^2*x^2 + 1)*x), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^4 + x^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}^2(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asinh}(a\,x)^2}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^2/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.289 $\int \frac{\sinh^{-1}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=135

$$-\frac{a\sinh^{-1}(ax)}{x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2}+a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)+a^2$$

```
[Out] -a*arcsinh(a*x)/x+a^2*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-a^2*arc
tanh((a^2*x^2+1)^(1/2))+a^2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-
a^2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(a^2*x
^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsinh(a*x)^2*(a^2*x
^2+1)^(1/2)/x^2
```

Rubi [A]

time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5809, 5816, 4267, 2611, 2320, 6724, 5776, 272, 65, 214}

$$a^2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-a^2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-a^2\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)+a^2\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2x^2}-a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\frac{a\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((a*ArcSinh[a*x])/x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) + a^2*Ar
cSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^2*ArcTanh[Sqrt[1 + a^2*x^2]] + a^2
*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^2*ArcSinh[a*x]*PolyLog[2, E^A
rcSinh[a*x]] - a^2*PolyLog[3, -E^ArcSinh[a*x]] + a^2*PolyLog[3, E^ArcSinh[a
*x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
```

```
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2}+a\int \frac{\sinh^{-1}(ax)}{x^2}\,dx-\frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a\sinh^{-1}(ax)}{x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2}-\frac{1}{2}a^2\text{Subst}\left(\int x^2\text{csch}(x)\,dx,x,\sinh^{-1}(a \right. \\
&= -\frac{a\sinh^{-1}(ax)}{x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2}+a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2} \\
&= -\frac{a\sinh^{-1}(ax)}{x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2}+a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+a^2 \\
&= -\frac{a\sinh^{-1}(ax)}{x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2}+a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-a^2 \\
&= -\frac{a\sinh^{-1}(ax)}{x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2}+a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-a^2
\end{aligned}$$

Mathematica [A]
time = 0.95, size = 188, normalized size = 1.39

$$\frac{1}{8}a^2\left(-4\sinh^{-1}(ax)\coth\left(\frac{1}{2}\sinh^{-1}(ax)\right)-\sinh^{-1}(ax)^2\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(ax)\right)-4\sinh^{-1}(ax)^2\log\left(1-e^{-\sinh^{-1}(ax)}\right)+4\sinh^{-1}(ax)^2\log\left(1+e^{-\sinh^{-1}(ax)}\right)+8\log\left(\tanh\left(\frac{1}{2}\sinh^{-1}(ax)\right)\right)-8\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)+8\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)-8\text{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right)+8\text{PolyLog}\left(3,e^{-\sinh^{-1}(ax)}\right)-\sinh^{-1}(ax)^2\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(ax)\right)+4\sinh^{-1}(ax)\tanh\left(\frac{1}{2}\sinh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-4*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]^2*Csch[ArcSinh[a*
x]/2]^2 - 4*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]^2*Lo
g[1 + E^(-ArcSinh[a*x])] + 8*Log[Tanh[ArcSinh[a*x]/2]] - 8*ArcSinh[a*x]*Pol
yLog[2, -E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x])]
- 8*PolyLog[3, -E^(-ArcSinh[a*x])] + 8*PolyLog[3, E^(-ArcSinh[a*x])] - ArcS
inh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 + 4*ArcSinh[a*x]*Tanh[ArcSinh[a*x]/2]))/8
```

Maple [A]
time = 4.99, size = 233, normalized size = 1.73

method	result
default	$-\frac{\operatorname{arcsinh}(ax)\left(x^{2} \operatorname{arcsinh}(ax) a^{2}+2\sqrt{a^{2}x^{2}+1}\, ax+\operatorname{arcsinh}(ax)\right)}{2\sqrt{a^{2}x^{2}+1}\, x^{2}}-\frac{a^{2} \operatorname{arcsinh}(ax)^{2} \ln\left(1-ax-\sqrt{a^{2}x^{2}+1}\right)}{2}-a^{2} \operatorname{arcsi}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)*(x^2*arcsinh(a*x)*a^2+2*(a^2*x^2+1)
^(1/2)*a*x+arcsinh(a*x))-1/2*a^2*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))
-a^2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x
^2+1)^(1/2))+1/2*a^2*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+a^2*arcsinh
(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(a^2*x^2+1)^(1/2
))-2*a^2*arctanh(a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{2}(ax)}{x^{3}\sqrt{a^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^2}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(x^3*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^2/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

3.290 $\int \frac{x^5(a+b\sinh^{-1}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=383

$$-\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}}+\frac{298b^2(1+c^2x^2)}{225c^6\sqrt{d+c^2dx^2}}-\frac{76b^2(1+c^2x^2)^2}{675c^6\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)^3}{125c^6\sqrt{d+c^2dx^2}}-\frac{16b^2x\sqrt{1+c^2x^2}\sinh^{-1}(c}{15c^5\sqrt{d+c^2dx^2}}$$

```
[Out] 298/225*b^2*(c^2*x^2+1)/c^6/(c^2*d*x^2+d)^(1/2)-76/675*b^2*(c^2*x^2+1)^2/c^
6/(c^2*d*x^2+d)^(1/2)+2/125*b^2*(c^2*x^2+1)^3/c^6/(c^2*d*x^2+d)^(1/2)-16/15
*a*b*x*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)-16/15*b^2*x*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)+8/45*b*x^3*(a+b*arcsinh(c*x))*(c^2
*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-2/25*b*x^5*(a+b*arcsinh(c*x))*(c^2*x^
2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+8/15*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1
/2)/c^6/d-4/15*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/5*x^4*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.38, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5812, 5798, 5772, 267, 5776, 272, 45}

$$-\frac{2bx^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{c^2dx^2+d}}+\frac{x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d}+\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^6d}-\frac{16abx\sqrt{c^2x^2+1}}{15c^5\sqrt{c^2dx^2+d}}-\frac{4x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^4d}+\frac{8bx^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{45c^3\sqrt{c^2dx^2+d}}+\frac{2b^2(c^2x^2+1)^3}{125c^6\sqrt{c^2dx^2+d}}-\frac{76b^2(c^2x^2+1)^2}{675c^6\sqrt{c^2dx^2+d}}+\frac{298b^2(c^2x^2+1)}{225c^6\sqrt{c^2dx^2+d}}-\frac{16b^2x\sqrt{c^2x^2+1}\sinh^{-1}(cx)}{15c^5\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-16*a*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (298*b^2*(1 +
c^2*x^2))/(225*c^6*Sqrt[d + c^2*d*x^2]) - (76*b^2*(1 + c^2*x^2)^2)/(675*c^6
*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^3)/(125*c^6*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(15*c^5*Sqrt[d + c^2*d*x^2])
 + (8*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3*Sqrt[d + c^2*d*
x^2]) - (2*b*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c*Sqrt[d + c^2
*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^6*d) - (4*x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4*d) + (x^4*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(5*c^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d} - \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{5c^2} - \frac{\left(2b\sqrt{1+c^2x^2}\right.}{}$$

$$= -\frac{2bx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{d+c^2dx^2}} - \frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^4d} +$$

$$= \frac{8bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{d+c^2dx^2}} + \frac{8}{}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{8bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{25c^6\sqrt{d+c^2dx^2}} - \frac{4b^2(1+c^2x^2)^2}{75c^6\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2}{125c^6\sqrt{d+c^2}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{298b^2(1+c^2x^2)}{225c^6\sqrt{d+c^2dx^2}} - \frac{76b^2(1+c^2x^2)^2}{675c^6\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2}{125c^6\sqrt{d+}}$$

Mathematica [A]
time = 0.23, size = 230, normalized size = 0.60

$$\frac{-30abcx\sqrt{1+c^2x^2}\,(120-20c^2x^2+9c^4x^4)+225a^2(8+4c^2x^2-c^4x^4+3c^6x^6)+2b^2(2072+1936c^2x^2-109c^4x^4+27c^6x^6)+30b\left(bcx\sqrt{1+c^2x^2}\,(-120+20c^2x^2-9c^4x^4)+15a(8+4c^2x^2-c^4x^4+3c^6x^6)\right)\sinh^{-1}(cx)+225b^2(8+4c^2x^2-c^4x^4+3c^6x^6)\sinh^{-1}(cx)^2}{3375c^6\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-30*a*b*c*x*Sqrt[1 + c^2*x^2]*(120 - 20*c^2*x^2 + 9*c^4*x^4) + 225*a^2*(8
+ 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6) + 2*b^2*(2072 + 1936*c^2*x^2 - 109*c^4*x
^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4
*x^4) + 15*a*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6))*ArcSinh[c*x] + 225*b^2*
(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*ArcSinh[c*x]^2)/(3375*c^6*Sqrt[d + c^
2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. 2(335) = 670.
time = 3.99, size = 1227, normalized size = 3.20

method	result	size
default	Expression too large to display	1227

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)
^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(1
6*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*
c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x
)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)
^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2-6*arc
sinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2
+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2+2*ar
csinh(c*x)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c
^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x
)^2+6*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*(16*x^
6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+
13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)
/c^6/d/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*
x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(
c^2*x^2+1)^(1/2)*c*x+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+
1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^
(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)/c^6/d/(c^2*x^2+1)+
5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x
))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(
1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(1+3*arcsinh(c*x))/c^6/d/
(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^
5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2
)*c*x+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

Maxima [A]
time = 0.29, size = 353, normalized size = 0.92

$$\frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)b^2\operatorname{arsinh}(cx)^2+\frac{2}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)ab\operatorname{arsinh}(cx)+\frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)a^2+\frac{2}{3375}b^2\left(\frac{27\sqrt{c^2x^2+1}\,c^2x^4-136\sqrt{c^2x^2+1}\,x^2+\frac{2072\sqrt{c^2x^2+1}}{c^2}}{c^4\sqrt{d}}-\frac{15\left(9c^4x^5-20c^2x^3+120x\right)\operatorname{arsinh}(cx)}{c^5\sqrt{d}}\right)-\frac{2\left(9c^4x^5-20c^2x^3+120x\right)ab}{225c^5\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d)
 + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*b^2*arcsinh(c*x)^2 + 2/15*(3*sqrt(c^2*d*x
^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2
+ d)/(c^6*d))*a*b*arcsinh(c*x) + 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) -
```

```
4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*a^2 + 2/
3375*b^2*((27*sqrt(c^2*x^2 + 1)*c^2*x^4 - 136*sqrt(c^2*x^2 + 1)*x^2 + 2072*
sqrt(c^2*x^2 + 1)/c^2)/(c^4*sqrt(d)) - 15*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*
arcsinh(c*x)/(c^5*sqrt(d))) - 2/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*a*b/(c
^5*sqrt(d))
```

Fricas [A]
time = 0.38, size = 319, normalized size = 0.83

$$\frac{225\,(3\,b^2c^6x^6 - b^2c^4x^4 + 4\,b^2c^2x^2 + 8\,b^2)\sqrt{c^2dx^2 + d}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + 30\left(45\,abc^6x^6 - 15\,abc^4x^4 + 60\,abc^2x^2 + 120\,ab - (9\,b^2c^5x^5 - 20\,b^2c^3x^3 + 120\,b^2cx)\sqrt{c^2x^2 + 1}\right)\sqrt{c^2dx^2 + d}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \left(27\,(25\,a^2 + 2\,b^2)c^6x^6 - (225\,a^2 + 218\,b^2)c^4x^4 + 4\,(225\,a^2 + 968\,b^2)c^2x^2 + 1800\,a^2 + 4144\,b^2 - 30\,(9\,abc^5x^5 - 20\,abc^3x^3 + 120\,abcx)\sqrt{c^2x^2 + 1}\right)\sqrt{c^2dx^2 + d}}{3375\,(c^8dx^2 + c^6d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 - b^2*c^4*x^4 + 4*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 - 15*a*b*c^4
*x^4 + 60*a*b*c^2*x^2 + 120*a*b - (9*b^2*c^5*x^5 - 20*b^2*c^3*x^3 + 120*b^2
*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
 (27*(25*a^2 + 2*b^2)*c^6*x^6 - (225*a^2 + 218*b^2)*c^4*x^4 + 4*(225*a^2 +
968*b^2)*c^2*x^2 + 1800*a^2 + 4144*b^2 - 30*(9*a*b*c^5*x^5 - 20*a*b*c^3*x^3
 + 120*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b\,\mathrm{asinh}\,(cx))^2}{\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(a + b \operatorname{asinh}(c\, x)\right)^2}{\sqrt{d\, c^2\, x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.291 $\int \frac{x^4 (a+b \sinh^{-1}(cx))^2}{\sqrt{d+c^2 d x^2}} \, dx$

Optimal. Leaf size=323

$$-\frac{15b^2x(1+c^2x^2)}{64c^4\sqrt{d+c^2dx^2}}+\frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}}+\frac{15b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{64c^5\sqrt{d+c^2dx^2}}+\frac{3bx^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{8c^3\sqrt{d+c^2dx^2}}-\frac{bx}{}$$

```
[Out] -15/64*b^2*x*(c^2*x^2+1)/c^4/(c^2*d*x^2+d)^(1/2)+1/32*b^2*x^3*(c^2*x^2+1)/c
^2/(c^2*d*x^2+d)^(1/2)+15/64*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*
x^2+d)^(1/2)+3/8*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+
d)^(1/2)-1/8*b*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/
2)+1/8*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/(c^2*d*x^2+d)^(1/2)-3/8
*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/4*x^3*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.30, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5812, 5783, 5776, 327, 221}

$$-\frac{bx^4\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{8c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))^2}{4c^2d}+\frac{\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))^3}{8bc^5\sqrt{c^2dx^2+d}}-\frac{3x\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))^2}{8c^4d}+\frac{3bx^2\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{8c^3\sqrt{c^2dx^2+d}}+\frac{b^2x^3(c^2x^2+1)}{32c^2\sqrt{c^2dx^2+d}}+\frac{15b^2\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{64c^5\sqrt{c^2dx^2+d}}-\frac{15b^2x(c^2x^2+1)}{64c^4\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-15*b^2*x*(1 + c^2*x^2))/(64*c^4*Sqrt[d + c^2*d*x^2]) + (b^2*x^3*(1 + c^2*
x^2))/(32*c^2*Sqrt[d + c^2*d*x^2]) + (15*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(64*c^5*Sqrt[d + c^2*d*x^2]) + (3*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x]))/(8*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(8*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
c^2*d) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(8*b*c^5*Sqrt[d + c^2*d
*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right.}{}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{8c^4d} +$$

$$= \frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}\left(a\right.}{8c\sqrt{d+}}$$

$$= -\frac{15b^2x(1+c^2x^2)}{64c^4\sqrt{d+c^2dx^2}} + \frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{8c^3\sqrt{d+c^2dx^2}}$$

$$= -\frac{15b^2x(1+c^2x^2)}{64c^4\sqrt{d+c^2dx^2}} + \frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}} + \frac{15b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{64c^5\sqrt{d+c^2dx^2}} + \frac{3bx}{}$$

Mathematica [A]
time = 0.55, size = 268, normalized size = 0.83

$$\frac{32a^2c\sqrt{d}\,x(1+c^2x^2)\left(-3+2c^2x^2\right)+96a^2\sqrt{d+c^2dx^2}\,\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)+b^2\sqrt{d}\,\sqrt{1+c^2x^2}\left(32\sinh^{-1}(cx)^3-4\sinh^{-1}(cx)\left(-16\cosh\left(2\sinh^{-1}(cx)\right)+\cosh\left(4\sinh^{-1}(cx)\right)\right)-32\sinh\left(2\sinh^{-1}(cx)\right)+\sinh\left(4\sinh^{-1}(cx)\right)+8\sinh^{-1}(cx)^2\left(-8\sinh\left(2\sinh^{-1}(cx)\right)+\sinh\left(4\sinh^{-1}(cx)\right)\right)\right)+4ab\sqrt{d}\,\sqrt{1+c^2x^2}\left(16\cosh\left(2\sinh^{-1}(cx)\right)-\cosh\left(4\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\left(6\sinh^{-1}(cx)-8\sinh\left(2\sinh^{-1}(cx)\right)+\sinh\left(4\sinh^{-1}(cx)\right)\right)\right)}{256c^5\sqrt{d}\,\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (32*a^2*c*Sqrt[d]*x*(1 + c^2*x^2)*(-3 + 2*c^2*x^2) + 96*a^2*Sqrt[d + c^2*d*
x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*Sqrt[1 + c^2*x^
2]*(32*ArcSinh[c*x]^3 - 4*ArcSinh[c*x]*(-16*Cosh[2*ArcSinh[c*x]] + Cosh[4*A
rcSinh[c*x]]) - 32*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]] + 8*ArcSinh[
c*x]^2*(-8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + 4*a*b*Sqrt[d]*Sq
rt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh
[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(2
56*c^5*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 991 vs. 2(283) = 566.
time = 4.65, size = 992, normalized size = 3.07

method	result
default	$\frac{a^2x^3\sqrt{c^2d\,x^2+d}}{4c^2d}-\frac{3a^2x\sqrt{c^2d\,x^2+d}}{8c^4d}+\frac{3a^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^4\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d\,(c^2x^2+1)}\,\text{arcsinh}(c}{8\sqrt{c^2x^2+1}\,c^5d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3
/8*a^2/c^4*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*
(1/8*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^3+1/512*(d*
(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*
x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(
c*x)+1)/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(
c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+
1)/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x
^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)/c^
5/d/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2+1)^(1/2)*
x^4*c^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*
arcsinh(c*x)^2+4*arcsinh(c*x)+1)/c^5/d/(c^2*x^2+1))+2*a*b*(3/16*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^2+1/256*(d*(c^2*x^2+1))^(1/
2)*(8*x^5*c^5+8*(c^2*x^2+1)^(1/2)*x^4*c^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^
(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+1)-1/16*(
```

```
d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+
1)^(1/2))*(-1+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*
(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsin
h(c*x))/c^5/d/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*(c^2*x^2
+1)^(1/2)*x^4*c^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^
(1/2))*(1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/sqrt(c
^2*d*x^2 + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.292 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=265

$$\frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}-\frac{14b^2(1+c^2x^2)}{9c^4\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)^2}{27c^4\sqrt{d+c^2dx^2}}+\frac{4b^2x\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{3c^3\sqrt{d+c^2dx^2}}-\frac{2bx^3\sqrt{1+c^2x^2}\,(a+b}{9c\sqrt{d+c^2dx}}$$

```
[Out] -14/9*b^2*(c^2*x^2+1)/c^4/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(c^2*x^2+1)^2/c^4/(c
^2*d*x^2+d)^(1/2)+4/3*a*b*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)+4/3*b
^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*
arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/3*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1
/2)/c^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5812, 5798, 5772, 267, 5776, 272, 45}

$$\frac{x^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{2bx^3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d}+\frac{4abx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}+\frac{2b^2(c^2x^2+1)^2}{27c^4\sqrt{c^2dx^2+d}}-\frac{14b^2(c^2x^2+1)}{9c^4\sqrt{c^2dx^2+d}}+\frac{4b^2x\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{3c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (14*b^2*(1 + c^2*
x^2))/(9*c^4*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^4*Sqrt[d
+ c^2*d*x^2]) + (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^3*Sqrt[d + c^
2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c*Sqrt[d +
c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d) + (x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d} - \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{3c^2} - \frac{\left(2b\sqrt{1+c^2x \cdots\right.}{\cdots}$$

$$= -\frac{2bx^3\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d} + \cdots$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\,(a+\cdots}{3c^4d\cdots}$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} + \frac{4b^2x\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}\,(a+b\sinh\cdots}{9c\sqrt{d+c^2dx^2}\cdots}$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{14b^2(1+c^2x^2)}{9c^4\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^2}{27c^4\sqrt{d+c^2dx^2}} + \frac{4b^2x\sqrt{1+c^2x^2}\,\mathrm{s}\cdots}{3c^3\sqrt{d+c^2\cdots}}$$

Mathematica [A]
time = 0.17, size = 176, normalized size = 0.66

$$\frac{-6abcx(-6+c^2x^2)\sqrt{1+c^2x^2}+2b^2(-20-19c^2x^2+c^4x^4)+9a^2(-2-c^2x^2+c^4x^4)-6b\left(bcx(-6+c^2x^2)\sqrt{1+c^2x^2}+a(6+3c^2x^2-3c^4x^4)\right)\sinh^{-1}(cx)+9b^2(-2-c^2x^2+c^4x^4)\sinh^{-1}(cx)^2}{27c^4\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-6*a*b*c*x*(-6 + c^2*x^2)*Sqrt[1 + c^2*x^2] + 2*b^2*(-20 - 19*c^2*x^2 + c^
4*x^4) + 9*a^2*(-2 - c^2*x^2 + c^4*x^4) - 6*b*(b*c*x*(-6 + c^2*x^2)*Sqrt[1
+ c^2*x^2] + a*(6 + 3*c^2*x^2 - 3*c^4*x^4))*ArcSinh[c*x] + 9*b^2*(-2 - c^2*
x^2 + c^4*x^4)*ArcSinh[c*x]^2)/(27*c^4*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(231) = 462$.
time = 3.43, size = 706, normalized size = 2.66

method	result
default	$a^2\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d} - \frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right) + b^2\left(\frac{\sqrt{d\,(c^2x^2+1)}\,\left(4c^4x^4+4\sqrt{c^2x^2+1}\,x^3c^3+5c^2x^2+3\sqrt{c^2x\cdots}\right.}{216c^4d(c^2x^2+1)}\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b^2*(
1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^
2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/d/(c^2
*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsin
h(c*x)^2-2*arcsinh(c*x)+2)/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2
*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/d/(c^2*
x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5
*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4
/d/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^
(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*(-1+3*arcsinh(c*x))/c^4/
d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*(
arcsinh(c*x)-1)/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x
^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*
c*x+1)*(1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1))
```

Maxima [A]
time = 0.29, size = 243, normalized size = 0.92

$$\frac{1}{3}b^2\left(\frac{\sqrt{c^2dx^2+d}\,x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}(cx)^2+\frac{2}{3}ab\left(\frac{\sqrt{c^2dx^2+d}\,x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}(cx)+\frac{1}{3}a^2\left(\frac{\sqrt{c^2dx^2+d}\,x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)+\frac{2}{27}b^2\left(\frac{\sqrt{c^2x^2+1}\,x^2-\frac{20\sqrt{c^2x^2+1}}{c^2}}{c^2\sqrt{d}}-\frac{3(c^2x^3-6x)\operatorname{arsinh}(cx)}{c^3\sqrt{d}}\right)-\frac{2(c^2x^3-6x)ab}{9c^3\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b^2*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/(c^4*d))*a
rcsinh(c*x)^2 + 2/3*a*b*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2
 + d)/(c^4*d))*arcsinh(c*x) + 1/3*a^2*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*
sqrt(c^2*d*x^2 + d)/(c^4*d)) + 2/27*b^2*((sqrt(c^2*x^2 + 1)*x^2 - 20*sqrt(c
^2*x^2 + 1)/c^2)/(c^2*sqrt(d)) - 3*(c^2*x^3 - 6*x)*arcsinh(c*x)/(c^3*sqrt(d
))) - 2/9*(c^2*x^3 - 6*x)*a*b/(c^3*sqrt(d))
```

Fricas [A]
time = 0.39, size = 254, normalized size = 0.96

$$\frac{9(b^2c^4x^4-b^2c^2x^2-2b^2)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3abc^4x^4-3abc^2x^2-6ab-(b^2c^3x^3-6b^2cx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left((9a^2+2b^2)c^4x^4-(9a^2+38b^2)c^2x^2-18a^2-40b^2-6(abc^3x^3-6abcx)\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{27(c^6dx^2+c^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 - 3*a*b*c^2*x^2 - 6*a*b - (b^2*c^3*x
^3 - 6*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - (9*a^2 + 38*b^2)*c^2*x^2 - 18*a^2 - 4
```

```
0*b^2 - 6*(a*b*c^3*x^3 - 6*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))
/(c^6*d*x^2 + c^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}(c\,x)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.293 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{b^2x(1+c^2x^2)}{4c^2\sqrt{d+c^2dx^2}}-\frac{b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{4c^3\sqrt{d+c^2dx^2}}-\frac{bx^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}}+\frac{x\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(\right.}{2c^2d}$$

```
[Out] 1/4*b^2*x*(c^2*x^2+1)/c^2/(c^2*d*x^2+d)^(1/2)-1/4*b^2*arcsinh(c*x)*(c^2*x^2
+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/6*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^
3/(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$,
Rules used = {5812, 5783, 5776, 327, 221}

$$\frac{x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{6bc^3\sqrt{c^2dx^2+d}}+\frac{b^2x(c^2x^2+1)}{4c^2\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{4c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x])/(4*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^2)/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b
*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx &= \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{2c^2}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int}{c\sqrt{}}\\ &= -\frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}+\\ &= \frac{b^2x(1+c^2x^2)}{4c^2\sqrt{d+c^2dx^2}}-\frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}}+\frac{x\sqrt{d+c^2dx^2}\,(a+b}{2c^2d}\\ &= \frac{b^2x(1+c^2x^2)}{4c^2\sqrt{d+c^2dx^2}}-\frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^3\sqrt{d+c^2dx^2}}-\frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\right.}{2c\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 198, normalized size = 0.97

$$\frac{12a^2cx(d+c^2dx^2)-12a^2\sqrt{d}\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-6abd\sqrt{1+c^2x^2}\left(\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)-b^2d\sqrt{1+c^2x^2}\left(4\sinh^{-1}(cx)^3+6\sinh^{-1}(cx)\cosh\left(2\sinh^{-1}(cx)\right)-3\left(1+2\sinh^{-1}(cx)^2\right)\sinh\left(2\sinh^{-1}(cx)\right)\right)}{24c^3d\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (12*a^2*c*x*(d + c^2*d*x^2) - 12*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x
+ Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6*a*b*d*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[
c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])) - b^2*d*Sqrt[
1 + c^2*x^2]*(4*ArcSinh[c*x]^3 + 6*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - 3*(1
 + 2*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(24*c^3*d*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(178) = 356$.
time = 4.06, size = 506, normalized size = 2.48

method	result
default	$\frac{a^2 x \sqrt{c^2 d\,x^2+d}}{2c^2 d} - \frac{a^2 \ln\left(\frac{x\,c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{2c^2\sqrt{c^2 d}} + b^2\left(-\frac{\sqrt{d\,(c^2x^2+1)}\ \operatorname{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\ c^3 d} + \frac{\sqrt{d\,(c^2x^2+1}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a^2/c^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c
^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(-1/6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/c^3/d*arcsinh(c*x)^3+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^
2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*
x)+1)/c^3/d/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^
2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)
/c^3/d/(c^2*x^2+1))+2*a*b*(-1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3
/d*arcsinh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+
1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1)+1/1
6*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x
^2+1)^(1/2))*(1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/sqrt(c
^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b \operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a + b\,\mathrm{asinh}(c\,x)\right)}^2}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.294 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^2\sqrt{d+c^2dx^2}}-\frac{2b^2x\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{c\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}$$

```
[Out] 2*b^2*(c^2*x^2+1)/c^2/(c^2*d*x^2+d)^(1/2)-2*a*b*x*(c^2*x^2+1)^(1/2)/c/(c^2*
d*x^2+d)^(1/2)-2*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)
+(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5798, 5772, 267}

$$-\frac{2abx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}+\frac{2b^2(c^2x^2+1)}{c^2\sqrt{c^2dx^2+d}}-\frac{2b^2x\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-2*a*b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)
)/(c^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*S
qrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(a+b\sinh^{-1}(cx)\right)\,dx}{c\sqrt{d+c^2dx^2}}$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int s}{c\sqrt{d+c^2d}}$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{2b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{c^2d}$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^2\sqrt{d+c^2dx^2}} - \frac{2b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c}}{}$$

Mathematica [**A**]
time = 0.15, size = 127, normalized size = 0.92

$$\frac{\sqrt{d+c^2dx^2}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}-2b\left(bcx-a\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)+b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2\right)}{c^2d\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + 2*b^2*Sqrt[1 + c
^2*x^2] - 2*b*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + b^2*Sqrt[1 + c^2
*x^2]*ArcSinh[c*x]^2))/(c^2*d*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(126) = 252.
time = 1.31, size = 296, normalized size = 2.14

method	result
default	$\frac{a^2\sqrt{c^2d\,x^2+d}}{c^2d}+b^2\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\left(c^2x^2+\sqrt{c^2x^2+1}\,cx+1\right)\left(\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+2\right)}{2c^2d(c^2x^2+1)}+\frac{\sqrt{d\left(c^2x\right.}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/c^2/d*(c^2*d*x^2+d)^(1/2)+b^2*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*
x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1)+1/2
*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)^2+2*
```

```
arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1))+2*a*b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^
2+(c^2*x^2+1)^(1/2)*c*x+1)*(arcsinh(c*x)-1)/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*(1+arcsinh(c*x))/c^2/d/(c^2*
x^2+1))
```

Maxima [A]
time = 0.28, size = 125, normalized size = 0.91

$$-2\,b^{2}\left(\frac{x\operatorname{arsinh}\left(cx\right)}{c\sqrt{d}}-\frac{\sqrt{c^{2}x^{2}+1}}{c^{2}\sqrt{d}}\right)-\frac{2\,abx}{c\sqrt{d}}+\frac{\sqrt{c^{2}dx^{2}+d}\,b^{2}\operatorname{arsinh}\left(cx\right)^{2}}{c^{2}d}+\frac{2\,\sqrt{c^{2}dx^{2}+d}\,ab\operatorname{arsinh}\left(cx\right)}{c^{2}d}+\frac{\sqrt{c^{2}dx^{2}+d}\,a^{2}}{c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*b^2*(x*arcsinh(c*x)/(c*sqrt(d)) - sqrt(c^2*x^2 + 1)/(c^2*sqrt(d))) - 2*a
*b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b^2*arcsinh(c*x)^2/(c^2*d) + 2*sqrt(
c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2 + d)*a^2/(c^2*d)
```

Fricas [A]
time = 0.40, size = 179, normalized size = 1.30

$$\frac{\left(b^{2}c^{2}x^{2}+b^{2}\right)\sqrt{c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}+2\,\left(abc^{2}x^{2}-\sqrt{c^{2}x^{2}+1}\,b^{2}cx+ab\right)\sqrt{c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(\left(a^{2}+2\,b^{2}\right)c^{2}x^{2}-2\,\sqrt{c^{2}x^{2}+1}\,abcx+a^{2}+2\,b^{2}\right)\sqrt{c^{2}dx^{2}+d}}{c^{4}dx^{2}+c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2
*(a*b*c^2*x^2 - sqrt(c^2*x^2 + 1)*b^2*c*x + a*b)*sqrt(c^2*d*x^2 + d)*log(c*
x + sqrt(c^2*x^2 + 1)) + ((a^2 + 2*b^2)*c^2*x^2 - 2*sqrt(c^2*x^2 + 1)*a*b*c
*x + a^2 + 2*b^2)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{\sqrt{d\left(c^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.295 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {5783}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\ &= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 62, normalized size = 1.32

$$\frac{\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)\left(3a^2+3ab\sinh^{-1}(cx)+b^2\sinh^{-1}(cx)^2\right)}{3c\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(3*a^2 + 3*a*b*ArcSinh[c*x] + b^2*ArcSinh[c
*x]^2))/(3*c*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(41) = 82$.
time = 0.57, size = 120, normalized size = 2.55

method	result	size
default	$\frac{a^{2} \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d \,x^{2}+d}\right)}{\sqrt{c^{2} d}}+\frac{b^{2} \sqrt{d \left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(c x \right)^{3}}{3 \sqrt{c^{2} x^{2}+1}\, c d}+\frac{a b \sqrt{d \left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(c x \right)^{2}}{\sqrt{c^{2} x^{2}+1}\, c d}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^3+a*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

Maxima [A]
time = 0.29, size = 47, normalized size = 1.00

$$\frac{b^2 \operatorname{arsinh}(cx)^3}{3\,c\sqrt{d}} + \frac{ab \operatorname{arsinh}(cx)^2}{c\sqrt{d}} + \frac{a^2 \operatorname{arsinh}(cx)}{c\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*arcsinh(c*x)^3/(c*sqrt(d)) + a*b*arcsinh(c*x)^2/(c*sqrt(d)) + a^2*a
rcsinh(c*x)/(c*sqrt(d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(c^2*d*x^2 + d
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2), x)
```

3.296 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=223

$$-\frac{2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}}-\frac{2b\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

```
[Out] -2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c
^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*
(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b^2*polylog(3,-c
*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-2*b^2*polylog(3
,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5816, 4267, 2611, 2320, 6724}

$$-\frac{2b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{\sqrt{c^2dx^2+d}}+\frac{2b^2\sqrt{c^2x^2+1}\,\mathrm{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}-\frac{2b^2\sqrt{c^2x^2+1}\,\mathrm{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[
d + c^2*d*x^2] - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^
ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b^2*Sqrt[1 + c^2
*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (2*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{\sqrt{d}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{\sqrt{d}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{\sqrt{d}}$$

Mathematica [A]
time = 0.57, size = 266, normalized size = 1.19

$$\frac{a^2\log(cx)}{\sqrt{d}}-\frac{a^2\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}}+\frac{2ab\sqrt{1+c^2x^2}\left(\sinh^{-1}(cx)\left(\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\log\left(1+e^{-\sinh^{-1}(cx)}\right)\right)+\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\left(\sinh^{-1}(cx)^2\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)^2\log\left(1+e^{-\sinh^{-1}(cx)}\right)+2\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-2\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+2\mathrm{PolyLog}\left(3,-e^{-\sinh^{-1}(cx)}\right)-2\mathrm{PolyLog}\left(3,e^{-\sinh^{-1}(cx)}\right)\right)}{\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d]
 + (2*a*b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log
[1 + E^(-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-
ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2] + (b^2*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]
^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] +
 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyLog[2,
E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3, E^(-Ar
cSinh[c*x])]))/Sqrt[d + c^2*d*x^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(250) = 500$.
time = 2.57, size = 564, normalized size = 2.53

method	result
default	$-\frac{a^2\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)}{\sqrt{d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\operatorname{arcsinh}(cx)^2\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}+\frac{2b^2\sqrt{d\left(c^2x^2\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x
^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,c*x+
(c^2*x^2+1)^(1/2))-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*
x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+2*a*b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2
*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1
/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+
(c^2*x^2+1)^(1/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -a^2*arcsinh(1/(c*abs(x)))/sqrt(d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/(sqrt(c^2*d*x^2 + d)*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c
^2*d*x^2 + d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^3 + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{x\sqrt{d(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{x\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(1/2)), x)
```

3.297 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{c\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{2bc\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{\sqrt{d+c^2dx^2}}$$

```
[Out] c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b*c*(a+b*arc
sinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d
)^(1/2)-b^2*c*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2
*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A]
time = 0.18, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5800, 5775, 3797, 2221, 2317, 2438}

$$-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{c\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}}+\frac{2bc\sqrt{c^2x^2+1}\,\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{b^2c\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2] - (Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(d*x) + (2*b*c*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2] - (b^2*c*
Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{a+b\sinh^{-1}(cx)}{x}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\coth(x)\right.}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}-\frac{\left(4bc\sqrt{\right.}}{}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{2bc\sqrt{1}}{}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{2bc\sqrt{1}}{}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{2bc\sqrt{1}}{}$$

Mathematica [A]
time = 0.30, size = 168, normalized size = 1.01

$$\frac{b^2\left(-1-c^2x^2+cx\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2-2b\sinh^{-1}(cx)\left(a+ac^2x^2-bcx\sqrt{1+c^2x^2}\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-a\left(a+ac^2x^2-2bcx\sqrt{1+c^2x^2}\log(cx)\right)-b^2cx\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{x\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (b^2*(-1 - c^2*x^2 + c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - 2*b*ArcSinh[c*
x]*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c*x])]) -
 a*(a + a*c^2*x^2 - 2*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x]) - b^2*c*x*Sqrt[1 +
c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(x*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(171) = 342$.
time = 2.13, size = 526, normalized size = 3.15

method	result
default	$-\frac{a^2\sqrt{c^2d\,x^2+d}}{dx}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^2x\,c^2}{(c^2x^2+1)d}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^2c}{\sqrt{c^2x^2+1}\,d}-\frac{b^2\sqrt{d\left(c^2x^2+\right.}}{(c^2x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*
x^2+1)*x/d*c^2-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/d
*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)/x/d+2*b^2*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c
+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1
-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))*c-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)/d*arcsinh(c*x)*c-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)*
x/d*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/x/d+2*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -((-1)^(2*c^2*d*x^2 + 2*d)*sqrt(d)*log(2*c^2*d + 2*d/x^2) - sqrt(d)*log(x^2
 + 1/c^2))*a*b*c/d + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2
*d*x^2 + d)*x^2), x) - 2*sqrt(c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(d*x) - sqrt(
c^2*d*x^2 + d)*a^2/(d*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^4 + d*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}\left(cx\right)\right)^{2}}{x^{2}\sqrt{d\left(c^{2}x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^2\,\sqrt{d\,c^2\,x^2+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(1/2)), x)
```

3.298 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=360

$$-\frac{bc\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))^2}{2dx^2}+\frac{c^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^2\tanh}{\sqrt{d+c^2dx^2}}$$

```
[Out] -b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x/(c^2*d*x^2+d)^(1/2)+c^2*(a+b*ar
csinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d
)^(1/2)-b^2*c^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^
(1/2)+b*c^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1
)^(1/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2
+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-b^2*c^2*polylog(3,-c*x-(c^
2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+b^2*c^2*polylog(3,c*x
+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsinh(
c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A]
time = 0.29, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5809, 5816, 4267, 2611, 2320, 6724, 5776, 272, 65, 214}

$$\frac{bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{x\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))^2}{2dx^2}+\frac{c^2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))^2}{\sqrt{c^2dx^2+d}}-\frac{b^2c^2\sqrt{c^2x^2+1}\,\text{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}+\frac{b^2c^2\sqrt{c^2x^2+1}\,\text{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}-\frac{b^2c^2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[d + c^2*d*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (
b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[d + c^2*d*x^2] +
 (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[d + c^2*d*x^2] - (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (b^2*c^2*Sqrt[1 + c^2*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b^2*c^2*Sqrt[1 + c^2*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
```

```
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{1}{2}c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx + \frac{\left(bc\sqrt{1+\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{\left(c^2\sqrt{\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{\left(c^2\sqrt{\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1}}{}$$

Mathematica [A]

time = 3.85, size = 455, normalized size = 1.26

$$\frac{-\frac{4a^2\sqrt{d+c^2dx^2}}{x^2}-4a^2c^2\sqrt{d}\log(x)+4a^2c^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{2abc^2d^2(1+c^2x^2)^{3/2}\left(-2\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+4\sinh^{-1}(cx)\log\left(1+e^{-\sinh^{-1}(cx)}\right)-4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+2\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{(d+c^2dx^2)^{3/2}}+\frac{b^2c^2d^2(1+c^2x^2)^{3/2}\left(-4\sinh^{-1}(cx)\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\sinh^{-1}(cx)^2\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)^2\log\left(1-e^{-\sinh^{-1}(cx)}\right)+4\sinh^{-1}(cx)^2\log\left(1+e^{-\sinh^{-1}(cx)}\right)+8\log\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)-8\sinh^{-1}(cx)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+8\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-8\text{PolyLog}\left(3,-e^{-\sinh^{-1}(cx)}\right)+8\text{PolyLog}\left(3,e^{-\sinh^{-1}(cx)}\right)-\sinh^{-1}(cx)^2\text{sech}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)+4\sinh^{-1}(cx)\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}{(d+c^2dx^2)^{3/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 - 4*a^2*c^2*Sqrt[d]*Log[x] + 4*a^2*c^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*d^2*(1 + c^2*x^2)^(
3/2)*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c
*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])]
- ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d
*x^2)^(3/2) + (b^2*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-4*ArcSinh[c*x]*Coth[ArcSin
h[c*x]/2] - ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]^2*Log[1
- E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[
Tanh[ArcSinh[c*x]/2]] - 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + 8*A
rcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 8*PolyLog[3, -E^(-ArcSinh[c*x])
] + 8*PolyLog[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2
 + 4*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(373) = 746$.
time = 3.91, size = 901, normalized size = 2.50

method	result
default	$-\frac{a^2\sqrt{c^2d\,x^2+d}}{2d\,x^2}+\frac{a^2c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}-\frac{b^2\operatorname{arcsinh}(cx)^2\sqrt{d\left(c^2x^2+1\right)}\,c^2}{2d(c^2x^2+1)}-\frac{b^2\operatorname{arcsinh}(cx)\sqrt{}}{dx\sqrt{c^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a^2*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c
^2*d*x^2+d)^(1/2))/x)-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x
^2+1)*c^2-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)/d/x/(c^2*x^2+1)^(1/2)*c-1/
2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/d/x^2/(c^2*x^2+1)-1/2*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/
2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(
2,c*x+(c^2*x^2+1)^(1/2))*c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*
polylog(3,c*x+(c^2*x^2+1)^(1/2))*c^2+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2+b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2)
)*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,-c*x-(c^2*x^2
```

```
+1)^(1/2))*c^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arctanh(c*x+
(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)
*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/d/x/(c^2*x^2+1)^(1/2)*c-a*b*arcsinh(c*x)*(d*
(c^2*x^2+1))^(1/2)/d/x^2/(c^2*x^2+1)-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2+a*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^
2+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(d) - sqrt(c^2*d*x^2 + d)/(d*x^2))*a^2 +
 integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*d*x^2 + d)*x^3) + 2
*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^5 + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{x^3 \sqrt{d (c^2x^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2}{x^3\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(1/2)), x)
```

3.299 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=299

$$-\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}}-\frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\,(a+}{3dx}$$

```
[Out] -1/3*b^2*c^2*(c^2*x^2+1)/x/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)/x^2/(c^2*d*x^2+d)^(1/2)-2/3*c^3*(a+b*arcsinh(c*x))^2*(c^2*
x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-4/3*b*c^3*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(
c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2/3*b^2*c^3*poly
log(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/
3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x^3+2/3*c^2*(a+b*arcsinh(c*x))
^2*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A]
time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5809, 5800, 5775, 3797, 2221, 2317, 2438, 5776, 270}

$$\frac{2c^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3dx}-\frac{bc\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}-\frac{2c^3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2dx^2+d}}-\frac{4bc^3\sqrt{c^2x^2+1}\,\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2dx^2+d}}-\frac{b^2c^2(c^2x^2+1)}{3x\sqrt{c^2dx^2+d}}+\frac{2b^2c^3\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -1/3*(b^2*c^2*(1 + c^2*x^2))/(x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/(3*x^2*Sqrt[d + c^2*d*x^2]) - (2*c^3*Sqrt[1 + c^2*
x^2]*(a + b*ArcSinh[c*x])^2)/(3*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^2)/(3*d*x) - (4*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[
1 - E^(-2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2]) + (2*b^2*c^3*Sqrt[1 + c^2
*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}-\frac{1}{3}\left(2c^2\right)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}\,dx+\frac{\left(2bc\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{2c^2\sqrt{c}}{}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{3dx^3}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}}+\frac{2c^3\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}}+\frac{2c^3\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}}+\frac{2c^3\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}}+\frac{2c^3\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{3\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.48, size = 278, normalized size = 0.93

$$\frac{-a^2+a^2c^2x^2-b^2c^2x^2+2a^2c^4x^4-b^2c^4x^4-abcx\sqrt{1+c^2x^2}+b^2\left(-1+c^2x^2+2c^4x^4-2c^3x^3\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2-b\sinh^{-1}(cx)\left(bcx\sqrt{1+c^2x^2}-2a(-1+c^2x^2+2c^4x^4)+4bc^3x^3\sqrt{1+c^2x^2}\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-4abc^3x^3\sqrt{1+c^2x^2}\log(cx)+2b^2c^3x^3\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{3x^3\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-a^2 + a^2*c^2*x^2 - b^2*c^2*x^2 + 2*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x*S
qrt[1 + c^2*x^2] + b^2*(-1 + c^2*x^2 + 2*c^4*x^4 - 2*c^3*x^3*Sqrt[1 + c^2*x
^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(b*c*x*Sqrt[1 + c^2*x^2] - 2*a*(-1 + c
```

```
^2*x^2 + 2*c^4*x^4) + 4*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c
*x])]) - 4*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[c*x] + 2*b^2*c^3*x^3*Sqrt[1 +
c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2145 vs. $2(281) = 562$.
time = 3.98, size = 2146, normalized size = 7.18

method	result	size
default	Expression too large to display	2146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/x^3/d*arcsinh(c*x)-4/
3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^
2-1)*c^3-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^3/d*arcsin
h(c*x)*(c^2*x^2+1)*c^6-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*
arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4
+2*c^2*x^2-1)*x/d*arcsinh(c*x)^2*c^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3+4/3*b^2*(d*(c^2*x^
2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^5/d*arcsinh(c*x)*c^8-2/3*b^2*(d*(c^2*
x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x/d*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)
/(3*c^4*x^4+2*c^2*x^2-1)/x/d*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2
*c^2*x^2-1)/x^3/d*arcsinh(c*x)^2-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)
/(3*c^4*x^4+2*c^2*x^2-1)/d*(c^2*x^2+1)^(1/2)*c^3+4/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*c^3-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^5/d*c^8-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(
3*c^4*x^4+2*c^2*x^2-1)*x^3/d*c^6-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*(c^2*x^2+1)
)^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^3+b^2*
(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^2/d*(c^2*x^2+1)^(1/2)*c^5-2
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^3/d*(c^2*x^2+1)*c^6-
2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^2/d*arcsinh(c*x)^2*(c
^2*x^2+1)^(1/2)*c^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x
/d*arcsinh(c*x)*(c^2*x^2+1)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*
c^2*x^2-1)/x^2/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-4/3*a*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^3/d*(c^2*x^2+1)*c^6+4*a*b*(d*(c^2*x^2+1))^(1
/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^3/d*arcsinh(c*x)*c^6+2/3*a*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x/d*(c^2*x^2+1)*c^4+2/3*a*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x/d*arcsinh(c*x)*c^4+2*b^2*(d*(c^2*x^2+1))^(1/
2)/(3*c^4*x^4+2*c^2*x^2-1)*x^3/d*arcsinh(c*x)^2*c^6-4*a*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^2/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5+4/3*a
```

```
*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*c^3-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/x/d*arcsin
h(c*x)*c^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/x^2/d*c*(c
^2*x^2+1)^(1/2)-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x/d*a
rcsinh(c*x)*c^4+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^3/d
*arcsinh(c*x)*c^6-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/x/d
*arcsinh(c*x)^2*c^2+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsi
nh(c*x)*c^3+4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^5/d*c^8
+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x^3/d*c^6-2/3*a*b*(d
*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)*x/d*c^4-a*b*(d*(c^2*x^2+1))^(1/
2)/(3*c^4*x^4+2*c^2*x^2-1)/d*(c^2*x^2+1)^(1/2)*c^3+a^2*(-1/3/d/x^3*(c^2*d*x
^2+d)^(1/2)+2/3*c^2/d/x*(c^2*d*x^2+d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(4*c^2*log(x)/sqrt(d) + 1/(sqrt(d)*x^2))*a*b*c + 2/3*a*b*(2*sqrt(c^2*d
*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))*arcsinh(c*x) + 1/3*a^2*(
2*sqrt(c^2*d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3)) + b^2*integr
ate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^6 + d*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{x^4\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2}{x^4\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(1/2)), x)
```

3.300 $\int \frac{x^5 \left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=515

$$\frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}}-\frac{32b^2(1+c^2x^2)}{9c^6d\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)^2}{27c^6d\sqrt{d+c^2dx^2}}+\frac{16b^2x\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{3c^5d\sqrt{d+c^2dx^2}}-\frac{2bx\sqrt{1+c^2x^2}\,(a}{c^5d\sqrt{d+}}$$

```
[Out] -32/9*b^2*(c^2*x^2+1)/c^6/d/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(c^2*x^2+1)^2/c^6/
d/(c^2*d*x^2+d)^(1/2)-x^4*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)+16
/3*a*b*x*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)+16/3*b^2*x*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)-2*b*x*(a+b*arcsinh(c*x))*(c^2
*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*arcsinh(c*x))*(c^2*x
^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+4*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^
2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog
(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d*x^2+d)^(1/2)+
2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d
*x^2+d)^(1/2)-8/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^6/d^2+4/3*x^2*
(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A]
time = 0.51, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5810, 5812, 5798, 5772, 267, 5776, 272, 45, 5789, 4265, 2317, 2438}

$$\frac{4b\sqrt{c^2x^2+1}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^6d\sqrt{c^2dx^2+d}}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^6d^2}+\frac{16abx\sqrt{c^2x^2+1}}{3c^5d\sqrt{c^2dx^2+d}}-\frac{2bx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c^5d\sqrt{c^2dx^2+d}}+\frac{4x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2}-\frac{2bx^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}}+\frac{2b^2(c^2x^2+1)^2}{27c^6d\sqrt{c^2dx^2+d}}-\frac{32b^2(c^2x^2+1)}{9c^6d\sqrt{c^2dx^2+d}}+\frac{16b^2x\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{3c^5d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d*Sqrt[d + c^2*d*x^2]) - (32*b^2*(1 + c
^2*x^2))/(9*c^6*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^6*d*
Sqrt[d + c^2*d*x^2]) + (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d*S
qrt[d + c^2*d*x^2]) - (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9
*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d +
c^2*d*x^2]) - (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^2) +
(4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2) + (4*b*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^6*d*Sqrt[d + c
^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(c^6*d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^A
rcSinh[c*x]])/(c^6*d*Sqrt[d + c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^4(a+b}{1}}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^3d\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4x^2\sqrt{d+c^2dx^2}}{}$$

$$= -\frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x}{}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}} - \frac{2}{}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} + \frac{8b^2(1+c^2x^2)}{3c^6d\sqrt{d+c^2dx^2}} - \frac{2b^2(1+c^2x^2)^2}{9c^6d\sqrt{d+c^2dx^2}} + \frac{16b^2x\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+}}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} - \frac{32b^2(1+c^2x^2)}{9c^6d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^2}{27c^6d\sqrt{d+c^2dx^2}} + \frac{16b^2x\sqrt{1+c^2x}}{3c^5d\sqrt{d+}}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} - \frac{32b^2(1+c^2x^2)}{9c^6d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^2}{27c^6d\sqrt{d+c^2dx^2}} + \frac{16b^2x\sqrt{1+c^2x}}{3c^5d\sqrt{d+}}$$

Mathematica [A]
time = 0.38, size = 427, normalized size = 0.83

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-72*a^2 - 94*b^2 - 36*a^2*c^2*x^2 - 92*b^2*c^2*x^2 + 9*a^2*c^4*x^4 + 2*b^2
*c^4*x^4 + 90*a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] -
 144*a*b*ArcSinh[c*x] - 72*a*b*c^2*x^2*ArcSinh[c*x] + 18*a*b*c^4*x^4*ArcSin
h[c*x] + 90*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 +
 c^2*x^2]*ArcSinh[c*x] - 72*b^2*ArcSinh[c*x]^2 - 36*b^2*c^2*x^2*ArcSinh[c*x
]^2 + 9*b^2*c^4*x^4*ArcSinh[c*x]^2 + 108*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[
ArcSinh[c*x]/2]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^Ar
cSinh[c*x]] + (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh
[c*x]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (54
*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(27*c^6*d*Sqrt[d +
c^2*d*x^2])
```

Maple [A]
time = 3.96, size = 934, normalized size = 1.81

method	result
default	$a^2\left(\frac{x^4}{3c^2 d\sqrt{c^2 d\,x^2+d}}-\frac{4\left(\frac{x^2}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2 d\,x^2+d}}\right)}{3c^2}\right)+\frac{2ib^2\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{dilog}\left(1-i\left(cx\right.\right.}{\sqrt{c^2x^2+1}\,c^6d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/3*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3/c^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1/
2)+2/d/c^4/(c^2*d*x^2+d)^(1/2)))-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^6/d^2*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2/9*b^2*(d*(c^2*
x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3+10/3*b^2*(d*(c^2*x
^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x+2/27*b^2*(d*(c^2*x^2+
1))^(1/2)/c^2/d^2/(c^2*x^2+1)*x^4-92/27*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(
c^2*x^2+1)*x^2+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*dilo
g(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-I)-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^6/d
^2/(c^2*x^2+1)*arcsinh(c*x)^2-94/27*b^2*(d*(c^2*x^2+1))^(1/2)/c^6/d^2/(c^2*
x^2+1)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^4
-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^2+2*I*a
*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)
+I)-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*dilog(1+I*(c*x+
(c^2*x^2+1)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2
*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))-16/3*a*b*(d*(c^2*x^2+1))^(1/2
)/c^6/d^2/(c^2*x^2+1)*arcsinh(c*x)+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c
^2*x^2+1)*arcsinh(c*x)*x^4-2/9*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1
)^(1/2)*x^3-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*
x^2+10/3*a*b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a^2*(x^4/(sqrt(c^2*d*x^2 + d)*c^2*d) - 4*x^2/(sqrt(c^2*d*x^2 + d)*c^4*d
) - 8/(sqrt(c^2*d*x^2 + d)*c^6*d)) + 1/3*(b^2*c^4*sqrt(d)*x^4 - 4*b^2*c^2*s
qrt(d)*x^2 - 8*b^2*sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))^
```

```
2/(c^8*d^2*x^2 + c^6*d^2) + integrate(2/3*((4*b^2*c^3*x^3 + (3*a*b*c^5 - b^
2*c^5)*x^5 + 8*b^2*c*x)*(c^2*x^2 + 1) + (3*b^2*c^4*x^4 + (3*a*b*c^6 - b^2*c
^6)*x^6 + 12*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1))/(c^10*d^(3/2)*x^5 + 2*c^8*d^(3/2)*x^3 + c^6*d^(3/2)*x + (c^9*d^(3/2)
*x^4 + 2*c^7*d^(3/2)*x^2 + c^5*d^(3/2))*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
[Out] integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+b\,\mathrm{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**5*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(a+b\,\mathrm{asinh}(c\,x)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.301 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=400

$$\frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}}-\frac{b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}}-\frac{bx^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\cdots$$

```
[Out] 1/4*b^2*x*(c^2*x^2+1)/c^4/d/(c^2*d*x^2+d)^(1/2)-x^3*(a+b*arcsinh(c*x))^2/c^
2/d/(c^2*d*x^2+d)^(1/2)-1/4*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d
*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x
^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2
)-1/2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/d/(c^2*d*x^2+d)^(1/2)-2*
b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^5/
d/(c^2*d*x^2+d)^(1/2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)
^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)+3/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(
1/2)/c^4/d^2
```

Rubi [A]
time = 0.42, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5810, 5812, 5783, 5776, 327, 221, 5797, 3799, 2221, 2317, 2438}

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{2bc^5d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{c^5d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\,\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d\sqrt{c^2dx^2+d}}+\frac{3x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}-\frac{bx^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c^5d\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{4c^5d\sqrt{c^2dx^2+d}}+\frac{b^2x(c^2x^2+1)}{4c^4d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^4*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x])/(4*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(2*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x
])^2)/(c^2*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
^2)/(c^5*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x])^2)/(2*c^4*d^2) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 +
 E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^3(a+b\cdots}{1\cdots}}{cd\sqrt{d+c^2dx^2}\cdots}$$

$$= \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\,(\cdots}{2\cdots}$$

$$= -\frac{b^2x(1+c^2x^2)}{2c^4d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx\cdots\right.}{c^2d\sqrt{d+c^2dx^2}\cdots}$$

$$= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{2c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\cdots\right.}{2c^3d\sqrt{d+c^2dx^2}}\cdots$$

$$= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\cdots\right.}{2c^3d\sqrt{d+c^2dx^2}}\cdots$$

$$= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\cdots\right.}{2c^3d\sqrt{d+c^2dx^2}}\cdots$$

$$= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\cdots\right.}{2c^3d\sqrt{d+c^2dx^2}}\cdots$$

Mathematica [A]
time = 1.33, size = 288, normalized size = 0.72

$$\frac{4a^2c\sqrt{d}\,x(3+c^2x^2) - 12a^2\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + b^2\sqrt{d}\left(8cx\sinh^{-1}(cx)^2 + 8\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right) + \sqrt{1+c^2x^2}\left(-4\sinh^{-1}(cx)^3 - 2\sinh^{-1}(cx)\left(\cosh\left(2\sinh^{-1}(cx)\right) + 8\log\left(1+e^{-2\sinh^{-1}(cx)}\right)\right) + 2\sinh^{-1}(cx)^2\left(-4+\sinh\left(2\sinh^{-1}(cx)\right)\right) + \sinh\left(2\sinh^{-1}(cx)\right)\right)\right) + 2ab\sqrt{d}\left(8cx\sinh^{-1}(cx) - \sqrt{1+c^2x^2}\left(6\sinh^{-1}(cx)^2 + \cosh\left(2\sinh^{-1}(cx)\right) + 4\log(1+c^2x^2) - 2\sinh^{-1}(cx)\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{8c^5d^{3/2}\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a^2*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a^2*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*(8*c*x*ArcSinh[c*x]^2 + 8*Sqrt[1
+ c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*(-4*ArcSinh
[c*x]^3 - 2*ArcSinh[c*x]*(Cosh[2*ArcSinh[c*x]] + 8*Log[1 + E^(-2*ArcSinh[c*
x])]) + 2*ArcSinh[c*x]^2*(-4 + Sinh[2*ArcSinh[c*x]]) + Sinh[2*ArcSinh[c*x]]
)) + 2*a*b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(6*ArcSinh[c*x]^
2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(378) = 756.
time = 4.65, size = 816, normalized size = 2.04

method	result
default	$\frac{a^{2} x^{3}}{2 c^{2} d \sqrt{c^{2} d \,x^{2}+d}}+\frac{3 a^{2} x}{2 c^{4} d \sqrt{c^{2} d \,x^{2}+d}}-\frac{3 a^{2} \ln \left(\frac{x \,c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d \,x^{2}+d}\right)}{2 c^{4} d \sqrt{c^{2} d}}-\frac{b^{2} \sqrt{d \left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}(cx)}{2 c^{3} d^{2} \sqrt{c^{2} x^{2}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3
/2*a^2/c^4/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/
2*b^2*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*polylog(2,-(c*x+(c^2*x^2+1)
^(1/2))^2)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*x^3+1/4*b^2*(d
*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*x-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2
/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+3/2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*
x^2+1)*arcsinh(c*x)^2*x+b^2*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)
*arcsinh(c*x)^2-1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x
)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-3/2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/c^5/d^2*arcsinh(c*x)^2+a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)
*arcsinh(c*x)*x^3-1/2*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x
^2+3*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*x+2*a*b*(d*
(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/4*a*b*(d*(c^2*x
^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/c^5/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*a^2*(x^3/(sqrt(c^2*d*x^2 + d)*c^2*d) + 3*x/(sqrt(c^2*d*x^2 + d)*c^4*d)
- 3*arcsinh(c*x)/(c^5*d^(3/2))) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2
+ 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2
*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.302 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=383

$$-\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}}-\frac{4b^2x\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}}+\frac{2bx\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}}-\frac{x^2(\cdots}{c\cdots}$$

```
[Out] 2*b^2*(c^2*x^2+1)/c^4/d/(c^2*d*x^2+d)^(1/2)-x^2*(a+b*arcsinh(c*x))^2/c^2/d/
(c^2*d*x^2+d)^(1/2)-4*a*b*x*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-4*b
^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+2*b*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-4*b*(a+b*arcsinh(c*
x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^4/d/(c^2*d*x^2+d)^(1/
2)+2*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^4/d/(c
^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^
(1/2)/c^4/d/(c^2*d*x^2+d)^(1/2)+2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/
c^4/d^2
```

Rubi [A]
time = 0.31, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5810, 5798, 5772, 267, 5812, 5789, 4265, 2317, 2438}

$$-\frac{4b\sqrt{c^2x^2+1}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d\sqrt{c^2dx^2+d}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2}-\frac{4abx\sqrt{c^2x^2+1}}{c^3d\sqrt{c^2dx^2+d}}+\frac{2bx\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}}+\frac{2b^2(c^2x^2+1)}{c^4d\sqrt{c^2dx^2+d}}-\frac{4b^2x\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{c^3d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a*b*x*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*
x^2))/(c^4*d*Sqrt[d + c^2*d*x^2]) - (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(c^3*d*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[
d + c^2*d*x^2]) + (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^4*d^2)
- (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^4*
d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*Poly
Log[2, I*E^ArcSinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
```

```
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx &= -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+\cdots}{\cdots}}{cd\sqrt{d+c^2dx\cdots}} \\
&= \frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\sqrt{d+c^2dx^2}\cdots}{\cdots} \\
&= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}-\frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}}+\frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}}-\cdots \\
&= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}-\frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}}-\frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}}+\frac{2bx\sqrt{\cdots}}{\cdots} \\
&= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}}-\frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}}+\frac{2bx\sqrt{\cdots}}{\cdots} \\
&= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}}-\frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}}+\frac{2bx\sqrt{\cdots}}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 318, normalized size = 0.83

$$\frac{2a^2+2b^2+a^2c^2x^2+2b^2c^2x^2-2abcx\sqrt{1+c^2x^2}+4ab\sinh^{-1}(cx)+2abc^2x^2\sinh^{-1}(cx)-2b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)+2b^2\sinh^{-1}(cx)^2+b^2c^2x^2\sinh^{-1}(cx)^2-4ab\sqrt{1+c^2x^2}\,\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+2ib^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-2ib^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)+2ib^2\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-2ib^2\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a^2 + 2*b^2 + a^2*c^2*x^2 + 2*b^2*c^2*x^2 - 2*a*b*c*x*Sqrt[1 + c^2*x^2]
+ 4*a*b*ArcSinh[c*x] + 2*a*b*c^2*x^2*ArcSinh[c*x] - 2*b^2*c*x*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + 2*b^2*ArcSinh[c*x]^2 + b^2*c^2*x^2*ArcSinh[c*x]^2 - 4*a
*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*A
rcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog
[2, (-I)/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSi
nh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

Maple [A]
time = 3.40, size = 702, normalized size = 1.83

method	result
default	$a^2\left(\frac{x^2}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2 d\,x^2+d}}\right)+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}(cx)^2x^2}{c^2d^2(c^2x^2+1)}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsi}}{c^3d^2\sqrt{c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2/d/c^4/(c^2*d*x^2+d)^(1/2))+b^2*(d*(c^2
*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^2-2*b^2*(d*(c^2*x^2+1))
^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x+2*b^2*(d*(c^2*x^2+1))^(1/2)
/c^2/d^2/(c^2*x^2+1)*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*ar
csinh(c*x)^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)+2*I*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2
+1)^(1/2)))-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*arcsinh
(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/c^4/d^2*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*a*b
*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-2*a*b*(d*(c^2*x
^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x+4*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^
2/(c^2*x^2+1)*arcsinh(c*x)+2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
c^4/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-I)-2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/c^4/d^2*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] -2*a*b*c*(x/(c^4*d^(3/2)) + arctan(c*x)/(c^5*d^(3/2))) + 2*a*b*(x^2/(sqrt(c
^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))*arcsinh(c*x) + a^2*(x
^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d)) + b^2*((c^2
*x^2 + 2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*x^2 + 1)*c^4*d^(3/2)) -
integrate(2*(c^4*x^4 + 3*c^2*x^2 + (c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1) + 2)
*log(c*x + sqrt(c^2*x^2 + 1))/((c^5*d^(3/2)*x^2 + c^3*d^(3/2))*(c^2*x^2 + 1
) + (c^6*d^(3/2)*x^3 + c^4*d^(3/2)*x)*sqrt(c^2*x^2 + 1)), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.303 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d+c^2dx^2}}+\frac{2b\sqrt{1+c^2x^2}\,(a}{}$$

```
[Out] -x*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))^2*(c^2
*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1
)^(1/2)/b/c^3/d/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+b^2*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5810, 5783, 5797, 3799, 2221, 2317, 2438}

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{c^2dx^2+d}}+\frac{b^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c^3d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) - (Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^3)/(3*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^3*d*Sqrt[d + c
^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^3*d
*Sqrt[d + c^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\sinh}{1+c^2x}}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{si}\right.}{c^3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,(a+}{3bc^3d\sqrt{d}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,(a+}{3bc^3d\sqrt{d}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,(a+}{3bc^3d\sqrt{d}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,(a+}{3bc^3d\sqrt{d}}$$

Mathematica [A]
time = 0.65, size = 215, normalized size = 0.92

$$\frac{-3a^2cdx-3abd\left(2cx\sinh^{-1}(cx)-\sqrt{1+c^2x^2}\left(\sinh^{-1}(cx)^2+\log\left(1+c^2x^2\right)\right)\right)+3a^2\sqrt{d}\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+b^2d\left(\sinh^{-1}(cx)\left(-3cx\sinh^{-1}(cx)+\sqrt{1+c^2x^2}\left(\sinh^{-1}(cx)\left(3+\sinh^{-1}(cx)\right)+6\log\left(1+e^{-2\sinh^{-1}(cx)}\right)\right)\right)-3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)\right)}{3c^3d^2\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-3*a^2*c*d*x - 3*a*b*d*(2*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(ArcSinh[c*
x]^2 + Log[1 + c^2*x^2])) + 3*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*d*(ArcSinh[c*x]*(-3*c*x*ArcSinh[c*x] + Sq
rt[1 + c^2*x^2]*(ArcSinh[c*x]*(3 + ArcSinh[c*x]) + 6*Log[1 + E^(-2*ArcSinh[
c*x])])) - 3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^3*d^
2*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(231) = 462$.
time = 4.11, size = 478, normalized size = 2.05

method	result
default	$-\frac{a^2 x}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{a^2\ln\left(\frac{x\,c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{c^2 d\sqrt{c^2 d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b^2\sqrt{d\left(c^2x^2+1\right.}}{c^2d^2(c^2x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] -a^2*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a^2/c^2/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)^3-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^2/d^2/(c^2*x^2+1)*x-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^3/d^2/(c^2*x^2+1)^(1/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2/(c^2*x^2+1)*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d) - arcsinh(c*x)/(c^3*d^(3/2))) + integrate(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.304 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}-\frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-i\right.}{c^2d\sqrt{d+c^2dx^2}}$$

```
[Out] -(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)+4*b*(a+b*arcsinh(c*x))*arct
an(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^2/d/(c^2*d*x^2+d)^(1/2)-2*I*b
^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d/(c^2*d*x^2
+d)^(1/2)+2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^
2/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5798, 5789, 4265, 2317, 2438}

$$\frac{4b\sqrt{c^2x^2+1}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(c^2*d*Sqrt[d + c^2*d*x^2])) + (4*b*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^2*d*Sqrt[d + c^2*d*x^2]
) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d*Sq
rt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{a+b\sinh^{-1}(cx)}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{sech}(x)\,dx,x,\sinh^{-1}\right.}{c^2d\sqrt{d+c^2dx^2}}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}-\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}-\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}-
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 217, normalized size = 1.15

$$-\frac{a^2+2ab\sinh^{-1}(cx)+b^2\sinh^{-1}(cx)^2-4ab\sqrt{1+c^2x^2}\,\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+2ib^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-2ib^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)+2ib^2\sqrt{1+c^2x^2}\,\operatorname{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-2ib^2\sqrt{1+c^2x^2}\,\operatorname{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a^2 + 2*a*b*ArcSinh[c*x] + b^2*ArcSinh[c*x]^2 - 4*a*b*Sqrt[1 + c^2*x^2]*
ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log
[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I
/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*
x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c^2*d*Sqrt
[d + c^2*d*x^2]))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(197) = 394$.
time = 1.15, size = 446, normalized size = 2.37

method	result
default	$-\frac{a^{2}}{c^{2} d \sqrt{c^{2} d\,x^{2}+d}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)}\,\operatorname{arcsinh}(c x)^{2}}{c^{2} d^{2}\left(c^{2} x^{2}+1\right)}-\frac{2 i b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)}\,\operatorname{arcsinh}(c x) \ln\left(1+i\left(c x+\sqrt{c^{2} x^{2}+\right.\right.}{\sqrt{c^{2} x^{2}+1}\, c^{2} d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/c^2/d/(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1
)*arcsinh(c*x)^2-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ar
csinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/c^2/d^2*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*b^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*dilog(1+I*(c*x+(c^2*x^2+1
)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*dilog(1-I
*(c*x+(c^2*x^2+1)^(1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*a
rcsinh(c*x)+2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+
(c^2*x^2+1)^(1/2)+I)-2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^
2*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2*x^2 +
 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*
x^2 + d)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2 + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a + b\operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.305 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}$$

```
[Out] x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*x^2+
1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+
1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)-b^2*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5787, 5797, 3799, 2221, 2317, 2438}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{cd\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{cd\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])^2)/(c*d*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2]) -
(b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d
*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\sinh^{-1}\right.}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Sub}}{c}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\right.}{c}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\right.}{c}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\right.}{c}$$

Mathematica [A]
time = 0.26, size = 152, normalized size = 0.85

$$\frac{-b^2\left(-cx+\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2+2b\sinh^{-1}(cx)\left(acx-b\sqrt{1+c^2x^2}\log\left(1+e^{-2\sinh^{-1}(cx)}\right)\right)+a\left(acx-b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)\right)+b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*c
*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqrt[
1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*Ar
cSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2])
```

Maple [A]
time = 1.60, size = 343, normalized size = 1.92

method	result
default	$\frac{a^2 x}{d\sqrt{c^2 d\,x^2+d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}(cx)^2 x}{d^2\left(c^2x^2+1\right)}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}(cx)^2}{c\,d^2\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\,\text{ar}}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x/d/(c^2*d*x^2+d)^(1/2)+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/d^2/(c
^2*x^2+1)*x+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c/d^2/(c^2*x^2+1)^(1/2
)-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*arcsinh(c*x)*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/c/d^2*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^
2*x^2+1)*x-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2), x) + 2*
a*b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a^2*x/(sqrt(c^2*d*x^2 + d)*d)
- a*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2), x)
```

3.306 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=412

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}}-\frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\mathrm{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}}-\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}}$$

```
[Out] (a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-4*b*(a+b*arcsinh(c*x))*arctan(c*
x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh
(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(
1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(
1/2)/d/(c^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^
(1/2)))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*poly
log(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*b^2*
polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2
*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/
2)
```

Rubi [A]

time = 0.33, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5811, 5816, 4267, 2611, 2320, 6724, 5789, 4265, 2317, 2438}

$$-\frac{4b\sqrt{c^2x^2+1}\,\mathrm{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{2b^2\sqrt{c^2x^2+1}\,\mathrm{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{2b^2\sqrt{c^2x^2+1}\,\mathrm{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(d*Sqrt[d + c^2*d*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcS
inh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*
x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d*Sqrt[d
 + c^2*d*x^2])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{1+c^2x^2}\,d}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}\,dx}{d\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\mathrm{Subs}}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{}}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2}{}$$

<u>Mathematica</u> [**A**]
time = 1.08, size = 568, normalized size = 1.38

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*d + a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*x] - a^2*Sqrt[d]*Sqrt[d + c^
2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 2*a*b*d*(ArcSinh[c*x] - 2*S
qrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c
*x]*Log[1 - E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(
-ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1
 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])]) + b^2*d*(ArcSinh[c*x]^2 + Sqrt[1
 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] + (2*I)*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1
+ E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-Arc
Sinh[c*x])] + (2*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (2*
I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*Sqrt[1 + c^2*x^2]*Arc
```

```
Sinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E
^(-ArcSinh[c*x])] - 2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])]))/(d^
2*Sqrt[d + c^2*d*x^2])
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x\,(c^2d\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a^2*(arcsinh(1/(c*abs(x)))/d^(3/2) - 1/(sqrt(c^2*d*x^2 + d)*d)) + integrat
e(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x) + 2*a*b*log(
c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(cx))^2}{x\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{x\,(d\,c^2\,x^2+d)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(3/2)), x)
```

3.307 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=305

$$-\frac{(a+b\sinh^{-1}(cx))^2}{dx\sqrt{d+c^2dx^2}}-\frac{2c^2x(a+b\sinh^{-1}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{2c\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{4bc\sqrt{1+c^2x^2}\,(a+b\sin}{d\sqrt{}}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))^2/
d/(c^2*d*x^2+d)^(1/2)-2*c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/d/(c^2*d*x
^2+d)^(1/2)-4*b*c*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^
2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+4*b*c*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b^2*c*p
olylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5809, 5787, 5797, 3799, 2221, 2317, 2438, 5799, 5569, 4267}

$$-\frac{2c^2x(a+b\sinh^{-1}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2c\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\sinh^{-1}(cx))^2}{dx\sqrt{c^2dx^2+d}}+\frac{4bc\sqrt{c^2x^2+1}\,\log\left(e^{2\sinh^{-1}(cx)}+1\right)(a+b\sinh^{-1}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{4bc\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx))}{d\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSi
nh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (2*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (4*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2*d*x^2]) + (4*b*c*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c
^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*S
qrt[d + c^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x]
)])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a}{}}{d\sqrt{d+c^2d}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a\right.}{d}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\left(4bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a\right.}{d\sqrt{d}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\right.}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\right.}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\right.}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\right.}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.64, size = 296, normalized size = 0.97

$$-\frac{a^2+2a^2c^2x^2+2ab\sinh^{-1}(cx)+4abc^2x^2\sinh^{-1}(cx)+b^2\sinh^{-1}(cx)^2+2b^2c^2x^2\sinh^{-1}(cx)^2-2b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2-2b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1-e^{-2\sinh^{-1}(cx)}\right)-2b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1+e^{-2\sinh^{-1}(cx)}\right)-2abcx\sqrt{1+c^2x^2}\log(cx)-abcx\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)+b^2cx\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+b^2cx\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{dx\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a^2 + 2*a^2*c^2*x^2 + 2*a*b*ArcSinh[c*x] + 4*a*b*c^2*x^2*ArcSinh[c*x] +
b^2*ArcSinh[c*x]^2 + 2*b^2*c^2*x^2*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-
2*ArcSinh[c*x])] - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*A
rcSinh[c*x])] - 2*a*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x] - a*b*c*x*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSin
h[c*x])] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(d*x*
Sqrt[d + c^2*d*x^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(317) = 634$.
time = 2.08, size = 659, normalized size = 2.16

method	result
default	$a^{2} \left(-\frac{1}{d x \sqrt{c^{2} d \,x^{2}+d}}-\frac{2 c^{2} x}{d \sqrt{c^{2} d \,x^{2}+d}}\right)-\frac{2 b^{2} \sqrt{d \left(c^{2} x^{2}+1\right)}\, \mathrm{arcsinh}\left(c x \right)^{2} x \,c^{2}}{\left(c^{2} x^{2}+1\right) d^{2}}-\frac{2 b^{2} \sqrt{d \left(c^{2} x^{2}+1\right)}\, \mathrm{a}}{\sqrt{c^{2} x^{2}+1}\,}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/2))-2*b^2*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)*x/d^2*c^2-2*b^2*(d*(c^2*x^2+1))^
(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/d^2*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcs
inh(c*x)^2/(c^2*x^2+1)/x/d^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c+b^2*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c+2*b^2*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1
/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh
(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c-4*a*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c-4*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x
)/(c^2*x^2+1)*x/d^2*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1
)/x/d^2+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2+
1)^(1/2))^4-1)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] a*b*c*(log(c^2*x^2 + 1)/d^(3/2) + 2*log(x)/d^(3/2)) - 2*(2*c^2*x/(sqrt(c^2*
d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a*b*arcsinh(c*x) - (2*c^2*x/(s
qrt(c^2*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a^2 + b^2*integrate(lo
g(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}\left(cx\right)\right)^{2}}{x^{2}\left(d\left(c^{2}x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(3/2)), x)
```

3.308 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=573

$$-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}}+\frac{4bc^2\sqrt{1+c^2x^2}\left(a+b\sin\right.}{d\sqrt{d}}$$

```
[Out] -3/2*c^2*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsinh(c*x))^
2/d/x^2/(c^2*d*x^2+d)^(1/2)-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/d/x/(c
^2*d*x^2+d)^(1/2)+4*b*c^2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*
(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arcsinh(c*x))^2*arctanh(
c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b^2*c^2*arct
anh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*b*c^2*(a+b
*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d
*x^2+d)^(1/2)-2*I*b^2*c^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)
^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*I*b^2*c^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)
))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-3*b*c^2*(a+b*arcsinh(c*x))*polyl
og(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-3*b^2*c
^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2
)+3*b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2
+d)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5809, 5811, 5816, 4267, 2611, 2320, 6724, 5789, 4265, 2317, 2438, 272, 65, 214}

$$\frac{4bc^2\sqrt{c^2x^2+1}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{3bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{c^2dx^2+d}}+\frac{3c^2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}-\frac{2ib^2c^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{2ib^2c^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{3b^2c^2\sqrt{c^2x^2+1}\,\text{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{3b^2c^2\sqrt{c^2x^2+1}\,\text{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{b^2c^2\sqrt{c^2x^2+1}\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x*Sqrt[d + c^2*d*x^2])) -
 (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[
c*x])^2/(2*d*x^2*Sqrt[d + c^2*d*x^2]) + (4*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*c^2*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^
2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d*Sqrt[
d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2
, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*c
^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

```
- (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]
])/(d*Sqrt[d + c^2*d*x^2]) - (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^Arc
Sinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[
3, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
```

```
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int}{d\sqrt{d+c^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx\right.}{2dx^2\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx\right.}{2dx^2\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx\right.}{2dx^2\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx\right.}{2dx^2\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx\right.}{2dx^2\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx\right.}{2dx^2\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx\right.}{2dx^2\sqrt{d+c^2dx}}$$

Mathematica [A]

time = 7.00, size = 884, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/2*a^2/(d^2*x^2) - (a^2*c^2)/(d^2*(1 + c^2*x^2)))
- (3*a^2*c^2*Log[x])/(2*d^(3/2)) + (3*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c
^2*x^2)]])/(2*d^(3/2)) + (a*b*c^2*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*A
rcTan[Tanh[ArcSinh[c*x]/2]] - 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sq
rt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
*Log[1 + E^(-ArcSinh[c*x])] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[
c*x])] + 12*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh
[c*x]/2]))/(4*d*Sqrt[d*(1 + c^2*x^2)]) + (b^2*c^2*(-8*ArcSinh[c*x]^2 - 4*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[
1 - E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c
*x]] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*S
qrt[1 + c^2*x^2]*Log[Tanh[ArcSinh[c*x]/2]] - 24*Sqrt[1 + c^2*x^2]*ArcSinh[c
*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-
I)/E^ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]]
+ 24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 24*Sqrt
[1 + c^2*x^2]*PolyLog[3, -E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^2]*PolyLog
[3, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Sech[ArcSinh[c*x]
/2]^2 + 4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(8*d*Sqrt[d
*(1 + c^2*x^2)])
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{x^3 (c^2 d x^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/d^(3/2) - 3*c^2/(sqrt(c^2*d*x^2 + d)*d) -
1/(sqrt(c^2*d*x^2 + d)*d*x^2))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/((c^2*d*x^2 + d)^(3/2)*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c
^2*d*x^2 + d)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}\left(cx\right)\right)^{2}}{x^{3} \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^3\,{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(3/2)), x)
```

3.309 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=452

$$-\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x(a}{3d}$$

```
[Out] -1/3*b^2*c^2*(c^2*x^2+1)/d/x/(c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsinh(c*x))^2/d
/x^3/(c^2*d*x^2+d)^(1/2)+4/3*c^2*(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(1/
2)+8/3*c^4*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsin
h(c*x))*(c^2*x^2+1)^(1/2)/d/x^2/(c^2*d*x^2+d)^(1/2)+8/3*c^3*(a+b*arcsinh(c*
x))^2*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+20/3*b*c^3*(a+b*arcsinh(c*x))
*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
-16/3*b*c^3*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^
(1/2)/d/(c^2*d*x^2+d)^(1/2)-b^2*c^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(
c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-5/3*b^2*c^3*polylog(2,(c*x+(c^2*x^2+
1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5809, 5787, 5797, 3799, 2221, 2317, 2438, 5799, 5569, 4267, 270}

$$\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3dx\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\sqrt{c^2dx^2+d}}+\frac{8c^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3d\sqrt{c^2dx^2+d}}-\frac{16bc^3\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{c^2dx^2+d}}+\frac{20bc^3\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{c^2dx^2+d}}-\frac{b^2c^2(c^2x^2+1)}{3dx\sqrt{c^2dx^2+d}}-\frac{b^2c^3\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{5b^2c^3\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{3d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/3*(b^2*c^2*(1 + c^2*x^2))/(d*x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*
x^2]*(a + b*ArcSinh[c*x]))/(3*d*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c
*x])^2/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x])^2)/(3*d*
x*Sqrt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c^2
*d*x^2]) + (8*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c
^2*d*x^2]) + (20*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*
ArcSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) - (16*b*c^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) -
(b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2
*d*x^2]) - (5*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*
d*Sqrt[d + c^2*d*x^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
```

```
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} - \frac{1}{3}\left(4c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)}{3d\sqrt{d+}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(c\right.}{3dx\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} +$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} +$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} +$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} +$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} +$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} +$$

Mathematica [A]
time = 0.59, size = 438, normalized size = 0.97

$$\frac{-a^2+4a^2c^2x^2-b^2c^2x^2+8a^2c^4x^4-b^2c^4x^4-abcx\sqrt{1+c^2x^2}-2ab\sinh^{-1}(cx)+8abc^2x^2\sinh^{-1}(cx)+16abc^4x^4\sinh^{-1}(cx)-b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)-b^2\sinh^{-1}(cx)^2+4b^2c^2x^2\sinh^{-1}(cx)^2+8b^2c^4x^4\sinh^{-1}(cx)^2-8b^2c^3x^3\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2-10b^2c^3x^3\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1-e^{-2\sinh^{-1}(cx)}\right)-6b^2c^3x^3\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1+e^{-2\sinh^{-1}(cx)}\right)-10abc^3x^3\sqrt{1+c^2x^2}\log(cx)-3abc^3x^3\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)+3b^2c^3x^3\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+5b^2c^3x^3\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{3dx^3\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-a^2 + 4*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x
*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh[c*x] + 8*a*b*c^2*x^2*ArcSinh[c*x] + 16*a
*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b^2*ArcS
inh[c*x]^2 + 4*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSinh[c*x]^2 -
8*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - 10*b^2*c^3*x^3*Sqrt[1 + c^
2*x^2]*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 6*b^2*c^3*x^3*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 10*a*b*c^3*x^3*Sqrt[1 +
 c^2*x^2]*Log[c*x] - 3*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + 3*b
^2*c^3*x^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 5*b^2*c^3*x
```

```
^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*d*x^3*Sqrt[d + c^2
*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2607 vs. $2(438) = 876$.

time = 3.95, size = 2608, normalized size = 5.77

method	result	size
default	Expression too large to display	2608

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] -64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2*x^2+1)
*c^8-32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^
2+1)*c^6+128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*ar
csinh(c*x)*c^6+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*
(c^2*x^2+1)*c^4+16*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*
arcsinh(c*x)*c^4+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*
x^2*(c^2*x^2+1)^(1/2)*c^5+8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-
1)/d^2*x*arcsinh(c*x)^2*c^4-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*
x^2-1)/d^2*x*arcsinh(c*x)*c^4-4*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*
x^2-1)/d^2/x*arcsinh(c*x)^2*c^2-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3+8/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^3-
8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)*c^3-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh
(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3+64/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^7*arcsinh(c*x)*c^10-32/3
*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2*x^2+1)*c^8+
32*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*arcsinh(c*x)*c
^8+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c
*x)^2*c^6-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^
2*x^2+1)*c^6+8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*ar
csinh(c*x)*c^6-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x
^5*arcsinh(c*x)*(c^2*x^2+1)*c^8-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7
*c^2*x^2-1)/d^2*x^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^5+1/3*b^2*(d*(c^2*x^
2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*
c+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)*c^3-8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^
2/x*arcsinh(c*x)*c^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/
d^2/x^2*c*(c^2*x^2+1)^(1/2)+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*
x^2-1)/d^2*x*arcsinh(c*x)*(c^2*x^2+1)*c^4-32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8
*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)*(c^2*x^2+1)*c^6-128/3*a*b*(d*(c^
```

```
2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)*c^5+32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c
^3+64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^7*c^10+32*a
*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8+8*a*b*(d*(c^2*
x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*c^6-8/3*a*b*(d*(c^2*x^2+1))^(
1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*c^4-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4
*x^4+7*c^2*x^2-1)/d^2*(c^2*x^2+1)^(1/2)*c^3+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(
8*c^4*x^4+7*c^2*x^2-1)/d^2/x^3*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-10/3*a*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3+32/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^7*c^10+40/3*b^2*(d*(
c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8-7/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8
*c^4*x^4+7*c^2*x^2-1)/d^2/x*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*
c^2*x^2-1)/d^2/x^3*arcsinh(c*x)^2-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3-b^2*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-10/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
*c^3+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)^2*c^
3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*(c^2*x^2+1)^(1/
2)*c^3+a^2*(-1/3/d/x^3/(c^2*d*x^2+d)^(1/2)-4/3*c^2*(-1/d/x/(c^2*d*x^2+d)^(1
/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(8*c^4*x/(sqrt(c^2*d*x^2 + d)*d) + 4*c^2/(sqrt(c^2*d*x^2 + d)*d*x) - 1/
(sqrt(c^2*d*x^2 + d)*d*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1
))^2/((c^2*d*x^2 + d)^(3/2)*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2
*d*x^2 + d)^(3/2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{x^4 (d (c^2x^2 + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(3/2),x)

[Out] Integral((a + b*asinh(c*x))**2/(x**4*(d*(c**2*x**2 + 1))**(3/2)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^4), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{x^4\,(d\,c^2\,x^2 + d)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(3/2)),x)

[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(3/2)), x)

3.310 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=512

$$\frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}}-\frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^6d^2\sqrt{d+c^2dx^2}}-\frac{16b^2x\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{bx^3\left(a+b\sinh\right.}{3c^3d^2\sqrt{1+c^2x^2}}$$

```
[Out] -1/3*x^4*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2/c^6/d^2/(c^
2*d*x^2+d)^(1/2)+2*b^2*(c^2*x^2+1)/c^6/d^2/(c^2*d*x^2+d)^(1/2)-4/3*x^2*(a+b
*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*x^3*(a+b*arcsinh(c*x))/c
^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-16/3*a*b*x*(c^2*x^2+1)^(1/2)/c
^5/d^2/(c^2*d*x^2+d)^(1/2)-16/3*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d^
2/(c^2*d*x^2+d)^(1/2)+11/3*b*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/d^2
/(c^2*d*x^2+d)^(1/2)-22/3*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x^2+d)^(1/2)+11/3*I*b^2*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x^2+d)^(1/2)-11/3*I*
b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x
^2+d)^(1/2)+8/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^6/d^3
```

Rubi [A]
time = 0.60, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5810, 5798, 5772, 267, 5812, 5789, 4265, 2317, 2438, 272, 45}

$$-\frac{22b\sqrt{c^2x^2+1}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^6d^3}-\frac{16abx\sqrt{c^2x^2+1}}{3c^5d^2\sqrt{c^2dx^2+d}}+\frac{11bx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{4x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{11ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{11ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}+\frac{2b^2(c^2x^2+1)}{c^6d^2\sqrt{c^2dx^2+d}}+\frac{b^2}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{16b^2x\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{3c^5d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b^2/(3*c^6*d^2*Sqrt[d + c^2*d*x^2]) - (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2))/(c^6*d^2*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^
2]) - (b*x^3*(a + b*ArcSinh[c*x]))/(3*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2]) + (11*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^5*d^2*Sqrt
[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(3*c^2*d*(d + c^2*d*x^2)^(3
/2)) - (4*x^2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) + (8*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^3) - (22*b*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*c^6*d^2*Sqrt[d + c^2
*d*x^2]) + (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]
])/(c^6*d^2*Sqrt[d + c^2*d*x^2]) - (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c^6*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
```

```
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{(d+c^2dx^2)^{3/2}}\,dx}{3c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^4(a+b}{(1-}}{3cd^2\sqrt{d+c^2dx}}$$

$$= -\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{4x^2\left(a+b\sinh^{-1}(\right.}{3c^4d^2\sqrt{d+c^2dx}}$$

$$= -\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} + \frac{11bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4(a}{3c^2}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{11b^2(1+c^2x^2)}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} +$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{10b^2(1+c^2x^2)}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2}}{3c^5d^2\sqrt{d}}$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x}}{3c^5d^2\sqrt{d}}$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x}}{3c^5d^2\sqrt{d}}$$

Mathematica [**A**]
time = 1.16, size = 333, normalized size = 0.65

$$\frac{\sqrt{d+c^2dx^2}\left(a^2(8+12c^2x^2+3c^4x^4)+ab\left(2(8+12c^2x^2+3c^4x^4)\sinh^{-1}(cx)-\sqrt{1+c^2x^2}\left(cx(5+6c^2x^2)+22(1+c^2x^2)\operatorname{ArcTan}\left(\tanh\left(\tfrac{1}{2}\sinh^{-1}(cx)\right)\right)\right)\right)+b^2\left(cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)-6cx(1+c^2x^2)^{3/2}\sinh^{-1}(cx)-\sinh^{-1}(cx)^2+3(1+c^2x^2)^2\left(2+\sinh^{-1}(cx)^2\right)+(1+c^2x^2)\left(1+6\sinh^{-1}(cx)^2\right)+11i(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\left(\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-\log\left(1+ie^{-\sinh^{-1}(cx)}\right)\right)+11i(1+c^2x^2)^{3/2}\left(\operatorname{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-\operatorname{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\right)\right)\right)}{3c^6d^3(1+c^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a^2*(8 + 12*c^2*x^2 + 3*c^4*x^4) + a*b*(2*(8 + 12*c^2
*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(c*x*(5 + 6*c^2*x^2) + 2
2*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) + b^2*(c*x*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x] - 6*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] - ArcSinh[c*x]^2 + 3
*(1 + c^2*x^2)^2*(2 + ArcSinh[c*x]^2) + (1 + c^2*x^2)*(1 + 6*ArcSinh[c*x]^2
) + (11*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(Log[1 - I/E^ArcSinh[c*x]] - Lo
g[1 + I/E^ArcSinh[c*x]]) + (11*I)*(1 + c^2*x^2)^(3/2)*(PolyLog[2, (-I)/E^Ar
cSinh[c*x]] - PolyLog[2, I/E^ArcSinh[c*x]]))))/(3*c^6*d^3*(1 + c^2*x^2)^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(475) = 950.
time = 4.11, size = 1042, normalized size = 2.04

method	result
default	$a^2\left(\frac{x^4}{c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{c^2d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2}{3d\,c^4\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{11ib^2\sqrt{d\,(c^2x^2+1)}\,\operatorname{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{\right.\right.}}{3\sqrt{c^2x^2+1}\,c^6d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(x^4/c^2/d/(c^2*d*x^2+d)^(3/2)-4/c^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/
3/d/c^4/(c^2*d*x^2+d)^(3/2)))-11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^6/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*b^2*(d*(c^2*x^
2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)+b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+
1)*arcsinh(c*x)^2*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arc
sinh(c*x)^2*x^2+11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*
arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))-11/3*I*b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)/c^6/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*b^2*(d*(c
^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x
^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^5*arcsinh(c*x)*x-11/3*I*a*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)+2*b^2*(d*
(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*x^2+b^2*(d*(c^2*x^2+1))^(1/2)/c^6/d^
3/(c^2*x^2+1)*arcsinh(c*x)^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^
2/c^6+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*x^2+5/3*b^2*(d*(c
^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsinh(c*x)^2+2*a*b*(d*(c^2*x^2+1))
^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(c*x)*x^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/c^5
/d^3/(c^2*x^2+1)^(1/2)*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*ar
csinh(c*x)+4*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arcsinh(c*x)*x
^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^5*x+10/3*a*b*(d*(c
^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsinh(c*x)+11/3*I*b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+11/3
*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(
1/2)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a^2*(3*x^4/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 12*x^2/((c^2*d*x^2 + d)^(3/2
)*c^4*d) + 8/((c^2*d*x^2 + d)^(3/2)*c^6*d)) + 1/3*(3*b^2*c^4*sqrt(d)*x^4 +
```

```
12*b^2*c^2*sqrt(d)*x^2 + 8*b^2*sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^
2*x^2 + 1))^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + integrate(-2/3*((1
2*b^2*c^3*x^3 - 3*(a*b*c^5 - b^2*c^5)*x^5 + 8*b^2*c*x)*(c^2*x^2 + 1) + (15*
b^2*c^4*x^4 - 3*(a*b*c^6 - b^2*c^6)*x^6 + 20*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^12*d^(5/2)*x^7 + 3*c^10*d^(5/2)*x
^5 + 3*c^8*d^(5/2)*x^3 + c^6*d^(5/2)*x + (c^11*d^(5/2)*x^6 + 3*c^9*d^(5/2)*
x^4 + 3*c^7*d^(5/2)*x^2 + c^5*d^(5/2))*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b\,\mathrm{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(a+b\,\mathrm{asinh}(c\,x)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.311 $\int \frac{x^4 \left(a+b \sinh ^{-1}(c x)\right)^2}{\left(d+c^2 d x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=398

$$-\frac{b^2 x}{3 c^4 d^2 \sqrt{d+c^2 d x^2}}+\frac{b^2 \sqrt{1+c^2 x^2} \sinh ^{-1}(c x)}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}-\frac{b x^2 \left(a+b \sinh ^{-1}(c x)\right)}{3 c^3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b \sinh ^{-1}(c x)\right)^2}{3 c^2 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{x (a}{c^4}$$

```
[Out] -1/3*x^3*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2*x/c^4/d^2/(
c^2*d*x^2+d)^(1/2)-x*(a+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b
*x^2*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/3*b
^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-4/3*(a+b*arcs
inh(c*x))^2*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsinh(
c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/d^2/(c^2*d*x^2+d)^(1/2)+8/3*b*(a+b*arcsinh(
c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+
d)^(1/2)+4/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^
5/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5810, 5783, 5797, 3799, 2221, 2317, 2438, 294, 221}

$$-\frac{x^3 \left(a+b \sinh ^{-1}(c x)\right)^2}{3 c^2 d \left(c^2 d x^2+d\right)^{3/2}}+\frac{\sqrt{c^2 x^2+1} \left(a+b \sinh ^{-1}(c x)\right)^3}{3 b c^5 d^2 \sqrt{c^2 d x^2+d}}-\frac{4 \sqrt{c^2 x^2+1} \left(a+b \sinh ^{-1}(c x)\right)^2}{3 c^5 d^2 \sqrt{c^2 d x^2+d}}+\frac{8 b \sqrt{c^2 x^2+1} \log \left(e^{2 \sinh ^{-1}(c x)}+1\right) \left(a+b \sinh ^{-1}(c x)\right)}{3 c^5 d^2 \sqrt{c^2 d x^2+d}}-\frac{x \left(a+b \sinh ^{-1}(c x)\right)^2}{c^4 d^2 \sqrt{c^2 d x^2+d}}-\frac{b x^2 \left(a+b \sinh ^{-1}(c x)\right)}{3 c^3 d^2 \sqrt{c^2 x^2+1} \sqrt{c^2 d x^2+d}}+\frac{4 b^2 \sqrt{c^2 x^2+1} \text{Li}_2\left(-e^{2 \sinh ^{-1}(c x)}\right)}{3 c^5 d^2 \sqrt{c^2 d x^2+d}}+\frac{b^2 \sqrt{c^2 x^2+1} \sinh ^{-1}(c x)}{3 c^5 d^2 \sqrt{c^2 d x^2+d}}-\frac{b^2 x}{3 c^4 d^2 \sqrt{c^2 d x^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*(b^2*x)/(c^4*d^2*Sqrt[d + c^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (b*x^2*(a + b*ArcSinh[c*x]))/(3*c^
3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x])^2)
/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d^2*Sqrt
[d + c^2*d*x^2]) - (4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^5*d^2*
Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c^5*
d^2*Sqrt[d + c^2*d*x^2]) + (8*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[
1 + E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) + (4*b^2*Sqrt[1 +
c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx &= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{c^2d}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^3(a+b\,\mathrm{si}}{(1+c}}{3cd^2\sqrt{d+c^2dx^2}} \\
&= -\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx\right.}{c^4d^2\sqrt{d+c^2dx^2}} \\
&= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}}-\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} \\
&= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2d}} \\
&= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2d}} \\
&= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2d}} \\
&= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2d}}
\end{aligned}$$

Mathematica [A]
time = 0.83, size = 359, normalized size = 0.90

$$\frac{-a^2c\sqrt{d}\,x(3+4c^2x^2)+ab\sqrt{d}\left(\sqrt{1+c^2x^2}+2cx\sinh^{-1}(cx)-8cx(1+c^2x^2)\sinh^{-1}(cx)+(1+c^2x^2)^{3/2}\left(3\sinh^{-1}(cx)^2+4\log\left(1+c^2x^2\right)\right)\right)+3a^2(1+c^2x^2)\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-b^2\sqrt{d}\left(cx+c^3x^3-\sqrt{1+c^2x^2}\sinh^{-1}(cx)+3cx\sinh^{-1}(cx)^2+4c^3x^3\sinh^{-1}(cx)^2-4(1+c^2x^2)^{3/2}\sinh^{-1}(cx)^2-(1+c^2x^2)^{3/2}\sinh^{-1}(cx)^3-8(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\log\left(1+e^{-2\sinh^{-1}(cx)}\right)+4(1+c^2x^2)^{3/2}\mathrm{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)\right)}{3c^5d^{5/2}(1+c^2x^2)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*c*Sqrt[d]*x*(3 + 4*c^2*x^2)) + a*b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*
x*ArcSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*
ArcSinh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 3*a^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*
x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - b^2*Sqrt[d]*(c*x + c^3*x^3
- Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 3*c*x*ArcSinh[c*x]^2 + 4*c^3*x^3*ArcSinh
[c*x]^2 - 4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - (1 + c^2*x^2)^(3/2)*ArcSin
h[c*x]^3 - 8*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])]
+ 4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^5*d^(5/2)*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3704 vs. $2(370) = 740$.
time = 4.77, size = 3705, normalized size = 9.31

method	result	size
default	Expression too large to display	3705

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 64*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+
16)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6+168*a*b*(d*(c^2*x^2+1))^(1/2)/
(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*x^4+440/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*
c^4*x^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2+a*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)^2-16/3*a*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)-16/3*a*b*(d*(c^2*x^2+1
))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*x^7+16/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16
)/d^3*(c^2*x^2+1)*x^5-152*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+
118*c^4*x^4+71*c^2*x^2+16)/d^3*arcsinh(c*x)*x^5-40/3*a*b*(d*(c^2*x^2+1))^(1
/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*x^3-4*a*b*(d*
(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^
3*x+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^
2*x^2+16)/c^5/d^3*(c^2*x^2+1)^(1/2)+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/c^5/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-64*a*b*(d*(c^2*x^2+1))^(1/
2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*arcsinh(c*x)*x
^7+8*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^
2+16)/c/d^3*(c^2*x^2+1)^(1/2)*x^4+28/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^
8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*(c^2*x^2+1)*x^3-362/3*a*b*(
d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/
d^3*arcsinh(c*x)*x^3+13*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+11
8*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*(c^2*x^2+1)^(1/2)*x^2+4*a*b*(d*(c^2*x^2+1)
)^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*(c^2*x^2+
1)*x-32*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2
```

```
*x^2+16)/c^4/d^3*arcsinh(c*x)*x+128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8
+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^5/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)+220/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^
2*x^2+16)/c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2-1/3*a^2*x^3/c^2/d/(c
^2*d*x^2+d)^(3/2)-4*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^
4*x^4+71*c^2*x^2+16)/c^4/d^3*arcsinh(c*x)*x+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^5/d^3*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsi
nh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*
c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*arcsinh(c*x)*(c^2*x^2+1)*
x^5-32*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*
x^2+16)*c^2/d^3*arcsinh(c*x)^2*x^7-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x
^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*arcsinh(c*x)*x^7+8*b^2*(d*
(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c/d^3*
(c^2*x^2+1)^(1/2)*x^6+21*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+1
18*c^4*x^4+71*c^2*x^2+16)/c/d^3*(c^2*x^2+1)^(1/2)*x^4-181/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*arcsin
h(c*x)^2*x^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x
^4+71*c^2*x^2+16)/c^2/d^3*(c^2*x^2+1)*x^3-40/3*b^2*(d*(c^2*x^2+1))^(1/2)/(2
4*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*arcsinh(c*x)*x^3+55
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+
16)/c^3/d^3*(c^2*x^2+1)^(1/2)*x^2-16*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+
87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*arcsinh(c*x)^2*x-17*b^2*(d*(c
^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*x^5-
a^2/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a^2/c^4/d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2
*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+84*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*
c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x
^4+8*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^
2+16)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*(c^2*x^2+1)*x+6
4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2
+16)/c^5/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)+28/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*arcsinh(c*x)*(c
^2*x^2+1)*x^3+13*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x
^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2+32*b^2*(d*(c^2
*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c/d^3*arcs
inh(c*x)^2*(c^2*x^2+1)^(1/2)*x^6+4*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87
*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*arcsinh(c*x)*(c^2*x^2+1)*x-44/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16
)/d^3*x^5+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4
+71*c^2*x^2+16)/c^5/d^3*(c^2*x^2+1)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)^3-76*b^2*(d...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d)) + x/(sqrt(c^2*d*x^2 + d)*c^4*d^2) - 3*arcsinh(c*x)/(c^5*d^(5/2)))*a^2
+ integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2) +
2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.312 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}}-\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{10b\sqrt{1}}{}$$

```
[Out] -1/3*x^2*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2/c^4/d^2/(c^
2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b
*x*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+10/3*b*
(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^4/d^2/
(c^2*d*x^2+d)^(1/2)-5/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^
2+1)^(1/2)/c^4/d^2/(c^2*d*x^2+d)^(1/2)+5/3*I*b^2*polylog(2,I*(c*x+(c^2*x^2+
1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^4/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5810, 5798, 5789, 4265, 2317, 2438, 267}

$$\frac{10b\sqrt{c^2x^2+1}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{5ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{5ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{b^2}{3c^4d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*b^2/(c^4*d^2*Sqrt[d + c^2*d*x^2]) - (b*x*(a + b*ArcSinh[c*x]))/(3*c^3*
d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(
3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt
[d + c^2*d*x^2]) + (10*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^Ar
cSinh[c*x]])/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) - (((5*I)/3)*b^2*Sqrt[1 + c^2*
x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d + c^2*d*x^2]) + (((5*
I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d +
 c^2*d*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{(d+c^2dx^2)^{3/2}}\,dx}{3c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+}{(1}}{3cd^2\sqrt{d+c^2d}}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c\right.}{3c^4d^2\sqrt{d+c^2d}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right.}{3c^2d\,(d+c^2dx^2)^{3/2}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right.}{3c^2d\,(d+c^2dx^2)^{3/2}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right.}{3c^2d\,(d+c^2dx^2)^{3/2}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right.}{3c^2d\,(d+c^2dx^2)^{3/2}}$$

Mathematica [A]
time = 0.72, size = 301, normalized size = 0.98

$$\frac{-a^2(2+3c^2x^2)+ab\left(-2(2+3c^2x^2)\sinh^{-1}(cx)+\sqrt{1+c^2x^2}\left(-cx+10(1+c^2x^2)\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)\right)-b^2\left(1+c^2x^2+cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)+2\sinh^{-1}(cx)^2+3c^2x^2\sinh^{-1}(cx)^2+5i(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-5i(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)+5i(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-5i(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\right)}{3c^4d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*(2 + 3*c^2*x^2)) + a*b*(-2*(2 + 3*c^2*x^2)*ArcSinh[c*x] + Sqrt[1 + c
^2*x^2]*(-(c*x) + 10*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) - b^2*(1
+ c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 2*ArcSinh[c*x]^2 + 3*c^2*x
^2*ArcSinh[c*x]^2 + (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^ArcS
inh[c*x]] - (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]
] + (5*I)*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (5*I)*(1 +
c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^4*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(294) = 588.
time = 3.61, size = 704, normalized size = 2.29

method	result

default	$a^2\left(-\frac{x^2}{c^2 d(c^2 d x^2+d)^{\frac{3}{2}}}-\frac{2}{3 d c^4 (c^2 d x^2+d)^{\frac{3}{2}}}\right)-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}(cx)^2x^2}{d^3(c^2x^2+1)^2c^2}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}(cx)x}{3d^3(c^2x^2+1)^{\frac{3}{2}}c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(c^2*d*x^2+d)^(3/2))-b^2*(d*(
c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)^2*x^2-1/3*b^2*(d*(c^2*
x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*arcsinh(c*x)*x-1/3*b^2*(d*(c^2*x^2+
1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*x^2-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*
x^2+1)^2/c^4*arcsinh(c*x)^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2
/c^4-5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*arcsinh(c*x)
*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/c^4/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))-5/3*I*b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2
)))+5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1-I*(c*
x+(c^2*x^2+1)^(1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arc
sinh(c*x)*x^2-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*x-4/3
*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arcsinh(c*x)+5/3*I*a*b*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/
3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^
(1/2)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2))
) - 2/3*a*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)
*c^4*d))*arcsinh(c*x) - 1/3*a^2*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((
c^2*d*x^2 + d)^(3/2)*c^4*d)) + b^2*integrate(x^3*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\operatorname{asinh}(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}(c\,x)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.313 $\int \frac{x^2(a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=312

$$\frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}}-\frac{b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}}+\frac{bx^2(a+b\sinh^{-1}(cx))}{3cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}+\frac{x^3(a+b\sinh^{-1}(cx))^2}{3d(d+c^2dx^2)^{3/2}}+\frac{\sqrt{1+c}}{3}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*x/c^2/d^2/(c^2*d
*x^2+d)^(1/2)+1/3*b*x^2*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x
^2+d)^(1/2)-1/3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1
/2)+1/3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1/2)-
2/3*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/
c^3/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(
c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5800, 5810, 5797, 3799, 2221, 2317, 2438, 294, 221}

$$\frac{bx^2(a+b\sinh^{-1}(cx))}{3cd^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{x^3(a+b\sinh^{-1}(cx))^2}{3d(c^2dx^2+d)^{3/2}}+\frac{\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))^2}{3c^3d^2\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\,\log\left(e^{2\sinh^{-1}(cx)}+1\right)(a+b\sinh^{-1}(cx))}{3c^3d^2\sqrt{c^2dx^2+d}}+\frac{b^2x}{3c^2d^2\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{3c^3d^2\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{3c^3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b^2*x)/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) + (b*x^2*(a + b*ArcSinh[c*x]))/(3*c*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSinh[c*x])^2)/(3*d
*(d + c^2*d*x^2)^(3/2)) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^3
*d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c
^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\,(d+c^2dx^2)^{3/2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\,(d+c^2dx^2)^{3/2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int}{3d^2\sqrt{d+c^2d}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\,(d+c^2dx^2)^{3/2}} -$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.61, size = 280, normalized size = 0.90

$$\frac{b^2cx+a^2c^3x^3+b^2c^3x^3-ab\sqrt{1+c^2x^2}-b^2\left(-c^3x^3+\sqrt{1+c^2x^2}+c^2x^2\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2-b\sinh^{-1}(cx)\left(-2ac^3x^3+b\sqrt{1+c^2x^2}+2b(1+c^2x^2)^{3/2}\log\left(1+e^{-2\sinh^{-1}(cx)}\right)\right)-ab\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)-abc^2x^2\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)+b^2(1+c^2x^2)^{3/2}\mathrm{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)}{3c^3d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b^2*c*x + a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 + c^2*x^2] - b^2*(-(c^3*x
^3) + Sqrt[1 + c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*Arc
Sinh[c*x]*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)^(3/2)*Log
```

```
[1 + E^(-2*ArcSinh[c*x])]) - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - a*b*c
^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + b^2*(1 + c^2*x^2)^(3/2)*PolyLog
[2, -E^(-2*ArcSinh[c*x])])/(3*c^3*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3131 vs. $2(292) = 584$.
time = 4.19, size = 3132, normalized size = 10.04

method	result	size
default	Expression too large to display	3132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
/d^3*(c^2*x^2+1)*arcsinh(c*x)*x^3+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^
6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*x)^2*x^7+1/3*b^2*(d*(c^2*x^
2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*
x)*x^7-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+
1)*c^3/d^3*(c^2*x^2+1)^(1/2)*x^6+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9
*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*x^5-2*a*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^3/d^3*(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*x^6-4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*
c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-8/3*a*b*(d*(c
^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*x^2+4/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)
/c^3/d^3*arcsinh(c*x)+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10
*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+
9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5-1/3*a*b*(d*(c^2*x^2+1))^(1/2)
/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x^3+2/3*a*b*(
d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsi
nh(c*x)*x^3-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5
*c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)-2/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2
+1))^(1/2)/c^3/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*x^3+b^2*(d*(c^2*x^2+
1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)
^2*x^5-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*
x^2+1)*c^2/d^3*(c^2*x^2+1)*x^5+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6
*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)*x^5-a*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*x
^4-a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c
/d^3*(c^2*x^2+1)^(1/2)*x^2-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x
^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)-b^2*(d*(c
^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^3/d^3*(c^2*
x^2+1)^(1/2)*arcsinh(c*x)^2*x^6+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^
```

```
6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*x)*x^7-1/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*
arcsinh(c*x)*x^5-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^
4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*x^4-b^2*(d*(c^2*x^2+1
))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*x^4-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x
^4+5*c^2*x^2+1)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+2/3*b^2*(d*(c^2*x^
2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7+b^2*(d
*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^
5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1
)/c^3/d^3*(c^2*x^2+1)^(1/2)+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*
x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x^3+1/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(c*x)^2*x^3+1/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*
arcsinh(c*x)*x^3+2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c^3/d^3*ar
csinh(c*x)^2-1/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c^3/d^3*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*
c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*x^2+
2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*
c^2/d^3*arcsinh(c*x)*x^5-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+1
0*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*x^4-4/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*(c^2*x^2+1)^(1/2)*
x^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2
+1)/c^2/d^3*(c^2*x^2+1)*x-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^
6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-1/3*b^2*
(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*
(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1
/2)/c^3/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/3*a*b*(d*(c^2*x^
2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*x^3+a^2*(-1/2*
x/c^2/d/(c^2*d*x^2+d)^(3/2)+1/2/c^2*(1/3*x/d/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/
(c^2*d*x^2+d)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/
2))) + 2/3*a*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*
c^2*d))*arcsinh(c*x) + 1/3*a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d
*x^2 + d)^(3/2)*c^2*d)) + b^2*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2 + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,(a + b\operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.314 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\text{Arc}}{3c^2d^2\sqrt{d+c^2dx^2}}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2/c^2/d^2/(c^2*d*
x^2+d)^(1/2)+1/3*b*x*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+
d)^(1/2)+2/3*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)
^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)+1/3*I*b^2*polylog(2,I
*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5798, 5788, 5789, 4265, 2317, 2438, 267}

$$\frac{2b\sqrt{c^2x^2+1}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{b^2}{3c^2d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b^2/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^2*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*c^2*d*(d
+ c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^
ArcSinh[c*x]])/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - ((I/3)*b^2*Sqrt[1 + c^2*x^
2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d*x^2]) + ((I/3)*
b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d
*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{x}{(1+c^2x}}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(b\right.}{}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2b}{}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2b}{}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2b}{}$$

Mathematica [A]
time = 0.64, size = 254, normalized size = 0.94

$$\frac{-a^2+ab\left(-2\sinh^{-1}(cx)+\sqrt{1+c^2x^2}\left(cx+2(1+c^2x^2)\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)\right)+b^2\left(1+c^2x^2+cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)-\sinh^{-1}(cx)^2-i(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\log\left(1-ie^{-\sinh^{-1}(cx)}\right)+i(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)-i(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+i(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-a^2 + a*b*(-2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*(c*x + 2*(1 + c^2*x^2)*Arc
Tan[Tanh[ArcSinh[c*x]/2]])) + b^2*(1 + c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcS
inh[c*x] - ArcSinh[c*x]^2 - I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]
] - I*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] + I*(1 + c^2*x^2)
^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(261) = 522.
time = 1.44, size = 591, normalized size = 2.19

method	result
default	$-\frac{a^2}{3c^2d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)x}{3d^3\left(c^2x^2+1\right)^{\frac{3}{2}}c}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,x^2}{3d^3\left(c^2x^2+1\right)^2}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^2}{3d^3\left(c^2x^2+1\right)^2c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x
^2+1)^(3/2)/c*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^
2*x^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)^2+1/
3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2-1/3*I*b^2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)
))+1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*l
n(1-I*(c*x+(c^2*x^2+1)^(1/2)))-1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^2/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*I*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*a*b*
(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c*x-2/3*a*b*(d*(c^2*x^2+1))^(1/
2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)+1/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-1/3*I*a*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2
*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(
c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.315 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{d+c^2dx^2}}+\frac{2\sqrt{1+c^2}}{3c}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2*x/d^2/(c^2*d*x^2+d
)^(1/2)+2/3*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)+1/3*b*(a+b*arcsi
nh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2/3*(a+b*arcsinh(c*x))
^2*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)-4/3*b*(a+b*arcsinh(c*x))*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)-2/
3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*
x^2+d)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197}

$$\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3cd^2\sqrt{c^2dx^2+d}}-\frac{4b\sqrt{c^2x^2+1}\,\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2b^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{b^2x}{3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*(b^2*x)/(d^2*Sqrt[d + c^2*d*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*d^2*
Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/(3*d*(d
 + c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x
^2]) + (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c*d^2*Sqrt[d + c^2*d
*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c
*x])])/(3*c*d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2,
-E^(2*ArcSinh[c*x])])/(3*c*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}\right.}{\left(1+c^2x^2\right)}}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + 2$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + 2$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + 2$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + 2$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + 2$$

Mathematica [A]
time = 0.86, size = 236, normalized size = 0.81

$$\frac{a^2cx(3+2c^2x^2)+ab\left((6cx+4c^3x^3)\sinh^{-1}(cx)+\sqrt{1+c^2x^2}\,(1-2(1+c^2x^2)\log\left(1+c^2x^2\right))\right)-b^2\left(cx+c^3x^3-\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)-cx\sinh^{-1}(cx)^2-2cx(1+c^2x^2)\sinh^{-1}(cx)^2+2(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2\log\left(1+e^{-2\sinh^{-1}(cx)}\right)\right)-2(1+c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)\right)}{3d^2\left(c+c^3x^2\right)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (a^2*c*x*(3 + 2*c^2*x^2) + a*b*((6*c*x + 4*c^3*x^3)*ArcSinh[c*x] + Sqrt[1 +
 c^2*x^2]*(1 - 2*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - b^2*(c*x + c^3*x^3 - Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]^2 - 2*c*x*(1 + c^2*x^2)*Arc
Sinh[c*x]^2 + 2*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1 +
```

```
E^(-2*ArcSinh[c*x])]) - 2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x
])]))/(3*d^2*(c + c^3*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2727 vs. $2(274) = 548$.
time = 1.82, size = 2728, normalized size = 9.34

method	result	size
default	Expression too large to display	2728

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*a
rcsinh(c*x)*(c^2*x^2+1)*x^5-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^
4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2-28/3*a*b*(d*
(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)*x^2+4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11
*c^2*x^2+4)*c^4/d^3*(c^2*x^2+1)*x^5+4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+
10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x^5+10/3*a*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*x^3+34/3*a*b*
(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c
*x)*x^3+a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3
*(c^2*x^2+1)^(1/2)*x^2+10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4
+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^3-2*b^2*(d*(c^2*x^2+1))^(
1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)
^(1/2)*x^4-4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*
c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c
^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^
6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+
10*c^4*x^4+11*c^2*x^2+4)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)*x^2+a^2*(1/3*x/d/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/
2))-3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3
*x^5-13/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2
/d^3*x^3+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/
c/d^3*(c^2*x^2+1)^(1/2)+4/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d
^3*arcsinh(c*x)^2-2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*pol
ylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6
+10*c^4*x^4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*x-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*
c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*x+2*b^2*(d*(c^2*x^2+1))^(
1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*(c^2*x^2+1)*x-4/3
*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*arcsinh(c*x)*ln(1+(c*x+(
c^2*x^2+1)^(1/2))^2)+7/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11
*c^2*x^2+4)*c/d^3*(c^2*x^2+1)^(1/2)*x^2-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^
```

```
6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)+4/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+
11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*x^3-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6
*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*x^3+2/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*(c^2*x^2+1)*x^5-14/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsi
nh(c*x)*x^5+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c
^3/d^3*(c^2*x^2+1)^(1/2)*x^4+17/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c
^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)^2*x^3-4/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*arcsinh(c*x)*x^7+2*b^2*(d*(c
^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)^2
*x^5-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/
d^3*x^7+4*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3
*arcsinh(c*x)^2*x+8/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*arc
sinh(c*x)-4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)
*c^6/d^3*x^7-14/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^
2+4)*c^4/d^3*x^5-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^
2*x^2+4)*c^2/d^3*x^3+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c
^2*x^2+4)/d^3*(c^2*x^2+1)*x+8*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x
^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*x+4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^
6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)-4/3*a*b/(c^2*x^2+1)^(1/2
)*(d*(c^2*x^2+1))^(1/2)/c/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5
/2))) + 2/3*a*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d
))*arcsinh(c*x) + 1/3*a^2*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 +
d)^(3/2)*d)) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)
^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2), x)
```

3.316 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=518

$$-\frac{b^2}{3d^2\sqrt{d+c^2dx^2}}-\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\,(d+c^2dx^2)^{3/2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}}-\frac{14b\sqrt{1+c^2x^2}}{}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2/d^2/(c^2*d*x^2+d)^(1
/2)+(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*x*(a+b*arcsinh(c*x
))/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-14/3*b*(a+b*arcsinh(c*x))*arct
an(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*(a+b*
arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d
*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2
*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+7/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+
1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-7/3*I*b^2*polylog(2,I*
(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+2*b*(a+b
*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*
d*x^2+d)^(1/2)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^
2/(c^2*d*x^2+d)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1
/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5811, 5816, 4267, 2611, 2320, 6724, 5789, 4265, 2317, 2438, 5788, 267}

$$-\frac{14b\sqrt{c^2x^2+1}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{c^2dx^2+d}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\,(c^2dx^2+d)^{3/2}}+\frac{7ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{7ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2b^2\sqrt{c^2x^2+1}\,\text{Li}_3\left(-e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{2b^2\sqrt{c^2x^2+1}\,\text{Li}_3\left(e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{b^2}{3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*b^2/(d^2*Sqrt[d + c^2*d*x^2]) - (b*c*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c*x])^2/(3*d*(d + c^2
*d*x^2)^(3/2)) + (a + b*ArcSinh[c*x])^2/(d^2*Sqrt[d + c^2*d*x^2]) - (14*b*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*d^2*Sqrt[d
 + c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcS
inh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((7*I)/3)
*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d
*x^2]) - (((7*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^
2*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*
```

```
PolyLog[3, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx}{d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} + \cdots$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+\cdots}{d^2\cdots}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+\cdots}{d^2\cdots}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+\cdots}{d^2\cdots}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+\cdots}{d^2\cdots}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+\cdots}{d^2\cdots}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+\cdots}{d^2\cdots}$$

Mathematica [**A**]
time = 2.73, size = 547, normalized size = 1.06

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 3*a^2*Sqrt[d]*
Log[c*x] - 3*a^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (a*b*d^2*(1
 + c^2*x^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^
(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]
] + 6*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-
ArcSinh[c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSin
h[c*x])]))/(d + c^2*d*x^2)^(3/2) + (b^2*d^2*(1 + c^2*x^2)^(3/2)*(-(1/Sqrt[1
```

```
 + c^2*x^2]) - (c*x*ArcSinh[c*x])/(1 + c^2*x^2) + ArcSinh[c*x]^2/(1 + c^2*x
^2)^(3/2) + (3*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]^2*Log[1 -
 E^(-ArcSinh[c*x])] + (7*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (7*I)*
ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Log[1 + E^(-ArcSi
nh[c*x])] + 6*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + (7*I)*PolyLog[2
, (-I)/E^ArcSinh[c*x]] - (7*I)*PolyLog[2, I/E^ArcSinh[c*x]] - 6*ArcSinh[c*x
]*PolyLog[2, E^(-ArcSinh[c*x])] + 6*PolyLog[3, -E^(-ArcSinh[c*x])] - 6*Poly
Log[3, E^(-ArcSinh[c*x])]))/(d + c^2*d*x^2)^(3/2))/(3*d^3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x\,(c^2d\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a^2*(3*arcsinh(1/(c*abs(x)))/d^(5/2) - 3/(sqrt(c^2*d*x^2 + d)*d^2) - 1
/((c^2*d*x^2 + d)^(3/2)*d)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
((c^2*d*x^2 + d)^(5/2)*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2
+ d)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(cx))^2}{x\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{x\,\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(5/2)), x)
```

3.317 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=421

$$\frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\,(d+c^2dx^2)^{3/2}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\,(d+c^2dx^2)^{3/2}}-\frac{8c^2x\left(a+b\right.}{3d^2\sqrt{d}}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(3/2)-4/3*c^2*x*(a+b*arcsinh(c*x))^
2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*c^2*x/d^2/(c^2*d*x^2+d)^(1/2)-8/3*c^2*x*(a+
b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))/d^2/(c
^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-8/3*c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^
(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-4*b*c*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^
2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+16/3*b*c*(a+b*arcs
inh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+
d)^(1/2)+5/3*b^2*c*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/
d^2/(c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2
+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5809, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5811, 5799, 5569, 4267}

$$-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}-\frac{8c\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{16bc\sqrt{c^2x^2+1}\,\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{4bc\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\,(c^2dx^2+d)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\,(c^2dx^2+d)^{3/2}}+\frac{5b^2c\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{b^2c^2x}{3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(d*x*(d +
c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(
3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*c
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (4
*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d
^2*Sqrt[d + c^2*d*x^2]) + (16*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Lo
g[1 + E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c*Sqrt[1 +
c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (b^
2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(d^2*Sqrt[d + c^2*d*x
^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
```

```
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
```

```
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\left(4c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx+\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \cdots}{d^2\sqrt{d+c^2 \cdots}} \\
&= \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}}-\cdots \\
&= -\frac{b^2c^2x}{d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(\cdots}{3d\cdots} \\
&= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a\cdots}{3d(\cdots} \\
&= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a\cdots}{3d(\cdots} \\
&= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a\cdots}{3d(\cdots} \\
&= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a\cdots}{3d(\cdots} \\
&= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a\cdots}{3d(\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.24, size = 408, normalized size = 0.97

$$
-\frac{3a^2+12a^2c^2x^2-b^2c^2x^2+8a^2c^4x^4-b^2c^4x^4+abcx\sqrt{1+c^2x^2}+6ab\sinh^{-1}(cx)+24abc^2x^2\sinh^{-1}(cx)+16abc^4x^4\sinh^{-1}(cx)+b^2cx\sqrt{1+c^2x^2}\sinh^{-1}(cx)+3b^2\sinh^{-1}(cx)^2+12b^2c^2x^2\sinh^{-1}(cx)^2+8b^2c^4x^4\sinh^{-1}(cx)^2-8b^2cx(1+c^2x^2)^{3/2}\sinh^{-1}(cx)^2-6b^2cx(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\log\left(1-e^{-2\sinh^{-1}(cx)}\right)-10b^2cx(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\log\left(1+e^{-2\sinh^{-1}(cx)}\right)-6abcx(1+c^2x^2)^{3/2}\log(cx)-5abcx(1+c^2x^2)^{3/2}\log\left(1+c^2x^2\right)+5b^2cx(1+c^2x^2)^{3/2}\mathrm{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+3b^2cx(1+c^2x^2)^{3/2}\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{3dx\left(d+c^2dx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(3*a^2 + 12*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 +
a*b*c*x*Sqrt[1 + c^2*x^2] + 6*a*b*ArcSinh[c*x] + 24*a*b*c^2*x^2*ArcSinh[c*x
] + 16*a*b*c^4*x^4*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
3*b^2*ArcSinh[c*x]^2 + 12*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSin
h[c*x]^2 - 8*b^2*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - 6*b^2*c*x*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 10*b^2*c*x*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 6*a*b*c*x*(1 + c^2
*x^2)^(3/2)*Log[c*x] - 5*a*b*c*x*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2] + 5*b
^2*c*x*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 3*b^2*c*x*(1
+ c^2*x^2)^(3/2)*PolyLog[2, E^(-2*ArcSinh[c*x])])/(d*x*(d + c^2*d*x^2)^(3/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3512 vs. $2(411) = 822$.
time = 2.26, size = 3513, normalized size = 8.34

method	result	size
default	Expression too large to display	3513

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 136/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2
*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*
x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3+8*b
^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(
c*x)*(c^2*x^2+1)*c^2+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+2
6*c^2*x^2+9)/d^3*x^7*arcsinh(c*x)*(c^2*x^2+1)*c^8+160/3*b^2*(d*(c^2*x^2+1))
^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*(c^2*x^2+1)
*c^6+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3
*x^4*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^5+40*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*(c^2*x^2+1)*c^4-88*a*b
*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*
x)*c^2+48*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3
*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-32/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1)
)^(1/2)/d^3*arcsinh(c*x)*c-64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4
*x^4+26*c^2*x^2+9)/d^3*x^9*c^10-224/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+
25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8-280/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6
*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*c^6-48*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c
^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*c^4-8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*
c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-5*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-9*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^
6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*arcsinh(c*x)^2+2*b^2/(c^2*x^2+1)^(1/2)
*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c-3*b^2*(d*(c^2
*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^(1/2)-
```

16/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)^2*c-32/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^9*c^10-
40*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^
8-160/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x
^5*c^6-29*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3
*x^3*c^4+2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*c+5/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*pol
ylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c+64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*
x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^8+160/3*a*b*(d*(c^2*x^2+
1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-128/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcs
inh(c*x)*c^6+40*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+
9)/d^3*x^3*(c^2*x^2+1)*c^4-3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^
4+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^(1/2)-18*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6
*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*arcsinh(c*x)+10/3*a*b/(c^2*x^2+1)^(1/2)
(d(c^2*x^2+1))^(1/2)/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c+2*a*b/(c^2*x^2
+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c+128/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)*c^5+24*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c
^4*x^4+26*c^2*x^2+9)/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c-3*b^2*(d*(c^2*x
^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)*c+88/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+
9)/d^3*x^5*(c^2*x^2+1)*c^6-280/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^
4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*(c^2*x^2+1)^(1/2)*c^5+10/3*b^2/
(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2
+1)^(1/2))^2)*c+2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c
*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c-8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+8*b^2*(d*(c^2*x^2+1))^(1/2)/(
8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2+272/3*a*b*(d*(c^2*
x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)*c^3-112*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^
2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+
25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/2)*c^3+8*a*b*(d*(c^2*x^2+1)
)^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-64/3*b^2*
(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^9*arcsinh(c
*x)*c^10-56*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d
^3*x^3*arcsinh(c*x)^2*c^4+80/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*
x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-48*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4-17/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/2)*
c^3-44*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*
arcsinh(c*x)^2*c^2+32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^8-224/3*b^2*(d...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a^2*(8*c^2*x/(sqrt(c^2*d*x^2 + d)*d^2) + 4*c^2*x/((c^2*d*x^2 + d)^(3/2
)*d) + 3/((c^2*d*x^2 + d)^(3/2)*d*x)) + integrate(b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/
((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{x^2 (d (c^2x^2 + 1))^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{x^2\,(d\,c^2\,x^2+d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(5/2)), x)
```

3.318 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=687

$$\frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{6d\left(d+c^2dx^2\right)^{3/2}}-\frac{(a+}{2dx^2}$$

```
[Out] -5/6*c^2*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/2*(a+b*arcsinh(c*x))^
2/d/x^2/(c^2*d*x^2+d)^(3/2)+1/3*b^2*c^2/d^2/(c^2*d*x^2+d)^(1/2)-5/2*c^2*(a+
b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-b*c*(a+b*arcsinh(c*x))/d^2/x/(c^2
*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-2/3*b*c^3*x*(a+b*arcsinh(c*x))/d^2/(c^2*x
^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+26/3*b*c^2*(a+b*arcsinh(c*x))*arctan(c*x+(c
^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5*c^2*(a+b*arcsi
nh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+
d)^(1/2)-b^2*c^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^
2+d)^(1/2)+5*b*c^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^
2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-13/3*I*b^2*c^2*polylog(2,-I*(c*x+(c^
2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+13/3*I*b^2*c^2*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/
2)-5*b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^
(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-5*b^2*c^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(
c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5*b^2*c^2*polylog(3,c*x+(c^2*x^2+1
)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.79, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {5809, 5811, 5816, 4267, 2611, 2320, 6724, 5789, 4265, 2317, 2438, 5788, 267, 272, 53, 65, 214}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(d^2*x*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (2*b*c^3*x*(a + b*ArcSinh[c*x]))/(3
*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x])^2
)/(6*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x])^2/(2*d*x^2*(d + c^2*d*
x^2)^(3/2)) - (5*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^2*Sqrt[d + c^2*d*x^2]) +
(26*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3
*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2
*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2
```

```
*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d
 + c^2*d*x^2]) - (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d^2*Sqrt[d
+ c^2*d*x^2]) - (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(
d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSin
h[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
```

```
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int}{d^2\sqrt{d+}}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh\right.}{6d\left(d+c^2dx\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 7.34, size = 983, normalized size = 1.43

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/2*a^2/(d^3*x^2) - (a^2*c^2)/(3*d^3*(1 + c^2*x^2)^
2) - (2*a^2*c^2)/(d^3*(1 + c^2*x^2))) - (5*a^2*c^2*Log[x])/(2*d^(5/2)) + (5
*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/(2*d^(5/2)) + (a*b*c^2*((4
*c*x)/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[c*x])/(1 + c^2*x^2)
```

```
+ 104*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*Sqrt[1 + c^2*x^2]*
Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2
]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 60*Sqr
t[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*Sqrt[1 + c^2*x^
2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-Arc
Sinh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 6*S
qrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/(12*d^2*Sqrt[d*(1 + c^2*x^2)]) + (b
^2*c^2*(8 + (8*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x]^2 - (8
*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Coth[Arc
Sinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 -
60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - (104*I)*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + (104*I)*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + 60*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^2]*Log[Tanh[ArcSin
h[c*x]/2]] - 120*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x
])] - (104*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (104*I)*S
qrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] + 120*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 120*Sqrt[1 + c^2*x^2]*PolyLog[3, -E
^(-ArcSinh[c*x])] + 120*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])] - 3
*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 12*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(24*d^2*Sqrt[d*(1 + c^2*x^2)])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^3\,(c^2d\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a^2*(15*c^2*arcsinh(1/(c*abs(x)))/d^(5/2) - 15*c^2/(sqrt(c^2*d*x^2 + d)
*d^2) - 5*c^2/((c^2*d*x^2 + d)^(3/2)*d) - 3/((c^2*d*x^2 + d)^(3/2)*d*x^2))
+ integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^3)
+ 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)^2}{x^3\,\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(5/2)), x)
```

3.319 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=506

$$-\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}}-\frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}\right.}{dx\left(d+c^2dx\right.}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^2/d/x^3/(c^2*d*x^2+d)^(3/2)+2*c^2*(a+b*arcsinh(c*x)
)^2/d/x/(c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^
(3/2)-1/3*b^2*c^2/d^2/x/(c^2*d*x^2+d)^(1/2)-2/3*b^2*c^4*x/d^2/(c^2*d*x^2+d)
^(1/2)+16/3*c^4*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b
*arcsinh(c*x))/d^2/x^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+16/3*c^3*(a+b*
arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+32/3*b*c^3*(a+b*a
rcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*
d*x^2+d)^(1/2)-32/3*b*c^3*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2
)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*po
lylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2
)
```

Rubi [A]
time = 0.75, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5809, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5811, 5799, 5569, 4267, 277}

$$-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{16c^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}-\frac{32bc^3\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{32bc^3\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{b^2c^2}{3d^2x\sqrt{c^2dx^2+d}}-\frac{2b^2c^4x}{3d^2\sqrt{c^2dx^2+d}}-\frac{8b^2c^3\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{8b^2c^3\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(b^2*c^2)/(d^2*x*Sqrt[d + c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d +
c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[
d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*d*x^3*(d + c^2*d*x^2)^(3/2)) +
(2*c^2*(a + b*ArcSinh[c*x])^2)/(d*x*(d + c^2*d*x^2)^(3/2)) + (8*c^4*x*(a +
b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c
*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (16*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (32*b*c^3*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) -
 (32*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x]
)])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -
E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*
x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \cdots}{3d^2\sqrt{d+c^2 \cdots}}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} \cdots$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{8bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2d \cdots}}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} + \frac{2b^2c^4x}{d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{(a+b \cdots}{3dx^3 (\cdots}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{(a+ \cdots}{3dx^3 \cdots}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{(a+ \cdots}{3dx^3 \cdots}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{(a+ \cdots}{3dx^3 \cdots}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{(a+ \cdots}{3dx^3 \cdots}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{(a+ \cdots}{3dx^3 \cdots}$$

Mathematica [A]
time = 2.09, size = 417, normalized size = 0.82

$$\frac{\frac{a^2\left(-1+6c^2x^2+24c^4x^4+16c^6x^6\right)}{x^3} - \frac{ab\left(-2\left(-1+6c^2x^2+24c^4x^4+16c^6x^6\right)\sinh^{-1}(cx)+cx\sqrt{1+c^2x^2}\left(1+16\left(c^2x^2+c^4x^4\right)\log(cx)+8\left(c^2x^2+c^4x^4\right)\log\left(1+c^2x^2\right)\right)\right)}{x^3} + b^2c^3\left(1+c^2x^2\right)^{3/2}\left(-\frac{cx}{\sqrt{1+c^2x^2}} - \frac{\sqrt{1+c^2x^2}}{cx} - \frac{\sinh^{-1}(cx)}{c^2x^2} + \frac{\sinh^{-1}(cx)}{1+c^2x^2} - 16\sinh^{-1}(cx)^2 + \frac{cx\sinh^{-1}(cx)^2}{\left(1+c^2x^2\right)^{3/2}} + \frac{8cx\sinh^{-1}(cx)^2}{\sqrt{1+c^2x^2}} - \frac{\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2}{c^3x^3} + \frac{8\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2}{cx} - 16\sinh^{-1}(cx)\log\left(1-e^{-2\sinh^{-1}(cx)}\right) - 16\sinh^{-1}(cx)\log\left(1+e^{-2\sinh^{-1}(cx)}\right) + 8\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right) + 8\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6))/x^3 - (a*b*(-2*(-1 + 6*c^
2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x] + c*x*Sqrt[1 + c^2*x^2]*(1 +
16*(c^2*x^2 + c^4*x^4)*Log[c*x] + 8*(c^2*x^2 + c^4*x^4)*Log[1 + c^2*x^2])))
/x^3 + b^2*c^3*(1 + c^2*x^2)^(3/2)*(-((c*x)/Sqrt[1 + c^2*x^2]) - Sqrt[1 + c
^2*x^2]/(c*x) - ArcSinh[c*x]/(c^2*x^2) + ArcSinh[c*x]/(1 + c^2*x^2) - 16*Ar
cSinh[c*x]^2 + (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^(3/2) + (8*c*x*ArcSinh[c*
```

```
x]^2)/Sqrt[1 + c^2*x^2] - (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c^3*x^3) + (8
*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*x) - 16*ArcSinh[c*x]*Log[1 - E^(-2*Ar
cSinh[c*x])] - 16*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] + 8*PolyLog[2,
-E^(-2*ArcSinh[c*x])] + 8*PolyLog[2, E^(-2*ArcSinh[c*x])]))/(3*d*(d + c^2*d
*x^2)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4950 vs. $2(484) = 968$.
time = 5.00, size = 4951, normalized size = 9.78

method	result	size
default	Expression too large to display	4951

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1120/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x
^2-1)/d^3*x^7*c^10+560/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+3
5*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*c^8+64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*
x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*c^6-16/3*a*b*(d*(c^2*x^2+1)
)^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4+64/3*a*b/
(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)*c^3+2/3*a*b*(d*(c^
2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^3*arc
sinh(c*x)-16/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*ln((c*x+(c^2
*x^2+1)^(1/2))^4-1)*c^3-4*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+
35*c^4*x^4+10*c^2*x^2-1)/d^3*(c^2*x^2+1)^(1/2)*c^3+256/3*a*b*(d*(c^2*x^2+1)
)^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*c^14+896/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/
d^3*x^9*c^12+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^
4+10*c^2*x^2-1)/d^3/x^2*c*(c^2*x^2+1)^(1/2)-256/3*a*b*(d*(c^2*x^2+1))^(1/2)
/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2*x^2+1)*c^12+1
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1
)/d^3/x*c^2-16/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(2,
-c*x-(c^2*x^2+1)^(1/2))*c^3+32/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2
)/d^3*arcsinh(c*x)^2*c^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x
^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^3*arcsinh(c*x)^2-8/3*b^2/(c^2*x^2+1)^(1/2
)*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-2/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^
3*(c^2*x^2+1)^(1/2)*c^3-16/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^
3*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c
^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*c^14+224/3*b^2*(d*(c^2*
x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*c^12+
88*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1
)/d^3*x^7*c^10+100/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^
4*x^4+10*c^2*x^2-1)/d^3*x^5*c^8-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+
```

```
36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*c^6+16/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)*(c^2
*x^2+1)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4
+10*c^2*x^2-1)/d^3/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c+560/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsin
h(c*x)*c^8+16*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+1
0*c^2*x^2-1)/d^3*x^4*(c^2*x^2+1)^(1/2)*c^7+344/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)^2*c^6-
32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2
-1)/d^3*x^3*(c^2*x^2+1)*c^6+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c
^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)*c^6+22/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*(c^2*x
^2+1)^(1/2)*c^5+12*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*
x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)^2*c^4-3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*
c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4+24*a*b*(d*(c^2*x^2+1)
)^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)*
c^4+32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2
*x^2-1)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3+16/3*a*b*(d*(c^2*x^2+1))^(1/
2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*(c^2*x^2+1)*c^4-12
*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/
d^3/x*arcsinh(c*x)*c^2-256/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x
^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*arcsinh(c*x)*(c^2*x^2+1)*c^12-80/3*b^2*
(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x
^3*arcsinh(c*x)*(c^2*x^2+1)*c^6-176/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8
+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/
2)*c^5-640/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10
*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*(c^2*x^2+1)*c^10-160*b^2*(d*(c^2*x^2+1))^(
1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*x)*(
c^2*x^2+1)*c^8-128*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*
x^4+10*c^2*x^2-1)/d^3*x^4*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^7-80/3*a*b*(d*
(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*
(c^2*x^2+1)*c^6-640/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c
^4*x^4+10*c^2*x^2-1)/d^3*x^7*(c^2*x^2+1)*c^10+128*a*b*(d*(c^2*x^2+1))^(1/2)
/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*c^10-
160*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-
1)/d^3*x^5*(c^2*x^2+1)*c^8+320*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6
*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*x)*c^8+688/3*a*b*(d*(c^2*x^
2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(8*c^2*log(c^2*x^2 + 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2
*d^(5/2)*x^4 + d^(5/2)*x^2)) + 2/3*(16*c^4*x/(sqrt(c^2*d*x^2 + d)*d^2) + 8*
c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*d*x) - 1/((c
^2*d*x^2 + d)^(3/2)*d*x^3))*a*b*arcsinh(c*x) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^
2 + d)*d^2) + 8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3
/2)*d*x) - 1/((c^2*d*x^2 + d)^(3/2)*d*x^3))*a^2 + b^2*integrate(log(c*x + s
qrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{x^4 (d (c^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**4*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{x^4\,(d\,c^2\,x^2 + d)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(5/2)), x)
```

3.320 $\int \frac{\sinh^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=366

$$-\frac{x}{3c^3\sqrt{c+a^2cx^2}}-\frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}+\frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}}+\frac{4\sinh^{-1}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+}$$

```
[Out] 1/5*x*arcsinh(a*x)^2/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)^2/c^2/(a^2*c
*x^2+c)^(3/2)-1/3*x/c^3/(a^2*c*x^2+c)^(1/2)-1/30*x/c^3/(a^2*x^2+1)/(a^2*c*x
^2+c)^(1/2)+1/10*arcsinh(a*x)/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8
/15*x*arcsinh(a*x)^2/c^3/(a^2*c*x^2+c)^(1/2)+4/15*arcsinh(a*x)/a/c^3/(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+8/15*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a/c^3
/(a^2*c*x^2+c)^(1/2)-16/15*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^
2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)-8/15*polylog(2,-(a*x+(a^2*x^2+1)^(
1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 198}

$$-\frac{8\sqrt{a^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(ax)}\right)}{15ac^3\sqrt{a^2cx^2+c}}-\frac{x}{3c^3\sqrt{a^2cx^2+c}}-\frac{x}{30c^3\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)^2}{15c^3\sqrt{a^2cx^2+c}}+\frac{8\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{15ac^3\sqrt{a^2cx^2+c}}+\frac{4\sinh^{-1}(ax)}{15ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}+\frac{\sinh^{-1}(ax)}{10ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{16\sqrt{a^2x^2+1}\sinh^{-1}(ax)\log\left(e^{2\sinh^{-1}(ax)}+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{4x\sinh^{-1}(ax)^2}{15c^2\left(a^2cx^2+c\right)^{3/2}}+\frac{x\sinh^{-1}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/3*x/(c^3*Sqrt[c + a^2*c*x^2]) - x/(30*c^3*(1 + a^2*x^2)*Sqrt[c + a^2*c*x
^2]) + ArcSinh[a*x]/(10*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + (4
*ArcSinh[a*x])/(15*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSin
h[a*x]^2)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^2)/(15*c^2*(c + a
^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^2)/(15*c^3*Sqrt[c + a^2*c*x^2]) + (8*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(15*a*c^3*Sqrt[c + a^2*c*x^2]) - (16*Sqrt[
1 + a^2*x^2]*ArcSinh[a*x]*Log[1 + E^(2*ArcSinh[a*x])])/(15*a*c^3*Sqrt[c + a
^2*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(15*a*c^
3*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)^2}{5c\,(c+a^2cx^2)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx}{5c} - \frac{\left(2a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)}{(1+a^2x^2)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{\sinh^{-1}(ax)}{10ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^2}{5c\,(c+a^2cx^2)^{5/2}} + \frac{4x\sinh^{-1}(ax)^2}{15c^2\,(c+a^2cx^2)^{3/2}} + \frac{8\int \frac{\sinh}{(c+}}{}$$

$$= -\frac{x}{30c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\sinh^{-}}{15ac^3\sqrt{1+a^2x}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}$$

Mathematica [A]

time = 0.54, size = 178, normalized size = 0.49

$$\frac{ax\left(-10-\frac{1}{1+a^2x^2}\right)+\left(-16\sqrt{1+a^2x^2}+\frac{2ax(15+20a^2x^2+8a^4x^4)}{(1+a^2x^2)^2}\right)\sinh^{-1}(ax)^2+\frac{\sinh^{-1}(ax)\left(11+8a^2x^2-32(1+a^2x^2)^2\log\left(1+e^{-2\sinh^{-1}(ax)}\right)\right)}{(1+a^2x^2)^{3/2}}+16\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(ax)}\right)}{30ac^3\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (a*x*(-10 - (1 + a^2*x^2)^(-1)) + (-16*Sqrt[1 + a^2*x^2] + (2*a*x*(15 + 20*
a^2*x^2 + 8*a^4*x^4))/(1 + a^2*x^2)^2)*ArcSinh[a*x]^2 + (ArcSinh[a*x]*(11 +
 8*a^2*x^2 - 32*(1 + a^2*x^2)^2*Log[1 + E^(-2*ArcSinh[a*x])]))/(1 + a^2*x^2
)^(3/2) + 16*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[a*x])])/(30*a*c^3*
Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 2.74, size = 570, normalized size = 1.56

method	result
default	$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(8a^5x^5-8a^4\sqrt{a^2x^2+1}\,x^4+20a^3x^3-16\sqrt{a^2x^2+1}\,a^2x^2+15ax-8\sqrt{a^2x^2+1}\right)\left(-64\,\text{arcsinh}(ax)a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*(a^2*x^2+1)^(1/2)*x^4+20*a^3*x^
3-16*(a^2*x^2+1)^(1/2)*a^2*x^2+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*arcsinh(a*x
)*a^8*x^8-64*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*a^7*x^7-32*a^8*x^8-32*(a^2*x^2+
1)^(1/2)*a^7*x^7-280*arcsinh(a*x)*a^6*x^6-248*(a^2*x^2+1)^(1/2)*arcsinh(a*x
)*a^5*x^5-142*a^6*x^6-126*(a^2*x^2+1)^(1/2)*a^5*x^5+80*arcsinh(a*x)^2*a^4*x
^4-456*arcsinh(a*x)*a^4*x^4-340*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*a^3*x^3-265*
a^4*x^4-156*(a^2*x^2+1)^(1/2)*a^3*x^3+190*arcsinh(a*x)^2*a^2*x^2-328*arcsin
h(a*x)*a^2*x^2-165*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*a*x-235*a^2*x^2-62*(a^2*x
^2+1)^(1/2)*a*x+128*arcsinh(a*x)^2-88*arcsinh(a*x)-80)/(40*a^10*x^10+215*a^
8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4+16/15/(a^2*x^2+1)^(1/2)
*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^2-16/15/(a^2*x^2+1)^(1/2)*(c*(a^2
*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-8/15/(a^2
*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*polylog(2,-(a*x+(a^2*x^2+1)^(1/2)
)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(a^2*c*x^2 + c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^2/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{2}(ax)}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(c*(a**2*x**2 + 1))**(7/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,%%%{-2
,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{-2,[0,0,0]%%%},0,%%%
{1,[4,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{asinh}\left(a\,x\right)}^2}{{\left(c\,a^2\,x^2+c\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)^2/(c + a^2*c*x^2)^(7/2), x)
```

3.321 $\int x^m(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=936

$$\frac{10b^2c^2d^2x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3(6+m)}+\frac{2b^2c^2d^2(52+15m+m^2)\,x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^2(6+m)^3}+\frac{2b^2c^4d^2x^{5+m}\sqrt{d+c^2dx^2}}{(6+m)^3}-\frac{30bcd^2x}{(2+}$$

```
[Out] 5*d*x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/(4+m)/(6+m)+x^(1+m)*(c
^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/(6+m)+10*b^2*c^2*d^2*x^(3+m)*(c^2*d*
x^2+d)^(1/2)/(4+m)^3/(6+m)+2*b^2*c^2*d^2*(m^2+15*m+52)*x^(3+m)*(c^2*d*x^2+d
)^(1/2)/(4+m)^2/(6+m)^3+2*b^2*c^4*d^2*x^(5+m)*(c^2*d*x^2+d)^(1/2)/(6+m)^3+1
5*d^2*x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)-30
*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(4+m)/(6+m)
/(c^2*x^2+1)^(1/2)-10*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)/(6+m)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-2*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/(m^2+8*m+12)/(c^2*x^2+1)^(1/2)-10*b*c^3*d^2*x^(4+m)*(a+
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)/(c^2*x^2+1)^(1/2)-4*b*c^3
*d^2*x^(4+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)/(6+m)/(c^2*x^2+1)
^(1/2)-2*b*c^5*d^2*x^(6+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(6+m)^2/(
c^2*x^2+1)^(1/2)+10*b^2*c^2*d^2*(10+3*m)*x^(3+m)*hypergeom([1/2, 3/2+1/2*m]
,[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m)^3/(6+m)/(m^2+5*m+6)/(c^2*x
^2+1)^(1/2)+30*b^2*c^2*d^2*x^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-
c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(6+m)/(m^2+7*m+12)/(c^2*x^2+1)^(1/2)+2
*b^2*c^2*d^2*(15*m^2+130*m+264)*x^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2
*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)^3/(m^2+5*m+6)/(c^2*x^2+1)^(
1/2)+15*d^3*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)/(6
+m)/(m^2+6*m+8)
```

Rubi [A]
time = 0.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (10*b^2*c^2*d^2*x^(3 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^3*(6 + m)) + (2*b^2
*c^2*d^2*(52 + 15*m + m^2)*x^(3 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^2*(6 + m
)^3) + (2*b^2*c^4*d^2*x^(5 + m)*Sqrt[d + c^2*d*x^2])/(6 + m)^3 - (30*b*c*d^
2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*(4 + m)*(6
 + m)*Sqrt[1 + c^2*x^2]) - (10*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b
```

```
*ArcSinh[c*x]))/((6 + m)*(8 + 6*m + m^2)*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2*x^
(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((12 + 8*m + m^2)*Sqrt[1
+ c^2*x^2]) - (10*b*c^3*d^2*x^(4 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/((4 + m)^2*(6 + m)*Sqrt[1 + c^2*x^2]) - (4*b*c^3*d^2*x^(4 + m)*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) - (2
*b*c^5*d^2*x^(6 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((6 + m)^2*S
qrt[1 + c^2*x^2]) + (15*d^2*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/((6 + m)*(8 + 6*m + m^2)) + (5*d*x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a
+ b*ArcSinh[c*x])^2)/((4 + m)*(6 + m)) + (x^(1 + m)*(d + c^2*d*x^2)^(5/2)*(
a + b*ArcSinh[c*x])^2)/(6 + m) + (30*b^2*c^2*d^2*x^(3 + m)*Sqrt[d + c^2*d*x
^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)^2*(3
 + m)*(4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) + (10*b^2*c^2*d^2*(10 + 3*m)*x^(3
+ m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2
*x^2)])/((2 + m)*(3 + m)*(4 + m)^3*(6 + m)*Sqrt[1 + c^2*x^2]) + (2*b^2*c^2*
d^2*(264 + 130*m + 15*m^2)*x^(3 + m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[
1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)*(3 + m)*(4 + m)^2*(6 + m)^
3*Sqrt[1 + c^2*x^2]) + (15*d^3*Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt
[d + c^2*d*x^2], x])/((6 + m)*(8 + 6*m + m^2))
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A]
time = 1.96, size = 0, normalized size = 0.00

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^m \left(c^2d\,x^2+d\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\left(a+b\operatorname{asinh}(cx)\right)^2\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.322 $\int x^m(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=488

$$\frac{2b^2c^2dx^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3}-\frac{6bcdx^{2+m}\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{(2+m)^2(4+m)\sqrt{1+c^2x^2}}-\frac{2bcdx^{2+m}\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx)}{(8+6m+m^2)\sqrt{1+c^2x^2}}$$

```
[Out] x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/(4+m)+2*b^2*c^2*d*x^(3+m)*
(c^2*d*x^2+d)^(1/2)/(4+m)^3+3*d*x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/(m^2+6*m+8)-6*b*c*d*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2
+m)^2/(4+m)/(c^2*x^2+1)^(1/2)-2*b*c*d*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-2*b*c^3*d*x^(4+m)*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(c^2*x^2+1)^(1/2)+2*b^2*c^2*d*(10+3*m)*x^(3+m
)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m
)^3/(m^2+5*m+6)/(c^2*x^2+1)^(1/2)+6*b^2*c^2*d*x^(3+m)*hypergeom([1/2, 3/2+1
/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(m^2+7*m+12)/(c^2*x
^2+1)^(1/2)+3*d^2*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2)
,x)/(m^2+6*m+8)
```

Rubi [A]
time = 0.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (2*b^2*c^2*d*x^(3 + m)*Sqrt[d + c^2*d*x^2])/(4 + m)^3 - (6*b*c*d*x^(2 + m)*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*(4 + m)*Sqrt[1 + c^2*x
^2]) - (2*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((8 + 6
*m + m^2)*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d*x^(4 + m)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x]))/((4 + m)^2*Sqrt[1 + c^2*x^2]) + (3*d*x^(1 + m)*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(8 + 6*m + m^2) + (x^(1 + m)*(d + c^2*d*
x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(4 + m) + (6*b^2*c^2*d*x^(3 + m)*Sqrt[d
+ c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2
+ m)^2*(3 + m)*(4 + m)*Sqrt[1 + c^2*x^2]) + (2*b^2*c^2*d*(10 + 3*m)*x^(3 +
m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x
^2)])/((2 + m)*(3 + m)*(4 + m)^3*Sqrt[1 + c^2*x^2]) + (3*d^2*Defer[Int][(x^
m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x])/(8 + 6*m + m^2)
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx=\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{m} \left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
[Out] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.323 $\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=199

$$-\frac{2bcx^{2+m}\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))^2}{2+m}+\frac{2b^2c^2x^{3+m}\sqrt{d+c^2dx^2}\,2}{(2+m)^2(3+}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(2+m)-2*b*c*x^(2+m)*(a+b*a
rcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(c^2*x^2+1)^(1/2)+2*b^2*c^2*x^(3+m
)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m
)^2/(3+m)/(c^2*x^2+1)^(1/2)+d*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*
x^2+d)^(1/2),x)/(2+m)
```

Rubi [A]
time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-2*b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*Sqrt
[1 + c^2*x^2]) + (x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2
+ m) + (2*b^2*c^2*x^(3 + m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 +
 m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)^2*(3 + m)*Sqrt[1 + c^2*x^2]) + (d*D
efer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x])/(2 + m)
```

Rubi steps

$$\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx=\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{m}\sqrt{c^{2} d\,x^{2}+d}\, \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \sqrt{d \left(c^{2} x^{2} + 1\right)} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.324 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+b\,\mathrm{arcsinh}\,(cx))^2}{\sqrt{c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*x^m/sqrt(c^2*d*x^2
 + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\left(a + b \operatorname{asinh}\left(cx\right)\right)^{2}}{\sqrt{d\left(c^{2}x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.325 $\int \frac{x^m (a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m (a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m (a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m (a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^m (a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$$

Mathematica [A]
time = 2.95, size = 0, normalized size = 0.00

$$\int \frac{x^m (a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (a+b\,\mathrm{arcsinh}\,(cx))^2}{(c^2d\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.326 $\int \frac{x^m \left(a+b \sinh^{-1}(cx)\right)^2}{\left(d+c^2 d x^2\right)^{5/2}}\, dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m \left(a+b \sinh^{-1}(cx)\right)^2}{\left(d+c^2 d x^2\right)^{5/2}}, x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \left(a+b \sinh^{-1}(cx)\right)^2}{\left(d+c^2 d x^2\right)^{5/2}}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m \left(a+b \sinh^{-1}(cx)\right)^2}{\left(d+c^2 d x^2\right)^{5/2}}\, dx = \int \frac{x^m \left(a+b \sinh^{-1}(cx)\right)^2}{\left(d+c^2 d x^2\right)^{5/2}}\, dx$$

Mathematica [A]
time = 2.95, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(a+b \sinh^{-1}(cx)\right)^2}{\left(d+c^2 d x^2\right)^{5/2}}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (a+b \,\mathrm{arcsinh}\,(cx))^2}{(c^2 d\, x^2+d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (a + b \operatorname{asinh}(cx))^2}{(d (c^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.327 $\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}},x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Defer[Int][(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{asinh}^2(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)**2/sqrt(a**2*x**2 + 1), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.328 $\int (c+a^2cx^2)^3 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=359

$$-\frac{413312c^3\sqrt{1+a^2x^2}}{128625a}-\frac{30256c^3(1+a^2x^2)^{3/2}}{385875a}-\frac{2664c^3(1+a^2x^2)^{5/2}}{214375a}-\frac{6c^3(1+a^2x^2)^{7/2}}{2401a}+\frac{4322c^3x\sinh^{-1}(ax)}{1225}$$

```
[Out] -30256/385875*c^3*(a^2*x^2+1)^(3/2)/a-2664/214375*c^3*(a^2*x^2+1)^(5/2)/a-6
/2401*c^3*(a^2*x^2+1)^(7/2)/a+4322/1225*c^3*x*arcsinh(a*x)+1514/3675*a^2*c^
3*x^3*arcsinh(a*x)+702/6125*a^4*c^3*x^5*arcsinh(a*x)+6/343*a^6*c^3*x^7*arcs
inh(a*x)-8/35*c^3*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a-18/175*c^3*(a^2*x^2+1)
^(5/2)*arcsinh(a*x)^2/a-3/49*c^3*(a^2*x^2+1)^(7/2)*arcsinh(a*x)^2/a+16/35*c
^3*x*arcsinh(a*x)^3+8/35*c^3*x*(a^2*x^2+1)*arcsinh(a*x)^3+6/35*c^3*x*(a^2*x
^2+1)^2*arcsinh(a*x)^3+1/7*c^3*x*(a^2*x^2+1)^3*arcsinh(a*x)^3-413312/128625
*c^3*(a^2*x^2+1)^(1/2)/a-48/35*c^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A]
time = 0.52, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {5786, 5772, 5798, 267, 5784, 455, 45, 200, 12, 1261, 712, 1813, 1864}

$$\frac{6}{343}a^6c^3x^7\sinh^{-1}(ax)+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{1514a^2c^3x^3\sinh^{-1}(ax)}{3675}-\frac{6c^3(a^2x^2+1)^{7/2}}{2401a}-\frac{2664c^3(a^2x^2+1)^{5/2}}{214375a}-\frac{30256c^3(a^2x^2+1)^{3/2}}{385875a}-\frac{413312c^3\sqrt{a^2x^2+1}}{128625a}+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\sinh^{-1}(ax)^3+\frac{6}{35}c^3x\left(a^2x^2+1\right)^2\sinh^{-1}(ax)^3+\frac{8}{35}c^3x\left(a^2x^2+1\right)\sinh^{-1}(ax)^3-\frac{3c^3(a^2x^2+1)^{7/2}\sinh^{-1}(ax)^2}{49a}-\frac{18c^3(a^2x^2+1)^{5/2}\sinh^{-1}(ax)^2}{175a}-\frac{8c^3(a^2x^2+1)^{3/2}\sinh^{-1}(ax)^2}{35a}-\frac{48c^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{35a}+\frac{16}{35}c^3x\sinh^{-1}(ax)^3+\frac{4322c^3x\sinh^{-1}(ax)}{1225}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

```
[Out] (-413312*c^3*Sqrt[1 + a^2*x^2])/(128625*a) - (30256*c^3*(1 + a^2*x^2)^(3/2)
)/(385875*a) - (2664*c^3*(1 + a^2*x^2)^(5/2))/(214375*a) - (6*c^3*(1 + a^2*
x^2)^(7/2))/(2401*a) + (4322*c^3*x*ArcSinh[a*x])/1225 + (1514*a^2*c^3*x^3*A
rcSinh[a*x])/3675 + (702*a^4*c^3*x^5*ArcSinh[a*x])/6125 + (6*a^6*c^3*x^7*Ar
cSinh[a*x])/343 - (48*c^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(35*a) - (8*c^3
*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(35*a) - (18*c^3*(1 + a^2*x^2)^(5/2)*A
rcSinh[a*x]^2)/(175*a) - (3*c^3*(1 + a^2*x^2)^(7/2)*ArcSinh[a*x]^2)/(49*a)
+ (16*c^3*x*ArcSinh[a*x]^3)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/35
+ (6*c^3*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcS
inh[a*x]^3)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3 \sinh^{-1}(ax)^3\,dx = \frac{1}{7}c^3x\left(1+a^2x^2\right)^3\sinh^{-1}(ax)^3+\frac{1}{7}(6c)\int \left(c+a^2cx^2\right)^2\sinh^{-1}(ax)^3\,dx-\frac{1}{7}$$

$$= -\frac{3c^3(1+a^2x^2)^{7/2}\sinh^{-1}(ax)^2}{49a}+\frac{6}{35}c^3x\left(1+a^2x^2\right)^2\sinh^{-1}(ax)^3+\frac{1}{7}c^3x(1$$

$$= \frac{6}{49}c^3x\sinh^{-1}(ax)+\frac{6}{49}a^2c^3x^3\sinh^{-1}(ax)+\frac{18}{245}a^4c^3x^5\sinh^{-1}(ax)+\frac{6}{343}a$$

$$= \frac{402c^3x\sinh^{-1}(ax)}{1225}+\frac{318a^2c^3x^3\sinh^{-1}(ax)}{1225}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{6}{34}$$

$$= \frac{962c^3x\sinh^{-1}(ax)}{1225}+\frac{1514a^2c^3x^3\sinh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{}{3}$$

$$= \frac{4322c^3x\sinh^{-1}(ax)}{1225}+\frac{1514a^2c^3x^3\sinh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+$$

$$= -\frac{960c^3\sqrt{1+a^2x^2}}{343a}-\frac{16c^3(1+a^2x^2)^{3/2}}{1715a}-\frac{36c^3(1+a^2x^2)^{5/2}}{8575a}-\frac{6c^3(1+a}{2401}$$

$$= -\frac{413312c^3\sqrt{1+a^2x^2}}{128625a}-\frac{30256c^3(1+a^2x^2)^{3/2}}{385875a}-\frac{2664c^3(1+a^2x^2)^{5/2}}{214375a}-$$

Mathematica [A]
time = 0.11, size = 169, normalized size = 0.47

$$\frac{c^3\left(-2\sqrt{1+a^2x^2}\,(22329151+747937a^2x^2+134541a^4x^4+16875a^6x^6)+210ax(226905+26495a^2x^2+7371a^4x^4+1125a^6x^6)\sinh^{-1}(ax)-11025\sqrt{1+a^2x^2}\,(2161+757a^2x^2+351a^4x^4+75a^6x^6)\sinh^{-1}(ax)^2+385875ax(35+35a^2x^2+21a^4x^4+5a^6x^6)\sinh^{-1}(ax)^3\right)}{13505625a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

```
[Out] (c^3*(-2*Sqrt[1 + a^2*x^2]*(22329151 + 747937*a^2*x^2 + 134541*a^4*x^4 + 16
875*a^6*x^6) + 210*a*x*(226905 + 26495*a^2*x^2 + 7371*a^4*x^4 + 1125*a^6*x^
6)*ArcSinh[a*x] - 11025*Sqrt[1 + a^2*x^2]*(2161 + 757*a^2*x^2 + 351*a^4*x^4
 + 75*a^6*x^6)*ArcSinh[a*x]^2 + 385875*a*x*(35 + 35*a^2*x^2 + 21*a^4*x^4 +
5*a^6*x^6)*ArcSinh[a*x]^3))/(13505625*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^3 \operatorname{arcsinh}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x)
```

Maxima [A]
time = 0.28, size = 276, normalized size = 0.77

$$-\frac{1}{1225}\left(75\sqrt{a^2x^2+1}\,a^4c^3x^6+351\sqrt{a^2x^2+1}\,a^2c^3x^4+757\sqrt{a^2x^2+1}\,c^3x^2+\frac{2161\sqrt{a^2x^2+1}\,c^3}{a^2}\right)a\,\mathrm{arsinh}\left(ax\right)^2+\frac{1}{35}\left(5\,a^6c^3x^7+21\,a^4c^3x^5+35\,a^2c^3x^3+35\,c^3x\right)\mathrm{arsinh}\left(ax\right)^3-\frac{2}{13505625}\left(16875\sqrt{a^2x^2+1}\,a^4c^3x^6+134541\sqrt{a^2x^2+1}\,a^2c^3x^4+747937\sqrt{a^2x^2+1}\,c^3x^2+\frac{22329151\sqrt{a^2x^2+1}\,c^3}{a^2}-\frac{105\left(1125\,a^6c^3x^7+7371\,a^4c^3x^5+26495\,a^2c^3x^3+226905\,c^3x\right)\mathrm{arsinh}\left(ax\right)}{a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/1225*(75*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 351*sqrt(a^2*x^2 + 1)*a^2*c^3*x
^4 + 757*sqrt(a^2*x^2 + 1)*c^3*x^2 + 2161*sqrt(a^2*x^2 + 1)*c^3/a^2)*a*arcs
inh(a*x)^2 + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3
*x)*arcsinh(a*x)^3 - 2/13505625*(16875*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 1345
41*sqrt(a^2*x^2 + 1)*a^2*c^3*x^4 + 747937*sqrt(a^2*x^2 + 1)*c^3*x^2 + 22329
151*sqrt(a^2*x^2 + 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 + 7371*a^4*c^3*x^5 +
26495*a^2*c^3*x^3 + 226905*c^3*x)*arcsinh(a*x)/a)*a
```

Fricas [A]
time = 0.40, size = 248, normalized size = 0.69

$$\frac{385875\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-11025\left(75\,a^6c^3x^6+351\,a^4c^3x^4+757\,a^2c^3x^2+2161\,c^3\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2+210\left(1125\,a^7c^3x^7+7371\,a^5c^3x^5+26495\,a^3c^3x^3+226905\,ac^3x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-2\left(16875\,a^6c^3x^6+134541\,a^4c^3x^4+747937\,a^2c^3x^2+22329151\,c^3\right)\sqrt{a^2x^2+1}}{13505625\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/13505625*(385875*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*
c^3*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 11025*(75*a^6*c^3*x^6 + 351*a^4*c^3
*x^4 + 757*a^2*c^3*x^2 + 2161*c^3)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 + 1))^2 + 210*(1125*a^7*c^3*x^7 + 7371*a^5*c^3*x^5 + 26495*a^3*c^3*x^3 + 2
26905*a*c^3*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(16875*a^6*c^3*x^6 + 134541
*a^4*c^3*x^4 + 747937*a^2*c^3*x^2 + 22329151*c^3)*sqrt(a^2*x^2 + 1))/a
```

Sympy [A]
time = 1.48, size = 355, normalized size = 0.99

$$\begin{cases}\frac{a^6c^3x^7\operatorname{asinh}^3(ax)}{7}+\frac{6a^6c^3x^7\operatorname{asinh}(ax)}{343}-\frac{3a^5c^3x^6\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{49}-\frac{6a^5c^3x^6\sqrt{a^2x^2+1}}{2401}+\frac{3a^4c^3x^5\operatorname{asinh}^3(ax)}{5}+\frac{702a^4c^3x^5\operatorname{asinh}(ax)}{6125}-\frac{351a^3c^3x^4\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{1225}-\frac{29898a^3c^3x^4\sqrt{a^2x^2+1}}{1500625}+a^2c^3x^3\operatorname{asinh}^3(ax)+\frac{1514a^2c^3x^3\operatorname{asinh}(ax)}{3675}-\frac{757ac^3x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{1225}-\frac{1495874ac^3x^2\sqrt{a^2x^2+1}}{13505625}+c^3x\operatorname{asinh}^3(ax)+\frac{4322c^3x\operatorname{asinh}(ax)}{1225}-\frac{2161c^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{1225a}-\frac{44658302c^3\sqrt{a^2x^2+1}}{13505625a} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**6*c**3*x**7*asinh(a*x)**3/7 + 6*a**6*c**3*x**7*asinh(a*x)/343
 - 3*a**5*c**3*x**6*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/49 - 6*a**5*c**3*x**6
*sqrt(a**2*x**2 + 1)/2401 + 3*a**4*c**3*x**5*asinh(a*x)**3/5 + 702*a**4*c**
3*x**5*asinh(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)*
```

```
*2/1225 - 29898*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)/1500625 + a**2*c**3*x**3
*asinh(a*x)**3 + 1514*a**2*c**3*x**3*asinh(a*x)/3675 - 757*a*c**3*x**2*sqrt
(a**2*x**2 + 1)*asinh(a*x)**2/1225 - 1495874*a*c**3*x**2*sqrt(a**2*x**2 + 1
)/13505625 + c**3*x*asinh(a*x)**3 + 4322*c**3*x*asinh(a*x)/1225 - 2161*c**3
*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(1225*a) - 44658302*c**3*sqrt(a**2*x**2
+ 1)/(13505625*a), Ne(a, 0)), (0, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{asinh}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.329 $\int (c+a^2cx^2)^2 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=265

$$-\frac{4144c^2\sqrt{1+a^2x^2}}{1125a}-\frac{272c^2(1+a^2x^2)^{3/2}}{3375a}-\frac{6c^2(1+a^2x^2)^{5/2}}{625a}+\frac{298}{75}c^2x\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{12}$$

```
[Out] -272/3375*c^2*(a^2*x^2+1)^(3/2)/a-6/625*c^2*(a^2*x^2+1)^(5/2)/a+298/75*c^2*
x*arcsinh(a*x)+76/225*a^2*c^2*x^3*arcsinh(a*x)+6/125*a^4*c^2*x^5*arcsinh(a*
x)-4/15*c^2*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a-3/25*c^2*(a^2*x^2+1)^(5/2)*a
rcsinh(a*x)^2/a+8/15*c^2*x*arcsinh(a*x)^3+4/15*c^2*x*(a^2*x^2+1)*arcsinh(a*
x)^3+1/5*c^2*x*(a^2*x^2+1)^2*arcsinh(a*x)^3-4144/1125*c^2*(a^2*x^2+1)^(1/2)
/a-8/5*c^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A]
time = 0.30, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {5786, 5772, 5798, 267, 5784, 455, 45, 200, 12, 1261, 712}

$$\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)-\frac{6c^2(a^2x^2+1)^{5/2}}{625a}-\frac{272c^2(a^2x^2+1)^{3/2}}{3375a}-\frac{4144c^2\sqrt{a^2x^2+1}}{1125a}+\frac{1}{5}c^2x(a^2x^2+1)^2\sinh^{-1}(ax)^3+\frac{4}{15}c^2x(a^2x^2+1)\sinh^{-1}(ax)^3-\frac{3c^2(a^2x^2+1)^{5/2}\sinh^{-1}(ax)^2}{25a}-\frac{4c^2(a^2x^2+1)^{3/2}\sinh^{-1}(ax)^2}{15a}-\frac{8c^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{5a}+\frac{8}{15}c^2x\sinh^{-1}(ax)^3+\frac{298}{75}c^2x\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```

```
[Out] (-4144*c^2*Sqrt[1 + a^2*x^2])/(1125*a) - (272*c^2*(1 + a^2*x^2)^(3/2))/(337
5*a) - (6*c^2*(1 + a^2*x^2)^(5/2))/(625*a) + (298*c^2*x*ArcSinh[a*x])/75 +
(76*a^2*c^2*x^3*ArcSinh[a*x])/225 + (6*a^4*c^2*x^5*ArcSinh[a*x])/125 - (8*c
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(5*a) - (4*c^2*(1 + a^2*x^2)^(3/2)*Arc
Sinh[a*x]^2)/(15*a) - (3*c^2*(1 + a^2*x^2)^(5/2)*ArcSinh[a*x]^2)/(25*a) + (
8*c^2*x*ArcSinh[a*x]^3)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/15 + (c
^2*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
```

```
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^3\,dx = \frac{1}{5}c^2x\left(1+a^2x^2\right)^2\sinh^{-1}(ax)^3+\frac{1}{5}(4c)\int \left(c+a^2cx^2\right)\sinh^{-1}(ax)^3\,dx-\frac{1}{5}(3c$$

$$= -\frac{3c^2\left(1+a^2x^2\right)^{5/2}\sinh^{-1}(ax)^2}{25a}+\frac{4}{15}c^2x\left(1+a^2x^2\right)\sinh^{-1}(ax)^3+\frac{1}{5}c^2x(1+$$

$$= \frac{6}{25}c^2x\sinh^{-1}(ax)+\frac{4}{25}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)-\frac{4c^2(1+}{}$$

$$= \frac{58}{75}c^2x\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)-\frac{8c^2\sqrt{}}{}$$

$$= \frac{298}{75}c^2x\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)-\frac{8c^2}{}$$

$$= -\frac{16c^2\sqrt{1+a^2x^2}}{5a}+\frac{298}{75}c^2x\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{125}a^4c^2$$

$$= -\frac{4144c^2\sqrt{1+a^2x^2}}{1125a}-\frac{272c^2\left(1+a^2x^2\right)^{3/2}}{3375a}-\frac{6c^2\left(1+a^2x^2\right)^{5/2}}{625a}+\frac{298}{75}c^2x\sin$$

Mathematica [A]
time = 0.08, size = 137, normalized size = 0.52

$$\frac{c^2\left(-2\sqrt{1+a^2x^2}\left(31841+842a^2x^2+81a^4x^4\right)+30ax\left(2235+190a^2x^2+27a^4x^4\right)\sinh^{-1}(ax)-225\sqrt{1+a^2x^2}\left(149+38a^2x^2+9a^4x^4\right)\sinh^{-1}(ax)^2+1125ax\left(15+10a^2x^2+3a^4x^4\right)\sinh^{-1}(ax)^3\right)}{16875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```

```
[Out] (c^2*(-2*Sqrt[1 + a^2*x^2]*(31841 + 842*a^2*x^2 + 81*a^4*x^4) + 30*a*x*(223
5 + 190*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] - 225*Sqrt[1 + a^2*x^2]*(149 + 3
```

```
8*a^2*x^2 + 9*a^4*x^4)*ArcSinh[a*x]^2 + 1125*a*x*(15 + 10*a^2*x^2 + 3*a^4*x
^4)*ArcSinh[a*x]^3))/(16875*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{2} \operatorname{arcsinh}\left(a x \right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x)
```

Maxima [A]
time = 0.26, size = 210, normalized size = 0.79

$$-\frac{1}{75} \left(9 \sqrt{a^2x^2+1}\,a^2c^2x^4+38 \sqrt{a^2x^2+1}\,c^2x^2+\frac{149 \sqrt{a^2x^2+1}\,c^2}{a^2}\right) a\operatorname{arsinh}\left(ax\right)^2+\frac{1}{15} \left(3\,a^4c^2x^5+10\,a^2c^2x^3+15\,c^2x\right) \operatorname{arsinh}\left(ax\right)^3-\frac{2}{16875} \left(81 \sqrt{a^2x^2+1}\,a^2c^2x^4+842 \sqrt{a^2x^2+1}\,c^2x^2-\frac{15 \left(27\,a^4c^2x^5+190\,a^2c^2x^3+2235\,c^2x\right) \operatorname{arsinh}\left(ax\right)}{a}+\frac{31841 \sqrt{a^2x^2+1}\,c^2}{a^2}\right) a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/75*(9*sqrt(a^2*x^2 + 1)*a^2*c^2*x^4 + 38*sqrt(a^2*x^2 + 1)*c^2*x^2 + 149
*sqrt(a^2*x^2 + 1)*c^2/a^2)*a*arcsinh(a*x)^2 + 1/15*(3*a^4*c^2*x^5 + 10*a^2
*c^2*x^3 + 15*c^2*x)*arcsinh(a*x)^3 - 2/16875*(81*sqrt(a^2*x^2 + 1)*a^2*c^2
*x^4 + 842*sqrt(a^2*x^2 + 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 + 190*a^2*c^2*x^3
 + 2235*c^2*x)*arcsinh(a*x)/a + 31841*sqrt(a^2*x^2 + 1)*c^2/a^2)*a
```

Fricas [A]
time = 0.41, size = 204, normalized size = 0.77

$$\frac{1125 \left(3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x\right) \log\left(ax+\sqrt{a^2x^2+1}\right)^3-225 \left(9\,a^4c^2x^4+38\,a^2c^2x^2+149\,c^2\right) \sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2+30 \left(27\,a^5c^2x^5+190\,a^3c^2x^3+2235\,ac^2x\right) \log\left(ax+\sqrt{a^2x^2+1}\right)-2 \left(81\,a^4c^2x^4+842\,a^2c^2x^2+31841\,c^2\right) \sqrt{a^2x^2+1}}{16875\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(1125*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(
a^2*x^2 + 1))^3 - 225*(9*a^4*c^2*x^4 + 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x
^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 30*(27*a^5*c^2*x^5 + 190*a^3*c^2*x
^3 + 2235*a*c^2*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(81*a^4*c^2*x^4 + 842*a
^2*c^2*x^2 + 31841*c^2)*sqrt(a^2*x^2 + 1))/a
```

Sympy [A]
time = 0.71, size = 262, normalized size = 0.99

$$\begin{cases} \frac{a^{4} c^{2} x^{5} \operatorname{asinh}^{3}{\left(a x \right)}}{5} + \frac{6 a^{4} c^{2} x^{5} \operatorname{asinh}{\left(a x \right)}}{125} - \frac{3 a^{3} c^{2} x^{4} \sqrt{a^{2} x^{2} + 1} \operatorname{asinh}^{2}{\left(a x \right)}}{25} - \frac{6 a^{3} c^{2} x^{4} \sqrt{a^{2} x^{2} + 1}}{625} + \frac{2 a^{2} c^{2} x^{3} \operatorname{asinh}^{3}{\left(a x \right)}}{3} + \frac{76 a^{2} c^{2} x^{3} \operatorname{asinh}{\left(a x \right)}}{225} - \frac{38 a c^{2} x^{2} \sqrt{a^{2} x^{2} + 1} \operatorname{asinh}^{2}{\left(a x \right)}}{75} - \frac{1684 a c^{2} x^{2} \sqrt{a^{2} x^{2} + 1}}{16875} + c^{2} x \operatorname{asinh}^{3}{\left(a x \right)} + \frac{298 c^{2} x \operatorname{asinh}{\left(a x \right)}}{75} - \frac{149 c^{2} \sqrt{a^{2} x^{2} + 1} \operatorname{asinh}^{2}{\left(a x \right)}}{75 a} - \frac{63682 c^{2} \sqrt{a^{2} x^{2} + 1}}{16875 a} & \text{for}\: a \neq 0 \\0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2*c*x**2+c)**2*asinh(a*x)**3,x)

[Out] Piecewise((a**4*c**2*x**5*asinh(a*x)**3/5 + 6*a**4*c**2*x**5*asinh(a*x)/125 - 3*a**3*c**2*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/25 - 6*a**3*c**2*x**4*sqrt(a**2*x**2 + 1)/625 + 2*a**2*c**2*x**3*asinh(a*x)**3/3 + 76*a**2*c**2*x**3*asinh(a*x)/225 - 38*a*c**2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/75 - 1684*a*c**2*x**2*sqrt(a**2*x**2 + 1)/16875 + c**2*x*asinh(a*x)**3 + 298*c**2*x*asinh(a*x)/75 - 149*c**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(75*a) - 63682*c**2*sqrt(a**2*x**2 + 1)/(16875*a), Ne(a, 0)), (0, True))

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{asinh}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^2,x)

[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^2, x)

3.330 $\int (c+a^2cx^2)\sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=153

$$-\frac{40c\sqrt{1+a^2x^2}}{9a}-\frac{2c(1+a^2x^2)^{3/2}}{27a}+\frac{14}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-\frac{2c\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a}-\frac{c(1}{}$$

```
[Out] -2/27*c*(a^2*x^2+1)^(3/2)/a+14/3*c*x*arcsinh(a*x)+2/9*a^2*c*x^3*arcsinh(a*x
)-1/3*c*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a+2/3*c*x*arcsinh(a*x)^3+1/3*c*x*(
a^2*x^2+1)*arcsinh(a*x)^3-40/9*c*(a^2*x^2+1)^(1/2)/a-2*c*arcsinh(a*x)^2*(a^
2*x^2+1)^(1/2)/a
```

Rubi [A]
time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5786, 5772, 5798, 267, 5784, 455, 45}

$$\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-\frac{2c(a^2x^2+1)^{3/2}}{27a}-\frac{40c\sqrt{a^2x^2+1}}{9a}+\frac{1}{3}cx(a^2x^2+1)\sinh^{-1}(ax)^3-\frac{c(a^2x^2+1)^{3/2}\sinh^{-1}(ax)^2}{3a}-\frac{2c\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{a}+\frac{2}{3}cx\sinh^{-1}(ax)^3+\frac{14}{3}cx\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-40*c*Sqrt[1 + a^2*x^2])/(9*a) - (2*c*(1 + a^2*x^2)^(3/2))/(27*a) + (14*c*
x*ArcSinh[a*x])/3 + (2*a^2*c*x^3*ArcSinh[a*x])/9 - (2*c*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^2)/a - (c*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(3*a) + (2*c*x*Ar
cSinh[a*x]^3)/3 + (c*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)\sinh^{-1}(ax)^3\,dx = \frac{1}{3}cx\left(1+a^2x^2\right)\sinh^{-1}(ax)^3+\frac{1}{3}(2c)\int \sinh^{-1}(ax)^3\,dx-(ac)\int x\sqrt{1+a^2x}$$

$$= -\frac{c\left(1+a^2x^2\right)^{3/2}\sinh^{-1}(ax)^2}{3a}+\frac{2}{3}cx\sinh^{-1}(ax)^3+\frac{1}{3}cx\left(1+a^2x^2\right)\sinh^{-1}(a$$

$$= \frac{2}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-\frac{2c\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a}-\frac{c(1+}{}$$

$$= \frac{14}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-\frac{2c\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a}-\frac{c(1}{}$$

$$= -\frac{4c\sqrt{1+a^2x^2}}{a}+\frac{14}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-\frac{2c\sqrt{1+a^2x^2}}{a}$$

$$= -\frac{40c\sqrt{1+a^2x^2}}{9a}-\frac{2c\left(1+a^2x^2\right)^{3/2}}{27a}+\frac{14}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(a$$

Mathematica [A]
time = 0.07, size = 99, normalized size = 0.65

$$\frac{c\left(-2\sqrt{1+a^2x^2}\left(61+a^2x^2\right)+6ax\left(21+a^2x^2\right)\sinh^{-1}(ax)-9\sqrt{1+a^2x^2}\left(7+a^2x^2\right)\sinh^{-1}(ax)^2+9ax\left(3+a^2x^2\right)\sinh^{-1}(ax)^3\right)}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c*(-2*Sqrt[1 + a^2*x^2]*(61 + a^2*x^2) + 6*a*x*(21 + a^2*x^2)*ArcSinh[a*x]
 - 9*Sqrt[1 + a^2*x^2]*(7 + a^2*x^2)*ArcSinh[a*x]^2 + 9*a*x*(3 + a^2*x^2)*A
rcSinh[a*x]^3))/(27*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)\operatorname{arcsinh}\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arcsinh(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)*arcsinh(a*x)^3,x)
```

Maxima [A]
time = 0.28, size = 124, normalized size = 0.81

$$-\frac{1}{3}\left(\sqrt{a^2x^2+1}\,cx^2+\frac{7\sqrt{a^2x^2+1}\,c}{a^2}\right)a\operatorname{arsinh}\left(ax\right)^2+\frac{1}{3}\left(a^2cx^3+3\,cx\right)\operatorname{arsinh}\left(ax\right)^3-\frac{2}{27}\left(\sqrt{a^2x^2+1}\,cx^2-\frac{3\left(a^2cx^3+21\,cx\right)\operatorname{arsinh}\left(ax\right)}{a}+\frac{61\sqrt{a^2x^2+1}\,c}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(a^2*x^2 + 1)*c*x^2 + 7*sqrt(a^2*x^2 + 1)*c/a^2)*a*arcsinh(a*x)^2
 + 1/3*(a^2*c*x^3 + 3*c*x)*arcsinh(a*x)^3 - 2/27*(sqrt(a^2*x^2 + 1)*c*x^2 -
 3*(a^2*c*x^3 + 21*c*x)*arcsinh(a*x)/a + 61*sqrt(a^2*x^2 + 1)*c/a^2)*a
```

Fricas [A]
time = 0.42, size = 140, normalized size = 0.92

$$\frac{9\left(a^{3}cx^{3}+3\,acx\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}-9\left(a^{2}cx^{2}+7\,c\right)\sqrt{a^{2}x^{2}+1}\,\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}+6\left(a^{3}cx^{3}+21\,acx\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)-2\left(a^{2}cx^{2}+61\,c\right)\sqrt{a^{2}x^{2}+1}}{27\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*(a^3*c*x^3 + 3*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*(a^2*c*x^2
 + 7*c)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*c*x^3 + 2
1*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(a^2*c*x^2 + 61*c)*sqrt(a^2*x^2 +
 1))/a
```

Sympy [A]
time = 0.31, size = 150, normalized size = 0.98

$$\begin{cases} \frac{a^{2}cx^{3}\operatorname{asinh}^{3}\left(ax\right)}{3}+\frac{2a^{2}cx^{3}\operatorname{asinh}\left(ax\right)}{9}-\frac{acx^{2}\sqrt{a^{2}x^{2}+1}\operatorname{asinh}^{2}\left(ax\right)}{3}-\frac{2acx^{2}\sqrt{a^{2}x^{2}+1}}{27}+cx\operatorname{asinh}^{3}\left(ax\right)+\frac{14cx\operatorname{asinh}\left(ax\right)}{3}-\frac{7c\sqrt{a^{2}x^{2}+1}\operatorname{asinh}^{2}\left(ax\right)}{3a}-\frac{122c\sqrt{a^{2}x^{2}+1}}{27a} & \text{for}\: a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**2*c*x**3*asinh(a*x)**3/3 + 2*a**2*c*x**3*asinh(a*x)/9 - a*c*x
**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/3 - 2*a*c*x**2*sqrt(a**2*x**2 + 1)/27
 + c*x*asinh(a*x)**3 + 14*c*x*asinh(a*x)/3 - 7*c*sqrt(a**2*x**2 + 1)*asinh(
a*x)**2/(3*a) - 122*c*sqrt(a**2*x**2 + 1)/(27*a), Ne(a, 0)), (0, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2),x)
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2), x)
```

3.331 $\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2} dx$

Optimal. Leaf size=174

$$\frac{2\sinh^{-1}(ax)^3\text{ArcTan}\left(e^{\sinh^{-1}(ax)}\right)}{ac}-\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{ac}$$

```
[Out] 2*arcsinh(a*x)^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c-3*I*arcsinh(a*x)^2*polyl
og(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c+3*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a
^2*x^2+1)^(1/2)))/a/c+6*I*arcsinh(a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2))
)/a/c-6*I*arcsinh(a*x)*polylog(3,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c-6*I*polylog
(4,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c+6*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1/2)))
/a/c
```

Rubi [A]
time = 0.09, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5789, 4265, 2611, 6744, 2320, 6724}

$$\frac{2\sinh^{-1}(ax)^3\text{ArcTan}\left(e^{\sinh^{-1}(ax)}\right)}{ac}-\frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{6i\sinh^{-1}(ax)\text{Li}_3\left(-ie^{\sinh^{-1}(ax)}\right)}{ac}-\frac{6i\sinh^{-1}(ax)\text{Li}_3\left(ie^{\sinh^{-1}(ax)}\right)}{ac}-\frac{6i\text{Li}_4\left(-ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{6i\text{Li}_4\left(ie^{\sinh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c) - ((3*I)*ArcSinh[a*x]^2*Pol
yLog[2, (-I)*E^ArcSinh[a*x]])/(a*c) + ((3*I)*ArcSinh[a*x]^2*PolyLog[2, I*E^
ArcSinh[a*x]])/(a*c) + ((6*I)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])
/(a*c) - ((6*I)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c) - ((6*I)*P
olyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c) + ((6*I)*PolyLog[4, I*E^ArcSinh[a*x]]
)/(a*c)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2}\,dx = \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \sinh^{-1}(ax)\right)}{ac}$$
$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{(3i)\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \sinh^{-1}(ax)\right)}{ac} + \frac{(3$$
$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{L}}{ac}$$
$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{L}}{ac}$$
$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{L}}{ac}$$
$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{L}}{ac}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 454 vs. $2(174) = 348$.
time = 0.17, size = 454, normalized size = 2.61

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] ((-1/64*I)*(7*Pi^4 + (8*I)*Pi^3*ArcSinh[a*x] + 24*Pi^2*ArcSinh[a*x]^2 - (32
*I)*Pi*ArcSinh[a*x]^3 - 16*ArcSinh[a*x]^4 + (8*I)*Pi^3*Log[1 + I/E^ArcSinh[
a*x]] + 48*Pi^2*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] - (96*I)*Pi*ArcSinh[
a*x]^2*Log[1 + I/E^ArcSinh[a*x]] - 64*ArcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*
x]] - 48*Pi^2*ArcSinh[a*x]*Log[1 - I*E^ArcSinh[a*x]] + (96*I)*Pi*ArcSinh[a*
x]^2*Log[1 - I*E^ArcSinh[a*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 64*
ArcSinh[a*x]^3*Log[1 + I*E^ArcSinh[a*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*A
rcSinh[a*x])/4]] - 48*(Pi - (2*I)*ArcSinh[a*x])^2*PolyLog[2, (-I)/E^ArcSinh
[a*x]] + 192*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]] - 48*Pi^2*PolyL
og[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*ArcSinh[a*x]*PolyLog[2, I*E^ArcSinh[a*
x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*x]] + 384*ArcSinh[a*x]*PolyLog
[3, (-I)/E^ArcSinh[a*x]] - 384*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]]
 - (192*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]] + 384*PolyLog[4, (-I)/E^ArcSinh[
a*x]] + 384*PolyLog[4, (-I)*E^ArcSinh[a*x]]))/(a*c)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^3}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{asinh}^{3}(ax)}{a^{2}x^{2}+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c),x)
[Out] Integral(asinh(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2),x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2), x)
```

3.332 $\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=294

$$\frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}}+\frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\frac{6\sinh^{-1}(ax)\text{ArcTan}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2}+\frac{\sinh^{-1}(ax)^3\text{ArcTan}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2}+\frac{3i\text{P}}{}$$

```
[Out] 1/2*x*arcsinh(a*x)^3/c^2/(a^2*x^2+1)-6*arcsinh(a*x)*arctan(a*x+(a^2*x^2+1)^
(1/2))/a/c^2+arcsinh(a*x)^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^2+3*I*polylog
(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3/2*I*arcsinh(a*x)^2*polylog(2,-I*(a*x
+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3
/2*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3*I*arcsinh(
a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*arcsinh(a*x)*polylog(3
,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(4,-I*(a*x+(a^2*x^2+1)^(1/2)))
/a/c^2+3*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3/2*arcsinh(a*x)^2/a/
c^2/(a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {5788, 5789, 4265, 2611, 6744, 2320, 6724, 5798, 2317, 2438}

$$\frac{x\sinh^{-1}(ax)^3}{2c^2\left(a^2x^2+1\right)}+\frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{a^2x^2+1}}+\frac{\sinh^{-1}(ax)^3\text{ArcTan}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2}-\frac{6\sinh^{-1}(ax)\text{ArcTan}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2}-\frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)\text{Li}_3\left(-ie^{\sinh^{-1}(ax)}\right)}{ac^2}-\frac{3i\sinh^{-1}(ax)\text{Li}_3\left(ie^{\sinh^{-1}(ax)}\right)}{ac^2}+\frac{3i\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac^2}-\frac{3i\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right)}{ac^2}-\frac{3i\text{Li}_4\left(-ie^{\sinh^{-1}(ax)}\right)}{ac^2}+\frac{3i\text{Li}_4\left(ie^{\sinh^{-1}(ax)}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*ArcSinh[a*x]^2)/(2*a*c^2*Sqrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(2*c^2*
(1 + a^2*x^2)) - (6*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + (ArcSinh
[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*PolyLog[2, (-I)*E^ArcSinh[
a*x]])/(a*c^2) - (((3*I)/2)*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]])
/(a*c^2) - ((3*I)*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + (((3*I)/2)*ArcSin
h[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*ArcSinh[a*x]*PolyLo
g[3, (-I)*E^ArcSinh[a*x]])/(a*c^2) - ((3*I)*ArcSinh[a*x]*PolyLog[3, I*E^Arc
Sinh[a*x]])/(a*c^2) - ((3*I)*PolyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c^2) + ((3
*I)*PolyLog[4, I*E^ArcSinh[a*x]])/(a*c^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx = \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{(3a)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2}\,dx}{2c}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{3\int \frac{\sinh^{-1}(ax)}{1+a^2x^2}\,dx}{c^2} + \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \sinh^{-1}\right.}{2ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} - \frac{(3i)\text{Subst}\left(\int x^2\right.}{}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\,\text{ta}}{}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\,\text{ta}}{}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\,\text{ta}}{}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\,\text{ta}}{}$$

Mathematica [A]
time = 1.60, size = 568, normalized size = 1.93

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcSinh[a*x] + 24*Pi^2*ArcSinh[a*x]^2 + ((
192*I)*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2] - (32*I)*Pi*ArcSinh[a*x]^3 + ((64*
I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 16*ArcSinh[a*x]^4 - 384*ArcSinh[a*x]
*Log[1 - I/E^ArcSinh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcSinh[a*x]] + 384*Arc
Sinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 48*Pi^2*ArcSinh[a*x]*Log[1 + I/E^ArcS
inh[a*x]] - (96*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^ArcSinh[a*x]] - 64*ArcSinh
[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 48*Pi^2*ArcSinh[a*x]*Log[1 - I*E^ArcSin
h[a*x]] + (96*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^ArcSinh[a*x]] - (8*I)*Pi^3*L
og[1 + I*E^ArcSinh[a*x]] + 64*ArcSinh[a*x]^3*Log[1 + I*E^ArcSinh[a*x]] + (8
*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]] - 48*(8 + Pi^2 - (4*I)*Pi*Ar
cSinh[a*x] - 4*ArcSinh[a*x]^2)*PolyLog[2, (-I)/E^ArcSinh[a*x]] + 384*PolyLo
g[2, I/E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]]
 - 48*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*ArcSinh[a*x]*PolyLog[2
, I*E^ArcSinh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*x]] + 384*ArcS
inh[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x]] - 384*ArcSinh[a*x]*PolyLog[3, (-I)
*E^ArcSinh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]] + 384*PolyLog[4,
 (-I)/E^ArcSinh[a*x]] + 384*PolyLog[4, (-I)*E^ArcSinh[a*x]]))/(a*c^2)
```

Maple [F]
time = 2.05, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(a^2c\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{asinh}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**2,x)
[Out] Integral(asinh(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^2,x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^2, x)
```

3.333 $\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=409

$$-\frac{1}{4ac^3\sqrt{1+a^2x^2}}-\frac{x\sinh^{-1}(ax)}{4c^3\left(1+a^2x^2\right)}+\frac{\sinh^{-1}(ax)^2}{4ac^3\left(1+a^2x^2\right)^{3/2}}+\frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}}+\frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\sinh^{-1}(ax)^3}{8c^3\left(1+a^2x^2\right)}-$$

```
[Out] -1/4*x*arcsinh(a*x)/c^3/(a^2*x^2+1)+1/4*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^(3
/2)+1/4*x*arcsinh(a*x)^3/c^3/(a^2*x^2+1)^2+3/8*x*arcsinh(a*x)^3/c^3/(a^2*x^
2+1)-5*arcsinh(a*x)*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^3+3/4*arcsinh(a*x)^3*
arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^3-9/4*I*arcsinh(a*x)*polylog(3,I*(a*x+(a^
2*x^2+1)^(1/2)))/a/c^3-9/4*I*polylog(4,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+9/
4*I*arcsinh(a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-5/2*I*polylog(
2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+9/8*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a
^2*x^2+1)^(1/2)))/a/c^3+5/2*I*polylog(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-9
/8*I*arcsinh(a*x)^2*polylog(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+9/4*I*polyl
og(4,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-1/4/a/c^3/(a^2*x^2+1)^(1/2)+9/8*arcsi
nh(a*x)^2/a/c^3/(a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {5788, 5789, 4265, 2611, 6744, 2320, 6724, 5798, 2317, 2438, 267}

$$-\frac{1}{4ac^3\sqrt{a^2x^2+1}}+\frac{3x\sinh^{-1}(ax)^3}{8c^3\left(a^2x^2+1\right)}+\frac{x\sinh^{-1}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{a^2x^2+1}}+\frac{\sinh^{-1}(ax)^2}{4ac^3\left(a^2x^2+1\right)^{3/2}}-\frac{x\sinh^{-1}(ax)}{4c^3\left(a^2x^2+1\right)}+\frac{3\sinh^{-1}(ax)^3\text{ArcTan}\left(e^{\sinh^{-1}(ax)}\right)}{4ac^3}-\frac{5\sinh^{-1}(ax)\text{ArcTan}\left(e^{\sinh^{-1}(ax)}\right)}{ac^3}-\frac{9i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{8ac^3}+\frac{9i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right)}{8ac^3}+\frac{9i\sinh^{-1}(ax)\text{Li}_3\left(-ie^{\sinh^{-1}(ax)}\right)}{4ac^3}-\frac{9i\sinh^{-1}(ax)\text{Li}_3\left(ie^{\sinh^{-1}(ax)}\right)}{4ac^3}+\frac{5i\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{2ac^3}-\frac{5i\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right)}{2ac^3}-\frac{9i\text{Li}_4\left(-ie^{\sinh^{-1}(ax)}\right)}{4ac^3}+\frac{9i\text{Li}_4\left(ie^{\sinh^{-1}(ax)}\right)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/4*1/(a*c^3*Sqrt[1 + a^2*x^2]) - (x*ArcSinh[a*x])/(4*c^3*(1 + a^2*x^2)) +
 ArcSinh[a*x]^2/(4*a*c^3*(1 + a^2*x^2)^(3/2)) + (9*ArcSinh[a*x]^2)/(8*a*c^3
*Sqrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) + (3*x*Arc
Sinh[a*x]^3)/(8*c^3*(1 + a^2*x^2)) - (5*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]]
)/(a*c^3) + (3*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(4*a*c^3) + (((5*I)/2
)*PolyLog[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/8)*ArcSinh[a*x]^2*Poly
Log[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((5*I)/2)*PolyLog[2, I*E^ArcSinh[a*
x]])/(a*c^3) + (((9*I)/8)*ArcSinh[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c
^3) + (((9*I)/4)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])/(a*c^3) - ((
(9*I)/4)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/4)*Po
lyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c^3) + (((9*I)/4)*PolyLog[4, I*E^ArcSinh[
a*x]])/(a*c^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
```

```
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{x\sinh^{-1}(ax)^3}{4c^3\,(1+a^2x^2)^2} - \frac{(3a)\int \frac{x\sinh^{-1}(ax)^2}{(1+a^2x^2)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx}{4c}$$

$$= \frac{\sinh^{-1}(ax)^2}{4ac^3\,(1+a^2x^2)^{3/2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\,(1+a^2x^2)^2} + \frac{3x\sinh^{-1}(ax)^3}{8c^3\,(1+a^2x^2)} - \frac{\int \frac{\sinh^{-1}(ax)}{(1+a^2x^2)^2}\,dx}{2c^3} - \frac{(9a)\int \frac{x\sinh}{(1+a}}{8c}$$

$$= -\frac{x\sinh^{-1}(ax)}{4c^3\,(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\,(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\,(1+a^2x^2)^2} + \frac{3x\sinh}{8c^3\,(1-}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\,(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\,(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh}{4c^3\,(1}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\,(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\,(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh}{4c^3\,(1}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\,(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\,(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh}{4c^3\,(1}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\,(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\,(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh}{4c^3\,(1}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\,(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\,(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh}{4c^3\,(1}$$

Mathematica [A]
time = 4.66, size = 654, normalized size = 1.60

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((-1/512*I)*(21*Pi^4 - (128*I)/Sqrt[1 + a^2*x^2] + (24*I)*Pi^3*ArcSinh[a*x]
 - ((128*I)*a*x*ArcSinh[a*x])/(1 + a^2*x^2) + 72*Pi^2*ArcSinh[a*x]^2 + ((12
8*I)*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + ((576*I)*ArcSinh[a*x]^2)/Sqrt[1
+ a^2*x^2] - (96*I)*Pi*ArcSinh[a*x]^3 + ((128*I)*a*x*ArcSinh[a*x]^3)/(1 + a
^2*x^2)^2 + ((192*I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 48*ArcSinh[a*x]^4
- 1280*ArcSinh[a*x]*Log[1 - I/E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[1 + I/E^Arc
Sinh[a*x]] + 1280*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 144*Pi^2*ArcSinh
[a*x]*Log[1 + I/E^ArcSinh[a*x]] - (288*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^Arc
Sinh[a*x]] - 192*ArcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 144*Pi^2*ArcSin
```

```
h[a*x]*Log[1 - I*E^ArcSinh[a*x]] + (288*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^Ar
cSinh[a*x]] - (24*I)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^3*Lo
g[1 + I*E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]]
 - 16*(80 + 9*Pi^2 - (36*I)*Pi*ArcSinh[a*x] - 36*ArcSinh[a*x]^2)*PolyLog[2,
 (-I)/E^ArcSinh[a*x]] + 1280*PolyLog[2, I/E^ArcSinh[a*x]] + 576*ArcSinh[a*x
]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]] - 144*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]]
 + (576*I)*Pi*ArcSinh[a*x]*PolyLog[2, I*E^ArcSinh[a*x]] + (576*I)*Pi*PolyLo
g[3, (-I)/E^ArcSinh[a*x]] + 1152*ArcSinh[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x
]] - 1152*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]] - (576*I)*Pi*PolyLog
[3, I*E^ArcSinh[a*x]] + 1152*PolyLog[4, (-I)/E^ArcSinh[a*x]] + 1152*PolyLog
[4, (-I)*E^ArcSinh[a*x]]))/(a*c^3)
```

Maple [F]
time = 2.51, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(ax\right)^{3}}{\left(a^{2} c\,x^{2}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{asinh}^{3}{\left(ax \right)}}{a^{6}x^{6}+3a^{4}x^{4}+3a^{2}x^{2}+1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(asinh(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^3,x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^3, x)
```

3.334 $\int (c+a^2cx^2)^{5/2}\sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=509

$$-\frac{865ac^2x^2\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}}-\frac{65a^3c^2x^4\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}}-\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}}{216a}+\frac{245}{384}c^2x\sqrt{c+a^2cx^2}\,\sinh^{-1}(a$$

```
[Out] 5/24*c*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3+1/6*x*(a^2*c*x^2+c)^(5/2)*arcsi
nh(a*x)^3-1/216*c^2*(a^2*x^2+1)^(5/2)*(a^2*c*x^2+c)^(1/2)/a+245/384*c^2*x*a
rcsinh(a*x)*(a^2*c*x^2+c)^(1/2)+65/576*c^2*x*(a^2*x^2+1)*arcsinh(a*x)*(a^2*
c*x^2+c)^(1/2)+1/36*c^2*x*(a^2*x^2+1)^2*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)-5/
32*c^2*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-1/12*c^2*(a^2
*x^2+1)^(5/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+5/16*c^2*x*arcsinh(a*x)^
3*(a^2*c*x^2+c)^(1/2)-865/2304*a*c^2*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1
/2)-65/2304*a^3*c^2*x^4*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-115/768*c^2*a
rcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/32*a*c^2*x^2*arcsi
nh(a*x)^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+5/64*c^2*arcsinh(a*x)^4*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5786, 5785, 5783, 5776, 5812, 30, 5798, 14, 267}

$$-\frac{865ac^2x^2\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{c^2(a^2x^2+1)^{5/2}\sqrt{a^2cx^2+c}}{216a}-\frac{15ac^2x^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{32\sqrt{a^2x^2+1}}+\frac{5}{16}c^2x\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^3+\frac{245}{384}c^2x\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)+\frac{1}{36}c^2x(a^2x^2+1)^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)+\frac{65}{576}c^2x(a^2x^2+1)\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)+\frac{5c^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^4}{64a\sqrt{a^2x^2+1}}-\frac{c^2(a^2x^2+1)^{5/2}\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{12a}-\frac{5c^2(a^2x^2+1)^{3/2}\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{32a}-\frac{115c^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{768a\sqrt{a^2x^2+1}}+\frac{1}{6}x(a^2cx^2+c)^{5/2}\sinh^{-1}(ax)^3+\frac{5}{24}cx(a^2cx^2+c)^{3/2}\sinh^{-1}(ax)^3-\frac{65a^3c^2x^4\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-865*a*c^2*x^2*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (65*a^3*c^2
*x^4*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (c^2*(1 + a^2*x^2)^(5/
2)*Sqrt[c + a^2*c*x^2])/(216*a) + (245*c^2*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*
x])/384 + (65*c^2*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/576 + (
c^2*x*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/36 - (115*c^2*Sqrt[
c + a^2*c*x^2]*ArcSinh[a*x]^2)/(768*a*Sqrt[1 + a^2*x^2]) - (15*a*c^2*x^2*Sq
rt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*Sqrt[1 + a^2*x^2]) - (5*c^2*(1 + a^2*
x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*a) - (c^2*(1 + a^2*x^2)^
(5/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(12*a) + (5*c^2*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x]^3)/16 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3)/24 +
 (x*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3)/6 + (5*c^2*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x]^4)/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^3\,dx = \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^3+\frac{1}{6}(5c)\int \left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3\,dx -$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{12a}+\frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}($$

$$= \frac{1}{36}c^2x\left(1+a^2x^2\right)^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)-\frac{5c^2(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}{32a}$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}}{216a}+\frac{65}{576}c^2x\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sinh^{-1}($$

$$= -\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}}{216a}+\frac{245}{384}c^2x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)+\frac{65}{576}c^2$$

$$= -\frac{865ac^2x^2\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}}-\frac{65a^3c^2x^4\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}}-\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+}}{216a}$$

Mathematica [A]
time = 0.50, size = 177, normalized size = 0.35

$$\frac{c^2\sqrt{c+a^2cx^2}\left(4320\sinh^{-1}(ax)^4-9720\cosh\left(2\sinh^{-1}(ax)\right)-243\cosh\left(4\sinh^{-1}(ax)\right)-8\cosh\left(6\sinh^{-1}(ax)\right)-72\sinh^{-1}(ax)^2\left(270\cosh\left(2\sinh^{-1}(ax)\right)+27\cosh\left(4\sinh^{-1}(ax)\right)+2\cosh\left(6\sinh^{-1}(ax)\right)\right)+288\sinh^{-1}(ax)^3\left(45\sinh\left(2\sinh^{-1}(ax)\right)+9\sinh\left(4\sinh^{-1}(ax)\right)+\sinh\left(6\sinh^{-1}(ax)\right)\right)+12\sinh^{-1}(ax)\left(1620\sinh\left(2\sinh^{-1}(ax)\right)+81\sinh\left(4\sinh^{-1}(ax)\right)+4\sinh\left(6\sinh^{-1}(ax)\right)\right)\right)}{55296a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(4320*ArcSinh[a*x]^4 - 9720*Cosh[2*ArcSinh[a*x]] -
 243*Cosh[4*ArcSinh[a*x]] - 8*Cosh[6*ArcSinh[a*x]] - 72*ArcSinh[a*x]^2*(270
*Cosh[2*ArcSinh[a*x]] + 27*Cosh[4*ArcSinh[a*x]] + 2*Cosh[6*ArcSinh[a*x]]) +
 288*ArcSinh[a*x]^3*(45*Sinh[2*ArcSinh[a*x]] + 9*Sinh[4*ArcSinh[a*x]] + Sin
h[6*ArcSinh[a*x]]) + 12*ArcSinh[a*x]*(1620*Sinh[2*ArcSinh[a*x]] + 81*Sinh[4
*ArcSinh[a*x]] + 4*Sinh[6*ArcSinh[a*x]])))/(55296*a*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 2.37, size = 802, normalized size = 1.58

method	result
default	$\frac{5\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{arcsinh}\left(ax\right)^{4}c^{2}}{64\sqrt{a^{2}x^{2}+1}\,a}+\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\left(32a^{7}x^{7}+32\sqrt{a^{2}x^{2}+1}\,a^{6}x^{6}+64a^{5}x^{5}+48a^{4}\sqrt{a^{2}x^{2}+1}\,x^{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/64*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c^2+1/13824*(
c*(a^2*x^2+1))^(1/2)*(32*a^7*x^7+32*(a^2*x^2+1)^(1/2)*a^6*x^6+64*a^5*x^5+48
*a^4*(a^2*x^2+1)^(1/2)*x^4+38*a^3*x^3+18*(a^2*x^2+1)^(1/2)*a^2*x^2+6*a*x+(a
^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3-18*arcsinh(a*x)^2+6*arcsinh(a*x)-1)*c^2
/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5+8*a^4*(a^2*x^2+1)^(1
/2)*x^4+12*a^3*x^3+8*(a^2*x^2+1)^(1/2)*a^2*x^2+4*a*x+(a^2*x^2+1)^(1/2))*(32
*arcsinh(a*x)^3-24*arcsinh(a*x)^2+12*arcsinh(a*x)-3)*c^2/a/(a^2*x^2+1)+15/5
12*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3+2*(a^2*x^2+1)^(1/2)*a^2*x^2+2*a*x+(a^2*
x^2+1)^(1/2))*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c^2/a/(a
^2*x^2+1)+15/512*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*(a^2*x^2+1)^(1/2)*a^2*x
^2+2*a*x-(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*
x)+3)*c^2/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*(a^2*
x^2+1)^(1/2)*x^4+12*a^3*x^3-8*(a^2*x^2+1)^(1/2)*a^2*x^2+4*a*x-(a^2*x^2+1)^(
1/2))*(32*arcsinh(a*x)^3+24*arcsinh(a*x)^2+12*arcsinh(a*x)+3)*c^2/a/(a^2*x^
2+1)+1/13824*(c*(a^2*x^2+1))^(1/2)*(32*a^7*x^7-32*(a^2*x^2+1)^(1/2)*a^6*x^6
+64*a^5*x^5-48*a^4*(a^2*x^2+1)^(1/2)*x^4+38*a^3*x^3-18*(a^2*x^2+1)^(1/2)*a^
2*x^2+6*a*x-(a^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3+18*arcsinh(a*x)^2+6*arcsi
nh(a*x)+1)*c^2/a/(a^2*x^2+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arcsinh(a*
x)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*asinh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{asinh}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.335 $\int (c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=348

$$-\frac{51acx^2\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}}-\frac{3a^3cx^4\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}}+\frac{45}{64}cx\sqrt{c+a^2cx^2}\sinh^{-1}(ax)+\frac{3}{32}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sin$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3+45/64*c*x*arcsinh(a*x)*(a^2*c*x^2+
c)^(1/2)+3/32*c*x*(a^2*x^2+1)*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)-3/16*c*(a^2*
x^2+1)^(3/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arcsinh(a*x)^3*(a
^2*c*x^2+c)^(1/2)-51/128*a*c*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-3/12
8*a^3*c*x^4*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-27/128*c*arcsinh(a*x)^2*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-9/16*a*c*x^2*arcsinh(a*x)^2*(a^2*c*x
^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+3/32*c*arcsinh(a*x)^4*(a^2*c*x^2+c)^(1/2)/a/(
a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5786, 5785, 5783, 5776, 5812, 30, 5798, 14}

$$-\frac{51acx^2\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{9acx^2\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{16\sqrt{a^2x^2+1}}+\frac{1}{4}x(a^2cx^2+c)^{3/2}\sinh^{-1}(ax)^3+\frac{3}{8}cx\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^3+\frac{45}{64}cx\sqrt{a^2cx^2+c}\sinh^{-1}(ax)+\frac{3}{32}cx(a^2x^2+1)\sqrt{a^2cx^2+c}\sinh^{-1}(ax)+\frac{3c\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^4}{32a\sqrt{a^2x^2+1}}-\frac{3c(a^2x^2+1)^{3/2}\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{16a}-\frac{27c\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{128a\sqrt{a^2x^2+1}}-\frac{3a^3cx^4\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-51*a*c*x^2*Sqrt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) - (3*a^3*c*x^4*Sq
rt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) + (45*c*x*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x])/64 + (3*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/32
- (27*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(128*a*Sqrt[1 + a^2*x^2]) - (9*
a*c*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*Sqrt[1 + a^2*x^2]) - (3*c*(
1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*a) + (3*c*x*Sqrt
[c + a^2*c*x^2]*ArcSinh[a*x]^3)/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3
)/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^4)/(32*a*Sqrt[1 + a^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
```

```
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3\,dx = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3+\frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^3\,dx -$$

$$= -\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{16a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax$$

$$= \frac{3}{32}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax}{16\sqrt{1+a^2x^2}}$$

$$= \frac{45}{64}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)+\frac{3}{32}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)$$

$$= -\frac{51acx^2\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}}-\frac{3a^3cx^4\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}}+\frac{45}{64}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}$$

Mathematica [A]
time = 0.20, size = 136, normalized size = 0.39

$$\frac{c\sqrt{c+a^2cx^2}\left(96\sinh^{-1}(ax)^4-24\sinh^{-1}(ax)^2\left(16\cosh\left(2\sinh^{-1}(ax)\right)+\cosh\left(4\sinh^{-1}(ax)\right)\right)-3\left(64\cosh\left(2\sinh^{-1}(ax)\right)+\cosh\left(4\sinh^{-1}(ax)\right)\right)+32\sinh^{-1}(ax)^3\left(8\sinh\left(2\sinh^{-1}(ax)\right)+\sinh\left(4\sinh^{-1}(ax)\right)\right)+12\sinh^{-1}(ax)\left(32\sinh\left(2\sinh^{-1}(ax)\right)+\sinh\left(4\sinh^{-1}(ax)\right)\right)\right)}{1024a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(96*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*(16*Cosh[2*Ar
cSinh[a*x]] + Cosh[4*ArcSinh[a*x]]) - 3*(64*Cosh[2*ArcSinh[a*x]] + Cosh[4*A
rcSinh[a*x]]) + 32*ArcSinh[a*x]^3*(8*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[
a*x]]) + 12*ArcSinh[a*x]*(32*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]])))
/(1024*a*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 2.27, size = 484, normalized size = 1.39

method	result

default	$\frac{3\sqrt{c\left(a^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(a x\right)^{4} c}{32\sqrt{a^{2} x^{2}+1}\, a}+\frac{\sqrt{c\left(a^{2} x^{2}+1\right)}\, \left(8 a^{5} x^{5}+8 a^{4} \sqrt{a^{2} x^{2}+1}\, x^{4}+12 a^{3} x^{3}+8 \sqrt{a^{2} x^{2}+1}\, a^{2} x^{2}+4 a x\right.}{2048 a \left(a^{2} x^{2}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/32*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c+1/2048*(c*(
a^2*x^2+1))^(1/2)*(8*a^5*x^5+8*a^4*(a^2*x^2+1)^(1/2)*x^4+12*a^3*x^3+8*(a^2*
x^2+1)^(1/2)*a^2*x^2+4*a*x+(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3-24*arcsinh
(a*x)^2+12*arcsinh(a*x)-3)*c/a/(a^2*x^2+1)+1/32*(c*(a^2*x^2+1))^(1/2)*(2*a^
3*x^3+2*(a^2*x^2+1)^(1/2)*a^2*x^2+2*a*x+(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^
3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c/a/(a^2*x^2+1)+1/32*(c*(a^2*x^2+1))^(
1/2)*(2*a^3*x^3-2*(a^2*x^2+1)^(1/2)*a^2*x^2+2*a*x-(a^2*x^2+1)^(1/2))*(4*arc
sinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)*c/a/(a^2*x^2+1)+1/2048*(c*(a
^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*(a^2*x^2+1)^(1/2)*x^4+12*a^3*x^3-8*(a^2*x
^2+1)^(1/2)*a^2*x^2+4*a*x-(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3+24*arcsinh(
a*x)^2+12*arcsinh(a*x)+3)*c/a/(a^2*x^2+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}} \operatorname{asinh}^{3}\left(a x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{asinh}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.336 $\int \sqrt{c+a^2cx^2} \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=205

$$-\frac{3ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a^2x^2}}+\frac{3}{4}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)-\frac{3\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{8a\sqrt{1+a^2x^2}}-\frac{3ax^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{4\sqrt{1+a^2x^2}}-$$

```
[Out] 3/4*x*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)+1/2*x*arcsinh(a*x)^3*(a^2*c*x^2+c)^(
1/2)-3/8*a*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-3/8*arcsinh(a*x)^2*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/4*a*x^2*arcsinh(a*x)^2*(a^2*c*x^2+c)
^(1/2)/(a^2*x^2+1)^(1/2)+1/8*arcsinh(a*x)^4*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+
1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5785, 5783, 5776, 5812, 30}

$$-\frac{3ax^2\sqrt{a^2cx^2+c}}{8\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^4}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^3-\frac{3ax^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{4\sqrt{a^2x^2+1}}-\frac{3\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{8a\sqrt{a^2x^2+1}}+\frac{3}{4}x\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*a*x^2*Sqrt[c + a^2*c*x^2])/(8*Sqrt[1 + a^2*x^2]) + (3*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x])/4 - (3*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(8*a*Sqrt[1 +
 a^2*x^2]) - (3*a*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(4*Sqrt[1 + a^2*x
^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcS
inh[a*x]^4)/(8*a*Sqrt[1 + a^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^3\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^3 + \frac{\sqrt{c+a^2cx^2}\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(3a\sqrt{c+\right.}{}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{4\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^3 + \frac{\sqrt{c+a^2}}{8a}$$

$$= \frac{3}{4}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) - \frac{3ax^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{4\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a^2x^2}} + \frac{3}{4}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) - \frac{3\sqrt{c+a^2cx^2}\,\sinh^{-1}(}{8a\sqrt{1+a^2x^2}}$$

Mathematica [A]
time = 0.10, size = 86, normalized size = 0.42

$$\frac{\sqrt{c\,(1+a^2x^2)}\,\left(-3\left(1+2\sinh^{-1}(ax)^2\right)\cosh\left(2\sinh^{-1}(ax)\right)+2\sinh^{-1}(ax)\left(\sinh^{-1}(ax)^3+\left(3+2\sinh^{-1}(ax)^2\right)\sinh\left(2\sinh^{-1}(ax)\right)\right)\right)}{16a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-3*(1 + 2*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]] + 2*
ArcSinh[a*x]*(ArcSinh[a*x]^3 + (3 + 2*ArcSinh[a*x]^2)*Sinh[2*ArcSinh[a*x]])
))/(16*a*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 2.46, size = 231, normalized size = 1.13

method	result
default	$\frac{\sqrt{c\left(a^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(a x\right)^{4}}{8 \sqrt{a^{2} x^{2}+1}\, a}+\frac{\sqrt{c\left(a^{2} x^{2}+1\right)}\, \left(2 a^{3} x^{3}+2 \sqrt{a^{2} x^{2}+1}\, a^{2} x^{2}+2 a x+\sqrt{a^{2} x^{2}+1}\right) \left(4 \operatorname{arcsinh}\left(a x\right)^{3}\right.}{32 a\left(a^{2} x^{2}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4+1/32*(c*(a^2*x
^2+1))^(1/2)*(2*a^3*x^3+2*(a^2*x^2+1)^(1/2)*a^2*x^2+2*a*x+(a^2*x^2+1)^(1/2)
)*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)/a/(a^2*x^2+1)+1/32*(
c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*(a^2*x^2+1)^(1/2)*a^2*x^2+2*a*x-(a^2*x^2+
1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)/a/(a^2*x^2+1
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^{2} x^{2}+1\right)}\, \operatorname{asinh}^{3}\left(a x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{asinh}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.337 $\int \frac{\sinh^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^4}{4a\sqrt{c+a^2cx^2}}$$

```
[Out] 1/4*arcsinh(a*x)^4*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5783}

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^4}{4a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^4}{4a\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 1.05, size = 39, normalized size = 0.98

method	result	size
default	$\frac{\sqrt{c\left(a^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(a x\right)^{4}}{4 \sqrt{a^{2} x^{2}+1}\, a c}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c*arcsinh(a*x)^4
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.35

$$\frac{\operatorname{arsinh}\left(ax\right)^{4}}{4\,a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] 1/4*arcsinh(a*x)^4/(a*sqrt(c))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{3}\left(ax\right)}{\sqrt{c\left(a^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asinh}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2), x)
```

3.338 $\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=218

$$\frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}}-\frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}-\frac{3\sqrt{1+a^2x^2}\,\sinh}{}$$

```
[Out] x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(1/2)+arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a/c
/(a^2*c*x^2+c)^(1/2)-3*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^2*
x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-3*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^
2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)+3/2*polylog(3,-(a*
x+(a^2*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5787, 5797, 3799, 2221, 2611, 2320, 6724}

$$-\frac{3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\text{Li}_2\left(-e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{2\sinh^{-1}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\sinh^{-1}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{ac\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2\log\left(e^{2\sinh^{-1}(ax)}+1\right)}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x
]^3)/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1
+ E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) + (3*S
qrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(2*a*c*Sqrt[c + a^2*c*x^2
])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{1+a^2x^2}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \sinh^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \sin\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh}\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh}\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh}\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh}\right.}{ac\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.18, size = 133, normalized size = 0.61

$$\frac{2ax\sinh^{-1}(ax)^3 - 2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\left(\sinh^{-1}(ax)+3\log\left(1+e^{-2\sinh^{-1}(ax)}\right)\right)+6\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(ax)}\right)+3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{-2\sinh^{-1}(ax)}\right)}{2ac\sqrt{c\left(1+a^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*a*x*ArcSinh[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*(ArcSinh[a*x] +
3*Log[1 + E^(-2*ArcSinh[a*x])]) + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[
2, -E^(-2*ArcSinh[a*x])] + 3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*ArcSinh[a*
x])])/(2*a*c*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A]
time = 2.58, size = 262, normalized size = 1.20

method	result
default	$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(ax-\sqrt{a^2x^2+1}\right)\text{arcsinh}(ax)^3}{a\,c^2\left(a^2x^2+1\right)} + \frac{2\sqrt{c\left(a^2x^2+1\right)}\,\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}\,a\,c^2} - \frac{3\sqrt{c\left(a^2x^2+1\right)}\,\text{arcsin}}{\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2+1))^(1/2)*(a*x-(a^2*x^2+1)^(1/2))*arcsinh(a*x)^3/a/c^2/(a^2*x^2
+1)+2*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^2*arcsinh(a*x)^3-3*(c*(a^
2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1
)^(1/2))^2)-3*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^2*arcsinh(a*x)*po
lylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(
1/2)/a/c^2*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{3}(ax)}{(c(a^{2}x^{2}+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2), x)
```

3.339 $\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=363

$$-\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}+\frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax}{3ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/3*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(3/2)-x*arcsinh(a*x)/c^2/(a^2*c*x^2+c)
^(1/2)+2/3*x*arcsinh(a*x)^3/c^2/(a^2*c*x^2+c)^(1/2)+1/2*arcsinh(a*x)^2/a/c^
2/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2/3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2
)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2
)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/2*ln(a^2*x^2+1)*(a^2*x^2+1)
^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)
^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+polylog(3,-(a*x+(a^2
*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5788, 5787, 5797, 3799, 2221, 2611, 2320, 6724, 5798, 266}

$$-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\text{Li}_2\left(-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\,\log\left(a^2x^2+1\right)}{2ac^2\sqrt{a^2cx^2+c}}+\frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{3ac^2\sqrt{a^2cx^2+c}}+\frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}-\frac{x\sinh^{-1}(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2\log\left(e^{2\sinh^{-1}(ax)}+1\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{x\sinh^{-1}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*ArcSinh[a*x])/(c^2*Sqrt[c + a^2*c*x^2])) + ArcSinh[a*x]^2/(2*a*c^2*Sqr
t[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x]^3)/(3*c*(c + a^2*c*x^
2)^(3/2)) + (2*x*ArcSinh[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^3)/(3*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^
2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[c + a^2*c*x^2])
 + (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[
c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(a*c^
2*Sqrt[c + a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx}{3c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{(1+a^2x^2)^2}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\int}{c^2\sqrt{c+}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\sinh^{-1}(ax)}{3c^2\sqrt{c+a^2cx}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\sinh^{-1}(ax)}{3c^2\sqrt{c+a^2cx}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\sinh^{-1}(ax)}{3c^2\sqrt{c+a^2cx}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\sinh^{-1}(ax)}{3c^2\sqrt{c+a^2cx}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\sinh^{-1}(ax)}{3c^2\sqrt{c+a^2cx}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\sinh^{-1}(ax)}{3c^2\sqrt{c+a^2cx}}$$

Mathematica [A]
time = 0.37, size = 195, normalized size = 0.54

$$\frac{(1+a^2x^2)^{3/2}\left(-\frac{6ax\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}+\frac{3\sinh^{-1}(ax)^2}{1+a^2x^2}-4\sinh^{-1}(ax)^3+\frac{2ax\sinh^{-1}(ax)^3}{(1+a^2x^2)^{3/2}}+\frac{4ax\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}-12\sinh^{-1}(ax)^2\log\left(1+e^{-2\sinh^{-1}(ax)}\right)+3\log(1+a^2x^2)+12\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\sinh^{-1}(ax)}\right)+6\mathrm{PolyLog}\left(3,-e^{-2\sinh^{-1}(ax)}\right)\right)}{6ac\,(c+a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*((-6*a*x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2] + (3*ArcSinh[
a*x]^2)/(1 + a^2*x^2) - 4*ArcSinh[a*x]^3 + (2*a*x*ArcSinh[a*x]^3)/(1 + a^2*
x^2)^(3/2) + (4*a*x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2] - 12*ArcSinh[a*x]^2*L
og[1 + E^(-2*ArcSinh[a*x])] + 3*Log[1 + a^2*x^2] + 12*ArcSinh[a*x]*PolyLog[
2, -E^(-2*ArcSinh[a*x])] + 6*PolyLog[3, -E^(-2*ArcSinh[a*x])]))/(6*a*c*(c +
 a^2*c*x^2)^(3/2))
```

Maple [A]
time = 2.87, size = 550, normalized size = 1.52

method	result
default	$\frac{\sqrt{c\,(a^2x^2+1)}\,\left(2a^3x^3-2\sqrt{a^2x^2+1}\,a^2x^2+3ax-2\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)\left(-6\operatorname{arcsinh}(ax)a^4x^4-6\sqrt{a^2x^2+1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*(a^2*x^2+1)^(1/2)*a^2*x^2+3*a*x-2*(a
^2*x^2+1)^(1/2))*arcsinh(a*x)*(-6*arcsinh(a*x)*a^4*x^4-6*(a^2*x^2+1)^(1/2)*
arcsinh(a*x)*a^3*x^3-6*a^4*x^4-6*(a^2*x^2+1)^(1/2)*a^3*x^3+6*arcsinh(a*x)^2
*a^2*x^2-12*arcsinh(a*x)*a^2*x^2-9*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*a*x-18*a^
2*x^2-6*(a^2*x^2+1)^(1/2)*a*x+8*arcsinh(a*x)^2-6*arcsinh(a*x)-12)/(3*a^6*x^
6+10*a^4*x^4+11*a^2*x^2+4)/a/c^3+(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/
c^3*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-2*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/
2)/a/c^3*ln(a*x+(a^2*x^2+1)^(1/2))+4/3*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1
/2)/a/c^3*arcsinh(a*x)^3-2*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^3*ar
csinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-2*(c*(a^2*x^2+1))^(1/2)/(a^2*x
^2+1)^(1/2)/a/c^3*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+(c*(a^
2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^3*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 +
3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{3}(ax)}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,%%%{-2
,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{-2,[0,0,0]%%%},0,%%%
{1,[4,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{asinh}\left(a\,x\right)}^3}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2), x)
```

3.340 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=515

$$-\frac{1}{20ac^3\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}-\frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}}-\frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}+\frac{3\sinh^{-1}(ax)^2}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a}}$$

```
[Out] 1/5*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)^3/c^2/(a^2*c
*x^2+c)^(3/2)-x*arcsinh(a*x)/c^3/(a^2*c*x^2+c)^(1/2)-1/10*x*arcsinh(a*x)/c^
3/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2)+3/20*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^(3/
2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a*x)^3/c^3/(a^2*c*x^2+c)^(1/2)-1/20/a
/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2/5*arcsinh(a*x)^2/a/c^3/(a^2*x^
2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+8/15*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a/c^3/
(a^2*c*x^2+c)^(1/2)-8/5*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^2
*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)+1/2*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)
/a/c^3/(a^2*c*x^2+c)^(1/2)-8/5*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/
2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)+4/5*polylog(3,-(a*x+(a^2
*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5788, 5787, 5797, 3799, 2221, 2611, 2320, 6724, 5798, 266, 267}

$$-\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\text{Li}_2\left(-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}-\frac{1}{20ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\,\log\left(a^2x^2+1\right)}{2ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)^3}{15c^3\sqrt{a^2cx^2+c}}+\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{15ac^3\sqrt{a^2cx^2+c}}+\frac{2\sinh^{-1}(ax)^2}{5ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{3\sinh^{-1}(ax)^2}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{x\sinh^{-1}(ax)}{c^3\sqrt{a^2cx^2+c}}-\frac{x\sinh^{-1}(ax)}{10c^3\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}-\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2\log\left(e^{2\sinh^{-1}(ax)}+1\right)}{5ac^3\sqrt{a^2cx^2+c}}+\frac{4x\sinh^{-1}(ax)^3}{15c^2\left(a^2cx^2+c\right)^{3/2}}+\frac{x\sinh^{-1}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/20*1/(a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(c
^3*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(10*c^3*(1 + a^2*x^2)*Sqrt[c + a
^2*c*x^2]) + (3*ArcSinh[a*x]^2)/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*
c*x^2]) + (2*ArcSinh[a*x]^2)/(5*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]
) + (x*ArcSinh[a*x]^3)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^3)/(
15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^3)/(15*c^3*Sqrt[c + a^2*c
*x^2]) + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(15*a*c^3*Sqrt[c + a^2*c*x^2]
) - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(5*a*c
^3*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^3*Sqr
t[c + a^2*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcS
inh[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[1 + a^2*x^2]*PolyLog[3,
 -E^(2*ArcSinh[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
```

```
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{5c\,(c+a^2cx^2)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx}{5c} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{(1+a^2x^2)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{3\sinh^{-1}(ax)^2}{20ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{5c\,(c+a^2cx^2)^{5/2}} + \frac{4x\sinh^{-1}(ax)^3}{15c^2\,(c+a^2cx^2)^{3/2}} + \frac{8\int \frac{\sinh^{-1}}{(c+a^2}}{15}$$

$$= -\frac{x\sinh^{-1}(ax)}{10c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(ax)^2}{20ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\sinh^{-1}(}{5ac^3\sqrt{1+a^2x^2}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{}{2}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{}{2}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{}{2}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{}{2}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{}{2}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\,(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{}{2}$$

Mathematica [A]
time = 0.45, size = 297, normalized size = 0.58

$$\frac{-\frac{3}{\sqrt{1+a^2x^2}} - 60ax\sinh^{-1}(ax) - \frac{6ax\sinh^{-1}(ax)}{1+a^2x^2} + \frac{9\sinh^{-1}(ax)^2}{(1+a^2x^2)^{3/2}} + \frac{24\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} + 32ax\sinh^{-1}(ax)^3 + \frac{12ax\sinh^{-1}(ax)^3}{(1+a^2x^2)^2} + \frac{16ax\sinh^{-1}(ax)^3}{1+a^2x^2} - 32\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3 - 96\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2\log\left(1+e^{-2\sinh^{-1}(ax)}\right) + 30\sqrt{1+a^2x^2}\log(1+a^2x^2) + 96\sqrt{1+a^2x^2}\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\sinh^{-1}(ax)}\right) + 48\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,-e^{-2\sinh^{-1}(ax)}\right)}{60ac^3\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (-3/Sqrt[1 + a^2*x^2] - 60*a*x*ArcSinh[a*x] - (6*a*x*ArcSinh[a*x])/(1 + a^2
*x^2) + (9*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + (24*ArcSinh[a*x]^2)/Sqrt[1
 + a^2*x^2] + 32*a*x*ArcSinh[a*x]^3 + (12*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2)
^2 + (16*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 32*Sqrt[1 + a^2*x^2]*ArcSinh[a
*x]^3 - 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(-2*ArcSinh[a*x])] +
30*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2] + 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*P
```

```
olyLog[2, -E^(-2*ArcSinh[a*x])] + 48*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*Ar
cSinh[a*x])])/(60*a*c^3*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 2.85, size = 888, normalized size = 1.72

method	result
default	$\frac{\sqrt{c\left(a^{2} x^{2}+1\right)}\,\left(8 a^{5} x^{5}-8 a^{4} \sqrt{a^{2} x^{2}+1}\, x^{4}+20 a^{3} x^{3}-16 \sqrt{a^{2} x^{2}+1}\, a^{2} x^{2}+15 a x-8 \sqrt{a^{2} x^{2}+1}\right)\left(24+24 a^{8} x^{8}+96\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*(a^2*x^2+1)^(1/2)*x^4+20*a^3*x^
3-16*(a^2*x^2+1)^(1/2)*a^2*x^2+15*a*x-8*(a^2*x^2+1)^(1/2))*(24+24*a^8*x^8+9
6*a^6*x^6+45*(a^2*x^2+1)^(1/2)*a*x-264*arcsinh(a*x)^2+256*arcsinh(a*x)^3-37
2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*a*x+84*(a^2*x^2+1)^(1/2)*a^5*x^5+105*(a^2*
x^2+1)^(1/2)*a^3*x^3+144*a^4*x^4-480*arcsinh(a*x)+96*a^2*x^2-192*arcsinh(a*
x)^2*a^8*x^8-192*arcsinh(a*x)*a^8*x^8-840*arcsinh(a*x)^2*a^6*x^6-852*arcsin
h(a*x)*a^6*x^6+160*arcsinh(a*x)^3*a^4*x^4-1368*arcsinh(a*x)^2*a^4*x^4+380*a
rcsinh(a*x)^3*a^2*x^2-1590*arcsinh(a*x)*a^4*x^4-984*arcsinh(a*x)^2*a^2*x^2-
1410*arcsinh(a*x)*a^2*x^2+24*(a^2*x^2+1)^(1/2)*a^7*x^7-936*(a^2*x^2+1)^(1/2
)*arcsinh(a*x)*a^3*x^3-192*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2*a^7*x^7-192*(a^
2*x^2+1)^(1/2)*arcsinh(a*x)*a^7*x^7-744*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2*a^
5*x^5-756*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*a^5*x^5-1020*(a^2*x^2+1)^(1/2)*arc
sinh(a*x)^2*a^3*x^3-495*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2*a*x)/(40*a^10*x^10
+215*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-2*(c*(a^2*x^2+1)
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(a*x+(a^2*x^2+1)^(1/2))+(c*(a^2*x^2+1))^(
1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)+16/15*(c*(a^2*
x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)^3-8/5*(c*(a^2*x^2+1))^(1
/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-
8/5*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)*polylog(2,-(
a*x+(a^2*x^2+1)^(1/2))^2)+4/5*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4
*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{3}(ax)}{(c(a^{2}x^{2}+1))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(7/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,%%%{-2
,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{-2,[0,0,0]%%%},0,%%%
{1,[4,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{asinh}\left(a\,x\right)}^3}{{\left(c\,a^2\,x^2+c\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(7/2), x)
```

3.341 $\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}},x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
[Out] Defer[Int][(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
[Out] Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
[Out] int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{asinh}^3(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*asinh(a*x)**3/sqrt(a**2*x**2 + 1), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.342 $\int \frac{x^4 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=187

$$\frac{45x^2}{128a^3}-\frac{3x^4}{128a}-\frac{45x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{64a^4}+\frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a^2}+\frac{45\sinh^{-1}(ax)^2}{128a^5}+\frac{9x^2\sinh^{-1}(ax)^2}{16a^3}-$$

```
[Out] 45/128*x^2/a^3-3/128*x^4/a+45/128*arcsinh(a*x)^2/a^5+9/16*x^2*arcsinh(a*x)^
2/a^3-3/16*x^4*arcsinh(a*x)^2/a+3/32*arcsinh(a*x)^4/a^5-45/64*x*arcsinh(a*x
)*(a^2*x^2+1)^(1/2)/a^4+3/32*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2-3/8*x*a
rcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2
)/a^2
```

Rubi [A]
time = 0.33, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5812, 5783, 5776, 30}

$$\frac{3\sinh^{-1}(ax)^4}{32a^5}+\frac{45\sinh^{-1}(ax)^2}{128a^5}+\frac{45x^2}{128a^3}+\frac{9x^2\sinh^{-1}(ax)^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{4a^2}+\frac{3x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{32a^2}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{8a^4}-\frac{45x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{64a^4}-\frac{3x^4}{128a}-\frac{3x^4\sinh^{-1}(ax)^2}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (45*x^2)/(128*a^3) - (3*x^4)/(128*a) - (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
)/(64*a^4) + (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a^2) + (45*ArcSinh[
a*x]^2)/(128*a^5) + (9*x^2*ArcSinh[a*x]^2)/(16*a^3) - (3*x^4*ArcSinh[a*x]^2
)/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^4) + (x^3*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^3)/(4*a^2) + (3*ArcSinh[a*x]^4)/(32*a^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{3\int x^3\sinh^{-1}(ax)^2\,dx}{4a}$$

$$= -\frac{3x^4\sinh^{-1}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{4a^2} + \frac{3}{8}\int$$

$$= \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a^2} + \frac{9x^2\sinh^{-1}(ax)^2}{16a^3} - \frac{3x^4\sinh^{-1}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\,\mathrm{s}}{8a^4}$$

$$= -\frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{64a^4} + \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a^2} + \frac{9x^2\sinh^{-1}(ax}{16a^3}$$

$$= \frac{45x^2}{128a^3} - \frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{64a^4} + \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a^2} + \frac{45\,\mathrm{sinl}}{12}$$

Mathematica [A]
time = 0.06, size = 121, normalized size = 0.65

$$\frac{45a^2x^2 - 3a^4x^4 + 6ax\sqrt{1+a^2x^2}\,(-15+2a^2x^2)\sinh^{-1}(ax) + (45+72a^2x^2-24a^4x^4)\sinh^{-1}(ax)^2 + 16ax\sqrt{1+a^2x^2}\,(-3+2a^2x^2)\sinh^{-1}(ax)^3 + 12\sinh^{-1}(ax)^4}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (45*a^2*x^2 - 3*a^4*x^4 + 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*ArcSinh
[a*x] + (45 + 72*a^2*x^2 - 24*a^4*x^4)*ArcSinh[a*x]^2 + 16*a*x*Sqrt[1 + a^2
*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 12*ArcSinh[a*x]^4)/(128*a^5)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Fricas [A]
time = 0.41, size = 166, normalized size = 0.89

$$-\frac{3\,a^4x^4 - 16\,(2\,a^3x^3 - 3\,ax)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 45\,a^2x^2 - 12\,\log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 3\,(8\,a^4x^4 - 24\,a^2x^2 - 15)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 6\,(2\,a^3x^3 - 15\,ax)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(3*a^4*x^4 - 16*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 + 1))^3 - 45*a^2*x^2 - 12*log(a*x + sqrt(a^2*x^2 + 1))^4 + 3*(8*a^
4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(2*a^3*x^3 - 15
*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^5
```

Sympy [A]
time = 0.91, size = 185, normalized size = 0.99

$$\begin{cases} -\frac{3x^4\operatorname{asinh}^2(ax)}{16a} - \frac{3x^4}{128a} + \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a^2} + \frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a^2} + \frac{9x^2\operatorname{asinh}^2(ax)}{16a^3} + \frac{45x^2}{128a^3} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{8a^4} - \frac{45x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{64a^4} + \frac{3\operatorname{asinh}^4(ax)}{32a^5} + \frac{45\operatorname{asinh}^2(ax)}{128a^5} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x**4*asinh(a*x)**2/(16*a) - 3*x**4/(128*a) + x**3*sqrt(a**2*x
**2 + 1)*asinh(a*x)**3/(4*a**2) + 3*x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(32
*a**2) + 9*x**2*asinh(a*x)**2/(16*a**3) + 45*x**2/(128*a**3) - 3*x*sqrt(a**
```

```
2*x**2 + 1)*asinh(a*x)**3/(8*a**4) - 45*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(6
4*a**4) + 3*asinh(a*x)**4/(32*a**5) + 45*asinh(a*x)**2/(128*a**5), Ne(a, 0)
), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^4*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.343 $\int \frac{x^3 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=153

$$\frac{40x}{9a^3}-\frac{2x^3}{27a}-\frac{40\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^4}+\frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^2}+\frac{2x\sinh^{-1}(ax)^2}{a^3}-\frac{x^3\sinh^{-1}(ax)^2}{3a}-\frac{2\sqrt{1+}}{}$$

```
[Out] 40/9*x/a^3-2/27*x^3/a+2*x*arcsinh(a*x)^2/a^3-1/3*x^3*arcsinh(a*x)^2/a-40/9*
arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+2/9*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
^2-2/3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(a*x)^3*(a^2*x^2
+1)^(1/2)/a^2
```

Rubi [A]
time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5812, 5798, 5772, 8, 5776, 30}

$$\frac{40x}{9a^3}+\frac{2x\sinh^{-1}(ax)^2}{a^3}+\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{9a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{3a^4}-\frac{40\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{9a^4}-\frac{2x^3}{27a}-\frac{x^3\sinh^{-1}(ax)^2}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (40*x)/(9*a^3) - (2*x^3)/(27*a) - (40*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^
4) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^2) + (2*x*ArcSinh[a*x]^2)/
a^3 - (x^3*ArcSinh[a*x]^2)/(3*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*
a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\sinh^{-1}(ax)^2\,dx}{a} \\
&= -\frac{x^3\sinh^{-1}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{3a^2} + \frac{2}{3}\int \\
&= \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(ax)^2}{a^3} - \frac{x^3\sinh^{-1}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\,\sinh}{3a^4} \\
&= -\frac{2x^3}{27a} - \frac{40\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(ax)^2}{a^3} \\
&= \frac{40x}{9a^3} - \frac{2x^3}{27a} - \frac{40\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(}{a^3}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 0.64

$$\frac{-2ax(-60+a^2x^2)+6(-20+a^2x^2)\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)-9ax(-6+a^2x^2)\sinh^{-1}(ax)^2+9(-2+a^2x^2)\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{27a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-2*a*x*(-60 + a^2*x^2) + 6*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
- 9*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 + 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*
ArcSinh[a*x]^3)/(27*a^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.26, size = 127, normalized size = 0.83

$$\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^3+\frac{2}{27}\,a\left(\frac{3\left(\sqrt{a^2x^2+1}\,x^2-\frac{20\sqrt{a^2x^2+1}}{a^2}\right)\operatorname{arsinh}(ax)}{a^3}-\frac{a^2x^3-60\,x}{a^4}\right)-\frac{(a^2x^3-6\,x)\operatorname{arsinh}(ax)^2}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^3 +
2/27*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)*arcsinh(a*x)/a
^3 - (a^2*x^3 - 60*x)/a^4) - 1/3*(a^2*x^3 - 6*x)*arcsinh(a*x)^2/a^3
```

Fricas [A]
time = 0.38, size = 128, normalized size = 0.84

$$-\frac{2\,a^3x^3-9\,\sqrt{a^2x^2+1}\,(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+9\,(a^3x^3-6\,ax)\log\left(ax+\sqrt{a^2x^2+1}\right)^2-6\,\sqrt{a^2x^2+1}\,(a^2x^2-20)\log\left(ax+\sqrt{a^2x^2+1}\right)-120\,ax}{27\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(2*a^3*x^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2
 + 1))^3 + 9*(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^2*
x^2 + 1)*(a^2*x^2 - 20)*log(a*x + sqrt(a^2*x^2 + 1)) - 120*a*x)/a^4
```

Sympy [A]
time = 0.60, size = 148, normalized size = 0.97

$$\begin{cases} -\frac{x^3\operatorname{asinh}^2(ax)}{3a}-\frac{2x^3}{27a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^2}+\frac{2x\operatorname{asinh}^2(ax)}{a^3}+\frac{40x}{9a^3}-\frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{3a^4}-\frac{40\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3*asinh(a*x)**2/(3*a) - 2*x**3/(27*a) + x**2*sqrt(a**2*x**2
+ 1)*asinh(a*x)**3/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**2
) + 2*x*asinh(a*x)**2/a**3 + 40*x/(9*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*
x)**3/(3*a**4) - 40*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**4), Ne(a, 0)), (0,
 True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.344 $\int \frac{x^2 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=105

$$-\frac{3x^2}{8a}+\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}-\frac{3\sinh^{-1}(ax)^2}{8a^3}-\frac{3x^2\sinh^{-1}(ax)^2}{4a}+\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2a^2}-\frac{\sinh^{-1}(ax)^4}{8a^3}$$

```
[Out] -3/8*x^2/a-3/8*arcsinh(a*x)^2/a^3-3/4*x^2*arcsinh(a*x)^2/a-1/8*arcsinh(a*x)
^4/a^3+3/4*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2+1/2*x*arcsinh(a*x)^3*(a^2*x
^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5812, 5783, 5776, 30}

$$-\frac{\sinh^{-1}(ax)^4}{8a^3}-\frac{3\sinh^{-1}(ax)^2}{8a^3}+\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{2a^2}+\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{3x^2}{8a}-\frac{3x^2\sinh^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-3*x^2)/(8*a) + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) - (3*ArcSinh[
a*x]^2)/(8*a^3) - (3*x^2*ArcSinh[a*x]^2)/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^3)/(2*a^2) - ArcSinh[a*x]^4/(8*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2a^2} - \frac{\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{3\int x\sinh^{-1}(ax)^2\,dx}{2a}$$
$$= -\frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2a^2} - \frac{\sinh^{-1}(ax)^4}{8a^3} + \frac{3}{2}\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}$$
$$= \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} - \frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2a^2} - \frac{\sinh^{-1}}{8a}$$
$$= -\frac{3x^2}{8a} + \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} - \frac{3\sinh^{-1}(ax)^2}{8a^3} - \frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2}}{}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.79

$$-\frac{3a^2x^2 - 6ax\sqrt{1+a^2x^2}\,\sinh^{-1}(ax) + (3+6a^2x^2)\sinh^{-1}(ax)^2 - 4ax\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^4}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/8*(3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*Ar
cSinh[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + ArcSinh[a*x]^4)/a^3
```

Maple [A]
time = 2.59, size = 84, normalized size = 0.80

method	result
default	$-\frac{-4\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\,ax+6\operatorname{arcsinh}(ax)^2a^2x^2+\operatorname{arcsinh}(ax)^4-6\sqrt{a^2x^2+1}\,\operatorname{arcsinh}(ax)ax+3a^2x^2+3\operatorname{arcsinh}(ax)^2}{8a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)*a*x+6*arcsinh(a*x)^2*a^2*x^2+arcs
inh(a*x)^4-6*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+
3)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Fricas [A]
time = 0.41, size = 128, normalized size = 1.22

$$\frac{4\,\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3-3\,a^2x^2-\log\left(ax+\sqrt{a^2x^2+1}\right)^4+6\,\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,(2\,a^2x^2+1)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a^2*x^2 - l
og(a*x + sqrt(a^2*x^2 + 1))^4 + 6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*
x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A]
time = 0.47, size = 100, normalized size = 0.95

$$\begin{cases} -\frac{3x^2\operatorname{asinh}^2(ax)}{4a}-\frac{3x^2}{8a}+\frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{2a^2}+\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{4a^2}-\frac{\operatorname{asinh}^4(ax)}{8a^3}-\frac{3\operatorname{asinh}^2(ax)}{8a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x**2*asinh(a*x)**2/(4*a) - 3*x**2/(8*a) + x*sqrt(a**2*x**2 +
1)*asinh(a*x)**3/(2*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) - a
sinh(a*x)**4/(8*a**3) - 3*asinh(a*x)**2/(8*a**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.345 $\int \frac{x \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=64

$$-\frac{6x}{a}+\frac{6\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}-\frac{3x\sinh^{-1}(ax)^2}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2}$$

```
[Out] -6*x/a-3*x*arcsinh(a*x)^2/a+6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2+arcsinh(a*
x)^3*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5798, 5772, 8}

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{a^2}+\frac{6\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^2}-\frac{6x}{a}-\frac{3x\sinh^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-6*x)/a + (6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2 - (3*x*ArcSinh[a*x]^2)/a
+ (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2} - \frac{3\int \sinh^{-1}(ax)^2\,dx}{a}$$
$$= -\frac{3x\sinh^{-1}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2} + 6\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{6\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2} - \frac{3x\sinh^{-1}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2} - \frac{6\int 1\,dx}{a}$$
$$= -\frac{6x}{a} + \frac{6\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2} - \frac{3x\sinh^{-1}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.91

$$\frac{-6ax+6\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)-3ax\sinh^{-1}(ax)^2+\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-6*a*x + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] - 3*a*x*ArcSinh[a*x]^2 + Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

Maple [A]
time = 2.46, size = 90, normalized size = 1.41

method	result	siz
default	$\frac{\operatorname{arcsinh}(ax)^3 a^2x^2+\operatorname{arcsinh}(ax)^3-3\sqrt{a^2x^2+1}\,\operatorname{arcsinh}(ax)^2 ax+6x^2\operatorname{arcsinh}(ax)a^2+6\operatorname{arcsinh}(ax)-6\sqrt{a^2x^2+1}\,ax}{a^2\sqrt{a^2x^2+1}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^3*a^2*x^2+arcsinh(a*x)^3-3*(a^2*x^2+1
)^(1/2)*arcsinh(a*x)^2*a*x+6*arcsinh(a*x)*a^2*x^2+6*arcsinh(a*x)-6*(a^2*x^2
+1)^(1/2)*a*x)
```

Maxima [A]
time = 0.26, size = 61, normalized size = 0.95

$$-\frac{3\,x\,\operatorname{arsinh}(ax)^2}{a} + \frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)^3}{a^2} - \frac{6\left(x - \frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*x*arcsinh(a*x)^2/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a^2 - 6*(x - sqrt(
a^2*x^2 + 1)*arcsinh(a*x)/a)/a
```

Fricas [A]
time = 0.36, size = 92, normalized size = 1.44

$$-\frac{3\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3+6\,ax-6\,\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(3*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a
^2*x^2 + 1))^3 + 6*a*x - 6*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/
a^2
```

Sympy [A]
time = 0.34, size = 61, normalized size = 0.95

$$\begin{cases} -\frac{3x\operatorname{asinh}^2(ax)}{a}-\frac{6x}{a}+\frac{\sqrt{a^2x^2+1}\,\operatorname{asinh}^3(ax)}{a^2}+\frac{6\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{a^2} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x*asinh(a*x)**2/a - 6*x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**3
/a**2 + 6*sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True))
```

Giac [A]
time = 0.43, size = 101, normalized size = 1.58

$$\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2}-\frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt(
a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
/a^2))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.346 $\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

```
[Out] 1/4*arcsinh(a*x)^4/a
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5783}

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^4/(4*a)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^4}{4a}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^4/(4*a)
```

Maple [A]
time = 0.29, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^4}{4a}$	12
default	$\frac{\operatorname{arcsinh}(ax)^4}{4a}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arcsinh(a*x)^4/a
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.85

$$\frac{\operatorname{arsinh}(ax)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*arcsinh(a*x)^4/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 0.37, size = 23, normalized size = 1.77

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(a*x + sqrt(a^2*x^2 + 1))^4/a
```

Sympy [A]
time = 0.25, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^4(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Mupad [B]
time = 0.10, size = 11, normalized size = 0.85

$$\frac{\mathrm{asinh}(a\,x)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] asinh(a*x)^4/(4*a)
```

3.347 $\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=102

$$-2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh}\right.$$

```
[Out] -2*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))+3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+
6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,a
*x+(a^2*x^2+1)^(1/2))-6*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(
a^2*x^2+1)^(1/2))
```

Rubi [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5816, 4267, 2611, 6744, 2320, 6724}

$$-3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)+6\sinh^{-1}(ax)\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)-6\sinh^{-1}(ax)\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right)-6\text{Li}_4\left(-e^{\sinh^{-1}(ax)}\right)+6\text{Li}_4\left(e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 3*ArcSinh[a*x]^2*PolyLog[2, -E^
ArcSinh[a*x]] + 3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 6*ArcSinh[a*x
]*PolyLog[3, -E^ArcSinh[a*x]] - 6*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] -
 6*PolyLog[4, -E^ArcSinh[a*x]] + 6*PolyLog[4, E^ArcSinh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$

$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) +$$

$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\text{L}$$

$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\text{L}$$

$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\text{L}$$

$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\text{L}$$

Mathematica [A]
time = 0.09, size = 146, normalized size = 1.43

$$\frac{1}{8}\left(\pi^4-2\sinh^{-1}(ax)^4-8\sinh^{-1}(ax)^3\log\left(1+e^{-\sinh^{-1}(ax)}\right)+8\sinh^{-1}(ax)^3\log\left(1-e^{\sinh^{-1}(ax)}\right)+24\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)+24\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+48\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right)-48\sinh^{-1}(ax)\text{PolyLog}\left(3,e^{\sinh^{-1}(ax)}\right)+48\text{PolyLog}\left(4,-e^{-\sinh^{-1}(ax)}\right)+48\text{PolyLog}\left(4,e^{\sinh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (Pi^4 - 2*ArcSinh[a*x]^4 - 8*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] + 8*
ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24*ArcSinh[a*x]^2*PolyLog[2, -E^(-
ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 48*ArcSinh[
a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[
a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])] + 48*PolyLog[4, E^ArcSinh[a*x]])/
8
```

Maple [A]
time = 2.77, size = 197, normalized size = 1.93

method	result
default	$-\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)-3\operatorname{arcsinh}(ax)^2\operatorname{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right)+6$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(2,-a*x
-(a^2*x^2+1)^(1/2))+6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*poly
log(4,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*
arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,a*
x+(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^3 + x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3{(ax)}}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)**3/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{asinh}\left(a\,x\right)}^3}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^3/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.348 $\int \frac{\sinh^{-1}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=88

$$-a\sinh^{-1}(ax)^3-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x}+3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)+3a\sinh^{-1}(ax)\mathrm{PolyLog}\Big(2,$$

```
[Out] -a*arcsinh(a*x)^3+3*a*arcsinh(a*x)^2*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3*a*ar
csinh(a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*a*polylog(3,(a*x+(a^2*x
^2+1)^(1/2))^2)-arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5800, 5775, 3797, 2221, 2611, 2320, 6724}

$$-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{x}+3a\sinh^{-1}(ax)\mathrm{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}a\mathrm{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)-a\sinh^{-1}(ax)^3+3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -(a*ArcSinh[a*x]^3) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x + 3*a*ArcSinh[a*
x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 3*a*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh
[a*x])] - (3*a*PolyLog[3, E^(2*ArcSinh[a*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} + (3a)\int \frac{\sinh^{-1}(ax)^2}{x}\,dx$$

$$= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} + (3a)\text{Subst}\left(\int x^2\coth(x)\,dx, x, \sinh^{-1}(ax)\right)$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} - (6a)\text{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax\right.$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} + 3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) -$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} + 3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) +$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} + 3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) +$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} + 3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 97, normalized size = 1.10

$$\frac{1}{8}a\left(i\pi^3 - 8\sinh^{-1}(ax)^3 - \frac{8\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ax} + 24\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 24\sinh^{-1}(ax)\text{PolyLog}\left(2, e^{2\sinh^{-1}(ax)}\right) - 12\text{PolyLog}\left(3, e^{2\sinh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(a*x)
+ 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 24*ArcSinh[a*x]*PolyLog[2
, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*ArcSinh[a*x])]))/8
```

Maple [A]
time = 4.52, size = 187, normalized size = 2.12

method	result
default	$\frac{\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)^3}{x} - 2a\operatorname{arcsinh}(ax)^3 + 3a\operatorname{arcsinh}(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + 6a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^3-2*a*arcsinh(a*x)^3+3*a*arcsinh(a*x
)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)
```

```
^(1/2))-6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a*arcsinh(a*x)^2*ln(1-a*x-(
a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*a*pol
ylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2
+ sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(sqrt(a^2*x^2
 + 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^4 + x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{3}(ax)}{x^{2}\sqrt{a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^3}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^3/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.349 $\int \frac{\sinh^{-1}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=210

$$-\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+a^2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sin\right.$$

```
[Out] -3/2*a*arcsinh(a*x)^2/x-6*a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+a
^2*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^2*
x^2+1)^(1/2))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3*a^
2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(a*x)^2*polylog(2,a*x+(a^
2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*
arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a^2*x^2
+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsinh(a*x)^3*(a^2*x
^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.26, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5809, 5816, 4267, 2611, 6744, 2320, 6724, 5776, 2317, 2438}

$$\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-3a^2\sinh^{-1}(ax)\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)+3a^2\sinh^{-1}(ax)\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right)-3a^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+3a^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)+3a^2\text{Li}_4\left(-e^{\sinh^{-1}(ax)}\right)-3a^2\text{Li}_4\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{2x^2}+a^2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\frac{3a\sinh^{-1}(ax)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (-3*a*ArcSinh[a*x]^2)/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(2*x^2) -
6*a^2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + a^2*ArcSinh[a*x]^3*ArcTanh[E^A
rcSinh[a*x]] - 3*a^2*PolyLog[2, -E^ArcSinh[a*x]] + (3*a^2*ArcSinh[a*x]^2*Po
lyLog[2, -E^ArcSinh[a*x]])/2 + 3*a^2*PolyLog[2, E^ArcSinh[a*x]] - (3*a^2*Ar
cSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]])/2 - 3*a^2*ArcSinh[a*x]*PolyLog[3,
-E^ArcSinh[a*x]] + 3*a^2*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 3*a^2*Po
lyLog[4, -E^ArcSinh[a*x]] - 3*a^2*PolyLog[4, E^ArcSinh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
```

```
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\sinh^{-1}(ax)^2}{x^2}\,dx-\frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2}-\frac{1}{2}a^2\text{Subst}\left(\int x^3\text{csch}(x)\,dx,x,\sinh^{-1}(\right.\\
&= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2}+a^2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\\
&= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\\
&= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\\
&= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\\
&= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+
\end{aligned}$$

Mathematica [A]
time = 2.95, size = 304, normalized size = 1.45

$$\frac{a\left(-a\pi^4x+2ax\sinh^{-1}(ax)^4-12ax\sinh^{-1}(ax)^2\coth\left(\frac{1}{2}\sinh^{-1}(ax)\right)-2ax\sinh^{-1}(ax)^3\text{csch}^2\left(\frac{1}{2}\sinh^{-1}(ax)\right)+48ax\sinh^{-1}(ax)\log\left(1-e^{-\sinh^{-1}(ax)}\right)-48ax\sinh^{-1}(ax)\log\left(1+e^{-\sinh^{-1}(ax)}\right)+8ax\sinh^{-1}(ax)^3\log\left(1+e^{-\sinh^{-1}(ax)}\right)-8ax\sinh^{-1}(ax)^3\log\left(1-e^{\sinh^{-1}(ax)}\right)-24ax\left(-2+\sinh^{-1}(ax)^2\right)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)-48ax\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)-24ax\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-48ax\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right)+48ax\sinh^{-1}(ax)\text{PolyLog}\left(3,e^{\sinh^{-1}(ax)}\right)-48ax\text{PolyLog}\left(4,-e^{-\sinh^{-1}(ax)}\right)-48ax\text{PolyLog}\left(4,e^{\sinh^{-1}(ax)}\right)+12ax\sinh^{-1}(ax)^2\tanh\left(\frac{1}{2}\sinh^{-1}(ax)\right)-4\sinh^{-1}(ax)^3\tanh\left(\frac{1}{2}\sinh^{-1}(ax)\right)\right)}{16x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a*(-(a*Pi^4*x) + 2*a*x*ArcSinh[a*x]^4 - 12*a*x*ArcSinh[a*x]^2*Coth[ArcSinh
[a*x]/2] - 2*a*x*ArcSinh[a*x]^3*Csch[ArcSinh[a*x]/2]^2 + 48*a*x*ArcSinh[a*x
]*Log[1 - E^(-ArcSinh[a*x])] - 48*a*x*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x]
)] + 8*a*x*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 8*a*x*ArcSinh[a*x]^3
*Log[1 - E^ArcSinh[a*x]] - 24*a*x*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^(-Arc
Sinh[a*x])] - 48*a*x*PolyLog[2, E^(-ArcSinh[a*x])] - 24*a*x*ArcSinh[a*x]^2*
PolyLog[2, E^ArcSinh[a*x]] - 48*a*x*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*
x])] + 48*a*x*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 48*a*x*PolyLog[4, -
E^(-ArcSinh[a*x])] - 48*a*x*PolyLog[4, E^ArcSinh[a*x]] + 12*a*x*ArcSinh[a*x
]^2*Tanh[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2]))/(16*x)
```

Maple [A]
time = 5.10, size = 377, normalized size = 1.80

method	result
default	$-\frac{\operatorname{arcsinh}(ax)^2\left(x^2 \operatorname{arcsinh}(ax)a^2+3\sqrt{a^2x^2+1}\,ax+\operatorname{arcsinh}(ax)\right)}{2\sqrt{a^2x^2+1}\,x^2}+\frac{a^2\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}+\frac{3a^2 \operatorname{arc}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)^2*(arcsinh(a*x)*a^2*x^2+3*(a^2*x^2+
1)^(1/2)*a*x+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2
))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a
*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a^2*x^2+1)^(1/2
))-1/2*a^2*arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(a*x)^
2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+3*a^2*arcsinh(a*x)*polylog(3,a*x+(a^2*x^
2+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x)*ln(1+
a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*arcsin
h(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)^3}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^3/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

3.350 $\int \frac{(c+a^2cx^2)^3}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=67

$$\frac{35c^3\text{Chi}(\sinh^{-1}(ax))}{64a}+\frac{21c^3\text{Chi}(3\sinh^{-1}(ax))}{64a}+\frac{7c^3\text{Chi}(5\sinh^{-1}(ax))}{64a}+\frac{c^3\text{Chi}(7\sinh^{-1}(ax))}{64a}$$

```
[Out] 35/64*c^3*Chi(arcsinh(a*x))/a+21/64*c^3*Chi(3*arcsinh(a*x))/a+7/64*c^3*Chi(
5*arcsinh(a*x))/a+1/64*c^3*Chi(7*arcsinh(a*x))/a
```

Rubi [A]
time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5791, 3393, 3382}

$$\frac{35c^3\text{Chi}(\sinh^{-1}(ax))}{64a}+\frac{21c^3\text{Chi}(3\sinh^{-1}(ax))}{64a}+\frac{7c^3\text{Chi}(5\sinh^{-1}(ax))}{64a}+\frac{c^3\text{Chi}(7\sinh^{-1}(ax))}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

```
[Out] (35*c^3*CoshIntegral[ArcSinh[a*x]])/(64*a) + (21*c^3*CoshIntegral[3*ArcSinh
[a*x]])/(64*a) + (7*c^3*CoshIntegral[5*ArcSinh[a*x]])/(64*a) + (c^3*CoshInt
egral[7*ArcSinh[a*x]])/(64*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\sinh^{-1}(ax)}\,dx = \frac{c^3\text{Subst}\left(\int \frac{\cosh^7(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= \frac{c^3\text{Subst}\left(\int \left(\frac{35\cosh(x)}{64x}+\frac{21\cosh(3x)}{64x}+\frac{7\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right)\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= \frac{c^3\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a}+\frac{(7c^3)\,\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a}+\frac{(21$$
$$= \frac{35c^3\text{Chi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{21c^3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Chi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{c^3\text{Chi}\left(7\sin\right.}{64a}$$

Mathematica [A]
time = 0.07, size = 43, normalized size = 0.64

$$\frac{c^3\left(35\text{Chi}\left(\sinh^{-1}(ax)\right)+21\text{Chi}\left(3\sinh^{-1}(ax)\right)+7\text{Chi}\left(5\sinh^{-1}(ax)\right)+\text{Chi}\left(7\sinh^{-1}(ax)\right)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
[Out] (c^3*(35*CoshIntegral[ArcSinh[a*x]] + 21*CoshIntegral[3*ArcSinh[a*x]] + 7*C
oshIntegral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]]))/(64*a)
```

Maple [A]
time = 2.28, size = 42, normalized size = 0.63

method	result
derivativedivides	$\frac{c^3(35\,\text{hyperbolicCosineIntegral}(\text{arcsinh}(ax))+21\,\text{hyperbolicCosineIntegral}(3\,\text{arcsinh}(ax))+7\,\text{hyperbolicCosineIntegral}(5}{64a}$
default	$\frac{c^3(35\,\text{hyperbolicCosineIntegral}(\text{arcsinh}(ax))+21\,\text{hyperbolicCosineIntegral}(3\,\text{arcsinh}(ax))+7\,\text{hyperbolicCosineIntegral}(5}{64a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/64/a*c^3*(35*Chi(arcsinh(a*x))+21*Chi(3*arcsinh(a*x))+7*Chi(5*arcsinh(a*x
))+Chi(7*arcsinh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int\frac{3a^2x^2}{\operatorname{asinh}(ax)}\,dx+\int\frac{3a^4x^4}{\operatorname{asinh}(ax)}\,dx+\int\frac{a^6x^6}{\operatorname{asinh}(ax)}\,dx+\int\frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/asinh(a*x),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/asinh(a*x), x) + Integral(3*a**4*x**4/asinh(a*x)
, x) + Integral(a**6*x**6/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,a^2\,x^2+c\right)}^3}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/asinh(a*x),x)
```

```
[Out] int((c + a^2*c*x^2)^3/asinh(a*x), x)
```

3.351 $\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=50

$$\frac{5c^2\text{Chi}(\sinh^{-1}(ax))}{8a}+\frac{5c^2\text{Chi}(3\sinh^{-1}(ax))}{16a}+\frac{c^2\text{Chi}(5\sinh^{-1}(ax))}{16a}$$

```
[Out] 5/8*c^2*Chi(arcsinh(a*x))/a+5/16*c^2*Chi(3*arcsinh(a*x))/a+1/16*c^2*Chi(5*a
rcsinh(a*x))/a
```

Rubi [A]
time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5791, 3393, 3382}

$$\frac{5c^2\text{Chi}(\sinh^{-1}(ax))}{8a}+\frac{5c^2\text{Chi}(3\sinh^{-1}(ax))}{16a}+\frac{c^2\text{Chi}(5\sinh^{-1}(ax))}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

```
[Out] (5*c^2*CoshIntegral[ArcSinh[a*x]])/(8*a) + (5*c^2*CoshIntegral[3*ArcSinh[a*
x]])/(16*a) + (c^2*CoshIntegral[5*ArcSinh[a*x]])/(16*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)}\,dx = \frac{c^2\text{Subst}\left(\int \frac{\cosh^5(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= \frac{c^2\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8x}+\frac{5\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= \frac{c^2\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{16a} + \frac{(5c^2)\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{16a} + \frac{(5$$
$$= \frac{5c^2\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a} + \frac{5c^2\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a} + \frac{c^2\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 0.68

$$\frac{c^2\left(10\text{Chi}\left(\sinh^{-1}(ax)\right)+5\text{Chi}\left(3\sinh^{-1}(ax)\right)+\text{Chi}\left(5\sinh^{-1}(ax)\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

```
[Out] (c^2*(10*CoshIntegral[ArcSinh[a*x]] + 5*CoshIntegral[3*ArcSinh[a*x]] + Cosh
Integral[5*ArcSinh[a*x]]))/(16*a)
```

Maple [A]
time = 1.78, size = 33, normalized size = 0.66

method	result
derivativedivides	$\frac{c^2(10\,\text{hyperbolicCosineIntegral}(\text{arcsinh}(ax))+5\,\text{hyperbolicCosineIntegral}(3\,\text{arcsinh}(ax))+\text{hyperbolicCosineIntegral}(5}{16a}$
default	$\frac{c^2(10\,\text{hyperbolicCosineIntegral}(\text{arcsinh}(ax))+5\,\text{hyperbolicCosineIntegral}(3\,\text{arcsinh}(ax))+\text{hyperbolicCosineIntegral}(5}{16a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*c^2*(10*Chi(arcsinh(a*x))+5*Chi(3*arcsinh(a*x))+Chi(5*arcsinh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/asinh(a*x),x)
[Out] c**2*(Integral(2*a**2*x**2/asinh(a*x), x) + Integral(a**4*x**4/asinh(a*x),
x) + Integral(1/asinh(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^2}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/asinh(a*x),x)
[Out] int((c + a^2*c*x^2)^2/asinh(a*x), x)
```

3.352 $\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{3c\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{c\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

```
[Out] 3/4*c*Chi(arcsinh(a*x))/a+1/4*c*Chi(3*arcsinh(a*x))/a
```

Rubi [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5791, 3393, 3382}

$$\frac{3c\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{c\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

```
[Out] (3*c*CoshIntegral[ArcSinh[a*x]])/(4*a) + (c*CoshIntegral[3*ArcSinh[a*x]])/(
4*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)}\,dx = \frac{c\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= \frac{c\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right)\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= \frac{c\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a} + \frac{(3c)\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a}$$
$$= \frac{3c\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a} + \frac{c\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

Mathematica [**A**]
time = 0.02, size = 23, normalized size = 0.79

$$\frac{c\left(3\text{Chi}\left(\sinh^{-1}(ax)\right) + \text{Chi}\left(3\sinh^{-1}(ax)\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/ArcSinh[a*x],x]
[Out] (c*(3*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]))/(4*a)
```

Maple [**A**]
time = 1.64, size = 22, normalized size = 0.76

method	result	size
derivativedivides	$\frac{c(3\,\text{hyperbolicCosineIntegral}(\text{arcsinh}(ax))+\text{hyperbolicCosineIntegral}(3\,\text{arcsinh}(ax)))}{4a}$	22
default	$\frac{c(3\,\text{hyperbolicCosineIntegral}(\text{arcsinh}(ax))+\text{hyperbolicCosineIntegral}(3\,\text{arcsinh}(ax)))}{4a}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/4/a*c*(3*Chi(arcsinh(a*x))+Chi(3*arcsinh(a*x)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{a^2x^2}{\operatorname{asinh}(ax)}\,dx+\int\frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/asinh(a*x),x)
[Out] c*(Integral(a**2*x**2/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{c\,a^2\,x^2+c}{\operatorname{asinh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/asinh(a*x),x)
[Out] int((c + a^2*c*x^2)/asinh(a*x), x)
```

3.353 $\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx = \int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2c\,x^2+c)\,\mathrm{arcsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

```
[Out] int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^2\operatorname{asinh}(ax)+\operatorname{asinh}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/asinh(a*x),x)
```

```
[Out] Integral(1/(a**2*x**2*asinh(a*x) + asinh(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\operatorname{asinh}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)*(c + a^2*c*x^2)),x)
[Out] int(1/(asinh(a*x)*(c + a^2*c*x^2)), x)
```

3.354 $\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)} dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)} dx = \int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)} dx$$

Mathematica [A]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
[Out] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2c\,x^2+c)^2 \operatorname{arcsinh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4 \operatorname{asinh}(ax)+2a^2x^2 \operatorname{asinh}(ax)+\operatorname{asinh}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/asinh(a*x),x)
```

```
[Out] Integral(1/(a**4*x**4*asinh(a*x) + 2*a**2*x**2*asinh(a*x) + asinh(a*x)), x)
/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\operatorname{asinh}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(asinh(a*x)*(c + a^2*c*x^2)^2), x)
```

3.355 $\int \frac{x^4 \sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^5}-\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^5}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^5}+\frac{\log(a+$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^5-1/16*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^5+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^5+1/16*ln(a+b*arcsinh(c*x))/b/c^5+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2
*a/b)/b/c^5+1/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^5-1/32*Shi(6*(
a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b/c^5
```

Rubi [A]
time = 0.32, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^5}-\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^5}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^5}+\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^5}-\frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^5}+\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b*c^5) - (C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (Cosh[(
6*a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^5) + Log[a + b*Ar
cSinh[c*x]]/(16*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]
))/b])/(32*b*c^5) + (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b]
)/(16*b*c^5) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32
*b*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{16(a+bx)}-\frac{\cosh(2x)}{32(a+bx)}-\frac{\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^5}\\
&= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^5}-\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^5}+\frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,d\right.}{32c}\\
&= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^5}-\cosh\\
&= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^5}-\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^5}+\cosh\left(\right.
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 152, normalized size = 0.74

$$\frac{-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+2\log\left(a+b\sinh^{-1}(cx)\right)+\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+2\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{32bc^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]

[Out] (-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) - 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + ArcSinh[c*x])] + 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^5)

Maple [A]
time = 9.33, size = 199, normalized size = 0.97

method	result
default	$\frac{\ln(a+b\,\mathrm{arcsinh}(cx))}{16c^5b}-\frac{e^{\frac{6a}{b}}\,\mathrm{expIntegral}\left(1,6\,\mathrm{arcsinh}(cx)+\frac{6a}{b}\right)}{64c^5b}+\frac{e^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)}{32c^5b}+\frac{e^{\frac{2a}{b}}\,\mathrm{expIntegral}(1,2\,\mathrm{arcs}}{64c^5b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/16*ln(a+b*arcsinh(c*x))/c^5/b-1/64/c^5/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)+1/32/c^5/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^5/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^5/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+1/32/c^5/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^5/b*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c^2x^2+1}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**4*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^4*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.356 $\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=183

$$\frac{\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^4}+\frac{\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^4}-\frac{\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^4}-\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\cdots\right.}{8bc\cdots}$$

```
[Out] -1/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-1/16*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^4+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^4+1
/8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^4+1/16*Chi(3*(a+b*arcsinh(c*x))/
b)*sinh(3*a/b)/b/c^4-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^4
```

Rubi [A]
time = 0.32, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^4}-\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^4}-\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^4}+\frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b*c^4) + (CoshIntegral[
(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^4) - (CoshIntegral[(5*(a
 + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^4) - (Cosh[a/b]*SinhIntegral[
(a + b*ArcSinh[c*x])/b])/(8*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*
ArcSinh[c*x]))/b])/(16*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSi
nh[c*x]))/b])/(16*b*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(x)}{8(a+bx)} - \frac{\sinh(3x)}{16(a+bx)} + \frac{\sinh(5x)}{16(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^4} + \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^4} - \frac{\text{Sub}}{}$$

$$= -\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,d\right.}{16c^4}$$

$$= \frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^4} + \frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16bc^4} - \frac{\text{Chi}\left(\frac{5a}{b}+5\,\text{s}\right.}{}$$

Mathematica [A]
time = 0.21, size = 135, normalized size = 0.74

$$\frac{2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)-2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

[Out] (2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)/b] - 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])])/(16*b*c^4)

Maple [A]
time = 6.25, size = 178, normalized size = 0.97

method	result
default	$\frac{e^{\frac{5a}{b}}\,\mathrm{expIntegral}\left(1,5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b}-\frac{e^{\frac{3a}{b}}\,\mathrm{expIntegral}\left(1,3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{32c^4b}-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{16c^4b}+\frac{e^{-\frac{a}{b}}\,\mathrm{expIn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/32/c^4/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/16/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/32/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/32/c^4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c^2x^2+1}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.357 $\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc^3}$$

```
[Out] 1/8*Chi(4*(a+b*arcsinh(c*x))/b)*cosh(4*a/b)/b/c^3-1/8*ln(a+b*arcsinh(c*x))/
b/c^3-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3
```

Rubi [A]
time = 0.16, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c^3) - Log[a
 + b*ArcSinh[c*x]]/(8*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh
[c*x]))/b])/(8*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^3}\\
&= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3}\\
&= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3}-\frac{\sin}{}\\
&= \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\right.}{8bc^3}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 65, normalized size = 0.79

$$\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\log\left(a+b\sinh^{-1}(cx)\right)-\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{8bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*x
]] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])])/(8*b*c^3)
```

Maple [A]
time = 4.72, size = 79, normalized size = 0.96

method	result	size
default	$-\frac{\ln(a+b\,\mathrm{arcsinh}(cx))}{8b\,c^3}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^3b}-\frac{\mathrm{e}^{-\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)}{16c^3b}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(a+b*arcsinh(c*x))/b/c^3-1/16/c^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4
*a/b)-1/16/c^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c^2x^2+1}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.358 $\int \frac{x\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=121

$$-\frac{\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^2}-\frac{\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^2}$$

```
[Out] 1/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+1/4*cosh(3*a/b)*Shi(3*(a+b*ar
csinh(c*x))/b)/b/c^2-1/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^2-1/4*Chi(
3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^2
```

Rubi [A]

time = 0.19, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^2}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/4*(CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b*c^2) - (CoshIntegr
al[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b*c^2) + (Cosh[a/b]*SinhIn
tegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^2) + (Cosh[(3*a)/b]*SinhIntegral[(3*
(a + b*ArcSinh[c*x]))/b])/(4*b*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}+\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}\\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx,\right.}{4c^2}\\
&= -\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^2}-\frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Sl}}{}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 91, normalized size = 0.75

$$\frac{-\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + ArcS
inh[c*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Cos
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^2)
```

Maple [A]
time = 4.19, size = 118, normalized size = 0.98

method	result
default	$\frac{e^{\frac{3a}{b}}\,\mathrm{expIntegral}\left(1,3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}+\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{8c^2b}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\,\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{8c^2b}-\frac{e^{-\frac{3a}{b}}\,\mathrm{expI}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/8/c^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-1/8/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c^2x^2+1}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)

[Out] Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.359 $\int \frac{\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc}+\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+1/2*ln(a+b*arcsinh(c*x))/b/
c-1/2*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c
```

Rubi [A]
time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5791, 3393, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc}+\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c) + Log[a +
b*ArcSinh[c*x]]/(2*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(2*b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c}$$
$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c} - \frac{\sinh}{}$$
$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc} + \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\right.}{2bc}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.77

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\log\left(a+b\sinh^{-1}(cx)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Log[a + b*ArcSinh[c*x
]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c)
```

Maple [A]
time = 4.98, size = 79, normalized size = 0.96

method	result	size
default	$\frac{\ln(a+b\,\mathrm{arcsinh}(cx))}{2bc}-\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)}{4cb}-\frac{\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)}{4cb}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(a+b*arcsinh(c*x))/b/c-1/4/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-
1/4/c/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x)),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x)), x)
```

3.360 $\int \frac{\sqrt{1+c^2x^2}}{x(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=78

$$-\frac{\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b}+\mathrm{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b-Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b
+Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/b) + (Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/b + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \int \left(\frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}+\frac{c^2x}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\right)dx\\
&= c^2\int \frac{x}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx+\int \frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx\\
&= \int \frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx+\mathrm{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)\\
&= \cosh\left(\frac{a}{b}\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\mathrm{Subst}\left(\int \frac{\mathrm{cosh}
\end{aligned}$$

$$= -\frac{\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b}+\int \frac{}{x\sqrt{1+c}}$$

Mathematica [A]
time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2+1}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))), x)
```

3.361 $\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=45

$$\frac{c\log\left(a+b\sinh^{-1}(cx)\right)}{b}+\text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

[Out] c*ln(a+b*arcsinh(c*x))/b+Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]

[Out] (c*Log[a + b*ArcSinh[c*x]])/b + Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \int \left(\frac{c^2}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}+\frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\right)dx\\
&= c^2\int \frac{1}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx+\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\\
&= \frac{c\log\left(a+b\sinh^{-1}(cx)\right)}{b}+\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx
\end{aligned}$$

Mathematica [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\,\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))), x)
```

3.362 $\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 3.20, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))), x)
```

3.363 $\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))), x)
```

3.364 $\int \frac{x^3 (1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$\frac{3\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64bc^4}+\frac{3\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64bc^4}-\frac{\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64bc^4}-\frac{\text{Chi}\left(\frac{7(a+b\sin}{}\right.}{}$$

```
[Out] -3/64*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-3/64*cosh(3*a/b)*Shi(3*(a+b
*arcsinh(c*x))/b)/b/c^4+1/64*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^4+
1/64*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^4+3/64*Chi((a+b*arcsinh(c*
x))/b)*sinh(a/b)/b/c^4+3/64*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^4-1
/64*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^4-1/64*Chi(7*(a+b*arcsinh(c
*x))/b)*sinh(7*a/b)/b/c^4
```

Rubi [A]
time = 0.35, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^4}+\frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64bc^4}-\frac{\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{64bc^4}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64bc^4}+\frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64bc^4}+\frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{64bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(64*b*c^4) + (3*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(64*b*c^4) - (CoshIntegral[
(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(64*b*c^4) - (CoshIntegral[(7*(a
 + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(64*b*c^4) - (3*Cosh[a/b]*SinhIntegra
l[(a + b*ArcSinh[c*x])/b])/(64*b*c^4) - (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(64*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*
ArcSinh[c*x]))/b])/(64*b*c^4) + (Cosh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSi
nh[c*x]))/b])/(64*b*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^4}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{3\sinh(x)}{64(a+bx)}-\frac{3\sinh(3x)}{64(a+bx)}+\frac{\sinh(5x)}{64(a+bx)}+\frac{\sinh(7x)}{64(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^4}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{64c^4}+\frac{\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{64c^4}-\frac{3\text{Subs}}{}\\
&= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{64c^4}-\frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh(}{a}\right.}{64c}\\
&= \frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64bc^4}+\frac{3\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{64bc^4}-\frac{\text{Chi}\left(\frac{5a}{b}+\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.48, size = 179, normalized size = 0.73

$$
\frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+3\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)-\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{7a}{b}\right)-3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{64bc^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)
/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] - 3*Cosh[a/b]*Sinh
Integral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSin
h[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(7*a)/
b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^4)
```

Maple [A]
time = 7.88, size = 238, normalized size = 0.97

method	result
default	$\frac{e^{\frac{7a}{b}} \operatorname{expIntegral}\left(1,7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)}{128c^4b} + \frac{e^{\frac{5a}{b}} \operatorname{expIntegral}\left(1,5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{128c^4b} - \frac{3\,e^{\frac{3a}{b}} \operatorname{expIntegral}\left(1,3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{128c^4b} - \frac{3\,e^{\frac{a}{b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/c^4/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+1/128/c^4/b*exp(5*a/b)*Ei
(1,5*arcsinh(c*x)+5*a/b)-3/128/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-
3/128/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/128/c^4/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)+3/128/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/128/c^
4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/128/c^4/b*exp(-7*a/b)*Ei(1,-7
*arcsinh(c*x)-7*a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(c^2x^2+1)^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c^2\,x^2+1\right)}^{3/2}}{a+b\,\operatorname{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

3.365 $\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}-\frac{\log\left(a+\right.}{}$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3+1/16*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^3+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^3-1/16*ln(a+b*arcsinh(c*x))/b/c^3+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2
*a/b)/b/c^3-1/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3-1/32*Shi(6*(
a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b/c^3
```

Rubi [A]
time = 0.28, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 5556, 3384, 3379, 3382}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^3}-\frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b*c^3) + (C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c^3) + (Cosh[(
6*a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) - Log[a + b*Ar
cSinh[c*x]]/(16*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]
))/b])/(32*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b]
)/(16*b*c^3) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32
*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{16(a+bx)} - \frac{\cosh(2x)}{32(a+bx)} + \frac{\cosh(4x)}{16(a+bx)} + \frac{\cosh(6x)}{32(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$
$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^3} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^3} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\right.}{3}$$
$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^3} + \frac{\cos}{}$$
$$= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^3} + \frac{\cosh\left(\frac{6}{}\right.}{}$$

Mathematica [A]
time = 0.35, size = 152, normalized size = 0.74

$$\frac{-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-2\log\left(a+b\sinh^{-1}(cx)\right)+\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-2\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{32bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) + 2*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] - 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/
b + ArcSinh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] -
 Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^3)
```

Maple [A]
time = 8.98, size = 199, normalized size = 0.97

method	result
default	$-\frac{\ln(a+b\,\mathrm{arcsinh}(cx))}{16b\,c^3}-\frac{\mathrm{e}^{\frac{6a}{b}}\,\mathrm{expIntegral}\left(1,6\,\mathrm{arcsinh}(cx)+\frac{6a}{b}\right)}{64c^3b}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)}{32c^3b}+\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}(1,2\,\mathrm{ar}}{64c^3b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*ln(a+b*arcsinh(c*x))/b/c^3-1/64/c^3/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+
6*a/b)-1/32/c^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^3/b*exp(2*a/
b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2
*a/b)-1/32/c^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^3/b*exp(-6*
a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c^2\,x^2+1\right)}^{3/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

3.366 $\int \frac{x(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=183

$$-\frac{\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^2}-\frac{3\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^2}-\frac{\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Sh}}{}$$

[Out] 1/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+3/16*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b/c^2+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^2-1/8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^2-3/16*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^2-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^2

Rubi [A]

time = 0.25, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^2}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^2}+\frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^2}$$

Antiderivative was successfully verified.

[In] Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]

[Out] -1/8*(CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b*c^2) - (3*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^2) - (CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^2) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^2) + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^2) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^2)

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)} + \frac{3\sinh(3x)}{16(a+bx)} + \frac{\sinh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^2} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^2} + \frac{3\text{Subst}}{}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^2} + \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\right.}{16c^2}$$
$$= -\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^2} - \frac{3\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16bc^2} - \frac{\text{Chi}\left(\frac{5a}{b}+\right.}{}$$

Mathematica [A]
time = 0.32, size = 136, normalized size = 0.74

$$\frac{-2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right) - 3\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) - \text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right) + 2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

[Out] (-2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a
)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSi
nh[c*x])])/(16*b*c^2)

Maple [A]
time = 6.12, size = 178, normalized size = 0.97

method	result
default	$\frac{e^{\frac{5a}{b}} \operatorname{expIntegral}\left(1,5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^2b}+\frac{3\,e^{\frac{3a}{b}} \operatorname{expIntegral}\left(1,3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{32c^2b}+\frac{e^{\frac{a}{b}} \operatorname{expIntegral}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{16c^2b}-\frac{e^{-\frac{a}{b}} \operatorname{exp}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/32/c^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3/32/c^2/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)+1/16/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/16/c^2/
b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-3/32/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-1/32/c^2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(3/2)*x/(b*arcsinh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c^2x^2+1)^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(c^2\,x^2+1\right)}^{3/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

3.367 $\int \frac{(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=144

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc}+\frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+1/8*Chi(4*(a+b*arcsinh(c*x)
)/b)*cosh(4*a/b)/b/c+3/8*ln(a+b*arcsinh(c*x))/b/c-1/2*Shi(2*(a+b*arcsinh(c*
x))/b)*sinh(2*a/b)/b/c-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c
```

Rubi [A]
time = 0.16, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5791, 3393, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc}+\frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c) + (Cosh[(4
*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c) + (3*Log[a + b*Arc
Sinh[c*x]])/(8*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/
b])/(2*b*c) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b
*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\right.}{2}$$
$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c} + \frac{\cosh}{}$$
$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc} + \frac{3\log\left(a\right.}{}$$

Mathematica [A]
time = 0.19, size = 109, normalized size = 0.76

$$\frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+3\log\left(a+b\sinh^{-1}(cx)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{8bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*CoshI
ntegral[4*(a/b + ArcSinh[c*x])] + 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(2*a)/
b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b
 + ArcSinh[c*x])])/(8*b*c)
```

Maple [A]
time = 9.43, size = 139, normalized size = 0.97

method	result
default	$\frac{3\ln(a+b\,\mathrm{arcsinh}(cx))}{8bc}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)}{16cb}-\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)}{4cb}-\frac{\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{expIntegral}(1,-2}{4cb}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 3/8*ln(a+b*arcsinh(c*x))/b/c-1/16/c/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)
-1/4/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b*exp(-2*a/b)*Ei(1,-2*
arcsinh(c*x)-2*a/b)-1/16/c/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)

[Out] Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c^2\,x^2+1\right)^{3/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x)),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x)), x)
```

3.368 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=139

$$-\frac{5\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b}-\frac{\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b}+\frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}}{}$$

```
[Out] 5/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b+1/4*cosh(3*a/b)*Shi(3*(a+b*arcsin
h(c*x))/b)/b-5/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b-1/4*Chi(3*(a+b*arcsi
nh(c*x))/b)*sinh(3*a/b)/b+Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(
1/2),x)
```

Rubi [A]
time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b) - (CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b) + (5*Cosh[a/b]*SinhIntegral
[(a + b*ArcSinh[c*x])/b])/(4*b) + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*Arc
Sinh[c*x]))/b])/(4*b) + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \left(\frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)} + \frac{2c^2x}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)} + \frac{}{\sqrt{1}\ldots}\right.$$

$$= (2c^2)\int \frac{x}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx + c^4\int \frac{x^3}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(c\ldots\right.}$$

$$= 2\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + \int \frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,d\ldots$$

$$= i\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right) + \left(2\cosh\left(\frac{a}{b}\right)\right)\text{Su}\ldots$$

$$= -\frac{2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b} + \frac{1}{4}\text{Subst}\ldots$$

$$= -\frac{2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b} - \frac{1}{4}\left(3\cos\ldots\right.$$

$$= -\frac{5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b} - \frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4b} + \frac{5\cosh\ldots}{}$$

Mathematica [A]
time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x*arcsinh(c*x) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\operatorname{asinh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))), x)
```

3.369 $\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=106

$$\frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2b}+\frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{2b}-\frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2b}+\text{Int}\left(\frac{}{x^2\sqrt{1+c}}\right.$$

```
[Out] 1/2*c*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b+3/2*c*ln(a+b*arcsinh(c*x))/
b-1/2*c*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b+Unintegrable(1/x^2/(a+b*a
rcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b) + (3*c*Log
[a + b*ArcSinh[c*x]])/(2*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSi
nh[c*x]))/b])/(2*b) + Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \left(\frac{2c^2}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)} + \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)} + \frac{}{\sqrt{}}\right.$$

$$= (2c^2)\int \frac{1}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx + c^4\int \frac{x^2}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(\right.}$$

$$= \frac{2c\log\left(a+b\sinh^{-1}(cx)\right)}{b} + c\text{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + \int \frac{}{x^2\sqrt{}}$$

$$= \frac{2c\log\left(a+b\sinh^{-1}(cx)\right)}{b} - c\text{Subst}\left(\int \left(\frac{1}{2(a+bx)} - \frac{\cosh(2x)}{2(a+bx)}\right)\,dx, x, \sinh\right.$$

$$= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{2b} + \frac{1}{2}c\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + \int \frac{}{x^2}$$

$$= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{2b} + \frac{1}{2}\left(c\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x\right.$$

$$= \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2b} + \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{2b} - \frac{c\sinh\left(\frac{2a}{b}\right)}{}$$

Mathematica [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2\left(a+b\,\text{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2\,(a+b\,\mathrm{asinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))), x)
```

3.370 $\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\sinh^{-1}(cx))} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A]
time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\,\text{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\,\mathrm{asinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c^2 x^2 + 1)^{3/2}}{x^3 \, (a + b \operatorname{asinh}(c x))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))), x)
```

3.371 $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^4\,(a+b\sinh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [A]

time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\,(a+b\sinh^{-1}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^4\,(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A]

time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [A]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4\,(a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4\,(a+b\,\mathrm{asinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**4*(a + b*asinh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c^2 x^2 + 1\right)^{3/2}}{x^4 \left(a + b \operatorname{asinh}\left(c x\right)\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))), x)
```

3.372 $\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$\frac{3\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{128bc^4}+\frac{\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{3a}{b}\right)}{32bc^4}-\frac{3\text{Chi}\left(\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{7a}{b}\right)}{256bc^4}-\frac{\text{Chi}\left(\frac{9(a+b}{}\right.}{}$$

```
[Out] -3/128*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-1/32*cosh(3*a/b)*Shi(3*(a+
b*arcsinh(c*x))/b)/b/c^4+3/256*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^
4+1/256*cosh(9*a/b)*Shi(9*(a+b*arcsinh(c*x))/b)/b/c^4+3/128*Chi((a+b*arcsin
h(c*x))/b)*sinh(a/b)/b/c^4+1/32*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c
^4-3/256*Chi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/b/c^4-1/256*Chi(9*(a+b*arc
sinh(c*x))/b)*sinh(9*a/b)/b/c^4
```

Rubi [A]
time = 0.35, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^4}-\frac{3\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{256bc^4}-\frac{\sinh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{256bc^4}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128bc^4}-\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^4}+\frac{3\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{256bc^4}+\frac{\cosh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{256bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(128*b*c^4) + (CoshInteg
ral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(32*b*c^4) - (3*CoshIntegral
[(7*(a + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(256*b*c^4) - (CoshIntegral[(9*
(a + b*ArcSinh[c*x]))/b]*Sinh[(9*a)/b])/(256*b*c^4) - (3*Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(128*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*
(a + b*ArcSinh[c*x]))/b])/(32*b*c^4) + (3*Cosh[(7*a)/b]*SinhIntegral[(7*(a
+ b*ArcSinh[c*x]))/b])/(256*b*c^4) + (Cosh[(9*a)/b]*SinhIntegral[(9*(a + b*
ArcSinh[c*x]))/b])/(256*b*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{3\sinh(x)}{128(a+bx)} - \frac{\sinh(3x)}{32(a+bx)} + \frac{3\sinh(7x)}{256(a+bx)} + \frac{\sinh(9x)}{256(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(9x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{256c^4} + \frac{3\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{256c^4} - \frac{3\text{Subs}}{}$$

$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{128c^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\right.}{32c^4}$$

$$= \frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{128bc^4} + \frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{32bc^4} - \frac{3\text{Chi}\left(\frac{7a}{b}+7\right.}{}$$

Mathematica [A]
time = 0.72, size = 180, normalized size = 0.73

$$\frac{6\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+8\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{7a}{b}\right)-\text{Chi}\left(9\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{9a}{b}\right)-6\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-8\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+3\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{9a}{b}\right)\text{Shi}\left(9\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{256bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (6*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 8*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] - 3*CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*
a)/b] - CoshIntegral[9*(a/b + ArcSinh[c*x])]*Sinh[(9*a)/b] - 6*Cosh[a/b]*Si
nhIntegral[a/b + ArcSinh[c*x]] - 8*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])] + 3*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + Cosh[(9
*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(256*b*c^4)
```

Maple [A]
time = 6.16, size = 238, normalized size = 0.97

method	result
default	$\frac{e^{\frac{9a}{b}} \operatorname{expIntegral}\left(1,9 \operatorname{arcsinh}(cx)+\frac{9a}{b}\right)}{512c^4b}+\frac{3\,e^{\frac{7a}{b}} \operatorname{expIntegral}\left(1,7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)}{512c^4b}-\frac{e^{\frac{3a}{b}} \operatorname{expIntegral}\left(1,3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{64c^4b}-\frac{3\,e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/512/c^4/b*exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)+3/512/c^4/b*exp(7*a/b)*Ei
(1,7*arcsinh(c*x)+7*a/b)-1/64/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3
/256/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/256/c^4/b*exp(-a/b)*Ei(1,-arcs
inh(c*x)-a/b)+1/64/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-3/512/c^4/
b*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-1/512/c^4/b*exp(-9*a/b)*Ei(1,-9*a
rcsinh(c*x)-9*a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(c^2x^2+1)^{\frac{5}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c^2\,x^2+1\right)}^{5/2}}{a+b\,\operatorname{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

3.373 $\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=268

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\cosh\left(\right.}{}$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3+1/32*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^3+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^3+1/128*Chi(8*(a+b*arcsinh(c*x))/b)*cosh(8*a/b)/b/c^3-5/128*ln(a+b*arcsin
h(c*x))/b/c^3+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c^3-1/32*Shi(4
*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3-1/32*Shi(6*(a+b*arcsinh(c*x))/b)*s
inh(6*a/b)/b/c^3-1/128*Shi(8*(a+b*arcsinh(c*x))/b)*sinh(8*a/b)/b/c^3
```

Rubi [A]
time = 0.33, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{8a}{b}\right)\text{Chi}\left(\frac{8(a+b\sinh^{-1}(cx))}{b}\right)}{128bc^3}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}-\frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}-\frac{\sinh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8(a+b\sinh^{-1}(cx))}{b}\right)}{128bc^3}-\frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{128bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b*c^3) + (C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) + (Cosh[(
6*a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) + (Cosh[(8*a)/
b]*CoshIntegral[(8*(a + b*ArcSinh[c*x]))/b])/(128*b*c^3) - (5*Log[a + b*Arc
Sinh[c*x]])/(128*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(32*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b
])/(32*b*c^3) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(3
2*b*c^3) - (Sinh[(8*a)/b]*SinhIntegral[(8*(a + b*ArcSinh[c*x]))/b])/(128*b*
c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{5}{128(a+bx)} - \frac{\cosh(2x)}{32(a+bx)} + \frac{\cosh(4x)}{32(a+bx)} + \frac{\cosh(6x)}{32(a+bx)} + \frac{\cosh(8x)}{128(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3} \\
&= -\frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{128bc^3} + \frac{\text{Subst}\left(\int \frac{\cosh(8x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{128c^3} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\right.}{3} \\
&= -\frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{128bc^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^3} + \frac{\cos}{} \\
&= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{32bc^3} + \frac{\cosh\left(\frac{6a}{b}\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.63, size = 197, normalized size = 0.74

$$\frac{-4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+4\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+4\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{8a}{b}\right)\text{Chi}\left(8\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-5\log\left(a+b\sinh^{-1}(cx)\right)+4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{8a}{b}\right)\text{Shi}\left(8\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{128bc^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]

[Out] (-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 4*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] + 4*Cosh[(6*a)/b]*CoshIntegral[6*(a/b + ArcSinh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + ArcSinh[c*x])] - 5*Log[a + b*ArcSinh[c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - Sinh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])])/(128*b*c^3)

Maple [A]
time = 12.27, size = 259, normalized size = 0.97

method	result
default	$-\frac{5\ln(a+b\,\mathrm{arcsinh}(cx))}{128 b\,c^3}-\frac{\mathrm{e}^{\frac{8a}{b}}\,\mathrm{expIntegral}\left(1,8\,\mathrm{arcsinh}(cx)+\frac{8a}{b}\right)}{256c^3 b}-\frac{\mathrm{e}^{\frac{6a}{b}}\,\mathrm{expIntegral}\left(1,6\,\mathrm{arcsinh}(cx)+\frac{6a}{b}\right)}{64c^3 b}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{expIntegral}(1}{64}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] -5/128*ln(a+b*arcsinh(c*x))/b/c^3-1/256/c^3/b*exp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)-1/64/c^3/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-1/64/c^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)-1/64/c^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^3/b*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)-1/256/c^3/b*exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

```
[Out] integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c^2x^2+1)^{\frac{5}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c^2\,x^2+1\right)}^{5/2}}{a+b\,\operatorname{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

3.374 $\int \frac{x\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$-\frac{5\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64bc^2}-\frac{9\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64bc^2}-\frac{5\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64bc^2}-\frac{\text{Chi}\left(\frac{7(}{}\right.}{}$$

```
[Out] 5/64*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+9/64*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^2+5/64*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^2+1
/64*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^2-5/64*Chi((a+b*arcsinh(c*x
))/b)*sinh(a/b)/b/c^2-9/64*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^2-5/
64*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^2-1/64*Chi(7*(a+b*arcsinh(c*
x))/b)*sinh(7*a/b)/b/c^2
```

Rubi [A]
time = 0.34, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5819, 5556, 3384, 3379, 3382}

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^2}-\frac{9\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^2}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^2}-\frac{\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^2}+\frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^2}+\frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(64*b*c^2) - (9*CoshInt
egral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(64*b*c^2) - (5*CoshIntegr
al[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(64*b*c^2) - (CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(64*b*c^2) + (5*Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(64*b*c^2) + (9*Cosh[(3*a)/b]*SinhIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(64*b*c^2) + (5*Cosh[(5*a)/b]*SinhIntegral[(5*(a
 + b*ArcSinh[c*x]))/b])/(64*b*c^2) + (Cosh[(7*a)/b]*SinhIntegral[(7*(a + b*
ArcSinh[c*x]))/b])/(64*b*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64(a+bx)}+\frac{9\sinh(3x)}{64(a+bx)}+\frac{5\sinh(5x)}{64(a+bx)}+\frac{\sinh(7x)}{64(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{5\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{5\text{Subst}
\end{aligned}
$$

$$
= \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{\left(9\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3\right.}{a+bx}\right.}{64c^2}
$$

$$
= -\frac{5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64bc^2} - \frac{9\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{64bc^2} - \frac{5\text{Chi}\left(\frac{5a}{b}\right.}{}
$$

Mathematica [A]
time = 0.59, size = 180, normalized size = 0.73

$$
\frac{-5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-9\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-5\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)-\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{7a}{b}\right)+5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+9\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+5\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{64bc^2}
$$

Antiderivative was successfully verified.

[In] Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]

[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 9*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 5*CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] + 5*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 9*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 5*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^2)

Maple [A]
time = 7.04, size = 238, normalized size = 0.97

method	result
default	$\frac{e^{\frac{7a}{b}} \operatorname{expIntegral}\left(1,7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)}{128c^2b}+\frac{5\,e^{\frac{5a}{b}} \operatorname{expIntegral}\left(1,5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{128c^2b}+\frac{9\,e^{\frac{3a}{b}} \operatorname{expIntegral}\left(1,3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{128c^2b}+\frac{5}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/128/c^2/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+5/128/c^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+9/128/c^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+5/128/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/128/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-9/128/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-5/128/c^2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/128/c^2/b*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(5/2)*x/(b*arcsinh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c^2x^2+1)^{\frac{5}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(c^2\,x^2+1\right)}^{5/2}}{a+b\,\operatorname{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

3.375 $\int \frac{(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$\frac{15\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc}+\frac{\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc}+\frac{5\log}{}$$

```
[Out] 15/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+3/16*Chi(4*(a+b*arcsinh(c
*x))/b)*cosh(4*a/b)/b/c+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/c+5/
16*ln(a+b*arcsinh(c*x))/b/c-15/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b
/c-3/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c-1/32*Shi(6*(a+b*arcsinh
(c*x))/b)*sinh(6*a/b)/b/c
```

Rubi [A]
time = 0.21, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5791, 3393, 3384, 3379, 3382}

$$\frac{15\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc}+\frac{\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc}-\frac{15\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc}-\frac{3\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc}-\frac{\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc}+\frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{16bc}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(32*b*c) + (3*C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c) + (Cosh[(6*
a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c) + (5*Log[a + b*Arc
Sinh[c*x]])/(16*b*c) - (15*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(32*b*c) - (3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b
])/(16*b*c) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*
b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^6(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c}\\ &= \frac{\text{Subst}\left(\int \left(\frac{5}{16(a+bx)}+\frac{15\cosh(2x)}{32(a+bx)}+\frac{3\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c}\\ &= \frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{16bc}+\frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{32c}+\frac{3\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\right.}{16}\\ &= \frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{16bc}+\frac{\left(15\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{32c}+\\ &= \frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc}+\frac{\cosh\left(\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 153, normalized size = 0.74

$$\frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+6\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+10\log\left(a+b\sinh^{-1}(cx)\right)-15\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-6\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{32bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

[Out] (15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 6*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + ArcSinh[c*x])] + 10*Log[a + b*ArcSinh[c*x]] - 15*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 6*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c)

Maple [A]
time = 12.64, size = 199, normalized size = 0.97

method	result
default	$\frac{5\ln(a+b\,\mathrm{arcsinh}(cx))}{16bc}-\frac{\mathrm{e}^{\frac{6a}{b}}\,\mathrm{expIntegral}\left(1,6\,\mathrm{arcsinh}(cx)+\frac{6a}{b}\right)}{64cb}-\frac{3\,\mathrm{e}^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)}{32cb}-\frac{15\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}(}{6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 5/16*ln(a+b*arcsinh(c*x))/b/c-1/64/c/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-3/32/c/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-15/64/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-15/64/c/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)-3/32/c/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c/b*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)

[Out] Integral((c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")

[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x)),x)

[Out] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x)), x)

3.376 $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=195

$$-\frac{11\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b}-\frac{7\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b}-\frac{\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b}+\frac{11\cosh}{}$$

```
[Out] 11/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b+7/16*cosh(3*a/b)*Shi(3*(a+b*arcs
inh(c*x))/b)/b+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b-11/8*Chi((a+b
*arcsinh(c*x))/b)*sinh(a/b)/b-7/16*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/
b-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b+Unintegrable(1/x/(a+b*arcs
inh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-11*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b) - (7*CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b) - (CoshIntegral[(5*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b) + (11*Cosh[a/b]*SinhIntegral[(a
+ b*ArcSinh[c*x])/b])/(8*b) + (7*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSi
nh[c*x]))/b])/(16*b) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))
/b])/(16*b) + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \left(\frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)} + \frac{3c^2x}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)} + \frac{}{\sqrt{1+}}\right.$$

$$= (3c^2)\int \frac{x}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx + (3c^4)\int \frac{x^3}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}\right.}$$

$$= 3\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + 3\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right.$$

$$= -\left(i\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8(a+bx)} - \frac{5i\sinh(3x)}{16(a+bx)} + \frac{i\sinh(5x)}{16(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)\right)$$

$$= -\frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b} + \frac{1}{16}\text{Subst}$$

$$= -\frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b} + \frac{1}{8}\left(5\cosh\right.$$

$$= -\frac{11\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b} - \frac{7\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16b} - \text{Chi}($$

Mathematica [A]
time = 1.41, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\left(a+b\,\text{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*
x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\operatorname{asinh}{(cx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x*(a + b*asinh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))), x)
```

3.377 $\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\sinh^{-1}(cx))} dx$

Optimal. Leaf size=159

$$\frac{c\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{b}+\frac{c\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8b}+\frac{15c\log\left(a+b\sinh^{-1}(cx)\right)}{8b}-\frac{c\sinh\left(\frac{2a}{b}\right)}{}$$

```
[Out] c*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b+1/8*c*Chi(4*(a+b*arcsinh(c*x))/
b)*cosh(4*a/b)/b+15/8*c*ln(a+b*arcsinh(c*x))/b-c*Shi(2*(a+b*arcsinh(c*x))/b
)*sinh(2*a/b)/b-1/8*c*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b+Unintegrabl
e(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/b + (c*Cosh[(4*a
)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b) + (15*c*Log[a + b*ArcS
inh[c*x]])/(8*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b
])/b - (c*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b) + D
efer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \left(\frac{3c^2}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)} + \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)} + \frac{}{\sqrt{1}}\right.$$
$$= (3c^2)\int \frac{1}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx + (3c^4)\int \frac{x^2}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}\right.}$$
$$= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{b} + c\text{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + (3c)\text{Subs}$$
$$= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{b} + c\text{Subst}\left(\int \left(\frac{3}{8(a+bx)} - \frac{\cosh(2x)}{2(a+bx)} + \frac{\cosh(4x)}{8(a+bx)}\right)\right.$$
$$= \frac{15c\log\left(a+b\sinh^{-1}(cx)\right)}{8b} + \frac{1}{8}c\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) - \frac{1}{2}c\text{Su}$$
$$= \frac{15c\log\left(a+b\sinh^{-1}(cx)\right)}{8b} - \frac{1}{2}\left(c\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x\right.$$
$$= \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b} + \frac{c\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8b} + \frac{15}{}$$

Mathematica [A]
time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\left(a+b\,\text{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) +
a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\,(a+b\operatorname{asinh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**2*(a + b*asinh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))), x)
```

3.378 $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 3.24, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) +
a*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{x^{3}\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**3*(a + b*asinh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c^2 x^2+1\right)^{5/2}}{x^3 \left(a+b \operatorname{asinh}\left(c x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))), x)
```

3.379 $\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\left(a+b\,\text{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) +
a*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**4*(a + b*asinh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c^2\,x^2+1)^{5/2}}{x^4\,\left(a+b\,\mathrm{asinh}\,(c\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))), x)
```

3.380 $\int \frac{x^4}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

```
[Out] -1/2*Chi(2*arcsinh(a*x))/a^5+1/8*Chi(4*arcsinh(a*x))/a^5+3/8*ln(arcsinh(a*x
))/a^5
```

Rubi [A]
time = 0.10, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5819, 3393, 3382}

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] -1/2*CoshIntegral[2*ArcSinh[a*x]]/a^5 + CoshIntegral[4*ArcSinh[a*x]]/(8*a^5
) + (3*Log[ArcSinh[a*x]])/(8*a^5)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x} - \frac{\cosh(2x)}{2x} + \frac{\cosh(4x)}{8x}\right)\,dx, x, \sinh^{-1}(ax)\right)}{a^5}$$
$$= \frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(2x}{x}\right.}{}$$
$$= -\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5} + \frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5} + \frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.76

$$\frac{-4\text{Chi}\left(2\sinh^{-1}(ax)\right) + \text{Chi}\left(4\sinh^{-1}(ax)\right) + 3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (-4*CoshIntegral[2*ArcSinh[a*x]] + CoshIntegral[4*ArcSinh[a*x]] + 3*Log[Arc
Sinh[a*x]])/(8*a^5)
```

Maple [A]
time = 2.95, size = 30, normalized size = 0.73

method	result	size
default	$\frac{3\ln(\text{arcsinh}(ax)) - 4\,\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(ax)) + \text{hyperbolicCosineIntegral}(4\,\text{arcsinh}(ax))}{8a^5}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(3*ln(arcsinh(a*x))-4*Chi(2*arcsinh(a*x))+Chi(4*arcsinh(a*x)))/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**4/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\operatorname{asinh}(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^4/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.381 $\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$-\frac{3\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}+\frac{\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}$$

```
[Out] -3/4*Shi(arcsinh(a*x))/a^4+1/4*Shi(3*arcsinh(a*x))/a^4
```

Rubi [A]
time = 0.10, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5819, 3393, 3379}

$$\frac{\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}-\frac{3\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[ArcSinh[a*x]])/(4*a^4) + SinhIntegral[3*ArcSinh[a*x]]/(4*a
^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= \frac{i\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x} - \frac{i\sinh(3x)}{4x}\right)\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^4} - \frac{3\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{3\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4} + \frac{\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}$$

Mathematica [A]
time = 0.06, size = 22, normalized size = 0.81

$$\frac{-3\text{Shi}\left(\sinh^{-1}(ax)\right) + \text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (-3*SinhIntegral[ArcSinh[a*x]] + SinhIntegral[3*ArcSinh[a*x]])/(4*a^4)
```

Maple [A]
time = 2.79, size = 23, normalized size = 0.85

method	result	size
default	$-\frac{3\,\text{hyperbolicSineIntegral}(\text{arcsinh}(ax)) - \text{hyperbolicSineIntegral}(3\,\text{arcsinh}(ax))}{4a^4}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(3*Shi(arcsinh(a*x))-Shi(3*arcsinh(a*x)))/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{\operatorname{asinh}(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^3/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.382 $\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

```
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

Rubi [A]
time = 0.09, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5819, 3393, 3382}

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^3}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)\,dx, x, \sinh^{-1}(ax)\right)}{a^3}\\ &= -\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{2a^3}\\ &= \frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 22, normalized size = 0.81

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)-\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

Maple [A]
time = 2.56, size = 21, normalized size = 0.78

method	result	size
default	$-\frac{\ln(\text{arcsinh}(ax))-\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(ax))}{2a^3}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(ln(arcsinh(a*x))-Chi(2*arcsinh(a*x)))/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\ \operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\operatorname{asinh}(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.383 $\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\operatorname{Chi}(2\sinh^{-1}(ax))}{2a^3}-\frac{\log(\sinh^{-1}(ax))}{2a^3}$$

```
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

Rubi [A]
time = 0.09, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5819, 3393, 3382}

$$\frac{\operatorname{Chi}(2\sinh^{-1}(ax))}{2a^3}-\frac{\log(\sinh^{-1}(ax))}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right)\,dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{2a^3}$$
$$= \frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3} - \frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Mathematica [**A**]
time = 0.02, size = 22, normalized size = 0.81

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right) - \log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

Maple [**A**]
time = 1.62, size = 21, normalized size = 0.78

method	result	size
default	$-\frac{\ln(\operatorname{arcsinh}(ax)) - \text{hyperbolicCosineIntegral}(2\operatorname{arcsinh}(ax))}{2a^3}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(ln(arcsinh(a*x))-Chi(2*arcsinh(a*x)))/a^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\operatorname{asinh}(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.384 $\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\operatorname{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

[Out] Shi(arcsinh(a*x))/a^2

Rubi [A]
time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5819, 3379}

$$\frac{\operatorname{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]

[Out] SinhIntegral[ArcSinh[a*x]]/a^2

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\ &= \frac{\operatorname{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 9, normalized size = 1.00

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] SinhIntegral[ArcSinh[a*x]]/a^2
```

Maple [A]
time = 2.37, size = 10, normalized size = 1.11

method	result	size
default	$\frac{\text{hyperbolicSineIntegral}(\text{arcsinh}(ax))}{a^2}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] Shi(arcsinh(a*x))/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{x}{\operatorname{asinh}(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.385 $\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

```
[Out] ln(arcsinh(a*x))/a
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5782}

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Log[ArcSinh[a*x]]/a
```

Rule 5782

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx = \frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Mathematica [A]
time = 0.02, size = 9, normalized size = 1.00

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Log[ArcSinh[a*x]]/a
```

Maple [A]
time = 0.28, size = 10, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\ln(\operatorname{arcsinh}(ax))}{a}$	10
default	$\frac{\ln(\operatorname{arcsinh}(ax))}{a}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(arcsinh(a*x))/a
```

Maxima [A]
time = 0.27, size = 9, normalized size = 1.00

$$\frac{\log\left(\operatorname{arsinh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] log(arcsinh(a*x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.
time = 0.34, size = 21, normalized size = 2.33

$$\frac{\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] log(log(a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [A]
time = 0.24, size = 7, normalized size = 0.78

$$\frac{\log\left(\operatorname{asinh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] log(asinh(a*x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [B]
time = 0.10, size = 9, normalized size = 1.00

$$\frac{\ln\left(\mathrm{asinh}(a\,x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] log(asinh(a*x))/a
```

3.386 $\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Defer[Int][1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx = \int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{arcsinh}\,(ax)\,\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)/((a^2*x^3 + x)*arcsinh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \operatorname{asinh}(a x) \sqrt{a^2 x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x*asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.387 $\int \frac{1}{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{1}{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Defer[Int][1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx=\int \frac{1}{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\,\mathrm{arcsinh}\,(ax)\,\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)/((a^2*x^4 + x^2)*arcsinh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{asinh}(a x) \sqrt{a^2 x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^2*asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.388 $\int \frac{x^5}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=183

$$-\frac{5\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^6}+\frac{5\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^6}-\frac{\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^6}+\frac{5\cosh\left(\frac{a}{b}\right)}{}$$

```
[Out] 5/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^6-5/16*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^6+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^6-5/
8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^6+5/16*Chi(3*(a+b*arcsinh(c*x))/b
)*sinh(3*a/b)/b/c^6-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^6
```

Rubi [A]
time = 0.27, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 3393, 3384, 3379, 3382}

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^6}+\frac{5\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^6}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^6}+\frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^6}-\frac{5\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^6}+\frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b*c^6) + (5*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^6) - (CoshIntegral[
(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^6) + (5*Cosh[a/b]*SinhIn
tegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^6) - (5*Cosh[(3*a)/b]*SinhIntegral[(
3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^6) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b*c^6)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^5(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^6} \\
&= -\frac{i\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8(a+bx)} - \frac{5i\sinh(3x)}{16(a+bx)} + \frac{i\sinh(5x)}{16(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^6} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^6} - \frac{5\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}\right.}{16c^6} \\
&= \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^6} - \frac{\left(5\cosh\left(\frac{3a}{b}\right)\right)\text{S}}{} \\
&= -\frac{5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^6} + \frac{5\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3}{}\right.}{16bc^6}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 136, normalized size = 0.74

$$-\frac{10\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right) - 5\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) + \text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right) - 10\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 5\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - \cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16bc^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -1/16*(10*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/
b + ArcSinh[c*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sin
```

```
h[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b
+ ArcSinh[c*x])])/(b*c^6)
```

Maple [A]
time = 5.89, size = 178, normalized size = 0.97

method	result
default	$\frac{e^{\frac{5a}{b}} \operatorname{expIntegral}\left(1,5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^6 b} - \frac{5\, e^{\frac{3a}{b}} \operatorname{expIntegral}\left(1,3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{32c^6 b} + \frac{5\, e^{\frac{a}{b}} \operatorname{expIntegral}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{16c^6 b} - \frac{5\, e^{-\frac{a}{b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/c^6/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-5/32/c^6/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)+5/16/c^6/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/16/c^6/
b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+5/32/c^6/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-1/32/c^6/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^5/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + b \operatorname{asinh}(cx)) \sqrt{c^2 x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x**5/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^5/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.389 $\int \frac{x^4}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=144

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc^5}+\frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\right.}{2b}$$

```
[Out] -1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^5+1/8*Chi(4*(a+b*arcsinh(c
*x))/b)*cosh(4*a/b)/b/c^5+3/8*ln(a+b*arcsinh(c*x))/b/c^5+1/2*Shi(2*(a+b*arc
sinh(c*x))/b)*sinh(2*a/b)/b/c^5-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)
/b/c^5
```

Rubi [A]
time = 0.22, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 3393, 3384, 3379, 3382}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^5}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8bc^5}+\frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -1/2*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b*c^5) + (Co
sh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c^5) + (3*Log[a
+ b*ArcSinh[c*x]])/(8*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSin
h[c*x]))/b])/(2*b*c^5) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]
))/b])/(8*b*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8(a+bx)} - \frac{\cosh(2x)}{2(a+bx)} + \frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^5}$$

$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^5} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^5} - \frac{\text{Su}}{}$$

$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^5} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh\right.}{2c^5}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c\right.}{8bc^5}$$

Mathematica [A]
time = 0.16, size = 109, normalized size = 0.76

$$-\frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - \cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 3\log\left(a+b\sinh^{-1}(cx)\right) - 4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{8bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -1/8*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Cosh[(4*a)/b]*
CoshIntegral[4*(a/b + ArcSinh[c*x])] - 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(
```

2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Sinh[(4*a)/b]*SinhIntegral[4 *(a/b + ArcSinh[c*x])])/(b*c^5)

Maple [A]
time = 10.38, size = 139, normalized size = 0.97

method	result
default	$\frac{3\ln(a+b\,\mathrm{arcsinh}(cx))}{8c^5b}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^5b}+\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)}{4c^5b}+\frac{\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{expIntegral}(1,-2}{4c^5b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*ln(a+b*arcsinh(c*x))/c^5/b-1/16/c^5/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*
a/b)+1/4/c^5/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/4/c^5/b*exp(-2*a/b)*
Ei(1,-2*arcsinh(c*x)-2*a/b)-1/16/c^5/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a
/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+b\,\mathrm{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x**4/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^4/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.390 $\int \frac{x^3}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=121

$$\frac{3\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^4}-\frac{\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^4}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^4}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^4}$$

```
[Out] -3/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4+1/4*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^4+3/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^4-1/4*Chi
(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^4
```

Rubi [A]

time = 0.23, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5819, 3393, 3384, 3379, 3382}

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^4}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^4}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^4}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b*c^4) - (CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b*c^4) - (3*Cosh[a/b]*SinhI
ntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^4) + (Cosh[(3*a)/b]*SinhIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(4*b*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{i\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3x)}{4(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^4} - \frac{3\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(\right.}{4c^4}$$

$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^4} + \frac{\cosh\left(\frac{3a}{b}\right)\text{Sub}}{}$$

$$= \frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^4} - \frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4bc^4}$$

Mathematica [A]
time = 0.16, size = 92, normalized size = 0.76

$$\frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right) - \text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) - 3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + \cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{4bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - CoshIntegral[3*(a/b + ArcSi
nh[c*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Co
sh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^4)
```

Maple [A]
time = 6.10, size = 118, normalized size = 0.98

method	result
default	$\frac{e^{\frac{3a}{b}} \operatorname{expIntegral}\left(1,3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^4b}-\frac{3\,e^{\frac{a}{b}} \operatorname{expIntegral}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{8c^4b}+\frac{3\,e^{-\frac{a}{b}} \operatorname{expIntegral}\left(1,-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{8c^4b}-\frac{e^{-\frac{3a}{b}} e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3/8/c^4/b*exp(a/b)*Ei(1,arc
sinh(c*x)+a/b)+3/8/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-1/8/c^4/b*exp(-3
*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x**3/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.391 $\int \frac{x^2}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^3}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3-1/2*ln(a+b*arcsinh(c*x))/
b/c^3-1/2*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c^3
```

Rubi [A]
time = 0.16, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 3393, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^3}-\frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3) - Log[a
+ b*ArcSinh[c*x]]/(2*b*c^3) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh
[c*x]))/b])/(2*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}\right)\,dx,x,\sinh^{-1}(cx)\right)}{c^3}$$
$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{2c^3}$$
$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh$$
$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\sinh\left(\right.}{}$$

Mathematica [A]
time = 0.14, size = 65, normalized size = 0.79

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\log\left(a+b\sinh^{-1}(cx)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*x
]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^3)
```

Maple [A]
time = 5.44, size = 79, normalized size = 0.96

method	result	size
default	$-\frac{\ln\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{2b\,c^{3}}-\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}\left(cx\right)+\frac{2a}{b}\right)}{4c^{3}b}-\frac{\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,-2\,\mathrm{arcsinh}\left(cx\right)-\frac{2a}{b}\right)}{4c^{3}b}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(a+b*arcsinh(c*x))/b/c^3-1/4/c^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*
a/b)-1/4/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.392 $\int \frac{x}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=54

$$-\frac{\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc^2}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5819, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b*c^2)) + (Cosh[a/b]*Si
nhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{c}{\ }\right.}{}$$
$$= -\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 0.85

$$-\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] -((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]])/(b*c^2))
```

Maple [A]
time = 2.80, size = 58, normalized size = 1.07

method	result	size
default	$\frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)}{2c^2b}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2c^2b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/c^2/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b\operatorname{asinh}(cx))\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(a + b\operatorname{asinh}(c\,x))\,\sqrt{c^2\,x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.393 $\int \frac{1}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=16

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

```
[Out] ln(a+b*arcsinh(c*x))/b/c
```

Rubi [A]
time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5782}

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

Rule 5782

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx=\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.00

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

Maple [A]
time = 0.28, size = 17, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\,\mathrm{arcsinh}(cx))}{bc}$	17
default	$\frac{\ln(a+b\,\mathrm{arcsinh}(cx))}{bc}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*arcsinh(c*x))/b/c
```

Maxima [A]
time = 0.27, size = 16, normalized size = 1.00

$$\frac{\log\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(b*arcsinh(c*x) + a)/(b*c)
```

Fricas [A]
time = 0.34, size = 28, normalized size = 1.75

$$\frac{\log\left(b\log\left(cx+\sqrt{c^2x^2+1}\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(b*log(c*x + sqrt(c^2*x^2 + 1)) + a)/(b*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.
time = 0.82, size = 26, normalized size = 1.62

$$\begin{cases} \frac{x}{a} & \text{for}\ c=0\wedge\left(b=0\vee c=0\right)\\ \frac{\operatorname{asinh}\left(cx\right)}{ac} & \text{for}\ b=0\\ \frac{\log\left(\frac{a}{b}+\operatorname{asinh}\left(cx\right)\right)}{bc} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x/a, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (asinh(c*x)/(a*c), Eq(b,
 0)), (log(a/b + asinh(c*x))/(b*c), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [**B**]
time = 0.14, size = 16, normalized size = 1.00

$$\frac{\ln\left(a+b\,\mathrm{asinh}(c\,x)\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] log(a + b*asinh(c*x))/(b*c)
```

3.394 $\int \frac{1}{x\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^3 + a*x + (b*c^2*x^3 + b*x)*arcsinh(c*x
)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a + b \operatorname{asinh}\left(cx\right)\right)\sqrt{c^{2}x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.395 $\int \frac{1}{x^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\,\mathrm{arcsinh}\,(cx)\right)\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^4 + a*x^2 + (b*c^2*x^4 + b*x^2)*arcsinh
(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right) \sqrt{c^{2} x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.396 $\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^2}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{x^2}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*
c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b\operatorname{asinh}(cx))(c^2x^2 + 1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(c^2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.397 $\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{x}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))},x\right)$$

[Out] Unintegrable(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

[In] Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]

Rubi steps

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx=\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]

[Out] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^
2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\operatorname{asinh}(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.398 $\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{1}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\text{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^2*
x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{asinh}(cx))(c^2x^2 + 1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\operatorname{asinh}(cx))\,(c^2x^2+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.399 $\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]

[Out] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^5 + 2*a*c^2*x^3 + a*x + (b*c^4*x^5 + 2*
b*c^2*x^3 + b*x)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a + b\operatorname{asinh}\left(cx\right)\right)\left(c^{2}x^{2} + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.400 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

`[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)`

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]`

`[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]`

Rubi steps

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]`

`[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]`

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^6 + 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 +
2*b*c^2*x^4 + b*x^2)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \operatorname{asinh}{\left(cx \right)}\right) \left(c^2x^2 + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.401 $\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)} dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{x^m(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)} dx = \int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)} dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(c^2x^2+1)^{\frac{5}{2}}}{a+b\operatorname{arcsinh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) +
a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \left(c^2 x^2+1\right)^{5/2}}{a+b \operatorname{asinh}\left(c x\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
[Out] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

3.402 $\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\mathrm{arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (c^2x^2+1)^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \left(c^2 x^2+1\right)^{3/2}}{a+b \operatorname{asinh}\left(c x\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
[Out] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

3.403 $\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx=\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{c^2x^2+1}}{a+b\,\mathrm{arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c^2x^2+1}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \sqrt{c^2 x^2 + 1}}{a + b \operatorname{asinh}(c x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.404 $\int \frac{x^m}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{x^m}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx = \int \frac{x^m}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\,(a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a + b\,\mathrm{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.405 $\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*
c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a + b\operatorname{asinh}(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(c^2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.406 $\int \frac{(c+a^2cx^2)^3}{\sinh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=94

$$-\frac{c^3(1+a^2x^2)^{7/2}}{a\sinh^{-1}(ax)}+\frac{35c^3\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{63c^3\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{35c^3\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(7\sinh^{-1}(ax)\right)}{64a}$$

```
[Out] -c^3*(a^2*x^2+1)^(7/2)/a/arcsinh(a*x)+35/64*c^3*Shi(arcsinh(a*x))/a+63/64*c
^3*Shi(3*arcsinh(a*x))/a+35/64*c^3*Shi(5*arcsinh(a*x))/a+7/64*c^3*Shi(7*arc
sinh(a*x))/a
```

Rubi [A]
time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5790, 5819, 5556, 3379}

$$-\frac{c^3(a^2x^2+1)^{7/2}}{a\sinh^{-1}(ax)}+\frac{35c^3\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{63c^3\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{35c^3\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(7\sinh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

```
[Out] -((c^3*(1 + a^2*x^2)^(7/2))/(a*ArcSinh[a*x])) + (35*c^3*SinhIntegral[ArcSin
h[a*x]])/(64*a) + (63*c^3*SinhIntegral[3*ArcSinh[a*x]])/(64*a) + (35*c^3*Si
nhIntegral[5*ArcSinh[a*x]])/(64*a) + (7*c^3*SinhIntegral[7*ArcSinh[a*x]])/(
64*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
```

```
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\sinh^{-1}(ax)^2}\,dx = -\frac{c^3(1+a^2x^2)^{7/2}}{a\sinh^{-1}(ax)} + (7ac^3)\int \frac{x(1+a^2x^2)^{5/2}}{\sinh^{-1}(ax)}\,dx$$

$$= -\frac{c^3(1+a^2x^2)^{7/2}}{a\sinh^{-1}(ax)} + \frac{(7c^3)\,\mathrm{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$

$$= -\frac{c^3(1+a^2x^2)^{7/2}}{a\sinh^{-1}(ax)} + \frac{(7c^3)\,\mathrm{Subst}\left(\int \left(\frac{5\sinh(x)}{64x} + \frac{9\sinh(3x)}{64x} + \frac{5\sinh(5x)}{64x} + \frac{\sinh(7x)}{64x}\right)\,dx, x, \sin\right.}{a}$$

$$= -\frac{c^3(1+a^2x^2)^{7/2}}{a\sinh^{-1}(ax)} + \frac{(7c^3)\,\mathrm{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a} + \frac{(35c^3)\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{x}\right.}{64a}$$

$$= -\frac{c^3(1+a^2x^2)^{7/2}}{a\sinh^{-1}(ax)} + \frac{35c^3\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{64a} + \frac{63c^3\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{64a} + \frac{35c^3\mathrm{Shi}\left(5\sinh^{-1}\right.}{64a}$$

Mathematica [A]
time = 0.25, size = 82, normalized size = 0.87

$$\frac{c^3\left(-64(1+a^2x^2)^{7/2} + 35\sinh^{-1}(ax)\mathrm{Shi}\left(\sinh^{-1}(ax)\right) + 63\sinh^{-1}(ax)\mathrm{Shi}\left(3\sinh^{-1}(ax)\right) + 35\sinh^{-1}(ax)\mathrm{Shi}\left(5\sinh^{-1}(ax)\right) + 7\sinh^{-1}(ax)\mathrm{Shi}\left(7\sinh^{-1}(ax)\right)\right)}{64a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

```
[Out] (c^3*(-64*(1 + a^2*x^2)^(7/2) + 35*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
+ 63*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 35*ArcSinh[a*x]*SinhIntegr
al[5*ArcSinh[a*x]] + 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]]))/(64*a*Ar
cSinh[a*x])
```

Maple [A]
time = 3.36, size = 106, normalized size = 1.13

method	result
derivativedivides	$\frac{c^3\left(7\,\text{hyperbolicSineIntegral}(7\,\text{arcsinh}(ax))\,\text{arcsinh}(ax)+63\,\text{hyperbolicSineIntegral}(3\,\text{arcsinh}(ax))\,\text{arcsinh}(ax)+35\,\text{hyper}\right.}{}$
default	$\frac{c^3\left(7\,\text{hyperbolicSineIntegral}(7\,\text{arcsinh}(ax))\,\text{arcsinh}(ax)+63\,\text{hyperbolicSineIntegral}(3\,\text{arcsinh}(ax))\,\text{arcsinh}(ax)+35\,\text{hyper}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/a*c^3*(7*Shi(7*arcsinh(a*x))*arcsinh(a*x)+63*Shi(3*arcsinh(a*x))*arcsi
nh(a*x)+35*Shi(5*arcsinh(a*x))*arcsinh(a*x)+35*Shi(arcsinh(a*x))*arcsinh(a*
x)-21*cosh(3*arcsinh(a*x))-7*cosh(5*arcsinh(a*x))-cosh(7*arcsinh(a*x))-35*(
a^2*x^2+1)^(1/2))/arcsinh(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x + (
a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*sqrt(a^2
*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2
+ 1))) + integrate((7*a^10*c^3*x^10 + 29*a^8*c^3*x^8 + 46*a^6*c^3*x^6 + 34*
a^4*c^3*x^4 + 11*a^2*c^3*x^2 + c^3 + (7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 + 18*a
^4*c^3*x^4 + 4*a^2*c^3*x^2 - c^3)*(a^2*x^2 + 1) + 7*(2*a^9*c^3*x^9 + 7*a^7*
c^3*x^7 + 9*a^5*c^3*x^5 + 5*a^3*c^3*x^3 + a*c^3*x)*sqrt(a^2*x^2 + 1))/((a^4
*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 +
 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x)^2
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{3a^{2}x^{2}}{\operatorname{asinh}^{2}(ax)}\,dx + \int \frac{3a^{4}x^{4}}{\operatorname{asinh}^{2}(ax)}\,dx + \int \frac{a^{6}x^{6}}{\operatorname{asinh}^{2}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}^{2}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/asinh(a*x)**2,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/asinh(a*x)**2, x) + Integral(3*a**4*x**4/asinh(a
*x)**2, x) + Integral(a**6*x**6/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-
2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{asinh}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/asinh(a*x)^2,x)
```

```
[Out] int((c + a^2*c*x^2)^3/asinh(a*x)^2, x)
```

3.407 $\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{c^2(1+a^2x^2)^{5/2}}{a\sinh^{-1}(ax)}+\frac{5c^2\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{15c^2\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{5c^2\mathrm{Shi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

```
[Out] -c^2*(a^2*x^2+1)^(5/2)/a/arcsinh(a*x)+5/8*c^2*Shi(arcsinh(a*x))/a+15/16*c^2
*Shi(3*arcsinh(a*x))/a+5/16*c^2*Shi(5*arcsinh(a*x))/a
```

Rubi [A]
time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5790, 5819, 5556, 3379}

$$-\frac{c^2(a^2x^2+1)^{5/2}}{a\sinh^{-1}(ax)}+\frac{5c^2\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{15c^2\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{5c^2\mathrm{Shi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

```
[Out] -((c^2*(1 + a^2*x^2)^(5/2))/(a*ArcSinh[a*x])) + (5*c^2*SinhIntegral[ArcSinh
[a*x]])/(8*a) + (15*c^2*SinhIntegral[3*ArcSinh[a*x]])/(16*a) + (5*c^2*SinhI
ntegral[5*ArcSinh[a*x]])/(16*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
```

```
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)^2}\,dx &= -\frac{c^2(1+a^2x^2)^{5/2}}{a\sinh^{-1}(ax)} + (5ac^2)\int \frac{x(1+a^2x^2)^{3/2}}{\sinh^{-1}(ax)}\,dx \\
&= -\frac{c^2(1+a^2x^2)^{5/2}}{a\sinh^{-1}(ax)} + \frac{(5c^2)\operatorname{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a} \\
&= -\frac{c^2(1+a^2x^2)^{5/2}}{a\sinh^{-1}(ax)} + \frac{(5c^2)\operatorname{Subst}\left(\int \left(\frac{\sinh(x)}{8x}+\frac{3\sinh(3x)}{16x}+\frac{\sinh(5x)}{16x}\right)dx, x, \sinh^{-1}(ax)\right)}{a} \\
&= -\frac{c^2(1+a^2x^2)^{5/2}}{a\sinh^{-1}(ax)} + \frac{(5c^2)\operatorname{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{16a} + \frac{(5c^2)\operatorname{Subst}\left(\int \frac{\sinh(x)}{x}\,d\right.}{8a} \\
&= -\frac{c^2(1+a^2x^2)^{5/2}}{a\sinh^{-1}(ax)} + \frac{5c^2\operatorname{Shi}(\sinh^{-1}(ax))}{8a} + \frac{15c^2\operatorname{Shi}(3\sinh^{-1}(ax))}{16a} + \frac{5c^2\operatorname{Shi}(5\sinh^{-1}(\right.}{16a}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 69, normalized size = 0.90

$$\frac{c^2\left(-16(1+a^2x^2)^{5/2}+10\sinh^{-1}(ax)\operatorname{Shi}(\sinh^{-1}(ax))+15\sinh^{-1}(ax)\operatorname{Shi}(3\sinh^{-1}(ax))+5\sinh^{-1}(ax)\operatorname{Shi}(5\sinh^{-1}(ax))\right)}{16a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

```
[Out] (c^2*(-16*(1 + a^2*x^2)^(5/2) + 10*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
+ 15*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 5*ArcSinh[a*x]*SinhIntegra
l[5*ArcSinh[a*x]]))/(16*a*ArcSinh[a*x])
```

Maple [A]
time = 3.43, size = 84, normalized size = 1.09

method	result
derivativedivides	$\frac{c^2\left(10\,\text{hyperbolicSineIntegral}(\text{arcsinh}(ax))\,\text{arcsinh}(ax)+15\,\text{hyperbolicSineIntegral}(3\,\text{arcsinh}(ax))\,\text{arcsinh}(ax)+5\,\text{hyperb}\right.}{16a\,\text{arcsinh}(a}$
default	$\frac{c^2\left(10\,\text{hyperbolicSineIntegral}(\text{arcsinh}(ax))\,\text{arcsinh}(ax)+15\,\text{hyperbolicSineIntegral}(3\,\text{arcsinh}(ax))\,\text{arcsinh}(ax)+5\,\text{hyperb}\right.}{16a\,\text{arcsinh}(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*c^2*(10*Shi(arcsinh(a*x))*arcsinh(a*x)+15*Shi(3*arcsinh(a*x))*arcsin
h(a*x)+5*Shi(5*arcsinh(a*x))*arcsinh(a*x)-5*cosh(3*arcsinh(a*x))-cosh(5*arc
sinh(a*x))-10*(a^2*x^2+1)^(1/2))/arcsinh(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x + (a^6*c^2*x^6 + 3*
a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*
x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^8*c^2*x^
8 + 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 + 8*a^2*c^2*x^2 + (5*a^6*c^2*x^6 + 9*a^
4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*(a^2*x^2 + 1) + c^2 + 5*(2*a^7*c^2*x^7 + 5
*a^5*c^2*x^5 + 4*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2*x^2 + 1))/((a^4*x^4 + (a^2
*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*lo
g(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/asinh(a*x)**2, x) + Integral(a**4*x**4/asinh(a*x
)**2, x) + Integral(asinh(a*x)**(-2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{asinh}\left(a\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/asinh(a*x)^2,x)
```

```
[Out] int((c + a^2*c*x^2)^2/asinh(a*x)^2, x)
```

3.408 $\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{c(1+a^2x^2)^{3/2}}{a\sinh^{-1}(ax)}+\frac{3c\mathrm{Shi}(\sinh^{-1}(ax))}{4a}+\frac{3c\mathrm{Shi}(3\sinh^{-1}(ax))}{4a}$$

```
[Out] -c*(a^2*x^2+1)^(3/2)/a/arcsinh(a*x)+3/4*c*Shi(arcsinh(a*x))/a+3/4*c*Shi(3*a
rcsinh(a*x))/a
```

Rubi [A]

time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5790, 5819, 5556, 3379}

$$-\frac{c(a^2x^2+1)^{3/2}}{a\sinh^{-1}(ax)}+\frac{3c\mathrm{Shi}(\sinh^{-1}(ax))}{4a}+\frac{3c\mathrm{Shi}(3\sinh^{-1}(ax))}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

```
[Out] -((c*(1 + a^2*x^2)^(3/2))/(a*ArcSinh[a*x])) + (3*c*SinhIntegral[ArcSinh[a*x
]])/(4*a) + (3*c*SinhIntegral[3*ArcSinh[a*x]])/(4*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)^2}\,dx &= -\frac{c(1+a^2x^2)^{3/2}}{a\sinh^{-1}(ax)}+(3ac)\int \frac{x\sqrt{1+a^2x^2}}{\sinh^{-1}(ax)}\,dx\\ &= -\frac{c(1+a^2x^2)^{3/2}}{a\sinh^{-1}(ax)}+\frac{(3c)\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a}\\ &= -\frac{c(1+a^2x^2)^{3/2}}{a\sinh^{-1}(ax)}+\frac{(3c)\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{4x}+\frac{\sinh(3x)}{4x}\right)\,dx,x,\sinh^{-1}(ax)\right)}{a}\\ &= -\frac{c(1+a^2x^2)^{3/2}}{a\sinh^{-1}(ax)}+\frac{(3c)\mathrm{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{4a}+\frac{(3c)\mathrm{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx,x\right.}{4a}\\ &= -\frac{c(1+a^2x^2)^{3/2}}{a\sinh^{-1}(ax)}+\frac{3c\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{3c\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 1.00

$$\frac{c\left(-4(1+a^2x^2)^{3/2}+3\sinh^{-1}(ax)\mathrm{Shi}\left(\sinh^{-1}(ax)\right)+3\sinh^{-1}(ax)\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)\right)}{4a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

```
[Out] (c*(-4*(1 + a^2*x^2)^(3/2) + 3*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]] + 3*
ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]]))/(4*a*ArcSinh[a*x])
```

Maple [A]
time = 2.39, size = 60, normalized size = 1.11

method	result

derivativedivides	$\frac{c\left(3\,\text{hyperbolicSineIntegral}(\text{arcsinh}(ax))\,\text{arcsinh}(ax)+3\,\text{hyperbolicSineIntegral}(3\,\text{arcsinh}(ax))\,\text{arcsinh}(ax)-\cosh(3\,\text{arcsi}\right.}{4a\,\text{arcsinh}(ax)}$
default	$\frac{c\left(3\,\text{hyperbolicSineIntegral}(\text{arcsinh}(ax))\,\text{arcsinh}(ax)+3\,\text{hyperbolicSineIntegral}(3\,\text{arcsinh}(ax))\,\text{arcsinh}(ax)-\cosh(3\,\text{arcsi}\right.}{4a\,\text{arcsinh}(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*c*(3*Shi(arcsinh(a*x))*arcsinh(a*x)+3*Shi(3*arcsinh(a*x))*arcsinh(a*x
)-cosh(3*arcsinh(a*x))-3*(a^2*x^2+1)^(1/2))/arcsinh(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 + 2*a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 +
 1))) + integrate((3*a^6*c*x^6 + 7*a^4*c*x^4 + 5*a^2*c*x^2 + (3*a^4*c*x^4 +
 2*a^2*c*x^2 - c)*(a^2*x^2 + 1) + 3*(2*a^5*c*x^5 + 3*a^3*c*x^3 + a*c*x)*sqr
t(a^2*x^2 + 1) + c)/((a^4*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*
x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{a^{2}x^{2}}{\operatorname{asinh}^{2}(ax)}\,dx+\int\frac{1}{\operatorname{asinh}^{2}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/asinh(a*x)**2,x)
```

```
[Out] c*(Integral(a**2*x**2/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c\,a^2\,x^2+c}{{\mathrm{asinh}\left(a\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/asinh(a*x)^2,x)
[Out] int((c + a^2*c*x^2)/asinh(a*x)^2, x)
```

3.409 $\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{ac\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}-\frac{a\text{Int}\left(\frac{x}{(1+a^2x^2)^{3/2}\sinh^{-1}(ax)},x\right)}{c}$$

```
[Out] -1/a/c/arcsinh(a*x)/(a^2*x^2+1)^(1/2)-a*Unintegrable(x/(a^2*x^2+1)^(3/2)/ar
csinh(a*x),x)/c
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
[Out] -(1/(a*c*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])) - (a*Defer[Int][x/((1 + a^2*x^2)^
(3/2)*ArcSinh[a*x]), x])/c
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)^2}\,dx=-\frac{1}{ac\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}-\frac{a\int \frac{x}{(1+a^2x^2)^{3/2}\sinh^{-1}(ax)}\,dx}{c}$$

Mathematica [A]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
[Out] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2c\,x^2+c)\,\mathrm{arcsinh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

```
[Out] int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a^2*x^2 + 1))/((a^3*c*x^2 + sqrt(a^2*x^2 + 1)*a^2*c*x + a*c)*l
og(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^4*x^4 + (a^2*x^2 + 1)^2 + (2*a^
3*x^3 + a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^6*c*x^6 + 3*a^4*c*x^4 + 3*a^2*c*x^2
 + (a^4*c*x^4 + a^2*c*x^2)*(a^2*x^2 + 1) + 2*(a^5*c*x^5 + 2*a^3*c*x^3 + a*c
*x)*sqrt(a^2*x^2 + 1) + c)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^2 \operatorname{asinh}^2(ax) + \operatorname{asinh}^2(ax)}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**2*asinh(a*x)**2 + asinh(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{asinh}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)),x)
[Out] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)), x)
```

3.410 $\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)^2} dx$

Optimal. Leaf size=58

$$-\frac{1}{ac^2 (1+a^2x^2)^{3/2} \sinh^{-1}(ax)} - \frac{3a\text{Int}\left(\frac{x}{(1+a^2x^2)^{5/2} \sinh^{-1}(ax)}, x\right)}{c^2}$$

```
[Out] -1/a/c^2/(a^2*x^2+1)^(3/2)/arcsinh(a*x)-3*a*Unintegrable(x/(a^2*x^2+1)^(5/2
)/arcsinh(a*x),x)/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x])) - (3*a*Defer[Int][x/((1 + a^2
*x^2)^(5/2)*ArcSinh[a*x]), x])/c^2
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)^2} dx = -\frac{1}{ac^2 (1+a^2x^2)^{3/2} \sinh^{-1}(ax)} - \frac{(3a) \int \frac{x}{(1+a^2x^2)^{5/2} \sinh^{-1}(ax)} dx}{c^2}$$

Mathematica [A]
time = 2.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 c\, x^2 + c)^2 \operatorname{arcsinh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

```
[Out] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a^2*x^2 + 1))/((a^5*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2 + (a^4*c^2
*x^3 + a^2*c^2*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))) - integr
ate((3*a^4*x^4 + 2*a^2*x^2 + (3*a^2*x^2 + 1)*(a^2*x^2 + 1) + 3*(2*a^3*x^3 +
 a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^8*c^2*x^8 + 4*a^6*c^2*x^6 + 6*a^4*c^2*x^4
+ 4*a^2*c^2*x^2 + (a^6*c^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a^2*x^2 + 1)
 + c^2 + 2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2
*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\operatorname{asinh}^2(ax)+2a^2x^2\operatorname{asinh}^2(ax)+\operatorname{asinh}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*asinh(a*x)**2 + 2*a**2*x**2*asinh(a*x)**2 + asinh(a*x
)**2), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.411 $\int \frac{x^3\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=213

$$-\frac{x^3(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a\cdots}{}\right)}{16b^2c^4}$$

```
[Out] -x^3*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))-1/8*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2/c^4-3/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+5/16*Chi(
5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^4+1/8*Shi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^4+3/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-5/16*
Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^4
```

Rubi [A]

time = 0.44, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5814, 5780, 5556, 3384, 3379, 3382}

$$-\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^4}+\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^4}+\frac{3\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^4}-\frac{5\sinh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^4}-\frac{x^3(c^2x^2+1)}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^4) + (Sinh[a/b]*SinhIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*
x]))/b])/(16*b^2*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rubi steps

$$\int \frac{x^3\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(5c)\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{5\text{Subst}\left(\right.}{}$$

$$= -\frac{x^3(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} +$$

$$= -\frac{x^3(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^4} + \frac{5\text{Subst}\left(\int \frac{\cosh(}{a+b}\right.}{}$$

$$= -\frac{x^3(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^4} - \frac{(}{}$$

$$= -\frac{x^3(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+\right.}{16b^2c^4}$$

Mathematica [A]
time = 0.48, size = 175, normalized size = 0.82

$$-\frac{\frac{16bc^3x^3}{a+b\sinh^{-1}(cx)} + \frac{16bc^5x^5}{a+b\sinh^{-1}(cx)} + 2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16b^2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*((16*b*c^3*x^3)/(a + b*ArcSinh[c*x]) + (16*b*c^5*x^5)/(a + b*ArcSinh[
c*x]) + 2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*Cosh
Integral[3*(a/b + ArcSinh[c*x])] - 5*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Ar
cSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 3*Sinh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])])/(b^2*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(201) = 402$.
time = 6.76, size = 633, normalized size = 2.97

method	result
default	$-\frac{16c^5x^5-16\sqrt{c^2x^2+1}\,c^4x^4+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^4b(a+b\,\text{arcsinh}(cx))} - \frac{5\,e^{\frac{5a}{b}}\,\text{expIntegral}\left(1, 5\,\text{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(16*c^5*x^5-16*(c^2*x^2+1)^(1/2)*c^4*x^4+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/32/c^4/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+1/32*(-4*c^2*x^2*(c^2*x^2+1)^(1/2)+4*
c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)/c^4/b/(a+b*arcsinh(c*x))+3/32/c^4/b^2*exp(
3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+1/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^4/b/(a+b*
arcsinh(c*x))+1/16/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1/16/c^4/b^2*(ar
csinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)
-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+1/32/c^4/b^2*(4*b*c^3
*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcsi
nh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*b*c*x+(c^2*x
^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^4/b^2*(16*b*c^5*x^5+16*(c^2*x^2+1)
^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*arcsinh(c*x)
*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c*x)-5*a/b)*ex
p(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b
*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^3
*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^4*x^6 + 11*c^2*x^4 + 3*x^2)*(c^
2*x^2 + 1) + (5*c^5*x^7 + 9*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*
x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c
^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*
x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c
^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\operatorname{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

3.412 $\int \frac{x^2\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{x^2(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\mathrm{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{2b^2c^3}$$

```
[Out] -x^2*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(
c*x))/b)/b^2/c^3-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c^3
```

Rubi [A]

time = 0.32, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5814, 5780, 5556, 12, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{2b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{2b^2c^3}-\frac{x^2(c^2x^2+1)}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(4*(a + b
*ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(2*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral
[(4*(a + b*ArcSinh[c*x]))/b])/(2*b^2*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rubi steps

$$\int \frac{x^2\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(4c)\int \frac{x^3}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{4\text{Subst}\left(\right.}{}$$

$$= -\frac{x^2(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{4\text{Subst}\left(\int \left(-\right.\right.}{}$$

$$= -\frac{x^2(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3}$$

$$= -\frac{x^2(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3} - \frac{\text{s}}{}$$

$$= -\frac{x^2(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}\right.}{2b}$$

Mathematica [**A**]
time = 0.23, size = 82, normalized size = 0.88

$$\frac{-\frac{2bc^2x^2(1+c^2x^2)}{a+b\sinh^{-1}(cx)} - \text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right) + \cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-2*b*c^2*x^2*(1 + c^2*x^2))/(a + b*ArcSinh[c*x]) - CoshIntegral[4*(a/b +
ArcSinh[c*x])]*Sinh[(4*a)/b] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c*x])])/(2*b^2*c^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(89) = 178$.
time = 5.50, size = 248, normalized size = 2.67

method	result
default	$\frac{1}{8c^3(a+b\,\text{arcsinh}(cx))b} - \frac{8c^4x^4-8\sqrt{c^2x^2+1}\,x^3c^3+8c^2x^2-4\sqrt{c^2x^2+1}\,cx+1}{16c^3(a+b\,\text{arcsinh}(cx))b} + \frac{e^{\frac{4a}{b}}\,\text{expIntegral}\left(1,4\,\text{arcsinh}(cx)+\frac{4a}{b}\right)}{4c^3b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c^3/(a+b*arcsinh(c*x))/b-1/16*(8*c^4*x^4-8*(c^2*x^2+1)^(1/2)*x^3*c^3+8*
c^2*x^2-4*(c^2*x^2+1)^(1/2)*c*x+1)/c^3/(a+b*arcsinh(c*x))/b+1/4/c^3/b^2*exp
(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-1/16/c^3/b^2*(8*b*c^4*x^4+8*(c^2*x^2+1)^
(1/2)*b*c^3*x^3+8*b*c^2*x^2+4*x*b*c*(c^2*x^2+1)^(1/2)+4*exp(-4*a/b)*arcsinh
(c*x)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/
b)*a+b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b
*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^3
*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^4*x^5 + 4*c^2*x^3 + x)*(c^2*x^2
+ 1) + (4*c^5*x^6 + 7*c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 +
(c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2
 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqr
t(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*
sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

3.413 $\int \frac{x\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=149

$$-\frac{x(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^2}$$

```
[Out] -x*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))+1/4*Chi((a+b*arcsinh(c*x))/b)*cosh(a/
b)/b^2/c^2+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2-1/4*Shi((a+b
*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*
a/b)/b^2/c^2
```

Rubi [A]
time = 0.25, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5814, 5774, 3384, 3379, 3382, 5780, 5556}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^2}-\frac{x(c^2x^2+1)}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(
a + b*ArcSinh[c*x])/b])/(4*b^2*c^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(4*b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[
c*x])/b])/(4*b^2*c^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x
]))/b])/(4*b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rubi steps

$$\int \frac{x\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(3c)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{b^2c^2} + \frac{3\text{Subst}\left(\right.}{}$$

$$= -\frac{x(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} +$$

$$= -\frac{x(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c}{b}\right.}{b^2c^2}$$

$$= -\frac{x(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c}{b}\right.}{b^2c^2}$$

$$= -\frac{x(1+c^2x^2)}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}\right.}{4b^2c}$$

Mathematica [A]
time = 0.27, size = 126, normalized size = 0.85

$$-\frac{\frac{4bcx}{a+b\sinh^{-1}(cx)} + \frac{4bc^3x^3}{a+b\sinh^{-1}(cx)} - \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{4b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/4*((4*b*c*x)/(a + b*ArcSinh[c*x]) + (4*b*c^3*x^3)/(a + b*ArcSinh[c*x]) -
 Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[
3*(a/b + ArcSinh[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Si
nh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(141) = 282$.
time = 5.34, size = 364, normalized size = 2.44

method	result
default	$-\frac{-4c^2x^2\sqrt{c^2x^2+1}+4c^3x^3-\sqrt{c^2x^2+1}+3cx}{8c^2b(a+b\,\text{arcsinh}(cx))} - \frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2} - \frac{-\sqrt{c^2x^2+1}+cx}{8c^2b(a+b\,\text{arcsinh}(cx))} - \mathrm{e}^{\frac{a}{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-4*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)/c^2/b
/(a+b*arcsinh(c*x))-3/8/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/8*(
-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(a+b*arcsinh(c*x))-1/8/c^2/b^2*exp(a/b)*Ei(1,
arcsinh(c*x)+a/b)-1/8/c^2/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/
b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*ar
csinh(c*x))-1/8/c^2/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsin
h(c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcs
inh(c*x)-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^3 + x)*(c^2*x^2 + 1) + (c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c
^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2
+ 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate((3*(c^2*x
^2 + 1)^(3/2)*c^3*x^3 + (6*c^4*x^4 + 5*c^2*x^2 + 1)*(c^2*x^2 + 1) + (3*c^5*
x^5 + 5*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*
b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^
2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c^2x^2+1}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

3.414 $\int \frac{\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}$$

```
[Out] (-c^2*x^2-1)/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)
/b^2/c-Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c
```

Rubi [A]
time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5790, 5780, 5556, 12, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}-\frac{c^2x^2+1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*ArcSi
nh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b
*ArcSinh[c*x]))/b])/(b^2*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(2c)\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc} - \text{s}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}\right.}{b}$$

Mathematica [A]
time = 0.13, size = 73, normalized size = 0.86

$$\frac{-\frac{b+bc^2x^2}{a+b\sinh^{-1}(cx)} - \text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) + \cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-((b + b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*
x])]*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b
^2*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(85) = 170.
time = 5.49, size = 192, normalized size = 2.26

method	result
default	$-\frac{1}{2bc(a+b\,\text{arcsinh}(cx))} - \frac{2c^2x^2-2\sqrt{c^2x^2+1}\,cx+1}{4c(a+b\,\text{arcsinh}(cx))b} + \frac{e^{\frac{2a}{b}}\,\text{expIntegral}\left(1,2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)}{2c\,b^2} - \frac{2b\,c^2x^2+2xbc\sqrt{c^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/c/(a+b*arcsinh(c*x))-1/4*(2*c^2*x^2-2*(c^2*x^2+1)^(1/2)*c*x+1)/c/(a+
b*arcsinh(c*x))/b+1/2/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b^2
*(2*b*c^2*x^2+2*x*b*c*(c^2*x^2+1)^(1/2)+2*exp(-2*a/b)*arcsinh(c*x)*Ei(1,-2*
arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*a
rcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x
 + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^2*x^2 - 1)*(c^2*x
^2 + 1)^(3/2) + 2*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*x^2
+ 1)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^
2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^
2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1
)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x))^2,x)
[Out] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x))^2, x)
```

3.415 $\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=103

$$-\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2}-\frac{\text{Int}\left(\frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}$$

```
[Out] (-c^2*x^2-1)/b/c/x/(a+b*arcsinh(c*x))+Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b
^2-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2-Unintegrable(1/x^2/(a+b*arcsinh(
c*x)),x)/b/c
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*x*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a
+ b*ArcSinh[c*x])/b])/b^2 - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b]
)/b^2 - Defer[Int][1/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{c\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{b}\\
&= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{b^2}-\frac{\int \frac{1}{x^2(}\\
&= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx\right.}{b^2}\\
&= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh}{b}\right.}{b^2}
\end{aligned}$$

Mathematica [A]
time = 7.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c
^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*x^3 - 2*c
*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 - c^2*x^2 - 1)*(c^2*x^2 + 1) + (c^5*x^
5 + c^3*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 +
2*a*b*c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^
2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a
^2*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x\,(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2+1}}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))^2), x)
```

3.416 $\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{1+c^2x^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\mathrm{Int}\left(\frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}$$

```
[Out] (-c^2*x^2-1)/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable(1/x^3/(a+b*arcsinh(c
*x)),x)/b/c
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][1/(x^3*(a +
 b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1+c^2x^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A]
time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate((3*(c^2*x^2
 + 1)^(3/2)*c*x + 2*(2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^3*x^3 + c*x)*sqrt(c^
2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 + 2*a*b*c^3*x^5 + a*b*
c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b^2*c^3*x^5 + b^2*c*x^
3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x)
 + a^2*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

3.417 $\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A]
time = 12.82, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((c^3*x^3 +
 4*c*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 + 7*c^2*x^2 + 3)*(c^2*x^2 + 1) + (
c^5*x^5 + 3*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1
)*a*b*c^3*x^6 + 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^
2*c^3*x^6 + 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*
sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x)
 + a^2*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

3.418 $\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]

time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]

time = 2.49, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((2*c^3*x^3
 + 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^4*x^4 + 5*c^2*x^2 + 2)*(c^2*x^2 + 1)
 + (2*c^5*x^5 + 5*c^3*x^3 + 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x
^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 +
 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2
*x^6)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x)
 + a^2*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

3.419 $\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=277

$$-\frac{x^3(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^4}-\frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(\ldots}{}\right)}{64b^2c}$$

```
[Out] -x^3*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))-3/64*Chi((a+b*arcsinh(c*x))/b)*co
sh(a/b)/b^2/c^4-9/64*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+5/64*C
hi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^4+7/64*Chi(7*(a+b*arcsinh(c*x)
)/b)*cosh(7*a/b)/b^2/c^4+3/64*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^4+9
/64*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-5/64*Shi(5*(a+b*arcsinh
(c*x))/b)*sinh(5*a/b)/b^2/c^4-7/64*Shi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/
b^2/c^4
```

Rubi [A]
time = 0.59, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5814, 5819, 5556, 3384, 3379, 3382}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^4}-\frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}+\frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}+\frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^4}+\frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}-\frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}-\frac{x^3(c^2x^2+1)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/(64*b^2*c^4) - (9*Cosh[(3*a)/b]*CoshIntegral[
(3*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(
5*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (7*Cosh[(7*a)/b]*CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a + b*
ArcSinh[c*x])/b])/(64*b^2*c^4) + (9*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*Ar
cSinh[c*x]))/b])/(64*b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*Arc
Sinh[c*x]))/b])/(64*b^2*c^4) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcS
inh[c*x]))/b])/(64*b^2*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\int \frac{x^2(1+c^2x^2)}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(7c)\int \frac{x^4(1+c^2x^2)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{7\text{Subst}\Big(}{}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \left(-\frac{\cosh(x)}{8(a+bx)} + \frac{\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)\,dx, x, \sinh}{bc^4}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{7\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^4} + \frac{7\text{Subst}\Big(\int \frac{\cosh(}{a+}}{}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(21\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^4} -$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^4} - \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\Big(\frac{3a}{b}}{64b^2c}$$

Mathematica [A]
time = 0.58, size = 399, normalized size = 1.44

$$\frac{-64bc^3x^3-128bc^5x^5-64bc^7x^7-3\left(a+b\sinh^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-9\left(a+b\sinh^{-1}(cx)\right)\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+5a\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+5b\sinh^{-1}(cx)\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+7a\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+7b\sinh^{-1}(cx)\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+3a\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+3b\sinh^{-1}(cx)\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+9a\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+9b\sinh^{-1}(cx)\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-5a\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-5b\sinh^{-1}(cx)\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-7a\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-7b\sinh^{-1}(cx)\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{64b^2c^4\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-64*b*c^3*x^3 - 128*b*c^5*x^5 - 64*b*c^7*x^7 - 3*(a + b*ArcSinh[c*x])*Cosh
[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + 5*a*Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcSinh[c*x])] + 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c*x])] + 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] +
 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] + 3*a*
Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*b*ArcSinh[c*x]*Sinh[a/b]*Sin
hIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[
c*x])] - 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 5*b*ArcSi
nh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Sinh[(7*a)
/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Sinh[(7*a)/b]*S
inhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b^2*c^4*(a + b*ArcSinh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(261) = 522$.
time = 7.76, size = 958, normalized size = 3.46

method	result	size
default	Expression too large to display	958

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(64*c^7*x^7-64*(c^2*x^2+1)^(1/2)*c^6*x^6+112*c^5*x^5-80*(c^2*x^2+1)^
(1/2)*c^4*x^4+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/
2))/c^4/b/(a+b*arcsinh(c*x))-7/128/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7
*a/b)-1/128*(16*c^5*x^5-16*(c^2*x^2+1)^(1/2)*c^4*x^4+20*c^3*x^3-12*c^2*x^2*
(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/128/c
^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3/128*(-4*c^2*x^2*(c^2*x^2+1)^
(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)/c^4/b/(a+b*arcsinh(c*x))+9/128/c^4
/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/128*(-(c^2*x^2+1)^(1/2)+c*x)/c
^4/b/(a+b*arcsinh(c*x))+3/128/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/128
/c^4/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-
arcsinh(c*x)-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+3/128/c^4
/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*exp(-3*a/b)*
Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3
*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/128/c^4/b^2*(16*b*c^5*x^5+
16*(c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+
5*arcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(
c*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1
/128/c^4/b^2*(64*b*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^6+112*b*c^5*x^5+80*
(c^2*x^2+1)^(1/2)*b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)^(1/2)*b*c^2*x^2+7*E
i(1,-7*arcsinh(c*x)-7*a/b)*arcsinh(c*x)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x
)-7*a/b)*exp(-7*a/b)*a+7*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^7 + 2*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^5*x^8 + 2*c^3*x^6 + c*x^4)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate(((7*c^5*x^7 + 9*c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (
14*c^6*x^8 + 27*c^4*x^6 + 16*c^2*x^4 + 3*x^2)*(c^2*x^2 + 1) + (7*c^7*x^9 +
18*c^5*x^7 + 15*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x
^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*
b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*
```

```
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c
^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcs
inh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(c^2x^2+1)^{\frac{3}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

3.420 $\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=219

$$-\frac{x^2(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3}-\frac{\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c^3}-\frac{3\text{Chi}\left(\frac{6(a+b\sinh^{-1}(}{b}\right.}{16b^2}$$

```
[Out] -x^2*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))-1/16*cosh(2*a/b)*Shi(2*(a+b*arcsi
nh(c*x))/b)/b^2/c^3+1/4*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^3+3/1
6*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*Chi(2*(a+b*arcsinh(c
*x))/b)*sinh(2*a/b)/b^2/c^3-1/4*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2
/c^3-3/16*Chi(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c^3
```

Rubi [A]

time = 0.42, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5814, 5819, 5556, 3384, 3379, 3382}

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{4b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^3}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{4b^2c^3}+\frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^3}-\frac{x^2(c^2x^2+1)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*(a + b*
ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c^3) - (3*CoshIntegral[(6*(a + b*Ar
cSinh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(
2*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*(
a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5814

Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1, 0] && IGtQ[m, -3]

Rule 5819

Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]

Rubi steps

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\int \frac{x(1+c^2x^2)}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(6c)\int \frac{x^3(1+c^2x^2)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{6\text{Subst}}{}$$

$$= -\frac{x^2(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4(a+bx)} + \frac{\sinh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^3} +$$

$$= -\frac{x^2(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^3} + \frac{\text{Subst}\left(\int \frac{\sinh(}{a+b}\right.}{}$$

$$= -\frac{x^2(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3} - $$

$$= -\frac{x^2(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{4a}{b}+4\sinh\right.}{4b}$$

Mathematica [**A**]
time = 0.53, size = 306, normalized size = 1.40

$$-\frac{16bc^2x^2+32bc^4x^4+16bc^6x^6-\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+4\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)+3a\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{6a}{b}\right)+3b\sinh^{-1}(cx)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{6a}{b}\right)+a\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+b\sinh^{-1}(cx)\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4a\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4b\sinh^{-1}(cx)\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-3a\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-3b\sinh^{-1}(cx)\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16b^2c^3\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c^2*x^2 + 32*b*c^4*x^4 + 16*b*c^6*x^6 - (a + b*ArcSinh[c*x])*Co
shIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 4*(a + b*ArcSinh[c*x])*C
oshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshIntegral[6*(a/b
 + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[
c*x])] + b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])]
- 4*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*b*ArcSinh[c*x]
*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*Sin
hIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhInte
gral[6*(a/b + ArcSinh[c*x])])/(b^2*c^3*(a + b*ArcSinh[c*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(207) = 414$.
time = 9.24, size = 704, normalized size = 3.21

method	result
default	$\frac{1}{16c^3(a+b\,\mathrm{arcsinh}(cx))b} - \frac{32x^6c^6-32\sqrt{c^2x^2+1}\,x^5c^5+48c^4x^4-32\sqrt{c^2x^2+1}\,x^3c^3+18c^2x^2-6\sqrt{c^2x^2+1}\,cx+1}{64c^3(a+b\,\mathrm{arcsinh}(cx))b} + \frac{3\,\mathrm{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/c^3/(a+b*arcsinh(c*x))/b-1/64*(32*x^6*c^6-32*(c^2*x^2+1)^(1/2)*x^5*c^5
+48*c^4*x^4-32*(c^2*x^2+1)^(1/2)*x^3*c^3+18*c^2*x^2-6*(c^2*x^2+1)^(1/2)*c*x
+1)/c^3/(a+b*arcsinh(c*x))/b+3/32/c^3/b^2*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*
a/b)-1/32*(8*c^4*x^4-8*(c^2*x^2+1)^(1/2)*x^3*c^3+8*c^2*x^2-4*(c^2*x^2+1)^(1
/2)*c*x+1)/c^3/(a+b*arcsinh(c*x))/b+1/8/c^3/b^2*exp(4*a/b)*Ei(1,4*arcsinh(c
*x)+4*a/b)+1/64*(2*c^2*x^2-2*(c^2*x^2+1)^(1/2)*c*x+1)/c^3/(a+b*arcsinh(c*x)
)/b-1/32/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^3/b^2*(2*b*c^
2*x^2+2*x*b*c*(c^2*x^2+1)^(1/2)+2*exp(-2*a/b)*arcsinh(c*x)*Ei(1,-2*arcsinh(
c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*arcsinh(c
*x))-1/32/c^3/b^2*(8*b*c^4*x^4+8*(c^2*x^2+1)^(1/2)*b*c^3*x^3+8*b*c^2*x^2+4*
x*b*c*(c^2*x^2+1)^(1/2)+4*exp(-4*a/b)*arcsinh(c*x)*Ei(1,-4*arcsinh(c*x)-4*a
/b)*b+4*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+b)/(a+b*arcsinh(c*x))-1/6
4/c^3/b^2*(32*b*c^6*x^6+32*(c^2*x^2+1)^(1/2)*b*c^5*x^5+48*b*c^4*x^4+32*(c^2
*x^2+1)^(1/2)*b*c^3*x^3+18*b*c^2*x^2+6*x*b*c*(c^2*x^2+1)^(1/2)+6*exp(-6*a/b
)*arcsinh(c*x)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b+6*exp(-6*a/b)*Ei(1,-6*arcsinh(
c*x)-6*a/b)*a+b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^6 + 2*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^5*x^7 + 2*c^3*x^5 + c*x^3)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate(((6*c^5*x^6 + 7*c^3*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(
6*c^6*x^7 + 11*c^4*x^5 + 6*c^2*x^3 + x)*(c^2*x^2 + 1) + 3*(2*c^7*x^8 + 5*c^
5*x^6 + 4*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*
a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 +
 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcs
inh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c^2x^2+1)^{\frac{3}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

3.421 $\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=213

$$-\frac{x(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a\cdots}{\cdots}\right)}{16b^2c^2}$$

```
[Out] -x*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))+1/8*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2/c^2+9/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2+5/16*Chi(
5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^2-1/8*Shi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^2-9/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^2-5/16*
Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^2
```

Rubi [A]
time = 0.42, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5814, 5791, 3393, 3384, 3379, 3382, 5819, 5556}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^2}-\frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}-\frac{x(c^2x^2+1)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^2) + (9*Cosh[(3*a)/b]*CoshIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^2) - (9*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^2) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*
x]))/b])/(16*b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\int \frac{1+c^2x^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(5c)\int \frac{x^2(1+c^2x^2)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{5\text{Subst}\left(\int \frac{\cosh^3(\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{x(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{5\text{S}\cdots}{\cdots}$$

$$= -\frac{x(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4bc^2} + \frac{5\text{Subst}\left(\int \frac{\cosh(3\cdots}{a+bx}\right.}{\cdots}$$

$$= -\frac{x(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^2} + \cdots$$

$$= -\frac{x(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+\cdots\right.}{16b^2c^2}$$

Mathematica [A]
time = 0.45, size = 295, normalized size = 1.38

$$-\frac{16bcx+32bc^3x^3+16bc^5x^5-2\left(a+b\sinh^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-9\left(a+b\sinh^{-1}(cx)\right)\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-5a\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-5b\sinh^{-1}(cx)\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+2a\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+2b\sinh^{-1}(cx)\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+9a\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+9b\sinh^{-1}(cx)\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+5a\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+5b\sinh^{-1}(cx)\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16b^2c^2\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c*x + 32*b*c^3*x^3 + 16*b*c^5*x^5 - 2*(a + b*ArcSinh[c*x])*Cosh
[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 5*a*Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcSinh[c*x])] - 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c*x])] + 2*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 2*b*Arc
Sinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*Si
nhIntegral[3*(a/b + ArcSinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcSinh[c*x])] + 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + Arc
Sinh[c*x])] + 5*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])])/(b^2*c^2*(a + b*ArcSinh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(201) = 402$.
time = 6.46, size = 633, normalized size = 2.97

method	result
default	$-\frac{16c^5x^5-16\sqrt{c^2x^2+1}\,c^4x^4+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{5\,\mathrm{e}^{\frac{5a}{b}}\,\mathrm{expIntegral}\left(1,5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(16*c^5*x^5-16*(c^2*x^2+1)^(1/2)*c^4*x^4+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-5/32/c^2/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32*(-4*c^2*x^2*(c^2*x^2+1)^(1/2)+4*
c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)/c^2/b/(a+b*arcsinh(c*x))-9/32/c^2/b^2*exp(
3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(a+b*
arcsinh(c*x))-1/16/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/16/c^2/b^2*(ar
csinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)
-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/32/c^2/b^2*(4*b*c^3
*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcsi
nh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*b*c*x+(c^2*x
^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^2/b^2*(16*b*c^5*x^5+16*(c^2*x^2+1)
^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*arcsinh(c*x)
*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c*x)-5*a/b)*ex
p(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^5 + 2*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^5*x^6 + 2*c^3*x^4 + c*x^2)*s
qrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2
*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)
)) + integrate((5*(c^5*x^5 + c^3*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^6*x^6 + 1
7*c^4*x^4 + 8*c^2*x^2 + 1)*(c^2*x^2 + 1) + (5*c^7*x^7 + 12*c^5*x^5 + 9*c^3*
x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 +
2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^
3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sq
rt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsin
h(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c^2x^2+1)^{\frac{3}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

3.422 $\int \frac{(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c}-\frac{\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a\cdots\right.}{}\right)}{b^2c}$$

```
[Out] -(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/
b)/b^2/c+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c-Chi(2*(a+b*arcsi
nh(c*x))/b)*sinh(2*a/b)/b^2/c-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b
^2/c
```

Rubi [A]
time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5790, 5819, 5556, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{b^2c}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{2b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{2b^2c}-\frac{(c^2x^2+1)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*Arc
Sinh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c) - (CoshIntegral[(4*(a + b*ArcSinh[c*x
]))/b]*Sinh[(4*a)/b])/(2*b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*Arc
Sinh[c*x]))/b])/(b^2*c) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x
]))/b])/(2*b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(4c)\int \frac{x(1+c^2x^2)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{4\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{4\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4(a+bx)} + \frac{\sinh(4x)}{8(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{\sinh(2x}{a+bx}\right.}{}$$

$$= -\frac{(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc} + \frac{c}{}$$

$$= -\frac{(1+c^2x^2)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c} - \frac{\text{Chi}\left(\frac{4a}{b}+4\sinh\right.}{2b}$$

Mathematica [A]
time = 0.35, size = 122, normalized size = 0.82

$$\frac{-\frac{2b(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)} - 2\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - \text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right) + 2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-2*b*(1 + c^2*x^2)^2)/(a + b*ArcSinh[c*x]) - 2*CoshIntegral[2*(a/b + ArcS
inh[c*x])]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/
b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*S
inhIntegral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(145) = 290.
time = 8.66, size = 420, normalized size = 2.82

method	result
default	$-\frac{3}{8bc(a+b\,\mathrm{arcsinh}(cx))} - \frac{8c^4x^4-8\sqrt{c^2x^2+1}\,x^3c^3+8c^2x^2-4\sqrt{c^2x^2+1}\,cx+1}{16c(a+b\,\mathrm{arcsinh}(cx))b} + \frac{e^{\frac{4a}{b}}\,\mathrm{expIntegral}(1,4\,\mathrm{arcsinh}(cx)+\frac{4a}{b})}{4c\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8/b/c/(a+b*arcsinh(c*x))-1/16*(8*c^4*x^4-8*(c^2*x^2+1)^(1/2)*x^3*c^3+8*c
^2*x^2-4*(c^2*x^2+1)^(1/2)*c*x+1)/c/(a+b*arcsinh(c*x))/b+1/4/c/b^2*exp(4*a/
b)*Ei(1,4*arcsinh(c*x)+4*a/b)-1/4*(2*c^2*x^2-2*(c^2*x^2+1)^(1/2)*c*x+1)/c/(
a+b*arcsinh(c*x))/b+1/2/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b
^2*(2*b*c^2*x^2+2*x*b*c*(c^2*x^2+1)^(1/2)+2*exp(-2*a/b)*arcsinh(c*x)*Ei(1,-
2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b
*arcsinh(c*x))-1/16/c/b^2*(8*b*c^4*x^4+8*(c^2*x^2+1)^(1/2)*b*c^3*x^3+8*b*c^
2*x^2+4*x*b*c*(c^2*x^2+1)^(1/2)+4*exp(-4*a/b)*arcsinh(c*x)*Ei(1,-4*arcsinh(
c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+b)/(a+b*arcsinh(c
*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c
^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)))
 + integrate(((4*c^4*x^4 + 3*c^2*x^2 - 1)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^5*x^
5 + 3*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (4*c^6*x^6 + 9*c^4*x^4 + 6*c^2*x^2 + 1
)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x
^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 +
 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
+ 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x))^2, x)
```

3.423 $\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=175

$$-\frac{(1+c^2x^2)^2}{bcx\,(a+b\sinh^{-1}(cx))}+\frac{9\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4b^2}-\frac{9\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a\text{-}}{}\right.}{4b^2}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x/(a+b*arcsinh(c*x))+9/4*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2-9/4*Shi((a+b*arcsi
nh(c*x))/b)*sinh(a/b)/b^2-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2-U
nintegrable((c^2*x^2+1)/x^2/(a+b*arcsinh(c*x)),x)/b/c
```

Rubi [A]
time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x*(a + b*ArcSinh[c*x]))) + (9*Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(4*b^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b
*ArcSinh[c*x]))/b])/(4*b^2) - (9*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x]
)/b])/(4*b^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(4*b^2) - Defer[Int][(1 + c^2*x^2)/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{(1+c^2x^2)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\sinh^{-1}(cx))}\,dx}{bc} + \frac{(3c)\int \frac{1+c^2x^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{(1+c^2x^2)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{\int \frac{1+c^2}{x^2(a+b\sin}}{b}$$

$$= -\frac{(1+c^2x^2)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{b}$$

$$= -\frac{(1+c^2x^2)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4b} + \frac{9\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(1+c^2x^2)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\sinh^{-1}(cx))}\,dx}{bc} + \frac{\left(9\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh}{a}\right.}{4b}$$

$$= -\frac{(1+c^2x^2)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{9\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Ch}}{}$$

Mathematica [A]
time = 5.22, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\left(a+b\,\text{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b
^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^
2 + 1))) + integrate(((3*c^5*x^5 + c^3*x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (
6*c^6*x^6 + 7*c^4*x^4 - 1)*(c^2*x^2 + 1) + 3*(c^7*x^7 + 2*c^5*x^5 + c^3*x^3
)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*c^3*x
^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3*x^4 +
 b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqr
t(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c^2 x^2 + 1)^{3/2}}{x (a + b \operatorname{asinh}(c x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))^2), x)
```

3.424 $\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=99

$$-\frac{(1+c^2x^2)^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\text{Int}\left(\frac{1+c^2x^2}{x^3(a+b\sinh^{-1}(cx))},x\right)}{bc}+\frac{2c\text{Int}\left(\frac{1+c^2x^2}{x(a+b\sinh^{-1}(cx))},x\right)}{b}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable((c^2*x^2+1)/x^3/(a
+b*arcsinh(c*x)),x)/b/c+2*c*Unintegrable((c^2*x^2+1)/x/(a+b*arcsinh(c*x)),x
)/b
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][(1 + c^2*
x^2)/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (2*c*Defer[Int][(1 + c^2*x^2)/
(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2} dx = -\frac{(1+c^2x^2)^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1+c^2x^2}{x^3(a+b\sinh^{-1}(cx))} dx}{bc}+\frac{(2c)\int \frac{1+c^2x^2}{x(a+b\sinh^{-1}(cx))} dx}{b}$$

Mathematica [A]
time = 2.95, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{2} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 +
(b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^
2*x^2 + 1))) + integrate(((2*c^5*x^5 - c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2)
 + 2*(2*c^6*x^6 + c^4*x^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (2*c^7*x^7 + 3*c
^5*x^5 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 +
 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 +
1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*
x) + a^2*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

3.425 $\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A]
time = 9.75, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 +
(b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^
2*x^2 + 1))) + integrate(((c^5*x^5 - 3*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2)
 + (2*c^6*x^6 - 3*c^4*x^4 - 8*c^2*x^2 - 3)*(c^2*x^2 + 1) + (c^7*x^7 - 3*c^3
*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3*x^6 +
 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^6 + 2*b
^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c^2*x^2 +
1)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*
x) + a^2*x^3), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{x^{3}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

3.426 $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=67

$$-\frac{(1+c^2x^2)^2}{bcx^4\,(a+b\sinh^{-1}(cx))}-\frac{4\text{Int}\left(\frac{1+c^2x^2}{x^5(a+b\sinh^{-1}(cx))},x\right)}{bc}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x^4/(a+b*arcsinh(c*x))-4*Unintegrable((c^2*x^2+1)/x^5/(a
+b*arcsinh(c*x)),x)/b/c
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^4*(a + b*ArcSinh[c*x]))) - (4*Defer[Int][(1 + c^2*
x^2)/(x^5*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\,(a+b\sinh^{-1}(cx))^2}\,dx=-\frac{(1+c^2x^2)^2}{bcx^4\,(a+b\sinh^{-1}(cx))}-\frac{4\int \frac{1+c^2x^2}{x^5(a+b\sinh^{-1}(cx))}\,dx}{bc}$$

Mathematica [A]
time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{4} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 +
(b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^
2*x^2 + 1))) - integrate((5*(c^3*x^3 + c*x)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^4*
x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + 3*(c^5*x^5 + 2*c^3*x^3 + c*x)*sqrt(c^2
*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c
*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5
 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*
x) + a^2*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}{x^{4} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**4*(a + b*asinh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

3.427 $\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=277

$$-\frac{x^3(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{32b^2c^4}+\frac{21\cosh\left(\frac{7a}{b}\right)\text{Chi}}{256}$$

```
[Out] -x^3*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))-3/128*Chi((a+b*arcsinh(c*x))/b)*c
osh(a/b)/b^2/c^4-3/32*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+21/25
6*Chi(7*(a+b*arcsinh(c*x))/b)*cosh(7*a/b)/b^2/c^4+9/256*Chi(9*(a+b*arcsinh(
c*x))/b)*cosh(9*a/b)/b^2/c^4+3/128*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/
c^4+3/32*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-21/256*Shi(7*(a+b*
arcsinh(c*x))/b)*sinh(7*a/b)/b^2/c^4-9/256*Shi(9*(a+b*arcsinh(c*x))/b)*sinh
(9*a/b)/b^2/c^4
```

Rubi [A]

time = 0.76, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5814, 5819, 5556, 3384, 3379, 3382}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{32b^2c^4}+\frac{21\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{256b^2c^4}+\frac{9\cosh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9(a+b\sinh^{-1}(cx))}{b}\right)}{256b^2c^4}+\frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128b^2c^4}+\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{32b^2c^4}-\frac{21\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{256b^2c^4}-\frac{9\sinh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\sinh^{-1}(cx))}{b}\right)}{256b^2c^4}-\frac{x^3(c^2x^2+1)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/(128*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral
[(3*(a + b*ArcSinh[c*x]))/b])/(32*b^2*c^4) + (21*Cosh[(7*a)/b]*CoshIntegral
[(7*(a + b*ArcSinh[c*x]))/b])/(256*b^2*c^4) + (9*Cosh[(9*a)/b]*CoshIntegral
[(9*(a + b*ArcSinh[c*x]))/b])/(256*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a
+ b*ArcSinh[c*x])/b])/(128*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(32*b^2*c^4) - (21*Sinh[(7*a)/b]*SinhIntegral[(7*(a +
 b*ArcSinh[c*x]))/b])/(256*b^2*c^4) - (9*Sinh[(9*a)/b]*SinhIntegral[(9*(a +
 b*ArcSinh[c*x]))/b])/(256*b^2*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\int \frac{x^2(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(9c)\int \frac{x^4(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \frac{\cosh^5(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{9\text{Subst}}{}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64(a+bx)} + \frac{\cosh(3x)}{64(a+bx)} + \frac{3\cosh(5x)}{64(a+bx)} + \frac{\cosh(7x)}{64(a+bx)}\right.\right.}{bc^4}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{9\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{256bc^4} + \frac{9\text{Subst}\left(\int \frac{\cos}{a}\right.}{}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(27\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{128bc^4}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{128b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}\right.}{32b}$$

Mathematica [A]
time = 0.96, size = 408, normalized size = 1.47

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/256*(256*b*c^3*x^3 + 768*b*c^5*x^5 + 768*b*c^7*x^7 + 256*b*c^9*x^9 + 6*(
a + b*ArcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 24*(a + b*
ArcSinh[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 21*a*Cos
h[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 21*b*ArcSinh[c*x]*Cosh[(7
*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 9*a*Cosh[(9*a)/b]*CoshIntegra
l[9*(a/b + ArcSinh[c*x])] - 9*b*ArcSinh[c*x]*Cosh[(9*a)/b]*CoshIntegral[9*(
a/b + ArcSinh[c*x])] - 6*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 6*b
*ArcSinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 24*a*Sinh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 24*b*ArcSinh[c*x]*Sinh[(3*a)/b]*S
inhIntegral[3*(a/b + ArcSinh[c*x])] + 21*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/
b + ArcSinh[c*x])] + 21*b*ArcSinh[c*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b +
ArcSinh[c*x])] + 9*a*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])] + 9
*b*ArcSinh[c*x]*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(b^2*c^
4*(a + b*ArcSinh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(261) = 522$.
time = 7.88, size = 1070, normalized size = 3.86

method	result	size
default	Expression too large to display	1070

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/512*(256*c^9*x^9-256*(c^2*x^2+1)^(1/2)*c^8*x^8+576*c^7*x^7-448*(c^2*x^2+
1)^(1/2)*c^6*x^6+432*c^5*x^5-240*(c^2*x^2+1)^(1/2)*c^4*x^4+120*c^3*x^3-40*c
^2*x^2*(c^2*x^2+1)^(1/2)+9*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-
9/512/c^4/b^2*exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)-3/512*(64*c^7*x^7-64*(c
^2*x^2+1)^(1/2)*c^6*x^6+112*c^5*x^5-80*(c^2*x^2+1)^(1/2)*c^4*x^4+56*c^3*x^3
-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c
*x))-21/512/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+1/64*(-4*c^2*x^2*
(c^2*x^2+1)^(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)/c^4/b/(a+b*arcsinh(c*x
))+3/64/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/256*(-(c^2*x^2+1)^(
1/2)+c*x)/c^4/b/(a+b*arcsinh(c*x))+3/256/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)
+a/b)+3/256/c^4/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-
a/b)*Ei(1,-arcsinh(c*x)-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x)
)+1/64/c^4/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*ex
p(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-
3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/512/c^4/b^2*(64*
b*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^6+112*b*c^5*x^5+80*(c^2*x^2+1)^(1/2)
*b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)^(1/2)*b*c^2*x^2+7*arcsinh(c*x)*exp(-
7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)*b+7*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a
/b)*a+7*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/512/c^4/b^2*(256*b*
c^9*x^9+256*(c^2*x^2+1)^(1/2)*b*c^8*x^8+576*b*c^7*x^7+448*(c^2*x^2+1)^(1/2)
*b*c^6*x^6+432*b*c^5*x^5+240*(c^2*x^2+1)^(1/2)*b*c^4*x^4+120*b*c^3*x^3+40*(
c^2*x^2+1)^(1/2)*b*c^2*x^2+9*arcsinh(c*x)*Ei(1,-9*arcsinh(c*x)-9*a/b)*exp(-
9*a/b)*b+9*Ei(1,-9*arcsinh(c*x)-9*a/b)*exp(-9*a/b)*a+9*b*c*x+(c^2*x^2+1)^(1
/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^9 + 3*c^4*x^7 + 3*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^7*x^10 + 3*c^5
*x^8 + 3*c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 +
```

```
1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*
log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((9*c^7*x^9 + 20*c^5*x^7 + 13*c^3
*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + 3*(6*c^8*x^10 + 17*c^6*x^8 + 17*c^4*x
^6 + 7*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (9*c^9*x^11 + 31*c^7*x^9 + 39*c^5*x^7
 + 21*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*
b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^
2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
+ 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(c^2x^2+1)^{\frac{5}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**3*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c^2\,x^2+1\right)}^{5/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

3.428 $\int \frac{x^2\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=281

$$-\frac{x^2(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3}-\frac{\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8b^2c^3}-\frac{3\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(\right.}{b}\right.}{16b^2}$$

```
[Out] -x^2*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))-1/16*cosh(2*a/b)*Shi(2*(a+b*arcsi
nh(c*x))/b)/b^2/c^3+1/8*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^3+3/1
6*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*cosh(8*a/b)*Shi(8*(a
+b*arcsinh(c*x))/b)/b^2/c^3+1/16*Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^
2/c^3-1/8*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c^3-3/16*Chi(6*(a+b*a
rcsinh(c*x))/b)*sinh(6*a/b)/b^2/c^3-1/16*Chi(8*(a+b*arcsinh(c*x))/b)*sinh(8
*a/b)/b^2/c^3
```

Rubi [A]

time = 0.62, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = $\{5814, 5819, 5556, 3384, 3379, 3382\}$

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{8a}{b}\right)\text{Chi}\left(\frac{8\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8b^2c^3}+\frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}+\frac{\cosh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}-\frac{x^2\left(c^2x^2+1\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*(a + b*
ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(8*b^2*c^3) - (3*CoshIntegral[(6*(a + b*Ar
cSinh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3) - (CoshIntegral[(8*(a + b*ArcSi
nh[c*x]))/b]*Sinh[(8*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(2*(
a + b*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a +
 b*ArcSinh[c*x]))/b])/(8*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b
*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(8*a)/b]*SinhIntegral[(8*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\int \frac{x(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(8c)\int \frac{x^3(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{8\text{Subst}}{}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} + \frac{\sinh(4x)}{8(a+bx)} + \frac{\sinh(6x)}{32(a+bx)}\right)dx, x, \sinh\right.}{bc^3}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^3} + \frac{\text{Subst}\left(\int \frac{\sinh(8}{a+bx}\right.}{}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(5\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^3}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{4a}{b}+4\sinh\right.}{8b}$$

Mathematica [A]
time = 0.83, size = 413, normalized size = 1.47

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c^2*x^2 + 48*b*c^4*x^4 + 48*b*c^6*x^6 + 16*b*c^8*x^8 - (a + b*A
rcSinh[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 2*(a + b*
ArcSinh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*Cosh
Integral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshInteg
ral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + a*CoshIntegral[8*(a/b + ArcSinh
[c*x])]*Sinh[(8*a)/b] + b*ArcSinh[c*x]*CoshIntegral[8*(a/b + ArcSinh[c*x])]
*Sinh[(8*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + b*A
rcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 2*a*Cosh[(
4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Cosh[(4*a)/
b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*
(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b
+ ArcSinh[c*x])] - a*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])] - b
*ArcSinh[c*x]*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])])/(b^2*c^3*
(a + b*ArcSinh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(265) = 530$.
time = 12.36, size = 1044, normalized size = 3.72

method	result	size
default	Expression too large to display	1044

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/128/c^3/(a+b*arcsinh(c*x))/b-1/256*(128*c^8*x^8-128*(c^2*x^2+1)^(1/2)*c^7
*x^7+256*x^6*c^6-192*(c^2*x^2+1)^(1/2)*x^5*c^5+160*c^4*x^4-80*(c^2*x^2+1)^(
1/2)*x^3*c^3+32*c^2*x^2-8*(c^2*x^2+1)^(1/2)*c*x+1)/c^3/(a+b*arcsinh(c*x))/b
+1/32/c^3/b^2*exp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)-1/64*(32*x^6*c^6-32*(c^
2*x^2+1)^(1/2)*x^5*c^5+48*c^4*x^4-32*(c^2*x^2+1)^(1/2)*x^3*c^3+18*c^2*x^2-6
*(c^2*x^2+1)^(1/2)*c*x+1)/c^3/(a+b*arcsinh(c*x))/b+3/32/c^3/b^2*exp(6*a/b)*
Ei(1,6*arcsinh(c*x)+6*a/b)-1/64*(8*c^4*x^4-8*(c^2*x^2+1)^(1/2)*x^3*c^3+8*c^
2*x^2-4*(c^2*x^2+1)^(1/2)*c*x+1)/c^3/(a+b*arcsinh(c*x))/b+1/16/c^3/b^2*exp(
4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64*(2*c^2*x^2-2*(c^2*x^2+1)^(1/2)*c*x+1
)/c^3/(a+b*arcsinh(c*x))/b-1/32/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/
b)+1/64/c^3/b^2*(2*b*c^2*x^2+2*x*b*c*(c^2*x^2+1)^(1/2)+2*exp(-2*a/b)*arcsin
h(c*x)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a
/b)*a+b)/(a+b*arcsinh(c*x))-1/64/c^3/b^2*(8*b*c^4*x^4+8*(c^2*x^2+1)^(1/2)*b
*c^3*x^3+8*b*c^2*x^2+4*x*b*c*(c^2*x^2+1)^(1/2)+4*exp(-4*a/b)*arcsinh(c*x)*E
i(1,-4*arcsinh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+b)
/(a+b*arcsinh(c*x))-1/64/c^3/b^2*(32*b*c^6*x^6+32*(c^2*x^2+1)^(1/2)*b*c^5*x
^5+48*b*c^4*x^4+32*(c^2*x^2+1)^(1/2)*b*c^3*x^3+18*b*c^2*x^2+6*x*b*c*(c^2*x^
2+1)^(1/2)+6*exp(-6*a/b)*arcsinh(c*x)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b+6*exp(-
6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a+b)/(a+b*arcsinh(c*x))-1/256/c^3/b^2*(1
28*b*c^8*x^8+128*(c^2*x^2+1)^(1/2)*b*c^7*x^7+256*b*c^6*x^6+192*(c^2*x^2+1)^
(1/2)*b*c^5*x^5+160*b*c^4*x^4+80*(c^2*x^2+1)^(1/2)*b*c^3*x^3+32*b*c^2*x^2+8
*x*b*c*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x)*exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*
a/b)*b+8*exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)*a+b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^8 + 3*c^4*x^6 + 3*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^7*x^9 + 3*c^5*
x^7 + 3*c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1
)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*l
```

```
og(c*x + sqrt(c^2*x^2 + 1))) + integrate(((8*c^7*x^8 + 17*c^5*x^6 + 10*c^3*
x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(8*c^8*x^9 + 22*c^6*x^7 + 21*c^4*x^5 +
 8*c^2*x^3 + x)*(c^2*x^2 + 1) + (8*c^9*x^10 + 27*c^7*x^8 + 33*c^5*x^6 + 17*
c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*
x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*
b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
+ 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c^2x^2+1)^{\frac{5}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**2*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c^2\,x^2+1\right)}^{5/2}}{{\left(a+b\,\operatorname{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

3.429 $\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=275

$$-\frac{x(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{5\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^2}+\frac{27\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}+\frac{25\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\ldots\right.}{64b\ldots}$$

```
[Out] -x*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))+5/64*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2/c^2+27/64*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2+25/64*C
hi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^2+7/64*Chi(7*(a+b*arcsinh(c*x)
)/b)*cosh(7*a/b)/b^2/c^2-5/64*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2-2
7/64*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^2-25/64*Shi(5*(a+b*arcsi
nh(c*x))/b)*sinh(5*a/b)/b^2/c^2-7/64*Shi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b
)/b^2/c^2
```

Rubi [A]
time = 0.57, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5814, 5791, 3393, 3384, 3379, 3382, 5819, 5556}

$$\frac{5\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^2}+\frac{27\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}+\frac{25\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}+\frac{7\cosh\left(\frac{7a}{b}\right)\mathrm{Chi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}-\frac{5\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^2}-\frac{27\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}-\frac{25\sinh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}-\frac{7\sinh\left(\frac{7a}{b}\right)\mathrm{Shi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}-\frac{x(c^2x^2+1)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegr
al[(a + b*ArcSinh[c*x])/b])/(64*b^2*c^2) + (27*Cosh[(3*a)/b]*CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) + (25*Cosh[(5*a)/b]*CoshIntegral[(
5*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) + (7*Cosh[(7*a)/b]*CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) - (5*Sinh[a/b]*SinhIntegral[(a + b*
ArcSinh[c*x])/b])/(64*b^2*c^2) - (27*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*A
rcSinh[c*x]))/b])/(64*b^2*c^2) - (25*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*A
rcSinh[c*x]))/b])/(64*b^2*c^2) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*Ar
cSinh[c*x]))/b])/(64*b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(1+c^2x^2)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\int \frac{(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(7c)\int \frac{x^2(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh^5(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{7\text{Subst}\left(\int \frac{\cosh^5(\ldots}{a\ldots}\right.}{\ldots}$$

$$= -\frac{x(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8(a+bx)} + \frac{5\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}\ldots\right.}{bc^2}$$

$$= -\frac{x(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^2} + \frac{7\text{Subst}\left(\int \frac{\cosh(3\ldots}{a+bx}\right.}{6\ldots}$$

$$= -\frac{x(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\left(35\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^2} + \ldots$$

$$= -\frac{x(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^2} + \frac{27\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}\ldots\right.}{64b^2\ldots}$$

Mathematica [A]
time = 0.73, size = 404, normalized size = 1.47

$$-\frac{64bcx+192bc^3x^3+192bc^5x^5+64bc^7x^7-5\left(a+b\sinh^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-27\left(a+b\sinh^{-1}(cx)\right)\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-25a\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-25b\sinh^{-1}(cx)\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-7a\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-7b\sinh^{-1}(cx)\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+5a\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+5b\sinh^{-1}(cx)\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+27a\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+27b\sinh^{-1}(cx)\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+25a\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+25b\sinh^{-1}(cx)\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+7a\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+7b\sinh^{-1}(cx)\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{64b^2c^2\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/64*(64*b*c*x + 192*b*c^3*x^3 + 192*b*c^5*x^5 + 64*b*c^7*x^7 - 5*(a + b*A
rcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 27*(a + b*ArcSinh
[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 25*a*Cosh[(5*a)
/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 25*b*ArcSinh[c*x]*Cosh[(5*a)/b]*
CoshIntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/
b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + A
rcSinh[c*x])] + 5*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*b*ArcSin
h[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 27*a*Sinh[(3*a)/b]*Sinh
Integral[3*(a/b + ArcSinh[c*x])] + 27*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhInte
gral[3*(a/b + ArcSinh[c*x])] + 25*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + Arc
```

```
Sinh[c*x])] + 25*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh
[c*x])] + 7*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + 7*b*ArcS
inh[c*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(b^2*c^2*(a +
b*ArcSinh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(259) = 518$.
time = 7.58, size = 958, normalized size = 3.48

method	result	size
default	Expression too large to display	958

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(64*c^7*x^7-64*(c^2*x^2+1)^(1/2)*c^6*x^6+112*c^5*x^5-80*(c^2*x^2+1)^
(1/2)*c^4*x^4+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/
2))/c^2/b/(a+b*arcsinh(c*x))-7/128/c^2/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7
*a/b)-5/128*(16*c^5*x^5-16*(c^2*x^2+1)^(1/2)*c^4*x^4+20*c^3*x^3-12*c^2*x^2*
(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-25/128/
c^2/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-9/128*(-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)/c^2/b/(a+b*arcsinh(c*x))-27/128/c
^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/128*(-(c^2*x^2+1)^(1/2)+c*x)
/c^2/b/(a+b*arcsinh(c*x))-5/128/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/1
28/c^2/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1
,-arcsinh(c*x)-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-9/128/c
^2/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*exp(-3*a/b
)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a
+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-5/128/c^2/b^2*(16*b*c^5*x^
5+16*(c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^
2+5*arcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsin
h(c*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
-1/128/c^2/b^2*(64*b*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^6+112*b*c^5*x^5+8
0*(c^2*x^2+1)^(1/2)*b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)^(1/2)*b*c^2*x^2+7
*arcsinh(c*x)*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)*b+7*exp(-7*a/b)*Ei(1,
-7*arcsinh(c*x)-7*a/b)*a+7*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^7 + 3*c^4*x^5 + 3*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^7*x^8 + 3*c^5*x^
6 + 3*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log
(c*x + sqrt(c^2*x^2 + 1))) + integrate((7*(c^7*x^7 + 2*c^5*x^5 + c^3*x^3)*(
c^2*x^2 + 1)^(3/2) + (14*c^8*x^8 + 37*c^6*x^6 + 33*c^4*x^4 + 11*c^2*x^2 + 1
)*(c^2*x^2 + 1) + (7*c^9*x^9 + 23*c^7*x^7 + 27*c^5*x^5 + 13*c^3*x^3 + 2*c*x
)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x
^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2
*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c^2x^2+1)^{\frac{5}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(c^2\,x^2+1\right)^{5/2}}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

3.430 $\int \frac{(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=216

$$-\frac{(1+c^2x^2)^3}{bc\,(a+b\sinh^{-1}(cx))}-\frac{15\mathrm{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c}-\frac{3\mathrm{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c}-\frac{3\mathrm{Chi}\left(\frac{6(a+b\sinh^{-1}}{b}\right.}{16}$$

```
[Out] -(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))+15/16*cosh(2*a/b)*Shi(2*(a+b*arcsinh(
c*x))/b)/b^2/c+3/4*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c+3/16*cosh(
6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c-15/16*Chi(2*(a+b*arcsinh(c*x))/b)*
sinh(2*a/b)/b^2/c-3/4*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c-3/16*Ch
i(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c
```

Rubi [A]

time = 0.28, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5790, 5819, 5556, 3384, 3379, 3382}

$$-\frac{15\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c}-\frac{3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{4b^2c}-\frac{3\sinh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c}+\frac{15\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c}+\frac{3\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{4b^2c}+\frac{3\cosh\left(\frac{6a}{b}\right)\mathrm{Shi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c}-\frac{(c^2x^2+1)^3}{bc\,(a+b\sinh^{-1}(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*(a + b*ArcSinh[c*x]))) - (15*CoshIntegral[(2*(a + b*
ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c) - (3*CoshIntegral[(4*(a + b*Arc
Sinh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c) - (3*CoshIntegral[(6*(a + b*ArcSinh
[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c) + (15*Cosh[(2*a)/b]*SinhIntegral[(2*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c) + (3*Cosh[(4*a)/b]*SinhIntegral[(4*(a +
b*ArcSinh[c*x]))/b])/(4*b^2*c) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(6c)\int \frac{x(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{6\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{6\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} + \frac{\sinh(4x)}{8(a+bx)} + \frac{\sinh(6x)}{32(a+bx)}\right)\,dx, x, \sinh^{-}}{bc}$$

$$= -\frac{(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc} + \frac{3\text{Subst}\left(\int \frac{\sinh(}{a+b}\right.}{}$$

$$= -\frac{(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(15\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc}$$

$$= -\frac{(1+c^2x^2)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{15\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c} - \frac{3\text{Chi}\left(\frac{4a}{b}+4\sin\right.}{}$$

Mathematica [**A**]
time = 0.52, size = 311, normalized size = 1.44

$$-\frac{16b+48bc^2x^2+48bc^4x^4+16bc^6x^6+15\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+12\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)+3a\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{6a}{b}\right)+3b\sinh^{-1}(cx)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{6a}{b}\right)-15a\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-15b\sinh^{-1}(cx)\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-12a\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-12b\sinh^{-1}(cx)\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-3a\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-3b\sinh^{-1}(cx)\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16b^2c\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b + 48*b*c^2*x^2 + 48*b*c^4*x^4 + 16*b*c^6*x^6 + 15*(a + b*ArcSin
h[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 12*(a + b*ArcS
inh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshInte
gral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[
6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] - 15*a*Cosh[(2*a)/b]*SinhIntegral[2*(
a/b + ArcSinh[c*x])] - 15*b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b
+ ArcSinh[c*x])] - 12*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])]
- 12*b*ArcSinh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*
a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cos
h[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(b^2*c*(a + b*ArcSinh[c*x]
))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(204) = 408$.
time = 11.49, size = 704, normalized size = 3.26

method	result
default	$-\frac{5}{16bc(a+b\,\mathrm{arcsinh}(cx))}-\frac{32x^6c^6-32\sqrt{c^2x^2+1}\,x^5c^5+48c^4x^4-32\sqrt{c^2x^2+1}\,x^3c^3+18c^2x^2-6\sqrt{c^2x^2+1}\,cx+1}{64c(a+b\,\mathrm{arcsinh}(cx))b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -5/16/b/c/(a+b*arcsinh(c*x))-1/64*(32*x^6*c^6-32*(c^2*x^2+1)^(1/2)*x^5*c^5+
48*c^4*x^4-32*(c^2*x^2+1)^(1/2)*x^3*c^3+18*c^2*x^2-6*(c^2*x^2+1)^(1/2)*c*x+
1)/c/(a+b*arcsinh(c*x))/b+3/32/c/b^2*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-
3/32*(8*c^4*x^4-8*(c^2*x^2+1)^(1/2)*x^3*c^3+8*c^2*x^2-4*(c^2*x^2+1)^(1/2)*c
*x+1)/c/(a+b*arcsinh(c*x))/b+3/8/c/b^2*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b
)-15/64*(2*c^2*x^2-2*(c^2*x^2+1)^(1/2)*c*x+1)/c/(a+b*arcsinh(c*x))/b+15/32/
c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-15/64/c/b^2*(2*b*c^2*x^2+2*x*b*
c*(c^2*x^2+1)^(1/2)+2*exp(-2*a/b)*arcsinh(c*x)*Ei(1,-2*arcsinh(c*x)-2*a/b)*
b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*arcsinh(c*x))-3/32/c/
b^2*(8*b*c^4*x^4+8*(c^2*x^2+1)^(1/2)*b*c^3*x^3+8*b*c^2*x^2+4*x*b*c*(c^2*x^2
+1)^(1/2)+4*exp(-4*a/b)*arcsinh(c*x)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b+4*exp(-4
*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+b)/(a+b*arcsinh(c*x))-1/64/c/b^2*(32*b*
c^6*x^6+32*(c^2*x^2+1)^(1/2)*b*c^5*x^5+48*b*c^4*x^4+32*(c^2*x^2+1)^(1/2)*b*
c^3*x^3+18*b*c^2*x^2+6*x*b*c*(c^2*x^2+1)^(1/2)+6*exp(-6*a/b)*arcsinh(c*x)*E
i(1,-6*arcsinh(c*x)-6*a/b)*b+6*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a+b)
/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c
*x + sqrt(c^2*x^2 + 1))) + integrate(((6*c^6*x^6 + 11*c^4*x^4 + 4*c^2*x^2 -
 1)*(c^2*x^2 + 1)^(3/2) + 6*(2*c^7*x^7 + 5*c^5*x^5 + 4*c^3*x^3 + c*x)*(c^2*
x^2 + 1) + (6*c^8*x^8 + 19*c^6*x^6 + 21*c^4*x^4 + 9*c^2*x^2 + 1)*sqrt(c^2*x
^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (
b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*
x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3
*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x))^2,x)
[Out] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x))^2, x)
```

3.431 $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=233

$$-\frac{(1+c^2x^2)^3}{bcx\,(a+b\sinh^{-1}(cx))}+\frac{25\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2}+\frac{25\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\mathrm{C}}{}$$

```
[Out] -(c^2*x^2+1)^3/b/c/x/(a+b*arcsinh(c*x))+25/8*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2+25/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2+5/16*Chi(5*(a+b
*arcsinh(c*x))/b)*cosh(5*a/b)/b^2-25/8*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/
b^2-25/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2-5/16*Shi(5*(a+b*arcsi
nh(c*x))/b)*sinh(5*a/b)/b^2-Unintegrable((c^2*x^2+1)^2/x^2/(a+b*arcsinh(c*x
)),x)/b/c
```

Rubi [A]
time = 0.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*x*(a + b*ArcSinh[c*x]))) + (25*Cosh[a/b]*CoshIntegra
l[(a + b*ArcSinh[c*x])/b])/(8*b^2) + (25*Cosh[(3*a)/b]*CoshIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(16*b^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2) - (25*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])
/b])/(8*b^2) - (25*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(16*b^2) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b
^2) - Defer[Int][(1 + c^2*x^2)^2/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{(1+c^2x^2)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{(1+c^2x^2)^2}{x^2(a+b\sinh^{-1}(cx))}\,dx}{bc} + \frac{(5c)\int \frac{(1+c^2x^2)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{(1+c^2x^2)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\text{Subst}\left(\int \frac{\cosh^5(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{\int \frac{(1+c^2x}{x^2(a+b\sinh}}{bc}$$

$$= -\frac{(1+c^2x^2)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8(a+bx)} + \frac{5\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right) dx, x, \sin}{b}$$

$$= -\frac{(1+c^2x^2)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16b} + \frac{25\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(1+c^2x^2)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{(1+c^2x^2)^2}{x^2(a+b\sinh^{-1}(cx))}\,dx}{bc} + \frac{\left(25\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh(}{a-}\right.}{8b}$$

$$= -\frac{(1+c^2x^2)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{25\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2} + \frac{25\cosh\left(\frac{3a}{b}\right)\text{Cl}}{}$$

Mathematica [A]
time = 6.37, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\left(a+b\,\text{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*
x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^7*x^7 + 8*c^5*x^5 + c^3*
x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (10*c^8*x^8 + 23*c^6*x^6 + 15*c^4*x^4 +
c^2*x^2 - 1)*(c^2*x^2 + 1) + 5*(c^9*x^9 + 3*c^7*x^7 + 3*c^5*x^5 + c^3*x^3)*
sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*c^3*x^4
 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3*x^4 + b
^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2
+ 2*a*b*x*arcsinh(c*x) + a^2*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{x \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x*(a + b*asinh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c^2 x^2 + 1)^{5/2}}{x (a + b \operatorname{asinh}(c x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))^2), x)
```

3.432 $\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=103

$$-\frac{(1+c^2x^2)^3}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\mathrm{Int}\left(\frac{(1+c^2x^2)^2}{x^3(a+b\sinh^{-1}(cx))},x\right)}{bc}+\frac{4c\mathrm{Int}\left(\frac{(1+c^2x^2)^2}{x(a+b\sinh^{-1}(cx))},x\right)}{b}$$

```
[Out] -(c^2*x^2+1)^3/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable((c^2*x^2+1)^2/x^3/
(a+b*arcsinh(c*x)),x)/b/c+4*c*Unintegrable((c^2*x^2+1)^2/x/(a+b*arcsinh(c*x
)),x)/b
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][(1 + c^2*
x^2)^2/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (4*c*Defer[Int][(1 + c^2*x^2
)^2/(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{(1+c^2x^2)^3}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{(1+c^2x^2)^2}{x^3(a+b\sinh^{-1}(cx))}\,dx}{bc}+\frac{(4c)\int \frac{(1+c^2x^2)^2}{x(a+b\sinh^{-1}(cx))}\,dx}{b}$$

Mathematica [A]
time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2 (a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*
c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^7*x^7 + 5*c^5*x^5 -
2*c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^8*x^8 + 8*c^6*x^6 + 3*c^4*x
^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (4*c^9*x^9 + 11*c^7*x^7 + 9*c^5*x^5 + c
^3*x^3 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 +
 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 +
1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^
2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2 (a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

3.433 $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 9.75, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*
c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((3*c^7*x^7 + 2*c^5*x^5 -
5*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2) + 3*(2*c^8*x^8 + 3*c^6*x^6 - c^4*x^4
 - 3*c^2*x^2 - 1)*(c^2*x^2 + 1) + (3*c^9*x^9 + 7*c^7*x^7 + 3*c^5*x^5 - 3*c^
3*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3*x^6
+ 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^6 + 2*
b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c^2*x^2 +
 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^
2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{x^{3} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

3.434 $\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]

[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*
c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^7*x^7 - c^5*x^5 - 8*
c^3*x^3 - 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^8*x^8 + c^6*x^6 - 6*c^4*x^4 -
 7*c^2*x^2 - 2)*(c^2*x^2 + 1) + (2*c^9*x^9 + 3*c^7*x^7 - 3*c^5*x^5 - 7*c^3*
x^3 - 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 +
2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^
2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^
2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{x^{4}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**4*(a + b*asinh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

3.435 $\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^6}-\frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\right.}{16b^2}$$

```
[Out] -x^5/b/c/(a+b*arcsinh(c*x))+5/8*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^6
-15/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^6+5/16*Chi(5*(a+b*arcs
inh(c*x))/b)*cosh(5*a/b)/b^2/c^6-5/8*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^
2/c^6+15/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^6-5/16*Shi(5*(a+b
*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^6
```

Rubi [A]

time = 0.29, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5818, 5780, 5556, 3384, 3379, 3382}

$$\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^6}-\frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}-\frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^6}+\frac{15\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}-\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^6) - (15*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[
c*x]))/b])/(16*b^2*c^6) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^6) - (5*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])
/(8*b^2*c^6) + (15*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(16*b^2*c^6) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(
16*b^2*c^6)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{5\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx}{bc} \\
&= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{5\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^6} \\
&= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{5\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)}-\frac{3\cosh(3x)}{16(a+bx)}+\frac{\cosh(5x)}{16(a+bx)}\right)\right.}{bc^6} \\
&= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{5\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16bc^6}+\frac{5\text{S}}{} \\
&= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh\right.}{8bc^6} \\
&= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^6}-\frac{15\cosh}{}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 158, normalized size = 0.77

$$
-\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{5\left(2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\right)}{16b^2c^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*(2*Cosh[a/b]*CoshIntegral[a/b + ArcS
inh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*
a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] + 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Sin
h[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])]))/(16*b^2*c^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(192) = 384.
time = 7.22, size = 633, normalized size = 3.10

method	result
default	$-\frac{16c^5x^5-16\sqrt{c^2x^2+1}\,c^4x^4+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^6b(a+b\,\text{arcsinh}(cx))}-\frac{5\,e^{\frac{5a}{b}}\,\text{expIntegral}\left(1,5\,\text{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^6b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(16*c^5*x^5-16*(c^2*x^2+1)^(1/2)*c^4*x^4+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))-5/32/c^6/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+5/32*(-4*c^2*x^2*(c^2*x^2+1)^(1/2)+4*
c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)/c^6/b/(a+b*arcsinh(c*x))+15/32/c^6/b^2*exp
(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^6/b/(a+b
*arcsinh(c*x))-5/16/c^6/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/16/c^6/b^2*(a
rcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x
)-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+5/32/c^6/b^2*(4*b*c^
3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcs
inh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*b*c*x+(c^2*
x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^6/b^2*(16*b*c^5*x^5+16*(c^2*x^2+1
)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*arcsinh(c*x
)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c*x)-5*a/b)*e
xp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^8 + c*x^6 + (c^2*x^7 + x^5)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((5*c^5*x^9 + 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 + 4*c*x^5)*(c^2*x^2
 + 1) + 5*(2*c^4*x^8 + 3*c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(
3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 +
1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5
*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1
)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+b\,\text{asinh}\,(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.436 $\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5}-\frac{\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a}{}\right.}{b^2c^5}$$

```
[Out] -x^4/b/c/(a+b*arcsinh(c*x))-cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^5
+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^5+Chi(2*(a+b*arcsinh(c*x
))/b)*sinh(2*a/b)/b^2/c^5-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c
^5
```

Rubi [A]
time = 0.24, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5818, 5780, 5556, 3384, 3379, 3382}

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^5}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^5}-\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^4/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a + b*ArcSinh[c*x]))/
b]*Sinh[(2*a)/b])/(b^2*c^5) - (CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b]*Sin
h[(4*a)/b])/(2*b^2*c^5) - (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(b^2*c^5) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b]
)/(2*b^2*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\int \frac{x^3}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^5}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)\,dx,x,\sinh$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{2bc^5}-\frac{\text{Sub}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5}-\frac{\text{Chi}\left(\frac{4}{}\right.$$

Mathematica [A]
time = 0.21, size = 117, normalized size = 0.83

$$\frac{-\frac{2bc^4x^4}{a+b\sinh^{-1}(cx)}+2\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)-2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2b^2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] ((-2*b*c^4*x^4)/(a + b*ArcSinh[c*x]) + 2*CoshIntegral[2*(a/b + ArcSinh[c*x]
)]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*C
osh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhInteg
ral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(137) = 274$.
time = 10.14, size = 420, normalized size = 2.98

method	result
default	$-\frac{3}{8c^5(a+b\,\mathrm{arcsinh}(cx))b}-\frac{8c^4x^4-8\sqrt{c^2x^2+1}\,x^3c^3+8c^2x^2-4\sqrt{c^2x^2+1}\,cx+1}{16c^5(a+b\,\mathrm{arcsinh}(cx))b}+\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right.}{4c^5b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8/c^5/(a+b*arcsinh(c*x))/b-1/16*(8*c^4*x^4-8*(c^2*x^2+1)^(1/2)*x^3*c^3+8
*c^2*x^2-4*(c^2*x^2+1)^(1/2)*c*x+1)/c^5/(a+b*arcsinh(c*x))/b+1/4/c^5/b^2*ex
p(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/4*(2*c^2*x^2-2*(c^2*x^2+1)^(1/2)*c*x+
1)/c^5/(a+b*arcsinh(c*x))/b-1/2/c^5/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/
b)+1/4/c^5/b^2*(2*b*c^2*x^2+2*x*b*c*(c^2*x^2+1)^(1/2)+2*exp(-2*a/b)*arcsinh
(c*x)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/
b)*a+b)/(a+b*arcsinh(c*x))-1/16/c^5/b^2*(8*b*c^4*x^4+8*(c^2*x^2+1)^(1/2)*b*
c^3*x^3+8*b*c^2*x^2+4*x*b*c*(c^2*x^2+1)^(1/2)+4*exp(-4*a/b)*arcsinh(c*x)*Ei
(1,-4*arcsinh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+b)/
(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^7 + c*x^5 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
```

```
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((4*c^5*x^8 + 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 + 3*c*x^4)*(c^2*x^2
+ 1) + 4*(2*c^4*x^7 + 3*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3
/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1
)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*
x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + b\operatorname{asinh}(cx))^2 \sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x**4/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{(a + b\operatorname{asinh}(c\,x))^2 \sqrt{c^2\,x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^4/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.437 $\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^4}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^4}+\frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a-}{}\right.}{4b^2c^4}$$

```
[Out] -x^3/b/c/(a+b*arcsinh(c*x))-3/4*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^4
+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+3/4*Shi((a+b*arcsinh(c
*x))/b)*sinh(a/b)/b^2/c^4-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c
^4
```

Rubi [A]
time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5818, 5780, 5556, 3384, 3379, 3382}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^4}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^4}+\frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^4}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^4}-\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshIntegral[(a + b*ArcSin
h[c*x])/b])/(4*b^2*c^4) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(4*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/
(4*b^2*c^4) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4
*b^2*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^4}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)\,dx,x,\sinh\right.}{bc^4}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{4bc^4}+\frac{3\text{Su}}{}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}\right.}{4bc^4}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^4}+\frac{3\cosh\left(\right.}{}$$

Mathematica [A]
time = 0.20, size = 113, normalized size = 0.80

$$-\frac{x^3}{bc\,(a+b\sinh^{-1}(cx))}+\frac{3\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\right)}{4b^2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) + (3*(-(Cosh[a/b]*CoshIntegral[a/b + ArcS
inh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])]))/(4*b^2*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(134) = 268.
time = 6.97, size = 364, normalized size = 2.56

method	result
default	$-\frac{-4c^2x^2\sqrt{c^2x^2+1}+4c^3x^3-\sqrt{c^2x^2+1}+3cx}{8c^4b(a+b\,\text{arcsinh}(cx))}-\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^4b^2}+\frac{-\frac{3\sqrt{c^2x^2+1}}{8}+\frac{3cx}{8}}{c^4b(a+b\,\text{arcsinh}(cx))}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-4*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)/c^4/b
/(a+b*arcsinh(c*x))-3/8/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/8*(
-(c^2*x^2+1)^(1/2)+c*x)/c^4/b/(a+b*arcsinh(c*x))+3/8/c^4/b^2*exp(a/b)*Ei(1,
arcsinh(c*x)+a/b)+3/8/c^4/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/
b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*ar
csinh(c*x))-1/8/c^4/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsin
h(c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcs
inh(c*x)-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((3*c^5*x^7 + 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 + 2*c*x^3)*(c^2*x^2
```

```
+ 1) + 3*(2*c^4*x^6 + 3*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3
/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1
)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*
x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.438 $\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\operatorname{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3}+\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^3}$$

```
[Out] -x^2/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^3
-Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^3
```

Rubi [A]
time = 0.16, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5818, 5780, 5556, 12, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^3}+\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^3}-\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*ArcSinh[c*x]))/
b]*Sinh[(2*a)/b])/(b^2*c^3) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh
[c*x]))/b])/(b^2*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}\right.}{bc^3}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3} + \cosh$$

Mathematica [A]
time = 0.27, size = 70, normalized size = 0.89

$$\frac{-\frac{bc^2x^2}{a+b\sinh^{-1}(cx)} - \text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) + \cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] (-((b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*x])]
*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b^2*c
^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(79) = 158.
time = 5.74, size = 192, normalized size = 2.43

method	result
default	$\frac{1}{2c^3(a+b\,\text{arcsinh}(cx))b} - \frac{2c^2x^2-2\sqrt{c^2x^2+1}\,cx+1}{4c^3(a+b\,\text{arcsinh}(cx))b} + \frac{e^{\frac{2a}{b}}\,\text{expIntegral}\left(1, 2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)}{2c^3b^2} - \frac{2b\,c^2x^2+2xbc\sqrt{c^2x^2+1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^3/(a+b*arcsinh(c*x))/b-1/4*(2*c^2*x^2-2*(c^2*x^2+1)^(1/2)*c*x+1)/c^3/
(a+b*arcsinh(c*x))/b+1/2/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/
c^3/b^2*(2*b*c^2*x^2+2*x*b*c*(c^2*x^2+1)^(1/2)+2*exp(-2*a/b)*arcsinh(c*x)*E
i(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)
/(a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^5 + c*x^3 + (c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((2*c^5*x^6 + 5*c^3*x^4 + 3*c*x^2 + (2*c^3*x^4 + c*x^2)*(c^2*x^2 +
1) + 2*(2*c^4*x^5 + 3*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*
a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3
/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4
+ 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
(a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b\operatorname{asinh}(cx))^2 \sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.439 $\int \frac{x}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}$$

```
[Out] -x/b/c/(a+b*arcsinh(c*x))+Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^2-Shi((
a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5818, 5774, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c
^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx\right.}{b^2c^2}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh\right.}{b^2c^2}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{S}}{}$$

Mathematica [**A**]
time = 0.10, size = 60, normalized size = 0.82

$$\frac{-\frac{bcx}{a+b\sinh^{-1}(cx)}+\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] (-((b*c*x)/(a + b*ArcSinh[c*x])) + Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x
]] - Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(73) = 146$.
time = 3.51, size = 151, normalized size = 2.07

method	result
default	$-\frac{-\sqrt{c^2x^2+1}+cx}{2c^2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2c^2b^2}-\frac{\mathrm{arcsinh}(cx)e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\,\mathrm{arcsinh}(cx)-\frac{a}{b}\right)b+e^{-\frac{a}{b}}\,\mathrm{expInteg}}{2c^2b^2(a+b\,\mathrm{arcsinh}(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(a+b*arcsinh(c*x))-1/2/c^2/b^2*exp(a/b)
*Ei(1,arcsinh(c*x)+a/b)-1/2/c^2/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c
*x)-a/b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(
a+b*arcsinh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2
*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + int
egrate((c^5*x^5 + (c^2*x^2 + 1)*c^3*x^3 + 3*c^3*x^3 + 2*c*x + (2*c^4*x^4 +
3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^2 + 2*(a*b
*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^2 + 2*
(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^
2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b
*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c*x
)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\,(cx)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.440 $\int \frac{1}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{bc\,(a+b\sinh^{-1}(cx))}$$

[Out] -1/b/c/(a+b*arcsinh(c*x))

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5783}

$$-\frac{1}{bc\,(a+b\sinh^{-1}(cx))}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]

[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^2}\,dx = -\frac{1}{bc\,(a+b\sinh^{-1}(cx))}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{bc\,(a+b\sinh^{-1}(cx))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]

```
[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))
```

Maple [A]
time = 0.28, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$-\frac{1}{bc(a+b\operatorname{arcsinh}(cx))}$	19
default	$-\frac{1}{bc(a+b\operatorname{arcsinh}(cx))}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/b/c/(a+b*arcsinh(c*x))
```

Maxima [A]
time = 0.26, size = 18, normalized size = 1.00

$$-\frac{1}{(b\operatorname{arsinh}(cx)+a)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/((b*arcsinh(c*x) + a)*b*c)
```

Fricas [A]
time = 0.38, size = 30, normalized size = 1.67

$$-\frac{1}{b^2c\log\left(cx+\sqrt{c^2x^2+1}\right)+abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/(b^2*c*log(c*x + sqrt(c^2*x^2 + 1)) + a*b*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.
time = 1.79, size = 36, normalized size = 2.00

$$\begin{cases} \frac{x}{a^2} & \text{for } b=0 \wedge c=0 \\ \frac{\operatorname{asinh}(cx)}{a^2c} & \text{for } b=0 \\ \frac{x}{a^2} & \text{for } c=0 \\ -\frac{1}{abc+b^2c\operatorname{asinh}(cx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), (asinh(c*x)/(a**2*c), Eq(b, 0)), (
x/a**2, Eq(c, 0)), (-1/(a*b*c + b**2*c*asinh(c*x)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [B]
time = 0.14, size = 18, normalized size = 1.00

$$-\frac{1}{c\,\mathrm{asinh}\left(c\,x\right)\,b^2+a\,c\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] -1/(b^2*c*asinh(c*x) + a*b*c)
```

3.441 $\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Int}\left(\frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}$$

```
[Out] -1/b/c/x/(a+b*arcsinh(c*x))-Unintegrable(1/x^2/(a+b*arcsinh(c*x)),x)/b/c
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*x*(a + b*ArcSinh[c*x]))) - Defer[Int][1/(x^2*(a + b*ArcSinh[c*x]))
, x]/(b*c)
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A]
time = 3.67, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x
^2 + 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate
((c^5*x^5 + c^3*x^3 + (c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*
x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^4 + 2*(a*b*c^4*x
^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^4 + 2*(b^2
*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 + b^2*
c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6 + 2*a
*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^3 + a^2*x + (b^2*c^2*x^3 + b^2*x)*arc
sinh(c*x)^2 + 2*(a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2} \sqrt{c^{2} x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.442 $\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\text{Int}\left(\frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}$$

```
[Out] -1/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable(1/x^3/(a+b*arcsinh(c*x)),x)/b/
c
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][1/(x^3*(a + b*ArcSinh[c
*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2} \sqrt{c^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x
^2 + 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integ
rate((2*c^5*x^5 + 3*c^3*x^3 + (2*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(
2*c^4*x^4 + 3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*
x^5 + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^
2*c^3*x^5 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^5*x^7 + 2*
b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
(a*b*c^5*x^7 + 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^4 + a^2*x^2 + (b^2*c^2*x^4 + b^2*x^2)
*arcsinh(c*x)^2 + 2*(a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2} \sqrt{c^{2} x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.443 $\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^3}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^3}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 5.37, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c^2x^2+1)^{\frac{3}{2}}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)

[Out] int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")

[Out] -(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((c^5*x^7 + 5*c^3*x^5 + 4*c*x^3 + (c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 + 7*c^2*x^4 + 3*x^2)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \operatorname{asinh}(cx))^2 (c^2x^2 + 1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)

[Out] Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.444 $\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=67

$$-\frac{x^2}{bc\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}+\frac{2\mathrm{Int}\left(\frac{x}{(1+c^2x^2)^2(a+b\sinh^{-1}(cx))},x\right)}{bc}$$

```
[Out] -x^2/b/c/(c^2*x^2+1)/(a+b*arcsinh(c*x))+2*Unintegrable(x/(c^2*x^2+1)^2/(a+b
*arcsinh(c*x)),x)/b/c
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^2/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) + (2*Defer[Int][x/((1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))^2}\,dx=-\frac{x^2}{bc\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}+\frac{2\int \frac{x}{(1+c^2x^2)^2(a+b\sinh^{-1}(cx))}\,dx}{bc}$$

Mathematica [A]
time = 2.45, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*
b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((3*c^3*x^4 +
(c^2*x^2 + 1)*c*x^2 + 3*c*x^2 + 2*(2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((a*b*
c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3
 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3
/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*
x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*s
qrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\,\mathrm{asinh}\,(cx))^2 (c^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.445 $\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 3.56, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^
2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^5 + (c^2
*x^2 + 1)*c^3*x^3 - c^3*x^3 - 2*c*x + (2*c^4*x^4 - c^2*x^2 - 1)*sqrt(c^2*x^
2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 +
 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c
^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 +
1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*
x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 +
2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2
+ a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2} \left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{(a+b\,\mathrm{asinh}\,(c\,x))^2\,(c^2\,x^2+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.446 $\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{bc\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}-\frac{2c\text{Int}\left(\frac{x}{(1+c^2x^2)^2(a+b\sinh^{-1}(cx))},x\right)}{b}$$

```
[Out] -1/b/c/(c^2*x^2+1)/(a+b*arcsinh(c*x))-2*c*Unintegrable(x/(c^2*x^2+1)^2/(a+b
*arcsinh(c*x)),x)/b
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) - (2*c*Defer[Int][x/((1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))^2}\,dx=-\frac{1}{bc\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}-\frac{(2c)\int\frac{x}{(1+c^2x^2)^2(a+b\sinh^{-1}(cx))}\,dx}{b}$$

Mathematica [A]
time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}} \left(a+b \operatorname{arcsinh}\left(cx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c^
2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
 + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^4*x^4 + c^2*x^
2 + (2*c^2*x^2 + 1)*(c^2*x^2 + 1) + 2*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) -
 1)/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^5*x^5 + 2*a
*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2
 + 1)^(3/2) + 2*(b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^
2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(c*x
 + sqrt(c^2*x^2 + 1)) + (a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)
*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 + 2*
b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2 +
a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b \operatorname{asinh}\left(cx\right)\right)^2 \left(c^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.447 $\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 5.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x^2 + 1)*b^2*
c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5
+ 3*c^3*x^3 + (3*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (6*c^4*x^4 + 5*c^2*x^2 +
1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*
(a*b*c^6*x^7 + 2*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b^2*c^5*x^6 +
 b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^7 + 2*b^2*c^4*x^5 + b^2*c^
2*x^3)*(c^2*x^2 + 1) + (b^2*c^7*x^8 + 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 + b^2*c
*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^8 + 3*a*
b*c^5*x^6 + 3*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^5 + 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4*
x^5 + 2*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^5 + 2*a*b*c^2*x^
3 + a*b*x)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.448 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\sinh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\sinh^{-1}(cx))^2} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\sinh^{-1}(cx))^2} dx = \int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\sinh^{-1}(cx))^2} dx$$

Mathematica [A]
time = 13.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\sinh^{-1}(cx))^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c^2x^2+1)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x^2 + 1)*b^2*
c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((4*c^5*
x^5 + 5*c^3*x^3 + (4*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(4*c^4*x^4 +
4*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^2 + 1
)^(3/2) + 2*(a*b*c^6*x^8 + 2*a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((b
^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^8 + 2*b^2*c^4*
x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^7*x^9 + 3*b^2*c^5*x^7 + 3*b^2*c^3
*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^
7*x^9 + 3*a*b*c^5*x^7 + 3*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^6 + 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c^
4*x^6 + 2*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^6 + 2*a*b*c^
2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \operatorname{asinh}\left(cx\right)\right)^2 \left(c^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.449 $\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 10.28, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a
*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^7 - 3*c^3*x^5 - 4
*c*x^3 + (c^3*x^5 - 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 - 5*c^2*x^4 - 3*x^2
)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2
+ 1)^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c
^2*x^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3
/2) + 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2
+ 1) + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^
7*x^6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b\,\mathrm{asinh}\,(cx))^2\,(c^2x^2 + 1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.450 $\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^2}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^2}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 3.95, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a
*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^5*x^6 - c^3*x^4 - 3
*c*x^2 + (2*c^3*x^4 - c*x^2)*(c^2*x^2 + 1) + 2*(2*c^4*x^5 - c^2*x^3 - x)*sq
rt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)
^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x
^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2)
+ 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1)
 + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^
6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + b \operatorname{asinh}\left(cx\right)\right)^2 \left(c^2x^2 + 1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.451 $\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A]
time = 7.39, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(c^2x^2+1)^{\frac{5}{2}}\left(a+b\,\text{arcsinh}\,(cx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + (
(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^
2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b
*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5 + 3*(c^2*x^2 +
1)*c^3*x^3 + c^3*x^3 - 2*c*x + (6*c^4*x^4 + c^2*x^2 - 1)*sqrt(c^2*x^2 + 1))
/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*
c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^8*x^7
 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^9*x^8
+ 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^6 + 6*a*b*c^5*x^
4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 +
 (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 +
 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2} \left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.452 $\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=62

$$-\frac{1}{bc\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{4c\text{Int}\left(\frac{x}{(1+c^2x^2)^3(a+b\sinh^{-1}(cx))},x\right)}{b}$$

```
[Out] -1/b/c/(c^2*x^2+1)^2/(a+b*arcsinh(c*x))-4*c*Unintegrable(x/(c^2*x^2+1)^3/(a
+b*arcsinh(c*x)),x)/b
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))) - (4*c*Defer[Int][x/((1 + c
^2*x^2)^3*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{1}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx = -\frac{1}{bc\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{(4c)\int \frac{x}{(1+c^2x^2)^3(a+b\sinh^{-1}(cx))}\,d}{b}$$

Mathematica [A]
time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c^2x^2+1)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2
*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3
*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((4*c^4*x^4 + 3*c^2*x^2 + (4*c^
2*x^2 + 1)*(c^2*x^2 + 1) + 4*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) - 1)/((a*b
*c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^7*x^
7 + 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^6*x^6
+ 2*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^7*x^7 + 3*b^2
*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^2*c^8*x^8 + 4*b^2*c^
6*x^6 + 6*b^2*c^4*x^4 + 4*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1)) + (a*b*c^8*x^8 + 4*a*b*c^6*x^6 + 6*a*b*c^4*x^4 + 4*a*b*c^
2*x^2 + a*b)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 + (
b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2
*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))^2 (c^2x^2+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.453 $\int \frac{1}{x(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 8.93, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b
^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^
2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a
*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((5*c^5*x^5 + 5*c^3*x^3
 + (5*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (10*c^4*x^4 + 7*c^2*x^2 + 1)*sqrt(c^
2*x^2 + 1))/((a*b*c^7*x^8 + 2*a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2
) + 2*(a*b*c^8*x^9 + 3*a*b*c^6*x^7 + 3*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2
+ 1) + ((b^2*c^7*x^8 + 2*b^2*c^5*x^6 + b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2
*(b^2*c^8*x^9 + 3*b^2*c^6*x^7 + 3*b^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1)
+ (b^2*c^9*x^10 + 4*b^2*c^7*x^8 + 6*b^2*c^5*x^6 + 4*b^2*c^3*x^4 + b^2*c*x^2
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^10 + 4*a*b*c
^7*x^8 + 6*a*b*c^5*x^6 + 4*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^7 + 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3 + a
^2*x + (b^2*c^6*x^7 + 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2
 + 2*(a*b*c^6*x^7 + 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a + b\operatorname{asinh}{\left(cx \right)}\right)^{2}\left(c^{2}x^{2} + 1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.454 $\int \frac{1}{x^2(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

`[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)`

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]`

`[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]`

Rubi steps

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 7.81, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]`

`[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]`

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b
^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 + b^
2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6 + 2
*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((6*c^5*x^5 + 7*c^3
*x^3 + 3*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + c*x + 2*(6*c^4*x^4 + 5*c^2*x^2 +
 1)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^9 + 2*a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^
2 + 1)^(3/2) + 2*(a*b*c^8*x^10 + 3*a*b*c^6*x^8 + 3*a*b*c^4*x^6 + a*b*c^2*x^
4)*(c^2*x^2 + 1) + ((b^2*c^7*x^9 + 2*b^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 +
1)^(3/2) + 2*(b^2*c^8*x^10 + 3*b^2*c^6*x^8 + 3*b^2*c^4*x^6 + b^2*c^2*x^4)*(
c^2*x^2 + 1) + (b^2*c^9*x^11 + 4*b^2*c^7*x^9 + 6*b^2*c^5*x^7 + 4*b^2*c^3*x^
5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x
^11 + 4*a*b*c^7*x^9 + 6*a*b*c^5*x^7 + 4*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x
^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^8 + 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4 + a
^2*x^2 + (b^2*c^6*x^8 + 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*
x)^2 + 2*(a*b*c^6*x^8 + 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*
x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2} \left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.455 $\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx = \int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(c^2x^2+1)^{\frac{5}{2}}}{(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^7*x^7 + 3*c^
5*x^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2
 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*
c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^7*(m + 6)*x^7 + c^5*(3*m +
 11)*x^5 + c^3*(3*m + 4)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^
8*(m + 6)*x^8 + c^6*(7*m + 30)*x^6 + 3*c^4*(3*m + 8)*x^4 + c^2*(5*m + 6)*x^
2 + m)*(c^2*x^2 + 1)*x^m + (c^9*(m + 6)*x^9 + c^7*(4*m + 19)*x^7 + 3*c^5*(2
*m + 7)*x^5 + c^3*(4*m + 9)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*
c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^
5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 +
b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x
^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^
2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(c^2\,x^2+1\right)}^{5/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

3.456 $\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{x^m(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{x^m(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^5*x^5 + 2*c^3*x^3 + c*x)
*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c
+ (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^
2 + 1))) + integrate((((c^5*(m + 4)*x^5 + c^3*(2*m + 3)*x^3 + c*(m - 1)*x)*(
c^2*x^2 + 1)^(3/2)*x^m + (2*c^6*(m + 4)*x^6 + c^4*(5*m + 12)*x^4 + 4*c^2*(m
 + 1)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^7*(m + 4)*x^7 + 3*c^5*(m + 3)*x^5 + 3
*c^3*(m + 2)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*
x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 +
1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^
2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
 a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (c^2x^2+1)^{\frac{3}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

3.457 $\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{c^2x^2+1}}{\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2*x^m + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2
 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b
^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*(m + 2)*x
^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^4*(m + 2)*x^4 + c^2*(3*m +
 2)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^5*(m + 2)*x^5 + c^3*(2*m + 3)*x^3 + c*(
m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 +
 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2
*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c
*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c^2x^2+1}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

3.458 $\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{x^m}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{m\mathrm{Int}\left(\frac{x^{-1+m}}{a+b\sinh^{-1}(cx)},x\right)}{bc}$$

```
[Out] -x^m/b/c/(a+b*arcsinh(c*x))+m*Unintegrable(x^(-1+m)/(a+b*arcsinh(c*x)),x)/b
/c
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^m/(b*c*(a + b*ArcSinh[c*x]))) + (m*Defer[Int][x^(-1 + m)/(a + b*ArcSinh
[c*x]), x])/(b*c)
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{x^m}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{m\int\frac{x^{-1+m}}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

Mathematica [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\ (a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*x^m + (c^3*x^3 + c*x)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x +
 ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integra
te(((c^3*m*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*m*x^4 + 3*c^2*m*x^
2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*m*x^5 + c^3*(2*m + 1)*x^3 + c*(m + 1)*x
)*x^m)/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*(c^
2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 + b^2*c^2*x^
2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a*b*c^3*x^3 + a*b*c*x)*s
qrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a+b\,\mathrm{asinh}\,(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**m/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.459 $\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

[Out] Unintegrable(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]

Rubi steps

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1
)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m -
2)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 2)*x^4 + c^2*(3*m - 2
)*x^2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 2)*x^5 + c^3*(2*m - 1)*x^3 + c
*(m + 1)*x)*x^m)/((a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b^2*c^5*x^5 + b^2*
c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^6 + 2*b^2*c^4*x^4 + b^2*c^2*x^2
)*(c^2*x^2 + 1) + (b^2*c^7*x^7 + 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*s
qrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^7 + 3*a*b*c^5*x
^5 + 3*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(a + b \operatorname{asinh}(cx)\right)^2 \left(c^2x^2 + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.460 $\int \frac{x^m}{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{x^m}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{x^m}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1)
 + ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2
+ b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2
*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m - 4)*x^3 + c*
(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 4)*x^4 + c^2*(3*m - 4)*x^2 + m)*
sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 4)*x^5 + c^3*(2*m - 3)*x^3 + c*(m + 1)*x)
*x^m)/((a*b*c^7*x^7 + 2*a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*
(a*b*c^8*x^8 + 3*a*b*c^6*x^6 + 3*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) +
 ((b^2*c^7*x^7 + 2*b^2*c^5*x^5 + b^2*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*
c^8*x^8 + 3*b^2*c^6*x^6 + 3*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2
*c^9*x^9 + 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 + 4*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^
2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^9 + 4*a*b*c^7*x^7 + 6
*a*b*c^5*x^5 + 4*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a+b\operatorname{asinh}(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^m/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.461 $\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

[Out] -1/2/a/arcsinh(a*x)^2

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5783}

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]

[Out] -1/2*1/(a*ArcSinh[a*x]^2)

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx = -\frac{1}{2a\sinh^{-1}(ax)^2}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]

[Out] -1/2*1/(a*ArcSinh[a*x]^2)

Maple [A]
time = 0.28, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{1}{2a\operatorname{arcsinh}(ax)^2}$	12
default	$-\frac{1}{2a\operatorname{arcsinh}(ax)^2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/a/arcsinh(a*x)^2
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.85

$$-\frac{1}{2\,a\,\operatorname{arsinh}\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/2/(a*arcsinh(a*x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 0.36, size = 23, normalized size = 1.77

$$-\frac{1}{2\,a\log\left(ax+\sqrt{a^2x^2+1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2/(a*log(a*x + sqrt(a^2*x^2 + 1))^2)
```

Sympy [A]
time = 0.40, size = 12, normalized size = 0.92

$$-\frac{1}{2a\operatorname{asinh}^2\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

[Out] -1/(2*a*asinh(a*x)**2)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3), x)

Mupad [B]
time = 0.09, size = 11, normalized size = 0.85

$$-\frac{1}{2\,a\,\mathrm{asinh}\left(a\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(asinh(a*x)^3*(a^2*x^2 + 1)^(1/2)),x)

[Out] -1/(2*a*asinh(a*x)^2)

3.462 $\int \frac{x^3(d+c^2dx^2)}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx}}{\sqrt{b}}\right.}{16b^{3/2}c^4}$$

```
[Out] -3/32*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)/c^4-3/32*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/32*d*exp(6*a/b)*erf(6^(1/2)*(a+b*ar
csinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4+1/32*d*erfi(6^(1/2)
*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)-
2*d*x^3*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]

time = 0.83, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5814, 5819, 5556, 3388, 2211, 2236, 2235}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}\,de^{\frac{6a}{b}}\text{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}\,de^{-\frac{6a}{b}}\text{Erfi}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{2dx^3(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^3*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^((2*
a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^4) + (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^4) - (3*d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b)) + (d*Sqrt[(3*Pi)/2]*Er
fi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((6*a)/b)
)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(d+c^2dx^2)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6d)\int \frac{x^2\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(12cd)\int \frac{x^4\sqrt{1}}{\sqrt{a+b\,\mathrm{s}}}}{b}$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6d)\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{(1}{}$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6d)\mathrm{Subst}\left(\int \left(-\frac{1}{8\sqrt{a+bx}} + \frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx, x, \mathrm{s}\right.}{bc^4}$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(3d)\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^4} + \frac{(3d)\mathrm{S}}{}$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(3d)\mathrm{Subst}\left(\int \frac{e^{-6x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^4} - \frac{(3d)\mathrm{S}}{}$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(3d)\mathrm{Subst}\left(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c^4} -$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{de^{\frac{6a}{b}}\sqrt{}}{}$$

Mathematica [A]
time = 1.41, size = 271, normalized size = 1.07

$$\frac{de^{-\frac{6a}{b}}\left(\sqrt{6}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{6(a+b\sinh^{-1}(cx))}{b}\right) - 3\sqrt{2}\,e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(cx))}{b}\right) - e^{\frac{6a}{b}}\left(64c^3x^3\sqrt{1+c^2x^2} - 3\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(cx))}{b}\right) + \sqrt{6}\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{6(a+b\sinh^{-1}(cx))}{b}\right) + 10\sinh\left(2\sinh^{-1}(cx)\right) - 8\sinh\left(4\sinh^{-1}(cx)\right) + 2\sinh\left(6\sinh^{-1}(cx)\right)\right)\right)}{32bc^4\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcSinh[c
*x]))/b] - 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-2*(a + b*ArcSinh[c*x]))/b] - E^((6*a)/b)*(64*c^3*x^3*Sqrt[1 + c^2*x^2] -
```

```
3*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSin
h[c*x]))/b] + Sqrt[6]*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a
 + b*ArcSinh[c*x]))/b] + 10*Sinh[2*ArcSinh[c*x]] - 8*Sinh[4*ArcSinh[c*x]] +
 2*Sinh[6*ArcSinh[c*x]])))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(c^{2} d\,x^{2}+d\right)}{\left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d \left(\int \frac{x^{3}}{a \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx + \int \frac{c^{2} x^{5}}{a \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

[Out] d*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x))

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(d\,c^2\,x^2+d\right)}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)

[Out] int((x^3*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)

3.463 $\int \frac{x^2(d+c^2dx^2)}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}-\frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}$$

```
[Out] 1/8*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3-1
/8*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1
/16*d*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/b^(3/2)/c^3+1/16*d*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-1/16*d*exp(5*a/b)*erf(5^(1/2)*(a+b*arcs
inh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/16*d*erfi(5^(1/2)*(
a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(5*a/b)-2*
d*x^2*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]

time = 0.86, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5814, 5819, 5556, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}\,de^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}+\frac{\sqrt{3\pi}\,de^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}+\frac{\sqrt{5\pi}\,de^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{2dx^2(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^2*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3) - (d*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/
Sqrt[b]])/(16*b^(3/2)*c^3) - (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcS
inh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) + (d*Sqrt[5*Pi]*Erfi[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d+c^2dx^2)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\int \frac{x\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(10cd)\int \frac{x^3\sqrt{}}{\sqrt{a+b}}}{b}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{(}{}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}} + \frac{\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \text{si}\right.}{bc^3}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(5d)\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^3} + \frac{(5d)}{}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(5d)\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^3} + \frac{(5d)}{}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(5d)\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c^3}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}}{}$$

Mathematica [A]
time = 0.99, size = 436, normalized size = 1.30

$$\frac{de^{-5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-e^{\frac{5a}{b}}-e^{\frac{5a}{b}+2\sinh^{-1}(cx)}+2e^{\frac{5a}{b}+4\sinh^{-1}(cx)}+2e^{\frac{5a}{b}+6\sinh^{-1}(cx)}-e^{\frac{5a}{b}+8\sinh^{-1}(cx)}-e^{\frac{5a}{b}+10\sinh^{-1}(cx)}-2e^{\frac{6a}{b}+5\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{5}e^{5\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\sinh^{-1}(cx))}{b}\right)+\sqrt{3}e^{\frac{2a}{b}+5\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)-2e^{\frac{4a}{b}+5\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)+\sqrt{3}e^{\frac{8a}{b}+5\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+\sqrt{5}e^{5\left(\frac{2a}{b}+\sinh^{-1}(cx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\right)}{16bc^3\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(-E^((5*a)/b) - E^((5*a)/b + 2*ArcSinh[c*x]) + 2*E^((5*a)/b + 4*ArcSinh[
c*x]) + 2*E^((5*a)/b + 6*ArcSinh[c*x]) - E^((5*a)/b + 8*ArcSinh[c*x]) - E^(
(5*a)/b + 10*ArcSinh[c*x]) - 2*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcS
```

```
inh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*x])*Sqrt[
-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + Sqrt[3
]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (
-3*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*
ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((8*a)/
b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c
*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*
Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c^3*E^(5*(a/b + ArcSinh[c*x]
))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c^2d\,x^2+d)}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{x^2}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx+\int \frac{c^2x^4}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(d\,c^2\,x^2+d\right)}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x^2*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

3.464 $\int \frac{x(d+c^2dx^2)}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}+$$

```
[Out] 1/4*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/c^2+1/4*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)+1/4*d*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2+1/4*d*erfi(2*(a+b*arcsinh(c*x))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)-2*d*x*(c^2*x^2+1)^(3/2)/b/c/(a+b*
arcsinh(c*x))^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5814, 5791, 3393, 3388, 2211, 2236, 2235, 5819, 5556}

$$\frac{\sqrt{\pi}\,de^{\frac{4a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}+\frac{\sqrt{\pi}\,de^{-\frac{4a}{b}}\mathrm{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}-\frac{2dx(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((4*a)/b
)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) + (d*
E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2*
b^(3/2)*c^2) + (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b
^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(d+c^2dx^2)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2d)\int\frac{\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc}+\frac{(8cd)\int\frac{x^2\sqrt{1+}}{\sqrt{a+b\sin}}}{b}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2d)\text{Subst}\left(\int\frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2}+\frac{(8d)\text{S}}{}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2d)\text{Subst}\left(\int\left(\frac{1}{2\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx,x,\sin\right.}{bc^2}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{d\text{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2}+\frac{d\text{Subst}\left(\right.}{}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{d\text{Subst}\left(\int\frac{e^{-4x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{2bc^2}+\frac{d\text{Subst}\left(\right.}{}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{d\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c^2}+\frac{d\text{S}}{}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{er}}{}$$

Mathematica [**A**]

time = 2.16, size = 321, normalized size = 1.36

$$\frac{de^{-\frac{4a}{b}}\left(2e^{\frac{6a}{b}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)+2e^{\frac{2a}{b}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)-\frac{\sqrt{b}\left(-\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\sinh^{-1}(cx))}{b}\right)+\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)+e^{\frac{4a}{b}}\left(8cx(1+c^2x^2)^{3/2}-\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(cx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{4(a+b\sinh^{-1}(cx))}{b}\right)\right)\right)}{\sqrt{a+b\sinh^{-1}(cx)}}\right)}{4b^{3/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(2*E^((6*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
] + 2*E^((2*a)/b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]] - (Sqrt[b]*(-(Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcS
inh[c*x]))/b]) + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcSinh[c*x]))/b] + E^((4*a)/b)*(8*c*x*(1 + c^2*x^2)^(3/2)
- Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh
[c*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcS
inh[c*x]))/b])))/Sqrt[a + b*ArcSinh[c*x]]))/(4*b^(3/2)*c^2*E^((4*a)/b))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(c^2d\,x^2+d)}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{x}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx+\int \frac{c^2x^3}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*
x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh
(c*x))*asinh(c*x)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(d\,c^2\,x^2+d\right)}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

3.465 $\int \frac{d+c^2dx^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=228

$$-\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{3de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

```
[Out] -3/4*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+3/
4*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*
d*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)
/b^(3/2)/c+1/4*d*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/b^(3/2)/c/exp(3*a/b)-2*d*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/
2)
```

Rubi [A]
time = 0.30, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5790, 5819, 5556, 3389, 2211, 2236, 2235}

$$-\frac{3\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{\sqrt{3\pi}\,de^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (d*E^((3*a)/b
)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c)
 + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c*E^(a/
b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^
(3/2)*c*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{d+c^2dx^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6cd)\int \frac{x\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{\sinh(3x)}{4\sqrt{a+bx}}\right)\,dx,x,\text{si}\right.}{bc}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(3d)\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{2bc}+\frac{(3d)}{}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(3d)\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{4bc}-\frac{(3d)}{}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(3d)\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{2b^2c}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{3de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{de^{\frac{3a}{b}}\sqrt{3\pi}}{}$$

Mathematica [A]

time = 0.53, size = 295, normalized size = 1.29

$$\frac{de^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-e^{\frac{3a}{b}}-3e^{\frac{3a}{b}+2\sinh^{-1}(cx)}-3e^{\frac{3a}{b}+4\sinh^{-1}(cx)}-e^{\frac{3a}{b}+6\sinh^{-1}(cx)}+3e^{\frac{4a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\,e^{3\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+3e^{\frac{2a}{b}+3\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)+\sqrt{3}\,e^{\frac{6a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\right)}{4bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(-E^((3*a)/b) - 3*E^((3*a)/b + 2*ArcSinh[c*x]) - 3*E^((3*a)/b + 4*ArcSin
h[c*x]) - E^((3*a)/b + 6*ArcSinh[c*x]) + 3*E^((4*a)/b + 3*ArcSinh[c*x])*Sqr
t[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcSinh
```

```
[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))
/b] + 3*E^((2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[
1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x])*Sqrt
[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(4*b*c*E^(3*(
a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{c^2 d\,x^2 + d}{(a + b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{c^2x^2}{a\sqrt{a + b\,\mathrm{asinh}\,(cx)} + b\sqrt{a + b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx + \int \frac{1}{a\sqrt{a + b\,\mathrm{asinh}\,(cx)} + b\sqrt{a + b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*
asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh
(c*x))*asinh(c*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d\,c^2\,x^2+d}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2),x)
[Out] int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

3.466 $\int \frac{d+c^2dx^2}{x(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}}+\frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c}{\sqrt{b}}\right.}{b^{3/2}}$$

```
[Out] 1/2*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)+1/2*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/exp(2*a/b)-2*d*(c^2*x^2+1)^(3/2)/b/c/x/(a+b*arcsinh(c*x))^(
1/2)-2*d*Unintegrable(1/x^2/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x)/b
/c
```

Rubi [A]
time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((2*a)/b
)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/b^(3/2) + (d*
Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2
*a)/b)) - (2*d*Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]
), x])/(b*c)
```

Rubi steps

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(2d)\int \frac{\sqrt{1+c^2x^2}}{x^2\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(4cd)\int \frac{\cdots}{\sqrt{a\cdots}}}{\cdots}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \cdots$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} + \frac{\cosh(2x)}{2\sqrt{a+bx}}\right) dx, \cdots\right)}{b}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \cdots$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} + \frac{d\text{Su}\cdots}{\cdots}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\cdots\right.}{b^2}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{de\cdots}{\cdots}$$

Mathematica [A]
time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{c^2 d\, x^2 + d}{x \left(a + b \operatorname{arcsinh}\left(c x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$d \left(\int \frac{c^{2} x^{2}}{a x \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx + \int \frac{1}{a x \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)/x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asinh(c*
x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,c^2\,x^2+d}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c^2*d*x^2)/(x*(a + b*asinh(c*x))^(3/2)),x)
```

```
[Out] int((d + c^2*d*x^2)/(x*(a + b*asinh(c*x))^(3/2)), x)
```

3.467 $\int \frac{x^3 (d+c^2 d x^2)^2}{(a+b \sinh^{-1}(cx))^{3/2}} \, dx$

Optimal. Leaf size=474

$$-\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}$$

```
[Out] -3/64*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(3/2)/c^4+1/64*d^2*exp(8*a/b)*erf(2*2^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4-3/64*d^2*erfi(2^(1/2)*(a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/64*d^2*er
fi(2*2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4
/exp(8*a/b)-1/32*d^2*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^
(1/2)/b^(3/2)/c^4-1/32*d^2*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2
)/b^(3/2)/c^4/exp(4*a/b)+1/64*d^2*exp(6*a/b)*erf(6^(1/2)*(a+b*arcsinh(c*x))
^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4+1/64*d^2*erfi(6^(1/2)*(a+b*arc
sinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)-2*d^2*x^3
*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]
time = 0.99, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5814, 5819, 5556, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\mathrm{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{\sqrt{\pi}d^2e^{-\frac{4a}{b}}\mathrm{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}d^2e^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{-\frac{8a}{b}}\mathrm{Erfi}\left(\frac{2\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{-\frac{6a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{2d^2x^3(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^3*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d^2*E^((
4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4
) - (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[Pi/2]*Erf[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((6*a)/b)*Sqrt[(3*
Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) -
(d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E
^((4*a)/b)) - (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi/2]*Erfi[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((8*a)/b)) + (d^2*Sqrt[(3*
Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^
((6*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(d+c^2dx^2)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6d^2)\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(16cd^2)\int \frac{x^4(1}{\sqrt{a+b}}}{b}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6d^2)\,\mathrm{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \cdots$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6d^2)\,\mathrm{Subst}\left(\int \left(-\frac{1}{16\sqrt{a+bx}} - \frac{\cosh(2x)}{32\sqrt{a+bx}} + \frac{c}{16\sqrt{}}\right.\right.}{bc^4}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\mathrm{Subst}\left(\int \frac{\cosh(8x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^4} - \frac{(3d^2)\,\mathrm{Su}}{}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\mathrm{Subst}\left(\int \frac{e^{-8x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^4} + \frac{d^2\mathrm{Subs}}{}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\mathrm{Subst}\left(\int e^{\frac{8a}{b}-\frac{8x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c^4} + \cdots$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{7}{2}}}{}$$

Mathematica [A]
time = 2.21, size = 490, normalized size = 1.03

$$-\frac{d^2e^{-\frac{8a}{b}}\left(128c^3e^{\frac{8a}{b}}x^3\sqrt{1+c^2x^2}-\sqrt{2}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{8(a+b\sinh^{-1}(cx))}{b}\right)-\sqrt{6}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{6(a+b\sinh^{-1}(cx))}{b}\right)+2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\sinh^{-1}(cx))}{b}\right)+3\sqrt{2}\,e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)-3\sqrt{2}\,e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(cx))}{b}\right)-2e^{\frac{12a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{4(a+b\sinh^{-1}(cx))}{b}\right)+\sqrt{6}\,e^{\frac{14a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{6(a+b\sinh^{-1}(cx))}{b}\right)+\sqrt{2}\,e^{\frac{16a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{8(a+b\sinh^{-1}(cx))}{b}\right)+26e^{\frac{8a}{b}}\sinh\left(2\sinh^{-1}(cx)\right)-18e^{\frac{8a}{b}}\sinh\left(4\sinh^{-1}(cx)\right)+2e^{\frac{8a}{b}}\sinh\left(6\sinh^{-1}(cx)\right)+e^{\frac{8a}{b}}\sinh\left(8\sinh^{-1}(cx)\right)\right)}{64bc^4\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -1/64*(d^2*(128*c^3*E^((8*a)/b)*x^3*Sqrt[1 + c^2*x^2] - Sqrt[2]*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, (-8*(a + b*ArcSinh[c*x]))/b] - Sqrt[6]*E^((
2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcSinh[c*x])
)/b] + 2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*
ArcSinh[c*x]))/b] + 3*Sqrt[2]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[2]*E^((10*a)/b)*Sqrt[a/b +
ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] - 2*E^((12*a)/b)*Sqrt[
a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] + Sqrt[6]*E^((14
*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[c*x]))/b] + Sq
rt[2]*E^((16*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (8*(a + b*ArcSinh[c*
x]))/b] + 26*E^((8*a)/b)*Sinh[2*ArcSinh[c*x]] - 18*E^((8*a)/b)*Sinh[4*ArcSi
nh[c*x]] + 2*E^((8*a)/b)*Sinh[6*ArcSinh[c*x]] + E^((8*a)/b)*Sinh[8*ArcSinh[
c*x]]))/(b*c^4*E^((8*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(c^2d\,x^2+d)^2}{(a+b\operatorname{arcsinh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{x^3}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{2c^2x^5}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{c^4x^7}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*as
inh(c*x)), x) + Integral(2*c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(d\,c^2\,x^2+d\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x^3*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

3.468 $\int \frac{x^2 \left(d+c^2 d x^2\right)^2}{\left(a+b \sinh^{-1}(cx)\right)^{3/2}} \, dx$

Optimal. Leaf size=457

$$-\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{5d^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right.}{64b^{3/2}c^3}$$

```
[Out] 5/64*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^
3-5/64*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(
a/b)-1/64*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-3/64*d^2*exp(5*a/b)*erf(5^(1/
2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3+3/64*d^2*
erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3
/exp(5*a/b)-1/64*d^2*exp(7*a/b)*erf(7^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(7^(1/2)*(a+b*arcsinh(c*x))^(1
/2)/b^(1/2))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(7*a/b)-2*d^2*x^2*(c^2*x^2+1)^
(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]
time = 1.18, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5814, 5819, 5556, 3389, 2211, 2236, 2235}

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{7\pi}\,d^2e^{\frac{7a}{b}}\mathrm{Erf}\left(\frac{\sqrt{7}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{5\sqrt{\pi}\,d^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{3\pi}\,d^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{3\sqrt{5\pi}\,d^2e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{7\pi}\,d^2e^{-\frac{7a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{7}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{2d^2x^2(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (5*d^2*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3) - (d
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*Ar
cSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (d^2*E^((7*a)/b)*Sqrt[7*Pi]*Erf[(
Sqrt[7]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (5*d^2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3*E^(a/b)) + (d^2
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c
^3*E^((3*a)/b)) + (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])
/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqrt[7*Pi]*Erfi[(Sqrt[7]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((7*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d+c^2dx^2)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d^2)\int \frac{x(1+c^2x^2)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(14cd^2)\int \frac{x^3 \ldots}{\sqrt{a+\ldots}}}{\ldots}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d^2)\,\mathrm{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \ldots$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d^2)\,\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\sin\ldots}{16\sqrt{a\ldots}}\right.\right.}{bc^3}\ldots$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(7d^2)\,\mathrm{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{32bc^3} + \frac{(7\ldots}{\ldots}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(7d^2)\,\mathrm{Subst}\left(\int \frac{e^{-7x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^3} - \frac{(7\ldots}{\ldots}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(7d^2)\,\mathrm{Subst}\left(\int e^{\frac{7a}{b}-\frac{7x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)\ldots}{32b^2c^3}\ldots$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{d^2e^{\frac{3a}{b}}\sqrt{3}\ldots}{\ldots}$$

Mathematica [A]
time = 1.75, size = 577, normalized size = 1.26

$$-\frac{d^2e^{-7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(e^{\frac{7a}{b}}+3e^{\frac{7a}{b}+2\sinh^{-1}(cx)}+e^{\frac{7a}{b}+4\sinh^{-1}(cx)}-5e^{\frac{7a}{b}+6\sinh^{-1}(cx)}-5e^{\frac{7a}{b}+8\sinh^{-1}(cx)}+e^{\frac{7a}{b}+10\sinh^{-1}(cx)}+3e^{\frac{7a}{b}+12\sinh^{-1}(cx)}+e^{\frac{7a}{b}+14\sinh^{-1}(cx)}+5e^{\frac{8a}{b}+7\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)-\sqrt{7}e^{7\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{7(a+b\sinh^{-1}(cx))}{b}\right)-3\sqrt{5}e^{\frac{2a}{b}+7\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\sinh^{-1}(cx))}{b}\right)-\sqrt{3}e^{\frac{4a}{b}+7\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+5e^{\frac{6a}{b}+7\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-\sqrt{3}e^{\frac{10a}{b}+7\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(cx))}{b}\right)-3\sqrt{5}e^{\frac{12a}{b}+7\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\sinh^{-1}(cx))}{b}\right)-\sqrt{7}e^{7\left(\frac{2a}{b}+\sinh^{-1}(cx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{7(a+b\sinh^{-1}(cx))}{b}\right)\right)}{64bc^3\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -1/64*(d^2*(E^((7*a)/b) + 3*E^((7*a)/b + 2*ArcSinh[c*x]) + E^((7*a)/b + 4*A
rcSinh[c*x]) - 5*E^((7*a)/b + 6*ArcSinh[c*x]) - 5*E^((7*a)/b + 8*ArcSinh[c*
x]) + E^((7*a)/b + 10*ArcSinh[c*x]) + 3*E^((7*a)/b + 12*ArcSinh[c*x]) + E^(
(7*a)/b + 14*ArcSinh[c*x]) + 5*E^((8*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] - Sqrt[7]*E^(7*ArcSinh[c*x])*Sqrt[
-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-7*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt
[5]*E^((2*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-5*(a + b*ArcSinh[c*x]))/b] - Sqrt[3]*E^((4*a)/b + 7*ArcSinh[c*x])*Sqrt[-
((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 5*E^((6
*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b
*ArcSinh[c*x])/b)] - Sqrt[3]*E^((10*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcSi
nh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*E^((12*a)/b + 7
*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))
/b] - Sqrt[7]*E^(7*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[1/2, (7*(a + b*ArcSinh[c*x]))/b]))/(b*c^3*E^(7*(a/b + ArcSinh[c*x]))*Sqrt[
a + b*ArcSinh[c*x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c^2d\,x^2+d)^2}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\frac{x^2}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int\frac{2c^2x^4}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int\frac{c^4x^6}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*as
inh(c*x)), x) + Integral(2*c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\,{\left(d\,c^2\,x^2+d\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x^2*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

3.469 $\int \frac{x(d+c^2dx^2)^2}{(a+b\sinh^{-1}(cx))^{3/2}} \, dx$

Optimal. Leaf size=358

$$-\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}$$

```
[Out] 5/32*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^2+5/32*d^2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))
*2^(1/2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arcs
inh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2+1/4*d^2*erfi(2*(a+b*arcsinh(c
*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)+1/32*d^2*exp(6*a/b)*erf
(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^2+1/3
2*d^2*erfi(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/
2)/c^2/exp(6*a/b)-2*d^2*x*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]

time = 0.84, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5814, 5791, 3393, 3388, 2211, 2236, 2235, 5819, 5556}

$$\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\mathrm{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\pi}d^2e^{-\frac{4a}{b}}\mathrm{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}d^2e^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{-\frac{6a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}-\frac{2d^2x(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) +
 (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt
[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (5*d^2*Sqrt[P
i/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((
2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
```

```
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(d+c^2dx^2)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d^2)\int \frac{(1+c^2x^2)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(12cd^2)\int \frac{x^2(1}{\sqrt{a+b}}}{b}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{(12c}{}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d^2)\,\mathrm{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}} + \frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{\cosh(4}{8\sqrt{a+}}\right.\right.}{bc^2}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\mathrm{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4bc^2} - \frac{(3d^2)\,\mathrm{Su}}{}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^2} + \frac{d^2\mathrm{Subs}}{}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\mathrm{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{4b^2c^2} + \frac{d}{}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 3.32, size = 475, normalized size = 1.33

$$\frac{d^2e^{-\frac{6a}{b}}\left(16e^{\frac{8a}{b}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)+16e^{\frac{4a}{b}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)-\frac{\sqrt{b}\left(64ce^{\frac{6a}{b}}x\sqrt{1+c^2x^2}+128c^3e^{\frac{6a}{b}}x^3\sqrt{1+c^2x^2}+64c^5e^{\frac{6a}{b}}x^5\sqrt{1+c^2x^2}-\sqrt{6}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{6(a+b\sinh^{-1}(cx))}{b}\right)-8e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\sinh^{-1}(cx))}{b}\right)+11\sqrt{2}e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)-11\sqrt{2}e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(cx))}{b}\right)+8e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{4(a+b\sinh^{-1}(cx))}{b}\right)+\sqrt{6}e^{\frac{12a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{6(a+b\sinh^{-1}(cx))}{b}\right)\right)}{\sqrt{a+b\sinh^{-1}(cx)}}\right)}{32b^{3/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d^2*(16*E^((8*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]] + 16*E^((4*a)/b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]] - (Sqrt[b]*(64*c*E^((6*a)/b)*x*Sqrt[1 + c^2*x^2] + 128*c^3*E^((6*a)/
b)*x^3*Sqrt[1 + c^2*x^2] + 64*c^5*E^((6*a)/b)*x^5*Sqrt[1 + c^2*x^2] - Sqrt[
6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcSinh[c*x]))/b]
- 8*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSi
nh[c*x]))/b] + 11*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma
[1/2, (-2*(a + b*ArcSinh[c*x]))/b] - 11*Sqrt[2]*E^((8*a)/b)*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + 8*E^((10*a)/b)*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] + Sqrt[6]*E^((12*a)/
b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[c*x]))/b]))/Sqrt[a
 + b*ArcSinh[c*x]]))/(32*b^(3/2)*c^2*E^((6*a)/b))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(c^2d\,x^2+d)^2}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{x}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{2c^2x^3}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{c^4x^5}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)

[Out] d**2*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh (c*x)), x) + Integral(2*c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b* asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect eur & l) Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(d\,c^2\,x^2+d\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)

[Out] int((x*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)

3.470 $\int \frac{(d+c^2dx^2)^2}{(a+b\sinh^{-1}(cx))^{3/2}} dx$

Optimal. Leaf size=346

$$-\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{5d^2e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right.}{16b^{3/2}c}$$

```
[Out] -5/8*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+
5/8*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-
5/16*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*P
i^(1/2)/b^(3/2)/c+5/16*d^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3
^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(3*a/b)-1/16*d^2*exp(5*a/b)*erf(5^(1/2)*(a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c+1/16*d^2*erfi(5^(1/2
)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(5*a/b)-2
*d^2*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5790, 5819, 5556, 3389, 2211, 2236, 2235}

$$-\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}-\frac{\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}+\frac{5\sqrt{3\pi}d^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{\sqrt{5\pi}d^2e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}-\frac{2d^2(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (5*d^2*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c) - (5*d^2*E^
((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b
^(3/2)*c) - (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]
]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) + (5*d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2*Sqrt[5*Pi]*E
rfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx = -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(10cd^2)\int \frac{x(1+c^2x^2)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(10d^2)\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(10d^2)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\text{s}\cdots}{16\sqrt{\cdots}}\right.\right.}{bc}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(5d^2)\,\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8bc} + \frac{(5\cdots}{\cdots}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(5d^2)\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{16bc} + \frac{(5\cdots}{\cdots}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(5d^2)\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)\cdots}{8b^2c}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{\frac{3a}{b}}\sqrt{\cdots}}{\cdots}$$

Mathematica [A]
time = 1.16, size = 440, normalized size = 1.27

$$\frac{d^2e^{-5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-e^{\frac{5a}{b}} - 5e^{\frac{5a}{b}+2\sinh^{-1}(cx)} - 10e^{\frac{5a}{b}+4\sinh^{-1}(cx)} - 10e^{\frac{5a}{b}+6\sinh^{-1}(cx)} - 5e^{\frac{5a}{b}+8\sinh^{-1}(cx)} - e^{\frac{5a}{b}+10\sinh^{-1}(cx)} + 10e^{\frac{6a}{b}+5\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right) + \sqrt{5}e^{5\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\sinh^{-1}(cx))}{b}\right) + 5\sqrt{3}e^{\frac{2a}{b}+5\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(cx))}{b}\right) + 10e^{\frac{4a}{b}+5\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right) + 5\sqrt{3}e^{\frac{8a}{b}+5\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(cx))}{b}\right) + \sqrt{5}e^{5\left(\frac{2a}{b}+\sinh^{-1}(cx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\right)}{16bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d^2*(-E^((5*a)/b) - 5*E^((5*a)/b + 2*ArcSinh[c*x]) - 10*E^((5*a)/b + 4*Arc
Sinh[c*x]) - 10*E^((5*a)/b + 6*ArcSinh[c*x]) - 5*E^((5*a)/b + 8*ArcSinh[c*x
]) - E^((5*a)/b + 10*ArcSinh[c*x]) + 10*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a
```

```
/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*
x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b]
 + 5*Sqrt[3]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 10*E^((4*a)/b + 5*ArcSinh[c*x])*Sq
rt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 5*Sqr
t[3]*E^((8*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a
 + b*ArcSinh[c*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b +
ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c*E^(5*(a/b +
ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} d\,x^{2}+d\right)^{2}}{\left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int \frac{2 c^{2} x^{2}}{a \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx + \int \frac{c^{4} x^{4}}{a \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx + \int \frac{1}{a \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(2*c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c
*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqr
t(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x))
+ b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,c^2\,x^2+d\right)^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2), x)
```

3.471 $\int \frac{(d+c^2dx^2)^2}{x(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=375

$$-\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}}+\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}-\frac{d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}}$$

```
[Out] 3/4*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)+3/4*d^2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)+1/4*d^2*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(
1/2))*Pi^(1/2)/b^(3/2)/exp(4*a/b)-2*d^2*(c^2*x^2+1)^(5/2)/b/c/x/(a+b*arcsin
h(c*x))^(1/2)-2*d^2*Unintegrable(1/x^2/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))
^(1/2),x)/b/c
```

Rubi [A]

time = 0.87, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+c^2dx^2)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)) - (d^
2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
2*b^(3/2)) + (d^2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/b^(3/2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*E^((4*a)/b)) - (d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) + (d^2*Sqrt[2*Pi]*Erfi[(Sq
rt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2*a)/b)) - (2*d^2*De
fer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(2d^2)\int \frac{(1+c^2x^2)^{3/2}}{x^2\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(8cd^2)\int -}{\sqrt{}}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(8d^2)\,\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} -$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(8d^2)\,\text{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}} + \frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{}{8\sqrt{}}\right.\right.}{b}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\sinh^{-1}(cx)}}{b^2} + \frac{d^2\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\right.}{b}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\sinh^{-1}(cx)}}{b^2} + \frac{d^2\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\right.}{2}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}}{}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}}{}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{\frac{2a}{b}}}{}$$

Mathematica [A]

time = 2.00, size = 0, normalized size = 0.00

$$\int \frac{(d+c^2dx^2)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2d\,x^2+d)^2}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^2/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{2c^2x^2}{ax\sqrt{a+b\,\mathrm{asinh}\,(cx)}+bx\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx+\int \frac{c^4x^4}{ax\sqrt{a+b\,\mathrm{asinh}\,(cx)}+bx\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx+\int \frac{1}{ax\sqrt{a+b\,\mathrm{asinh}\,(cx)}+bx\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2/x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(2*c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asi
nh(c*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*asinh(c*x)) +
 b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*as
inh(c*x)) + b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,c^2\,x^2+d\right)^2}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c^2*d*x^2)^2/(x*(a + b*asinh(c*x))^(3/2)),x)
```

```
[Out] int((d + c^2*d*x^2)^2/(x*(a + b*asinh(c*x))^(3/2)), x)
```

3.472 $\int \left(c+a^2cx^2\right)^{3/2} \sqrt{\sinh^{-1}(ax)}\, dx$

Optimal. Leaf size=319

$$\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}}+\frac{c\sqrt{\pi}\,\sqrt{c+a}}{2}$$

```
[Out] 1/4*c*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erf
(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^
2+1)^(1/2)-1/32*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*
x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/256*c*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)
*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/256*c*erfi(2*arcsinh(a*x)^(1/2))
*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/4*x*(a^2*c*x^2+c)^(3/2)
*arcsinh(a*x)^(1/2)+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5786, 5785, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236, 5819}

$$\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{3/2}}{4a\sqrt{a^2x^2+1}}+\frac{3}{8}cx\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (3*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/8 + (x*(c + a^2*c*x^2)^(3/2)
*Sqrt[ArcSinh[a*x]])/4 + (c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(4*a*Sq
rt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]
])/(256*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi]*Sqrt[c + a^2*
c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)} + \frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)} + $$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)} + $$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)} + $$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)} + $$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)} + $$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)} + $$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)} + $$

Mathematica [A]
time = 0.13, size = 142, normalized size = 0.45

$$\frac{c\sqrt{c+a^2cx^2}\left(-\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-4\sinh^{-1}(ax)\right)-8\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}(ax)\right)+\sqrt{\sinh^{-1}(ax)}\left(32\sinh^{-1}(ax)^{3/2}-8\sqrt{2}\,\Gamma\left(\frac{3}{2},2\sinh^{-1}(ax)\right)-\Gamma\left(\frac{3}{2},4\sinh^{-1}(ax)\right)\right)\right)}{128a\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-(Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]])
- 8*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[
a*x]]*(32*ArcSinh[a*x]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[a*x]] - Gamma
[3/2, 4*ArcSinh[a*x]])))/(128*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\operatorname{arcsinh}\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{asinh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(1/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\mathrm{asinh}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.473 $\int \sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=186

$$\frac{1}{2}x\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{1+a^2x^2}}-\frac{\sqrt{}}{}$$

```
[Out] 1/3*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*erf(2^(
1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)
^(1/2)-1/32*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)
^(1/2)/a/(a^2*x^2+1)^(1/2)+1/2*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5785, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh
[a*x]^(3/2))/(3*a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[
Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (Sqrt[Pi/2]*Sqrt[c
+ a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\int\frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}dx}{2\sqrt{1+a^2x^2}} - \frac{\big(a\sqrt{}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+}}{}$$

Mathematica [**A**]
time = 0.07, size = 104, normalized size = 0.56

$$\frac{\sqrt{c(1+a^2x^2)}\left(16\sinh^{-1}(ax)^2 - 3\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}(ax)\right) - 3\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},2\sinh^{-1}(ax)\right)\right)}{48a\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
[Out] (Sqrt[c*(1 + a^2*x^2)]*(16*ArcSinh[a*x]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*G
amma[3/2, -2*ArcSinh[a*x]] - 3*Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, 2*ArcS
inh[a*x]]))/(48*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2c\,x^2+c}\,\sqrt{\operatorname{arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^{2}x^{2}+1\right)}\,\sqrt{\operatorname{asinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\operatorname{asinh}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.474 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{c+a^2cx^2}}$$

```
[Out] 2/3*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 1.26, size = 36, normalized size = 0.86

method	result	size
default	$\frac{2 \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{a^2x^2+1}}{3a \sqrt{c\left(a^2x^2+1\right)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(asinh(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\mathrm{asinh}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.475 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sqrt{\sinh^{-1}(ax)}}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{1+a^2x^2}\,\mathrm{Int}\left(\frac{x}{(1+a^2x^2)\sqrt{\sinh^{-1}(ax)}},x\right)}{2c\sqrt{c+a^2cx^2}}$$

```
[Out] x*arcsinh(a*x)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x/(a^2*x^2+1)/arcsinh(a*x)^(1/2),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Defer
[Int][x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{x\sqrt{\sinh^{-1}(ax)}}{c\sqrt{c+a^2cx^2}}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int\frac{x}{(1+a^2x^2)\sqrt{\sinh^{-1}(ax)}}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(a x\right)}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}{\left(a x \right)}}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{asinh}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.476 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x\sqrt{\sinh^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\sqrt{\sinh^{-1}(ax)}}{3c^2\sqrt{c+a^2cx^2}}-\frac{a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)^2\sqrt{\sinh^{-1}(ax)}},x\right)}{6c^2\sqrt{c+a^2cx^2}}-\frac{a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a}\right.}{3c^2\sqrt{c}}$$

```
[Out] 1/3*x*arcsinh(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arcsinh(a*x)^(1/2)/c^2
/(a^2*c*x^2+c)^(1/2)-1/6*a*(a^2*x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/a
rcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+c)^(1/2)-1/3*a*(a^2*x^2+1)^(1/2)*Uninte
grable(x/(a^2*x^2+1)/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[a*x]
])/(3*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^
2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(6*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 +
 a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(3*c^2*Sqrt[
c + a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{x\sqrt{\sinh^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{3c}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^2\sqrt{\sinh^{-1}(ax}}}{6c^2\sqrt{c+a^2cx^2}}\\ &= \frac{x\sqrt{\sinh^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\sqrt{\sinh^{-1}(ax)}}{3c^2\sqrt{c+a^2cx^2}}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^2\sqrt{\sinh^{-1}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]

[Out] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2), x]

Maple [A]
time = 5.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)

[Out] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{asinh}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2), x)
```

3.477 $\int (c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=449

$$-\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c(1+a^2x^2)^{3/2}\,\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(a}}{32a}$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2)+3/8*c*x*arcsinh(a*x)^(3/2)*(a^
2*c*x^2+c)^(1/2)+3/20*c*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1
)^(1/2)+3/128*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2
+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1
/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/2048*c*erf(2*arcsinh
(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/2048*c*erfi
(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/3
2*c*(a^2*x^2+1)^(3/2)*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a-27/256*c*(a^
2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)-9/32*a*c*x^2*(a^2*c
*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/(a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5786, 5785, 5783, 5777, 5819, 3393, 3388, 2211, 2235, 2236, 5798, 5791}

$$\frac{3\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3c\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{5/2}}{20a\sqrt{a^2x^2+1}}+\frac{1}{4}x(a^2cx^2+c)^{3/2}\sinh^{-1}(ax)^{3/2}+\frac{3}{8}cx\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{3/2}-\frac{3c(a^2x^2+1)^{3/2}\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}}{32a}-\frac{9acx^2\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{a^2x^2+1}}-\frac{27c\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-27*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(256*a*Sqrt[1 + a^2*x^2]) -
(9*a*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*Sqrt[1 + a^2*x^2]) -
 (3*c*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*a) +
(3*c*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/8 + (x*(c + a^2*c*x^2)^(3/2)
*ArcSinh[a*x]^(3/2))/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2))/(20*a
*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[
a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqr
t[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (
3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*
Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
```

```
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2} + \frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}$$

$$= -\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\,\sinh^{-}$$

$$= -\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}} - \frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{si}}}{32a}$$

$$= -\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}} - \frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{si}}}{32a}$$

$$= \frac{9c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}} - \frac{3c}{}$$

$$= -\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}} -$$

$$= -\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}} -$$

$$= -\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}} -$$

$$= -\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}} -$$

Mathematica [A]
time = 0.22, size = 186, normalized size = 0.41

$$\frac{c\sqrt{c+a^2cx^2}\left(384\sinh^{-1}(ax)^3-480\sinh^{-1}(ax)\cosh\left(2\sinh^{-1}(ax)\right)+60\sqrt{2\pi}\,\sqrt{\sinh^{-1}(ax)}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)+60\sqrt{2\pi}\,\sqrt{\sinh^{-1}(ax)}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)+5\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-4\sinh^{-1}(ax)\right)-5\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},4\sinh^{-1}(ax)\right)+640\sinh^{-1}(ax)^2\sinh\left(2\sinh^{-1}(ax)\right)\right)}{2560a\sqrt{1+a^2x^2}\,\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
[Out] (c*Sqrt[c + a^2*c*x^2]*(384*ArcSinh[a*x]^3 - 480*ArcSinh[a*x]*Cosh[2*ArcSin
h[a*x]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]]
```

```
+ 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 5*Sqr
t[-ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]] - 5*Sqrt[ArcSinh[a*x]]*Gamma[5
/2, 4*ArcSinh[a*x]] + 640*ArcSinh[a*x]^2*Sinh[2*ArcSinh[a*x]]))/(2560*a*Sqr
t[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(3/2),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.478 $\int \sqrt{c+a^2cx^2} \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=271

$$-\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}}-\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}}+\frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}+\frac{\sqrt{c+a^2cx^2}}{5a\sqrt{1}}$$

```
[Out] 1/2*x*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)+1/5*arcsinh(a*x)^(5/2)*(a^2*c*
x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1
/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*erfi(2^(1/2)*arc
sinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3
/16*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/8*a*x^2*(a
^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/(a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5785, 5783, 5777, 5819, 3393, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}-\frac{3ax^2\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{a^2x^2+1}}-\frac{3\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(16*a*Sqrt[1 + a^2*x^2]) - (3*a
*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*Sqrt[1 + a^2*x^2]) + (x*Sqr
t[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^
(5/2))/(5*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c +
a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
```

```
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(3a\sqrt{c\right.}}{}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c}}{}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c}}{}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c}}{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{}$$

Mathematica [A]
time = 0.19, size = 126, normalized size = 0.46

$$\frac{\sqrt{c(1+a^2x^2)}\left(15\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)+15\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)+8\sqrt{\sinh^{-1}(ax)}\left(-15\cosh\left(2\sinh^{-1}(ax)\right)+4\sinh^{-1}(ax)\left(4\sinh^{-1}(ax)+5\sinh\left(2\sinh^{-1}(ax)\right)\right)\right)\right)}{640a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 15*
Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(-15*Cos
```

```
h[2*ArcSinh[a*x]] + 4*ArcSinh[a*x]*(4*ArcSinh[a*x] + 5*Sinh[2*ArcSinh[a*x]]
))))/(640*a*Sqrt[1 + a^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{a^2 c\, x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
[Out] int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\,(a^2x^2+1)}\, \operatorname{asinh}^{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{asinh}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.479 $\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}{5a\sqrt{c+a^2cx^2}}$$

```
[Out] 2/5*arcsinh(a*x)^(5/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}{5a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}{5a\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 1.74, size = 36, normalized size = 0.86

method	result	size
default	$\frac{2 \operatorname{arcsinh}(a x)^{\frac{5}{2}} \sqrt{a^{2} x^{2}+1}}{5 a \sqrt{c \left(a^{2} x^{2}+1\right)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*arcsinh(a*x)^(5/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}{\left(a x \right)}}{\sqrt{c \left(a^{2} x^{2}+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(asinh(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.480 $\int \frac{\sinh^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sinh^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{3a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x\sqrt{\sinh^{-1}(ax)}}{1+a^2x^2},x\right)}{2c\sqrt{c+a^2cx^2}}$$

```
[Out] x*arcsinh(a*x)^(3/2)/c/(a^2*c*x^2+c)^(1/2)-3/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x*arcsinh(a*x)^(1/2)/(a^2*x^2+1),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*a*Sqrt[1 + a^2*x^2]*Def
er[Int][(x*Sqrt[ArcSinh[a*x]])/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{x\sinh^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(3a\sqrt{1+a^2x^2}\right)\int\frac{x\sqrt{\sinh^{-1}(ax)}}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [**A**]
time = 5.12, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(a x\right)^{\frac{3}{2}}}{\left(a^{2} c \,x^{2}+c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
[Out] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}{\left(a x \right)}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(asinh(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{asinh}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.481 $\int (c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=514

$$\frac{225}{512}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(1+a^2x^2)\,\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}-\frac{45c\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax}{256a\sqrt{1+a^2x^2}}$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2)-5/32*c*(a^2*x^2+1)^(3/2)*arcsi
nh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c
)^(1/2)-45/256*c*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
-15/32*a*c*x^2*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+3/2
8*c*arcsinh(a*x)^(7/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/512*c*erf
(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^
2+1)^(1/2)-15/512*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*
c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/16384*c*erf(2*arcsinh(a*x)^(1/2))*Pi^
(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/16384*c*erfi(2*arcsinh(a*x
)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+225/512*c*x*(a^2*
c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)+15/256*c*x*(a^2*x^2+1)*(a^2*c*x^2+c)^(1/2
)*arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {5786, 5785, 5783, 5777, 5812, 5780, 5556, 12, 3389, 2211, 2235, 2236, 5798, 5819}

$$\frac{15\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{3c\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{7/2}}{28a\sqrt{a^2x^2+1}}+\frac{1}{4}x(a^2cx^2+c)^{3/2}\sinh^{-1}(ax)^{5/2}+\frac{3}{8}cx\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{5/2}-\frac{5c(a^2x^2+1)^{3/2}\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{3/2}}{32a}-\frac{15acx^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{3/2}}{32\sqrt{a^2x^2+1}}-\frac{45c\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{3/2}}{256a\sqrt{a^2x^2+1}}+\frac{225}{512}cx\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(a^2x^2+1)\,\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (225*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/512 + (15*c*x*(1 + a^2*x^2
)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/256 - (45*c*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(256*a*Sqrt[1 + a^2*x^2]) - (15*a*c*x^2*Sqrt[c + a^2*c*x
^2]*ArcSinh[a*x]^(3/2))/(32*Sqrt[1 + a^2*x^2]) - (5*c*(1 + a^2*x^2)^(3/2)*S
qrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(32*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(5/2))/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2))/4 + (3
*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(28*a*Sqrt[1 + a^2*x^2]) + (15*c
*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 +
a^2*x^2]) + (15*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[
2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi/2]*Sqrt[
c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
```

```
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{4}x(c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{5/2}+\frac{1}{4}(3c)\int\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2}\,dx$$

$$= -\frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}($$

$$= \frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{15acx^2\sqrt{c+a^2cx^2}\,\sinh}{32\sqrt{1+a^2x^2}}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\sin}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\sin}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\sin}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\sin}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\sin}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\sin}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\sin}$$

Mathematica [A]
time = 0.24, size = 201, normalized size = 0.39

$$\frac{c\sqrt{c+a^2cx^2}\left(1536\sinh^{-1}(ax)^4-4480\sinh^{-1}(ax)^2\cosh\left(2\sinh^{-1}(ax)\right)+420\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-420\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-7\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-4\sinh^{-1}(ax)\right)-7\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},4\sinh^{-1}(ax)\right)+3360\sinh^{-1}(ax)\sinh\left(2\sinh^{-1}(ax)\right)+3584\sinh^{-1}(ax)^3\sinh\left(2\sinh^{-1}(ax)\right)\right)}{14336a\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(1536*ArcSinh[a*x]^4 - 4480*ArcSinh[a*x]^2*Cosh[2*Ar
cSinh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] -
 7*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]] - 7*Sqrt[ArcSinh[a*x]]*G
amma[7/2, 4*ArcSinh[a*x]] + 3360*ArcSinh[a*x]*Sinh[2*ArcSinh[a*x]] + 3584*A
rcSinh[a*x]^3*Sinh[2*ArcSinh[a*x]]))/(14336*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSin
h[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{asinh}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.482 $\int \sqrt{c+a^2cx^2} \sinh^{-1}(ax)^{5/2} dx$

Optimal. Leaf size=298

$$\frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{5\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}}-\frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{8\sqrt{1+a^2x^2}}+\frac{1}{2}x\sqrt{c+a^2$$

```
[Out] 1/2*x*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)-5/16*arcsinh(a*x)^(3/2)*(a^2*c
*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-5/8*a*x^2*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c
)^(1/2)/(a^2*x^2+1)^(1/2)+1/7*arcsinh(a*x)^(7/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2
*x^2+1)^(1/2)+15/512*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*
c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/512*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*
2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/32*x*(a^2*c*x^2
+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5785, 5783, 5777, 5812, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{5/2}-\frac{5ax^2\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}{8\sqrt{a^2x^2+1}}-\frac{5\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}{16a\sqrt{a^2x^2+1}}+\frac{15}{32}x\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (15*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/32 - (5*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(16*a*Sqrt[1 + a^2*x^2]) - (5*a*x^2*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(3/2))/(8*Sqrt[1 + a^2*x^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[
a*x]^(5/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[1 + a^2*
x^2]) + (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])
/(256*a*Sqrt[1 + a^2*x^2]) - (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(5a\sqrt{c\cdots\right.}{}$$

$$= -\frac{5ax^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2} + \frac{\sqrt{c\cdots}}{}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5ax^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{\cdots}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c\cdots}}{}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c\cdots}}{}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c\cdots}}{}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c\cdots}}{}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c\cdots}}{}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c\cdots}}{}$$

Mathematica [A]
time = 0.20, size = 135, normalized size = 0.45

$$\frac{\sqrt{c\left(1+a^2x^2\right)}\left(105\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right) - 105\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right) + 8\sqrt{\sinh^{-1}(ax)}\left(64\sinh^{-1}(ax)^3 - 140\sinh^{-1}(ax)\cosh\left(2\sinh^{-1}(ax)\right) + 7\left(15+16\sinh^{-1}(ax)^2\right)\sinh\left(2\sinh^{-1}(ax)\right)\right)\right)}{3584a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 10
5*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(64*Ar
```

```
cSinh[a*x]^3 - 140*ArcSinh[a*x]*Cosh[2*ArcSinh[a*x]] + 7*(15 + 16*ArcSinh[a
*x]^2)*Sinh[2*ArcSinh[a*x]])))/(3584*a*Sqrt[1 + a^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcsinh}(ax)^{\frac{5}{2}} \sqrt{a^2c\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
[Out] int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.483 $\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}}$$

```
[Out] 2/7*arcsinh(a*x)^(7/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{5783\}$

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 1.22, size = 36, normalized size = 0.86

method	result	size
default	$\frac{2 \operatorname{arcsinh}(a x)^{\frac{7}{2}} \sqrt{a^{2} x^{2}+1}}{7 a \sqrt{c\left(a^{2} x^{2}+1\right)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/7*arcsinh(a*x)^(7/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.484 $\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sinh^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}}-\frac{5a\sqrt{1+a^2x^2}\,\mathrm{Int}\left(\frac{x\sinh^{-1}(ax)^{3/2}}{1+a^2x^2},x\right)}{2c\sqrt{c+a^2cx^2}}$$

```
[Out] x*arcsinh(a*x)^(5/2)/c/(a^2*c*x^2+c)^(1/2)-5/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x*arcsinh(a*x)^(3/2)/(a^2*x^2+1),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]) - (5*a*Sqrt[1 + a^2*x^2]*Def
er[Int][(x*ArcSinh[a*x]^(3/2))/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{x\sinh^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(5a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^{3/2}}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 4.97, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
[Out] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.485 $\int (a^2+x^2)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,dx$

Optimal. Leaf size=309

$$\frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}+\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}+\frac{a^3\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}}+\frac{a^3\sqrt{\pi}\sqrt{a^2+\cdots}}{2\cdots}$$

```
[Out] 1/4*a^3*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/32*a^3*erf(2
^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/
2)-1/32*a^3*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/
2)/(1+x^2/a^2)^(1/2)+1/256*a^3*erf(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2+x^2)
^(1/2)/(1+x^2/a^2)^(1/2)-1/256*a^3*erfi(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2
+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2)+3/8*
a^2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5786, 5785, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236, 5819}

$$\frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}+\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}+\frac{\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\text{Erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}-\frac{\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}}-\frac{\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}+\frac{a^3\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{\frac{x^2}{a^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (3*a^2*x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/8 + (x*(a^2 + x^2)^(3/2)*Sqrt[
ArcSinh[x/a]])/4 + (a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/(4*Sqrt[1 + x^2
/a^2]) + (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(256*Sqrt
[1 + x^2/a^2]) + (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[A
rcSinh[x/a]]])/(256*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Er
fi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
```

```
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\, dx = \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}\left(3a^2\right) \int \sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\, dx$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} - \frac{\left(3a\sqrt{\right.}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a}}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a}}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a}}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a}}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a}}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a}}{}$$

Mathematica **[A]**

time = 0.12, size = 156, normalized size = 0.50

$$\frac{a^3\sqrt{a^2+x^2}\left(-\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-4\sinh^{-1}\left(\frac{x}{a}\right)\right)-8\sqrt{2}\,\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}\left(\frac{x}{a}\right)\right)+\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\left(32\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}-8\sqrt{2}\,\Gamma\left(\frac{3}{2},2\sinh^{-1}\left(\frac{x}{a}\right)\right)-\Gamma\left(\frac{3}{2},4\sinh^{-1}\left(\frac{x}{a}\right)\right)\right)\right)}{128\sqrt{1+\frac{x^2}{a^2}}\,\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (a^3*Sqrt[a^2 + x^2]*(-(Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -4*ArcSinh[x/a]]) -
8*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -2*ArcSinh[x/a]] + Sqrt[ArcSinh[x/
a]]*(32*ArcSinh[x/a]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[x/a]] - Gamma[3
/2, 4*ArcSinh[x/a]])))/(128*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^2+x^2\right)^{\frac{3}{2}}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

```
[Out] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(3/2)*asinh(x/a)**(1/2),x)
[Out] Integral((a**2 + x**2)**(3/2)*sqrt(asinh(x/a)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
[Out] integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\, \left(a^2 + x^2\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2),x)
[Out] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2), x)
```

3.486 $\int \sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\, dx$

Optimal. Leaf size=176

$$\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}}+\frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}-\frac{a\sqrt{}}{}$$

```
[Out] 1/3*a*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/32*a*erf(2^(1/
2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-1
/32*a*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+
x^2/a^2)^(1/2)+1/2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5785, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}-\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^
(3/2))/(3*Sqrt[1 + x^2/a^2]) + (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*E
rfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,dx = \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{2\sqrt{1+\frac{x^2}{a^2}}} - \frac{\sqrt{a^2+}}{}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right.}{}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right.}{}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right.}{}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a\sqrt{a^2+x^2}\right.}{}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a\sqrt{a^2+x^2}\right.}{}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+}}{}$$

Mathematica [A]
time = 0.06, size = 110, normalized size = 0.62

$$\frac{a\sqrt{a^2+x^2}\left(16\sinh^{-1}\left(\frac{x}{a}\right)^2-3\sqrt{2}\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}\left(\frac{x}{a}\right)\right)-3\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},2\sinh^{-1}\left(\frac{x}{a}\right)\right)\right)}{48\sqrt{1+\frac{x^2}{a^2}}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (a*Sqrt[a^2 + x^2]*(16*ArcSinh[x/a]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma
[3/2, -2*ArcSinh[x/a]] - 3*Sqrt[2]*Sqrt[ArcSinh[x/a]]*Gamma[3/2, 2*ArcSinh[
x/a]]))/(48*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+x^2}\,\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

```
[Out] int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+x^2}\,\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(1/2)*asinh(x/a)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + x**2)*sqrt(asinh(x/a)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\mathrm{asinh}\left(\frac{x}{a}\right)}\,\sqrt{a^2+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2), x)
```

3.487 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

```
[Out] 2/3*a*arcsinh(x/a)^(3/2)*(1+x^2/a^2)^(1/2)/(a^2+x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {5783}

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx &= \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{\sqrt{a^2+x^2}}\\ &= \frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Maple [A]
time = 1.93, size = 34, normalized size = 0.87

method	result	size
default	$\frac{2\,\mathrm{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} a\sqrt{\frac{a^2+x^2}{a^2}}}{3\sqrt{a^2+x^2}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*arcsinh(x/a)^(3/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

Fricas [A]
time = 0.36, size = 20, normalized size = 0.51

$$\frac{2}{3}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*log((x + sqrt(a^2 + x^2))/a)^(3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(1/2),x)
[Out] Integral(sqrt(asinh(x/a))/sqrt(a**2 + x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2), x)
```

3.488 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\,\mathrm{Int}\left(\frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2+x^2}}$$

```
[Out] x*arcsinh(x/a)^(1/2)/a^2/(a^2+x^2)^(1/2)-1/2*(1+x^2/a^2)^(1/2)*Unintegrable
(x/(1+x^2/a^2)/arcsinh(x/a)^(1/2),x)/a^3/(a^2+x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(a^2*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defer[Int
][x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int\frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{2a^3\sqrt{a^2+x^2}}$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]

[Out] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2), x]

Maple [A]
time = 5.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^{2}+x^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)

[Out] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^{2}+x^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(3/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{{\left(a^2+x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2),x)
```

```
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2), x)
```

3.489 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=173

$$\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}+\frac{2x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\,\mathrm{Int}\left(\frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}},x\right)}{6a^5\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\,\mathrm{Int}\left(\frac{}{\left(1+\frac{x^2}{a^2}\right)\sqrt{s}}\right.}{3a^5\sqrt{a^2+x}}$$

```
[Out] 1/3*x*arcsinh(x/a)^(1/2)/a^2/(a^2+x^2)^(3/2)+2/3*x*arcsinh(x/a)^(1/2)/a^4/(
a^2+x^2)^(1/2)-1/6*(1+x^2/a^2)^(1/2)*Unintegrable(x/(1+x^2/a^2)^2/arcsinh(x
/a)^(1/2),x)/a^5/(a^2+x^2)^(1/2)-1/3*(1+x^2/a^2)^(1/2)*Unintegrable(x/(1+x^
2/a^2)/arcsinh(x/a)^(1/2),x)/a^5/(a^2+x^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(3*a^2*(a^2 + x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[x/a]])
/(3*a^4*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defer[Int][x/((1 + x^2/a^2)^2
*Sqrt[ArcSinh[x/a]]), x])/(6*a^5*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defe
r[Int][x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(3*a^5*Sqrt[a^2 + x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx &= \frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}+\frac{2\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx}{3a^2}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2+x^2}}\\ &= \frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}+\frac{2x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2+x^2}}-\end{aligned}$$

Mathematica [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]

[Out] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2), x]

Maple [A]
time = 7.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)

[Out] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(5/2),x)
[Out] Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{{\left(a^2+x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2),x)
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2), x)
```

3.490 $\int \left(a^2+x^2\right)^{3/2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} \, dx$

Optimal. Leaf size=433

$$-\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}}-\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}}-\frac{3(a^2+x^2)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}}+\frac{3}{8}a^2x\sqrt{a^2}$$

[Out] 1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2)+3/8*a^2*x*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)+3/20*a^3*arcsinh(x/a)^(5/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a^3*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a^3*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*erf(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*erfi(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-3/32*(a^2+x^2)^(5/2)*arcsinh(x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)-27/256*a^3*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)-9/32*a*x^2*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)

Rubi [A]
time = 0.38, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5786, 5785, 5783, 5777, 5819, 3393, 3388, 2211, 2235, 2236, 5798, 5791}

$$\frac{3}{8}a^2x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x(a^2+x^2)^{3/2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{3(a^2+x^2)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\text{Erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3a^3\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{\frac{x^2}{a^2}+1}}-\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{\frac{x^2}{a^2}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]

[Out] (-27*a^3*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(256*Sqrt[1 + x^2/a^2]) - (9*a*x^2*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(32*Sqrt[1 + x^2/a^2]) - (3*(a^2 + x^2)^(5/2)*Sqrt[ArcSinh[x/a]])/(32*a*Sqrt[1 + x^2/a^2]) + (3*a^2*x*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/8 + (x*(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2))/4 + (3*a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(20*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(2048*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[ArcSinh[x/a]]])/(2048*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
```

```
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(a^2+x^2\right)^{3/2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx = \frac{1}{4}x\left(a^2+x^2\right)^{3/2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}\left(3a^2\right) \int \sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx$$

$$= -\frac{3(a^2+x^2)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x(a$$

$$= -\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+x^2)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x$$

$$= -\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+x^2)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x$$

$$= \frac{9a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+}{}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a}{}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a}{}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a}{}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a}{}$$

Mathematica [A]
time = 0.20, size = 210, normalized size = 0.48

$$\frac{a^3\sqrt{a^2+x^2}\left(384\sinh^{-1}\left(\frac{x}{a}\right)^3-480\sinh^{-1}\left(\frac{x}{a}\right)\cosh\left(2\sinh^{-1}\left(\frac{x}{a}\right)\right)+60\sqrt{2\pi}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+60\sqrt{2\pi}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+5\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\Gamma\left(\frac{5}{2},-4\sinh^{-1}\left(\frac{x}{a}\right)\right)-5\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\Gamma\left(\frac{5}{2},4\sinh^{-1}\left(\frac{x}{a}\right)\right)+640\sinh^{-1}\left(\frac{x}{a}\right)^2\sinh\left(2\sinh^{-1}\left(\frac{x}{a}\right)\right)\right)}{2560\sqrt{1+\frac{x^2}{a^2}}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (a^3*Sqrt[a^2 + x^2]*(384*ArcSinh[x/a]^3 - 480*ArcSinh[x/a]*Cosh[2*ArcSinh[
x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] +
60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 5*Sqrt[
-ArcSinh[x/a]]*Gamma[5/2, -4*ArcSinh[x/a]] - 5*Sqrt[ArcSinh[x/a]]*Gamma[5/2
, 4*ArcSinh[x/a]] + 640*ArcSinh[x/a]^2*Sinh[2*ArcSinh[x/a]]))/(2560*Sqrt[1
+ x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

```
[Out] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(3/2)*asinh(x/a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}\left(a^2+x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2),x)
```

```
[Out] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2), x)
```

3.491 $\int \sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx$

Optimal. Leaf size=259

$$-\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}}-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}+\frac{a\sqrt{a^2+x^2}\ \mathrm{si}}{5\sqrt{1+}}$$

```
[Out] 1/2*x*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)+1/5*a*arcsinh(x/a)^(5/2)*(a^2+x^2)
^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi
^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a*erfi(2^(1/2)*arcsinh(x/a)^
(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-3/16*a*(a^2+x^2)^
(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)-3/8*x^2*(a^2+x^2)^(1/2)*arcsinh(
x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5785, 5783, 5777, 5819, 3393, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{\frac{x^2}{a^2}+1}}-\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{\frac{x^2}{a^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (-3*a*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(16*Sqrt[1 + x^2/a^2]) - (3*x^2*S
qrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(8*a*Sqrt[1 + x^2/a^2]) + (x*Sqrt[a^2 +
x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt
[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(64*Sqrt[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}\,dx=\frac{1}{2}x\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}+\frac{\sqrt{a^2+x^2}\int\frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{2\sqrt{1+\frac{x^2}{a^2}}}-\frac{\left(3\sqrt{a^2+x^2}\right.}{}$$

$$=-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}+\frac{a\sqrt{a^2+}}{5}$$

$$=-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}+\frac{a\sqrt{a^2+}}{5}$$

$$=-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}+\frac{a\sqrt{a^2+}}{5}$$

$$=-\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}}-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2}$$

$$=-\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}}-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2}$$

$$=-\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}}-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2}$$

$$=-\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}}-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2}$$

Mathematica [A]
time = 0.17, size = 133, normalized size = 0.51

$$\frac{a\sqrt{a^2+x^2}\left(15\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+8\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\left(16\sinh^{-1}\left(\frac{x}{a}\right)^2-15\cosh\left(2\sinh^{-1}\left(\frac{x}{a}\right)\right)+20\sinh^{-1}\left(\frac{x}{a}\right)\sinh\left(2\sinh^{-1}\left(\frac{x}{a}\right)\right)\right)\right)}{640\sqrt{1+\frac{x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (a*Sqrt[a^2 + x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 15*Sqrt
[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 8*Sqrt[ArcSinh[x/a]]*(16*ArcSinh[
x/a]^2 - 15*Cosh[2*ArcSinh[x/a]] + 20*ArcSinh[x/a]*Sinh[2*ArcSinh[x/a]])))/
(640*Sqrt[1 + x^2/a^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\sqrt{a^2+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

```
[Out] int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + x^2}\, \operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)*(a**2+x**2)**(1/2),x)
[Out] Integral(sqrt(a**2 + x**2)*asinh(x/a)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}\left(\frac{x}{a}\right)^{3/2} \sqrt{a^2 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2), x)
```

3.492 $\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

```
[Out] 2/5*a*arcsinh(x/a)^(5/2)*(1+x^2/a^2)^(1/2)/(a^2+x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {5783}

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
```

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx &= \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{\sqrt{a^2+x^2}}\\
&= \frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Maple [A]
time = 1.84, size = 34, normalized size = 0.87

method	result	size
default	$\frac{2\,\mathrm{arcsinh}\left(\frac{x}{a}\right)^{\frac{5}{2}} a\sqrt{\frac{a^2+x^2}{a^2}}}{5\sqrt{a^2+x^2}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arcsinh(x/a)^(5/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

Fricas [A]
time = 0.36, size = 20, normalized size = 0.51

$$\frac{2}{5}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2/5*log((x + sqrt(a^2 + x^2))/a)^(5/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] Integral(asinh(x/a)**(3/2)/sqrt(a**2 + x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\operatorname{asinh}\left(\frac{x}{a}\right)}^{3/2}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2),x)
```

```
[Out] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2), x)
```

3.493 $\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=84

$$\frac{x\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{1+\frac{x^2}{a^2}}\,\text{Int}\left(\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{1+\frac{x^2}{a^2}},x\right)}{2a^3\sqrt{a^2+x^2}}$$

```
[Out] x*arcsinh(x/a)^(3/2)/a^2/(a^2+x^2)^(1/2)-3/2*(1+x^2/a^2)^(1/2)*Unintegrable
(x*arcsinh(x/a)^(1/2)/(1+x^2/a^2),x)/a^3/(a^2+x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[x/a]^(3/2))/(a^2*Sqrt[a^2 + x^2]) - (3*Sqrt[1 + x^2/a^2]*Defer[I
nt][(x*Sqrt[ArcSinh[x/a]])/(1 + x^2/a^2), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx=\frac{x\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{\left(3\sqrt{1+\frac{x^2}{a^2}}\right)\int \frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{1+\frac{x^2}{a^2}} \, dx}{2a^3\sqrt{a^2+x^2}}$$

Mathematica [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
[Out] Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2), x]
```

Maple [A]
time = 4.87, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^{2}+x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
[Out] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}{\left(\frac{x}{a} \right)}}{\left(a^{2}+x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(3/2),x)
[Out] Integral(asinh(x/a)**(3/2)/(a**2 + x**2)**(3/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
[Out] integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{asinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2),x)
[Out] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

3.494 $\int \frac{x}{\sqrt{1+x^2}\sqrt{\sinh^{-1}(x)}}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\sqrt{\pi}\,\text{Erf}\left(\sqrt{\sinh^{-1}(x)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\sinh^{-1}(x)}\right)$$

```
[Out] -1/2*erf(arcsinh(x)^(1/2))*Pi^(1/2)+1/2*erfi(arcsinh(x)^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5819, 3389, 2211, 2235, 2236}

$$\frac{1}{2}\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\sinh^{-1}(x)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{Erf}\left(\sqrt{\sinh^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
```

```
[Out] -1/2*(Sqrt[Pi]*Erf[Sqrt[ArcSinh[x]]]) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[x]]])/2
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1+x^2}\sqrt{\sinh^{-1}(x)}}\,dx &= \text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)\\
&= -\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(x)}\right)+\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(x)}\right)\\
&= -\frac{1}{2}\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(x)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(x)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 1.03

$$\frac{1}{2}\left(\frac{\sqrt{-\sinh^{-1}(x)}\,\Gamma\left(\frac{1}{2},-\sinh^{-1}(x)\right)}{\sqrt{\sinh^{-1}(x)}}+\Gamma\left(\frac{1}{2},\sinh^{-1}(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
[Out] ((Sqrt[-ArcSinh[x]]*Gamma[1/2, -ArcSinh[x]])/Sqrt[ArcSinh[x]] + Gamma[1/2,
ArcSinh[x]])/2
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\,\sqrt{\operatorname{arcsinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

```
[Out] int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 + 1}\,\sqrt{\operatorname{asinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)**(1/2)/asinh(x)**(1/2),x)
[Out] Integral(x/(sqrt(x**2 + 1)*sqrt(asinh(x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{\operatorname{asinh}(x)}\,\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(asinh(x)^(1/2)*(x^2 + 1)^(1/2)),x)
[Out] int(x/(asinh(x)^(1/2)*(x^2 + 1)^(1/2)), x)
```

3.495 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=396

$$\frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}}+\frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{1+a^2x^2}}+\frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\right.}{64a\sqrt{1+a^2x^2}}$$

[Out] 1/384*c^2*erf(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/384*c^2*erfi(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/128*c^2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/128*c^2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/64*c^2*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/64*c^2*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+5/8*c^2*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5791, 3393, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{6}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{6}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{6}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{6}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{5c^2\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]

[Out] (5*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\int \frac{(1+a^2x^2)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^6(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5}{16\sqrt{x}}+\frac{15\cosh(2x)}{32\sqrt{x}}+\frac{3\cosh(4x)}{16\sqrt{x}}+\frac{\cosh(6x)}{32\sqrt{x}}\right)\,dx, x, \sinh^{-1}(a\right.}{a\sqrt{1+a^2x^2}}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(6x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right.}{32a\sqrt{1+a^2x^2}}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\sinh^{-1}(ax}\right.}{32a\sqrt{1+a^2x^2}}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{1+a^2x^2}} + \frac{15c^2\sqrt{}}{}$$

Mathematica [A]
time = 0.21, size = 197, normalized size = 0.50

$$\frac{c^2\sqrt{c+a^2cx^2}\left(240\sinh^{-1}(ax)+\sqrt{6}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\tfrac{1}{2},-6\sinh^{-1}(ax)\right)+18\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\tfrac{1}{2},-4\sinh^{-1}(ax)\right)+45\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\tfrac{1}{2},-2\sinh^{-1}(ax)\right)-45\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\tfrac{1}{2},2\sinh^{-1}(ax)\right)-18\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\tfrac{1}{2},4\sinh^{-1}(ax)\right)-\sqrt{6}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\tfrac{1}{2},6\sinh^{-1}(ax)\right)\right)}{384a\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(240*ArcSinh[a*x] + Sqrt[6]*Sqrt[-ArcSinh[a*x]]*Ga
mma[1/2, -6*ArcSinh[a*x]] + 18*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*
x]] + 45*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - 45*Sqrt[
2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] - 18*Sqrt[ArcSinh[a*x]]*Ga
mma[1/2, 4*ArcSinh[a*x]] - Sqrt[6]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[
a*x]]))/(384*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2 c\,x^2+c)^{\frac{5}{2}}}{\sqrt{\operatorname{arcsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
[Out] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^{5/2}}{\sqrt{\mathrm{asinh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(1/2),x)
```

```
[Out] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(1/2), x)
```

3.496 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=264

$$\frac{3c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}}+\frac{c\sqrt{\pi}\,\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{1+a^2x^2}}+\frac{c\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sin\right.}{4a\sqrt{1+a^2x^2}}$$

```
[Out] 1/8*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/
a/(a^2*x^2+1)^(1/2)+1/8*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)
*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erf(2*arcsinh(a*x)^(1/2))*P
i^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erfi(2*arcsinh(a*x)^
(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/4*c*(a^2*c*x^2+c)
^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5791, 3393, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{3c\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (3*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*S
qrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(32*a*Sqrt[1 + a^2*x
^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(
4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSin
h[a*x]]])/(32*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\left(c\sqrt{c+a^2cx^2}\right)\int \frac{(1+a^2x^2)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\frac{\cosh(2x)}{2\sqrt{x}}+\frac{\cosh(4x)}{8\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a\sqrt{1+a^2x^2}}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a\sqrt{1+a^2x^2}}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right.}{8a\sqrt{1+a^2x^2}}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\ldots}{}$$

Mathematica [A]
time = 0.13, size = 141, normalized size = 0.53

$$\frac{c\sqrt{c+a^2cx^2}\left(\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\sinh^{-1}(ax)\right)+4\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+\sqrt{\sinh^{-1}(ax)}\left(24\sqrt{\sinh^{-1}(ax)}-4\sqrt{2}\,\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)-\Gamma\left(\frac{1}{2},4\sinh^{-1}(ax)\right)\right)\right)}{32a\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] + 4
*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[a*x
]]*(24*Sqrt[ArcSinh[a*x]] - 4*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] - Gamma[1/
2, 4*ArcSinh[a*x]])))/(32*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/sqrt(asinh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\sqrt{\mathrm{asinh}\left(a\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(1/2), x)
```

3.497 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=156

$$\frac{\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2cx^2}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{1+a^2x^2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\sinh\right.}}{4a\sqrt{1+a^2x^2}}$$

```
[Out] 1/8*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/
(a^2*x^2+1)^(1/2)+1/8*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
/a/(a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5791, 3393, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\,\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^
2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*
a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\sqrt{c+a^2cx^2}\int \frac{\sqrt{1+a^2x^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cosh(2x)}{2\sqrt{x}}\right)\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+}}{}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a\sqrt{1+a^2x^2}} + -$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c}}{}$$

Mathematica [A]
time = 0.07, size = 101, normalized size = 0.65

$$\frac{\sqrt{c(1+a^2x^2)}\left(8\sinh^{-1}(ax)+\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)-\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)\right)}{8a\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(8*ArcSinh[a*x] + Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[
1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x
]]))/(8*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\sqrt{\text{arcsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(asinh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{asinh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(1/2), x)
```

3.498 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{c+a^2cx^2}}$$

```
[Out] 2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$\frac{2\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\frac{2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 1.27, size = 36, normalized size = 0.90

method	result	size
default	$\frac{2\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{a^2x^2+1}}{a\sqrt{c\,(a^2x^2+1)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,(a^2x^2+1)}\,\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\operatorname{asinh}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.499 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx=\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

Maple [A]
time = 5.11, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{3}{2}}\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\operatorname{asinh}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.500 $\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx=\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

Maple [A]
time = 5.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*sqrt(asinh(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\operatorname{asinh}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.501 $\int \frac{(c+a^2cx^2)^{5/2}}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{3c^2\sqrt{\pi}\,\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{1+a^2x^2}}-\frac{15c^2\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\right.}}{16a\sqrt{1+a^2x^2}}$$

```
[Out] -15/32*c^2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(
1/2)/a/(a^2*x^2+1)^(1/2)+15/32*c^2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)
*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/8*c^2*erf(2*arcsinh(a*x
)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/8*c^2*erfi(2*ar
csinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/32*c^2
*erf(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^
2*x^2+1)^(1/2)+1/32*c^2*erfi(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*c*x^2+c)^(5/2)*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5790, 5819, 5556, 3389, 2211, 2235, 2236}

$$-\frac{3\sqrt{\pi}\,c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{3\pi}{2}}\,c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{6}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}\,c^2\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{3\pi}{2}}\,c^2\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{6}\,\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\,(a^2cx^2+c)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(5/2))/(a*Sqrt[ArcSinh[a*x]]) - (3*c^
2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2
*x^2]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2*x^2]) + (1
5*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*
a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]
*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12ac^2\sqrt{c+a^2cx^2}\right)\int \frac{x(1+a^2x^2)^2}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}} + \frac{\text{si}}{32}\right.\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right.}{8a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{3c^2\sqrt{\pi}\,\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{1+a^2x^2}} - \frac{15c^2\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 0.71, size = 399, normalized size = 1.02

$$\frac{c^2e^{-6\sinh^{-1}(ax)}\sqrt{c+a^2cx^2}\left(-1-6e^{2\sinh^{-1}(ax)}+e^{4\sinh^{-1}(ax)}-52e^{6\sinh^{-1}(ax)}+e^{8\sinh^{-1}(ax)}-6e^{10\sinh^{-1}(ax)}-e^{12\sinh^{-1}(ax)}-64a^2e^{6\sinh^{-1}(ax)}x^2-16e^{6\sinh^{-1}(ax)}\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+16e^{6\sinh^{-1}(ax)}\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+\sqrt{6}e^{6\sinh^{-1}(ax)}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-6\sinh^{-1}(ax)\right)+12e^{6\sinh^{-1}(ax)}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\sinh^{-1}(ax)\right)-\sqrt{2}e^{6\sinh^{-1}(ax)}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)-\sqrt{2}e^{6\sinh^{-1}(ax)}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)+12e^{6\sinh^{-1}(ax)}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},4\sinh^{-1}(ax)\right)+\sqrt{6}e^{6\sinh^{-1}(ax)}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},6\sinh^{-1}(ax)\right)\right)}{32a\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-1 - 6*E^(2*ArcSinh[a*x]) + E^(4*ArcSinh[a*x]) -
52*E^(6*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]) - 6*E^(10*ArcSinh[a*x]) - E^(12*
ArcSinh[a*x]) - 64*a^2*E^(6*ArcSinh[a*x])*x^2 - 16*E^(6*ArcSinh[a*x])*Sqrt[
2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 16*E^(6*ArcSinh[
```

```
a*x])*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + Sqrt
[6]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -6*ArcSinh[a*x]] + 12
*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - Sqrt[
2]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqr
t[2]*E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] + 12*
E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]] + Sqrt[6]*
E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[a*x]]))/(32*a*E^
(6*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2 c\, x^2 + c)^{\frac{5}{2}}}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
[Out] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{asinh}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(3/2),x)
```

```
[Out] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(3/2), x)
```

3.502 $\int \frac{(c+a^2cx^2)^{3/2}}{\sinh^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=256

$$-\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{c\sqrt{\pi}\,\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{1+a^2x^2}}-\frac{c\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] -1/2*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)
/a/(a^2*x^2+1)^(1/2)+1/2*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2
)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/4*c*erf(2*arcsinh(a*x)^(1/2))*P
i^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/4*c*erfi(2*arcsinh(a*x)^(
1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*c*x^2+c)^(3/2
)*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5790, 5819, 5556, 3389, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\,(a^2cx^2+c)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(a*Sqrt[ArcSinh[a*x]]) - (c*Sq
rt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2
]) - (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*
Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*
x]]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8ac\sqrt{c+a^2cx^2}\right)\int \frac{x(1+a^2x^2)}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-}\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}\right)dx,\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a\sqrt{1+a^2x^2}} +$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c}}{}$$

Mathematica [A]

time = 0.25, size = 225, normalized size = 0.88

$$-\frac{ce^{-4\sinh^{-1}(ax)}\sqrt{c+a^2cx^2}\left(1+14e^{4\sinh^{-1}(ax)}+e^{8\sinh^{-1}(ax)}+16a^2e^{4\sinh^{-1}(ax)}x^2+4e^{4\sinh^{-1}(ax)}\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-4e^{4\sinh^{-1}(ax)}\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-2e^{4\sinh^{-1}(ax)}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\sinh^{-1}(ax)\right)-2e^{4\sinh^{-1}(ax)}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},4\sinh^{-1}(ax)\right)\right)}{8a\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] -1/8*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x])
 + 16*a^2*E^(4*ArcSinh[a*x])*x^2 + 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi]*Sqrt[Arc
Sinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi
]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 2*E^(4*ArcSinh[a*x]
```

```
)*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - 2*E^(4*ArcSinh[a*x])*Sq
rt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]]))/(a*E^(4*ArcSinh[a*x])*Sqrt[1
+ a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{\operatorname{arcsinh}\left(a x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{3}{2}}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{{\mathrm{asinh}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(3/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(3/2), x)
```

3.503 $\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] -1/2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a
/(a^2*x^2+1)^(1/2)+1/2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a
^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)^(1/
2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$,
Rules used = {5790, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[P
i/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x
^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a
*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4a\sqrt{c+a^2cx^2}\right)\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+}}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c}}{}$$

Mathematica [A]
time = 0.13, size = 115, normalized size = 0.76

$$-\frac{\sqrt{c+a^2cx^2}\left(4+4a^2x^2+\sqrt{2\pi}\,\sqrt{\sinh^{-1}(ax)}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)-\sqrt{2\pi}\,\sqrt{\sinh^{-1}(ax)}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\sinh^{-1}(ax)}\right)\right)}{2a\sqrt{1+a^2x^2}\,\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

```
[Out] -1/2*(Sqrt[c + a^2*c*x^2]*(4 + 4*a^2*x^2 + Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*
Sqrt[ArcSinh[a*x]]]))/(a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 c\,x^2+c}}{\operatorname{arcsinh}\left(a x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2} x^{2}+1\right)}}{\operatorname{asinh}^{\frac{3}{2}}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(3/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{{\mathrm{asinh}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(3/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(3/2), x)
```

3.504 $\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$-\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [A]
time = 1.23, size = 36, normalized size = 0.90

method	result	size
default	$-\frac{2\sqrt{a^{2} x^{2}+1}}{\sqrt{\operatorname{arcsinh}\left(a x\right)}\, a \sqrt{c \left(a^{2} x^{2}+1\right)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

Fricas [A]
time = 0.34, size = 57, normalized size = 1.42

$$-\frac{2 \sqrt{a^{2} c x^{2} + c} \sqrt{a^{2} x^{2} + 1}}{{\left(a^{3} c x^{2} + a c\right)} \sqrt{\log\left(a x + \sqrt{a^{2} x^{2} + 1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*sqrt(log(a*x + 
sqrt(a^2*x^2 + 1))))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c \left(a^{2} x^{2} + 1\right)} \operatorname{asinh}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{asinh}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.505 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{1+a^2x^2}}{a\,(c+a^2cx^2)^{3/2}\sqrt{\sinh^{-1}(ax)}}-\frac{4a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)^2\sqrt{\sinh^{-1}(ax)}},x\right)}{c\sqrt{c+a^2cx^2}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2)-4*a*(a^2*x^2+
1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/arcsinh(a*x)^(1/2),x)/c/(a^2*c*x^2+c)
^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]) - (4*a*
Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(c
*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a\,(c+a^2cx^2)^{3/2}\sqrt{\sinh^{-1}(ax)}}-\frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^2\sqrt{\sinh^{-1}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)), x]
```

Maple [A]
time = 4.89, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \operatorname{asinh}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.506 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{1+a^2x^2}}{a\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}-\frac{8a\sqrt{1+a^2x^2}\,\mathrm{Int}\left(\frac{x}{(1+a^2x^2)^3\sqrt{\sinh^{-1}(ax)}},x\right)}{c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2)-8*a*(a^2*x^2+
1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^3/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+
c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]) - (8*a*
Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^3*Sqrt[ArcSinh[a*x]]), x])/(c
^2*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}-\frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^3\sqrt{\sinh}}}{c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)), x]
```

Maple [A]
time = 5.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.507 $\int \frac{(c+a^2cx^2)^{3/2}}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}}+\frac{2c\sqrt{\pi}\,\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{1+a^2x^2}}+2c\sqrt{}$$

```
[Out] -2/3*(a^2*c*x^2+c)^(3/2)*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)+2/3*c*erf(2
*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+2/3*c
*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2
)+2/3*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2
)/a/(a^2*x^2+1)^(1/2)+2/3*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/
2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-16/3*c*x*(a^2*x^2+1)*(a^2*c*x^2+
c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5790, 5814, 5791, 3393, 3388, 2211, 2235, 2236, 5819, 5556}

$$\frac{2\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\,(a^2cx^2+c)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{16cx(a^2x^2+1)\sqrt{a^2cx^2+c}}{3\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(3*a*ArcSinh[a*x]^(3/2)) - (16
*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*c*Sqrt[
Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
+ (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh
[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{\left(8ac\sqrt{c+a^2cx^2}\right)\int \frac{x(1+a^2x^2)}{\sinh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\int}{3\sqrt{1+}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\mathrm{S}}{3}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\mathrm{S}}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\mathrm{Su}}{3}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4c\sqrt{c+a^2cx^2}\right)\mathrm{Su}}{3a}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\mathrm{Su}}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,(c+a^2cx^2)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\mathrm{er}}{3a\sqrt{1-}}$$

Mathematica [A]

time = 0.28, size = 262, normalized size = 0.89

$$-\frac{ce^{-4\sinh^{-1}(ax)}\sqrt{c+a^2cx^2}\left(1+14e^{4\sinh^{-1}(ax)}+e^{8\sinh^{-1}(ax)}+16a^2e^{4\sinh^{-1}(ax)}x^2-8\sinh^{-1}(ax)+8e^{8\sinh^{-1}(ax)}\sinh^{-1}(ax)+64ae^{4\sinh^{-1}(ax)}x\sqrt{1+a^2x^2}\sinh^{-1}(ax)+16e^{4\sinh^{-1}(ax)}\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-4\sinh^{-1}(ax)\right)+16\sqrt{2}e^{4\sinh^{-1}(ax)}\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+16\sqrt{2}e^{4\sinh^{-1}(ax)}\sinh^{-1}(ax)^{3/2}\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)+16e^{4\sinh^{-1}(ax)}\sinh^{-1}(ax)^{3/2}\Gamma\left(\frac{1}{2},4\sinh^{-1}(ax)\right)\right)}{24a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

```
[Out] -1/24*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]
) + 16*a^2*E^(4*ArcSinh[a*x])*x^2 - 8*ArcSinh[a*x] + 8*E^(8*ArcSinh[a*x])*A
```

```
rcSinh[a*x] + 64*a*E^(4*ArcSinh[a*x])*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + 16
*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]] + 16*
Sqrt[2]*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]
] + 16*Sqrt[2]*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a
*x]] + 16*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]])
)/(a*E^(4*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\operatorname{arcsinh}\left(a x \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{5}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{{\mathrm{asinh}\left(a\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2), x)
```

3.508 $\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}}+\frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{1+a^2x^2}}+\frac{2\sqrt{2\pi}\sqrt{c+a^2}}{}$$

```
[Out] 2/3*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/
(a^2*x^2+1)^(1/2)+2/3*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2/3*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)^(1
/2)/a/arcsinh(a*x)^(3/2)-8/3*x*(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5790, 5778, 3388, 2211, 2235, 2236}

$$\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}-\frac{8x\sqrt{a^2cx^2+c}}{3\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x*
Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[c + a^2*c*
x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*Sqrt[2*P
i]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*
x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{\left(4a\sqrt{c+a^2cx^2}\right)\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2\sqrt{2\pi}\,\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\,\sqrt{\sinh^{-1}}\right.}{3a\sqrt{1+a^2x^2}}$$

Mathematica [A]
time = 0.11, size = 122, normalized size = 0.67

$$-\frac{2\sqrt{c+a^2cx^2}\left(1+a^2x^2+4ax\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)+\sqrt{2}\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+\sqrt{2}\,\sinh^{-1}(ax)^{3/2}\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)\right)}{3a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2]*(1 + a^2*x^2 + 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
 + Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*ArcS
inh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]))/(3*a*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{asinh}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(5/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{{\mathrm{asinh}\left(a\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2), x)
```

3.509 $\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}$$

[Out] -2/3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]

[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$-\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))
```

Maple [A]
time = 1.72, size = 36, normalized size = 0.86

method	result	size
default	$-\frac{2\sqrt{a^{2}x^{2}+1}}{3\,\mathrm{arcsinh}(ax)^{\frac{3}{2}}\,a\sqrt{c\left(a^{2}x^{2}+1\right)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

Fricas [A]
time = 0.36, size = 57, normalized size = 1.36

$$-\frac{2\sqrt{a^{2}cx^{2}+c}\,\sqrt{a^{2}x^{2}+1}}{3\left(a^{3}cx^{2}+ac\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*log(a*x + sqr
t(a^2*x^2 + 1))^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{asinh}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.510 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{2\sqrt{1+a^2x^2}}{3a\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}-\frac{4a\sqrt{1+a^2x^2}\,\mathrm{Int}\left(\frac{x}{(1+a^2x^2)^2\sinh^{-1}(ax)^{3/2}},x\right)}{3c\sqrt{c+a^2cx^2}}$$

[Out] -2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2)-4/3*a*(a^2*x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/arcsinh(a*x)^(3/2),x)/c/(a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]

[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)) - (4*a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^2*ArcSinh[a*x]^(3/2)), x])/(3*c*Sqrt[c + a^2*c*x^2])

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3a\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}-\frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^2\sinh^{-1}(ax}}{3c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]

[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)), x]

Maple [A]
time = 4.94, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{asinh}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.511 $\int \frac{1}{(c+a^2cx^2)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{2\sqrt{1+a^2x^2}}{3a\,(c+a^2cx^2)^{5/2}\sinh^{-1}(ax)^{3/2}}-\frac{8a\sqrt{1+a^2x^2}\,\mathrm{Int}\left(\frac{x}{(1+a^2x^2)^3\sinh^{-1}(ax)^{3/2}},x\right)}{3c^2\sqrt{c+a^2cx^2}}$$

[Out] -2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2)-8/3*a*(a^2*x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^3/arcsinh(a*x)^(3/2),x)/c^2/(a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]

[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)) - (8*a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^3*ArcSinh[a*x]^(3/2)), x])/(3*c^2*Sqrt[c + a^2*c*x^2])

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3a\,(c+a^2cx^2)^{5/2}\sinh^{-1}(ax)^{3/2}}-\frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^3\sinh^{-1}(a}{3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]

[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)), x]

Maple [A]
time = 5.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{5}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{asinh}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.512 $\int x^2 \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=235

$$-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-2(3+n)}e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(1-}{c^3\sqrt{1+c^2x^2}}$$

```
[Out] -1/8*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^2+1)^(
1/2)+(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/(2^(6+2*n))/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1
/2)-exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*
d*x^2+d)^(1/2)/(2^(6+2*n))/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5819, 5556, 3388, 2212}

$$\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{8bc^3(n+1)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -1/8*(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(b*c^3*(1 + n)*Sqrt
[1 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (
-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^
2]*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c
^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx &= \frac{\sqrt{d+c^2dx^2}\int x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}\\
&= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\sinh^2(x)\,dx, x, \sinh^{-1}(c\right.}{c^3\sqrt{1+c^2x^2}}\\
&= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\left(-\frac{1}{8}(a+bx)^n+\frac{1}{8}(a+bx)^n\cosh(4x)\right)\,dx,\right.}{c^3\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int (a\right.}{8c}\\
&= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-}\right.}{16c^3}\\
&= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{4^{-3-n}e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\,(}{}\end{aligned}$$

Mathematica [A]
time = 0.68, size = 170, normalized size = 0.72

$$\frac{d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{8(a+b\sinh^{-1}(cx))}{b+bn}+4^{-n}e^{-\frac{4a}{b}}\left(-\frac{(a+b\sinh^{-1}(cx))^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{4(a+b\sinh^{-1}(cx))}{b}\right)-e^{\frac{8a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{4(a+b\sinh^{-1}(cx))}{b}\right)\right)\right)}{64c^3\sqrt{d\left(1+c^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b +
 b*n) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] -
 E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c
*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c^3*Sqr
t[d*(1 + c^2*x^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2}(a+b\,\mathrm{arcsinh}\,(cx))^{n}\sqrt{c^{2}d\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\,\mathrm{asinh}\,(cx)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

3.513 $\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=355

$$\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{1+c^2x^2}}+\frac{e^{-\frac{a}{b}}\sqrt{d+c^2dx^2}}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*d
*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+
1/8*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/
2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/8*exp(a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^
2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/8*3^(-1-n)*exp(3*a/b)*(a+b
*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/
(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5819, 5556, 3389, 2212}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}+\frac{e^{-\frac{a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}+\frac{e^{a/b}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}+\frac{3^{-n-1}e^{\frac{3a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (3^(-1 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcSinh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSi
nh[c*x])/b))^n) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
-((a + b*ArcSinh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcS
inh[c*x])/b))^n) + (E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (a + b*ArcSinh[c*x])/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[
c*x])/b)^n) + (3^(-1 - n)*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((
a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx = \frac{\sqrt{d+c^2dx^2}\int x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh(x)\,dx,x,\sinh^{-1}(cx)\right.}{c^2\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\left(\frac{1}{4}(a+bx)^n\sinh(x)+\frac{1}{4}(a+bx)^n\sinh(3x)\right)\right.}{c^2\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\sinh(x)\,dx,x,\sinh^{-1}(cx)\right)}{4c^2\sqrt{1+c^2x^2}}+\frac{\sqrt{d}}{}$$
$$= -\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\sinh^{-1}(cx)\right)}{8c^2\sqrt{1+c^2x^2}}-\frac{\sqrt{d+}}{}$$
$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma}{8c^2\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.69, size = 229, normalized size = 0.65

$$\frac{de^{-\frac{3a}{b}}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(3e^{\frac{4a}{b}}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^{-n}\Gamma\left(1+n,\frac{a}{b}+\sinh^{-1}(cx)\right)+\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\left(3^{-n}\Gamma\left(1+n,-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+3e^{\frac{2a}{b}}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(cx)}{b}\right)+3^{-n}e^{\frac{6a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{2n}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\right)\right)}{24c^2\sqrt{d\left(1+c^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((3*E^((4*a)/b)*Gamma[1 + n, a/
b + ArcSinh[c*x]])/(a/b + ArcSinh[c*x])^n + (Gamma[1 + n, (-3*(a + b*ArcSin
h[c*x]))/b]/3^n + 3*E^((2*a)/b)*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + (
E^((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSin
h[c*x]))/b])/(3^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n))/(-((a + b*ArcSinh[c*x
])/b))^n))/(24*c^2*E^((3*a)/b)*Sqrt[d*(1 + c^2*x^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x(a+b\,\mathrm{arcsinh}\,(cx))^{n}\,\sqrt{c^{2}d\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\,\mathrm{asinh}\,(cx)\right)^{n}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

3.514 $\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=235

$$\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(1+n,}{c\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)^(1/2
)+2^(-3-n)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b)*(c^2*d*x
^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-
3-n)*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*x))/b)*(c^2
*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.160$, Rules used = {5791, 3393, 3388, 2212}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{2bc(n+1)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(2*b*c*(1 + n)*Sqrt[1 +
c^2*x^2]) + (2^(-3 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
 b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2
*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\sqrt{d+c^2dx^2}\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$
$$=\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\,dx,x,\sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$
$$=\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\left(\frac{1}{2}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)\right)\,dx,x,\text{si}\right.}{c\sqrt{1+c^2x^2}}$$
$$=\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}}+\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+b\right.}{2c\sqrt{1}}$$
$$=\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}}+\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-2x}(a\right.}{4c\sqrt{1+}}$$
$$=\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}\left(a+\right.}{}$$

Mathematica [A]
time = 0.43, size = 160, normalized size = 0.68

$$\frac{d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{4a+4b\sinh^{-1}(cx)}{b+bn}+2^{-n}e^{-\frac{2a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)-2^{-n}e^{\frac{2a}{b}}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\sinh^{-1}(cx))}{b}\right)\right)}{8c\sqrt{d\left(1+c^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((4*a + 4*b*ArcSinh[c*x])/(b +
b*n) + Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b
```

```
*ArcSinh[c*x])/b))^n) - (E^((2*a)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/
b])/(2^n*(a/b + ArcSinh[c*x])^n)))/(8*c*Sqrt[d*(1 + c^2*x^2)])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arcsinh}\,(cx))^{n}\sqrt{c^{2}d\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\,(c^{2}x^{2}+1)}\,(a+b\,\mathrm{asinh}\,(cx))^{n}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

3.515 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=199

$$\frac{de^{-\frac{a}{b}}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}}+\frac{de^{a/b}\sqrt{1+c^2x^2}\left(a+b\sin\right.}{}$$

```
[Out] 1/2*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/
2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/2*d*exp(a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a
+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d*Unintegrable((a+b*arcsinh(c*x)
)^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c
*x])/b)])/(2*E^(a/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (
d*E^(a/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcS
inh[c*x])/b])/(2*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + d*Defer[
Int][(a + b*ArcSinh[c*x])^n/(x*Sqrt[d + c^2*d*x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx=\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
[Out] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^n \sqrt{c^2 d\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,(c^2x^2+1)}\,(a+b\,\mathrm{asinh}\,(cx))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x, x)
```

3.516 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=84

$$\frac{cd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{d+c^2dx^2}}+d\mathrm{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x^2\sqrt{d+c^2dx^2}},x\right)$$

```
[Out] c*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)^(1/2)+
d*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] (c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(b*(1 + n)*Sqrt[d + c^
2*d*x^2]) + d*Defer[Int][(a + b*ArcSinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x^2]),
x]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx=\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^{n} \sqrt{c^{2} d\,x^{2}+d}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima
")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas
")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,(c^{2}x^{2}+1)}\,(a+b\,\mathrm{asinh}\,(cx))^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x**2,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2 + d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

3.517 $\int x^2(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=616

$$-\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}}{c^3\sqrt{1+c^2x^2}}$$

```
[Out] -1/16*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^2+1
)^(1/2)+2^(-7-n)*3^(-1-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*arcsinh(
c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^
2*x^2+1)^(1/2)+2^(-7-2*n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(
c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^
2*x^2+1)^(1/2)-2^(-7-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*
x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*
x^2+1)^(1/2)+2^(-7-n)*d*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*ar
csinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+
1)^(1/2)-2^(-7-2*n)*d*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)
^(1/2)-2^(-7-n)*3^(-1-n)*d*exp(6*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,6*(a+b
*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x
^2+1)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {5819, 5556, 3388, 2212}

$$\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}+\frac{d2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{d2^{-n-7}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}+\frac{d2^{-n-7}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{d2^{-2n-7}e^{\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{d2^{-n-7}3^{-n-1}e^{\frac{6a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{16bc^3(n+1)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -1/16*(d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(b*c^3*(1 + n)*S
qrt[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((6*a)/b)*Sq
rt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - 2*n)*d*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])
/(c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-7
 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*
ArcSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x
])/b))^n) + (2^(-7 - n)*d*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a
+ b*ArcSinh[c*x])/b)^n) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1
```

```
 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)
/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSi
nh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2 \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)^n \, dx = \frac{\left(d\sqrt{d+c^2dx^2}\right) \int x^2(1+c^2x^2)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^n \, dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right) \operatorname{Subst}\left(\int (a+bx)^n \cosh^4(x)\sinh^2(x)\, dx, x, \sinh \ldots\right.}{c^3\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right) \operatorname{Subst}\left(\int \left(-\frac{1}{16}(a+bx)^n - \frac{1}{32}(a+bx)^n \cosh(2 \ldots\right.\right.}{c^3 \ldots}$$

$$= -\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}} - \frac{\left(d\sqrt{d+c^2dx^2}\right) \operatorname{Subs} \ldots}{\ldots}$$

$$= -\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right) \operatorname{Subs} \ldots}{6 \ldots}$$

$$= -\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d \ldots}}{\ldots}$$

Mathematica [A]
time = 2.16, size = 429, normalized size = 0.70

$$-\frac{2^{-7-2n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{(a+b\sinh^{-1}(cx))^2}{b^2}\right)^{-n}\left(-2^nb(1+n)\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{6(a+b\sinh^{-1}(cx))}{b}\right)-3^{1+n}be^{\frac{2a}{b}}(1+n)\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{4(a+b\sinh^{-1}(cx))}{b}\right)+2^n3^{1+n}be^{\frac{4a}{b}}(1+n)\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)-2^n3^{1+n}be^{\frac{8a}{b}}(1+n)\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{2(a+b\sinh^{-1}(cx))}{b}\right)+3^{1+n}be^{\frac{10a}{b}}(1+n)\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{4(a+b\sinh^{-1}(cx))}{b}\right)+2^ne^{\frac{6a}{b}}\left(2^{3+n}3^{1+n}\left(a+b\sinh^{-1}(cx)\right)\left(-\frac{(a+b\sinh^{-1}(cx))^2}{b^2}\right)^n+be^{\frac{6a}{b}}(1+n)\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{6(a+b\sinh^{-1}(cx))}{b}\right)\right)\right)}{bc^3(1+n)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -((2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-(
2^n*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))
/b]) - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b] + 2^n*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + A
rcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b] - 2^n*3^(1 + n)*b*
E^((8*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*Ar
cSinh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/
b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^n*E^((6*a)/b)*(2^(3 + n)
*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^2))^n + b*E^((6
*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh
[c*x]))/b]))))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcS
inh[c*x])^2/b^2))^n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(c^2 d\, x^2+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas
")
```

```
[Out] integral((c^2*d*x^4 + d*x^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b\,\mathrm{asinh}(c\,x))^n \left(d\,c^2\,x^2 + d\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

3.518 $\int x(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=542

$$\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}+\frac{3^{-n}de^{-\frac{3a}{b}}\sqrt{d+}}{}$$

```
[Out] 1/32*5^(-1-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(c^
2*d*x^2+d)^(1/2)/c^2/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+1/32*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^
2+d)^(1/2)/(3^n)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+1/16*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*d*x^2+
d)^(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/16*d*
exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)
^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/32*d*exp(3*a/b)*(
a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(
3^n)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/32*5^(-1-n)*d*exp(5
*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5819, 5556, 3389, 2212}

$$\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{de^{-\frac{a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{16c^2\sqrt{c^2x^2+1}}+\frac{de^{a/b}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\sinh^{-1}(cx)}{b}\right)}{16c^2\sqrt{c^2x^2+1}}+\frac{d3^{-n}e^{\frac{3a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{d5^{-n-1}e^{\frac{5a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{32c^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (5^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(
a + b*ArcSinh[c*x]))/b])/(32*c^2*E^((5*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*Ar
cSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*c^2*E^(a/b)*Sqrt[1 + c^2
*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])/(16*c^2*Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (d*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*S
qrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x
]))/b])/(32*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^4(x)\sinh(x)\,dx,x,\sinh\right.}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{8}(a+bx)^n\sinh(x)+\frac{3}{16}(a+bx)^n\text{si}\right.\right.}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh(5x)\,dx,x,\sinh^{-1}(cx)\right)}{16c^2\sqrt{1+c^2x^2}}$$

$$=-\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-5x}(a+bx)^n\,dx,x,\sinh^{-1}(cx)\right)}{32c^2\sqrt{1+c^2x^2}}+$$

$$=\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-}}{32c^2\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 1.24, size = 390, normalized size = 0.72

$$\frac{15^{-1-n}d^2e^{-\frac{5a}{b}}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{(a+b\sinh^{-1}(cx))^2}{b^2}\right)^{-2n}\left(2\,15^{1+n}e^{\frac{6a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^n\left(-\frac{(a+b\sinh^{-1}(cx))^2}{b^2}\right)^n\Gamma\left(1+n,\frac{a}{b}+\sinh^{-1}(cx)\right)+3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\left(3^n\left(-\frac{(a+b\sinh^{-1}(cx))^2}{b^2}\right)^n\Gamma\left(1+n,-\frac{5(a+b\sinh^{-1}(cx))}{b}\right)+5^{1+n}e^{\frac{2a}{b}}\left(-\frac{(a+b\sinh^{-1}(cx))^2}{b^2}\right)^n\Gamma\left(1+n,-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+2\,3^n5^{1+n}e^{\frac{4a}{b}}\left(-\frac{(a+b\sinh^{-1}(cx))^2}{b^2}\right)^n\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(cx)}{b}\right)+5^{1+n}e^{\frac{8a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{2n}\Gamma\left(1+n,\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+3^ne^{\frac{10a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{2n}\Gamma\left(1+n,\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\right)\right)}{32c^2\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (15^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2*15^(1 + n)*E^(
(6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Ga
mma[1 + n, a/b + ArcSinh[c*x]] + 3*(a/b + ArcSinh[c*x])^n*(3^n*(-((a + b*Ar
cSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b] + 5^(1 + n)
*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcS
inh[c*x]))/b] + 2*3^n*5^(1 + n)*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))
^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(-((a +
b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] + 3^n*E^
((10*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (5*(a + b*ArcSinh
[c*x]))/b])))/(32*c^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x
])^2/b^2))^(2*n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arcsinh}\left(cx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^2*d*x^3 + d*x)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

3.519 $\int \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=420

$$\frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-2(3+n)}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(1}{c\sqrt{1+c^2x^2}}$$

```
[Out] 3/8*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)^(1
/2)+d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)
^(1/2)/(2^(6+2*n))/c/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+2^(-3-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b)*(c^2*
d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2
^(-3-n)*d*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*x))/b)
*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-d*exp(4
*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/(2^(6+2*n))/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5791, 3393, 3388, 2212}

$$\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}-\frac{d2^{-n-3}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}-\frac{d2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{8bc(n+1)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt[
1 + c^2*x^2]) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-3 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*
Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*d*E^((2*a)/b
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh
[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(3 + n))*c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^4(x)\,dx,x,\sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{3}{8}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)+\right.\right.}{c\sqrt{1+c^2x^2}}$$

$$=\frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}}{}$$

$$=\frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}}{1}$$

$$=\frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{4^{-3-n}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}}{}$$

Mathematica [A]
time = 1.00, size = 287, normalized size = 0.68

$$\frac{d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{8\left(a+b\sinh^{-1}(cx)\right)}{b+bn}+8\left(\frac{4a+4b\sinh^{-1}(cx)}{b+bn}+2^{-n}e^{-\frac{2a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-2^{-n}e^{\frac{2a}{b}}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^{-n}\Gamma\left(1+n,\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)+4^{-n}e^{-\frac{4a}{b}}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-e^{\frac{8a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)\right)}{64c\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b
 + b*n) + 8*((4*a + 4*b*ArcSinh[c*x])/(b + b*n) + Gamma[1 + n, (-2*(a + b*A
rcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((2*a
)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(2^n*(a/b + ArcSinh[c*x])^n)
) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] - E^(
(8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x])
)/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c*Sqrt[d +
c^2*d*x^2])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

3.520 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))^n}{x}\,dx$

Optimal. Leaf size=390

$$\frac{3^{-1-n}d^2e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}+\frac{5d^2e^{-\frac{a}{b}}\sqrt{1+c^2x^2}}{}$$

```
[Out] 1/8*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c
^2*x^2+1)^(1/2)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+
5/8*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^(
1/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+5/8*d^2*exp(a
/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/
(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/8*3^(-1-n)*d^2*exp(3*a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/((
(a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d^2*Unintegrable((a+b*arcsinh(
c*x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(cx))^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] (3^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(
a + b*ArcSinh[c*x]))/b])/(8*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSi
nh[c*x])/b))^n) + (5*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 +
 n, -((a + b*ArcSinh[c*x])/b)])/(8*E^(a/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*Ar
cSinh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^
n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])/(8*Sqrt[d + c^2*d*x^2]*((a + b*ArcS
inh[c*x])/b)^n) + (3^(-1 - n)*d^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(8*Sqrt[d + c^2*d*x^2
]*((a + b*ArcSinh[c*x])/b)^n) + d^2*Defer[Int][(a + b*ArcSinh[c*x])^n/(x*Sq
rt[d + c^2*d*x^2]), x]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(cx))^n}{x}\,dx=\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\sinh^{-1}(cx))^n}{x}\,dx$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
[Out] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas")
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**n/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x, x)
```

3.521 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=273

$$\frac{3cd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{d+c^2dx^2}}+\frac{2^{-3-n}cd^2e^{-\frac{2a}{b}}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(\right.}{\sqrt{d+c^2dx^2}}$$

```
[Out] 3/2*c*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)^
(1/2)+2^(-3-n)*c*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b
)*(c^2*x^2+1)^(1/2)/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1
/2)-2^(-3-n)*c*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh
(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+
d^2*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] (3*c*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(2*b*(1 + n)*Sqrt[
d + c^2*d*x^2]) + (2^(-3 - n)*c*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^
n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*Sqrt[d + c^2*d*x^
2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*c*d^2*E^((2*a)/b)*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(
Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + d^2*Defer[Int][(a + b*Arc
Sinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x^2]), x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx=\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{n}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

```
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="fricas
")
```

```
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

3.522 $\int x^2(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=816

$$-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}}{c^3\sqrt{1+c^2x^2}}$$

```
[Out] -5/128*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^
2+1)^(1/2)+2^(-11-3*n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-8*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(8*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*
arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b
)^n)/(c^2*x^2+1)^(1/2)+d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x)
)/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a
+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arcsinh(c*x)
)/b)^n)/(c^2*x^2+1)^(1/2)+2^(-7-n)*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMM
A(1+n,2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/
b)^n)/(c^2*x^2+1)^(1/2)-d^2*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+
b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/(((a+b*arcsinh(c*x))
/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arcsinh(c*x)
)^n*GAMMA(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsin
h(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-11-3*n)*d^2*exp(8*a/b)*(a+b*arcsinh(c*x
))^n*GAMMA(1+n,8*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsi
nh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5819, 5556, 3388, 2212}

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (-5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[1 + c^2*x^2]) + (2^(-11 - 3*n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((8*a)/b)*Sqrt[
1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*3^(-1 - n)*d^2*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*
x]))/b])/(c^3*E^((6*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n)
+ (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
```

```
b*ArcSinh[c*x])/b))^n) - (2^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1
 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*d^2*E^((2*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x
]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d^2*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(4 + n))*c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^
n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2
]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])/(c
^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int x^2(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^6(x)\sinh^2(x)\,dx,x,\sin\right.}{c^3\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{5}{128}(a+bx)^n-\frac{1}{32}(a+bx)^n\cosh(\right.\right.}{}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{S}}{}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{S}}{}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d+}}{}$$

Mathematica [A]
time = 5.19, size = 667, normalized size = 0.82

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -((2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-
(3^(1 + n)*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh
[c*x]))/b]) - 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[
1 + n, (-6*(a + b*ArcSinh[c*x]))/b] - 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1
+ n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] + 3^(
1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n,
(-2*(a + b*ArcSinh[c*x]))/b] + E^((8*a)/b)*(5*2^(4 + 3*n)*3^(1 + n)*a*(-((a
 + b*ArcSinh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1 + n)*b*ArcSinh[c*x]*(-((a
 + b*ArcSinh[c*x])^2/b^2))^n - 3^(1 + n)*4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-
((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b] + 2^(3
 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1
 + n, (4*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6*a)/b)*(-((a + b*ArcSi
nh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6
*a)/b)*n*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x])
)/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (
```

```
8*(a + b*ArcSinh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*n*(-((a + b*ArcSinh[c*
x])/b))^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])))/(b*c^3*E^((8*a)/b)*(1
 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))^n))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
[Out] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas
")
[Out] integral((c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c^2*d*x^2 + d)*(b*arc
sinh(c*x) + a)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

3.523 $\int x(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=745

$$\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}+\frac{5^{-n}d^2e^{-\frac{5a}{b}}\sqrt{d}\cdots}{}$$

```
[Out] 1/128*7^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-7*(a+b*arcsinh(c*x))/b)*
(c^2*d*x^2+d)^(1/2)/c^2/exp(7*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^
(1/2)+1/128*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(c^
2*d*x^2+d)^(1/2)/(5^n)/c^2/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+
1)^(1/2)+1/128*3^(1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+5/128*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/
b)*(c^2*d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)
^(1/2)+5/128*d^2*exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))
/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/
128*3^(1-n)*d^2*exp(3*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*
x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
+1/128*d^2*exp(5*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b
)*(c^2*d*x^2+d)^(1/2)/(5^n)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2
)+1/128*7^(-1-n)*d^2*exp(7*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,7*(a+b*arcsi
nh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^
(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5819, 5556, 3389, 2212}

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (7^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcSinh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b
*ArcSinh[c*x])/b))^n) + (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gam
ma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)*Sqrt[1 + c
^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3^(1 - n)*d^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(128*c^2
*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5*d^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/
b)])/(128*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5
```

```
*d^2*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b
*ArcSinh[c*x])/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) +
 (3^(1 - n)*d^2*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*Ar
cSinh[c*x])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) + (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])/(128
*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^6(x)\sinh(x)\,dx,x,\sin\right.}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5}{64}(a+bx)^n\sinh(x)+\frac{9}{64}(a+bx)^n\right.\right.}{}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh(7x)\,dx,x,\sinh^{-1}(cx)\right)}{64c^2\sqrt{1+c^2x^2}}$$

$$=-\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-7x}(a+bx)^n\,dx,x,\sinh^{-1}(cx)\right)}{128c^2\sqrt{1+c^2x^2}}$$

$$=\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 2.00, size = 685, normalized size = 0.92

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (105^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(5^(2 + n)*21^(1
 + n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])
^2/b^2))^n*Gamma[1 + n, a/b + ArcSinh[c*x]] + 15^(1 + n)*(-((a + b*ArcSinh[
c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-7*(a + b*ArcSinh[c*x]))/b] + E^((2*a)/b)
*(5*21^(1 + n)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a +
b*ArcSinh[c*x]))/b] + 9*35^(1 + n)*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^
2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b] + 5^(2 + n)*21^(1 + n)*
E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a + b*Arc
Sinh[c*x])/b)] + 35^(1 + n)*E^((8*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*Ar
cSinh[c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] + 8*35^(1 +
n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/
b^2))^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] - 3^(2 + n)*7^(1 + n)*E^((
10*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 +
 n, (5*(a + b*ArcSinh[c*x]))/b] + 8*21^(1 + n)*E^((10*a)/b)*(-((a + b*ArcSi
nh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (5*(a + b
*ArcSinh[c*x]))/b] + 15^(1 + n)*E^((12*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a +
```

```
 b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])))/(128
*c^2*E^((7*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n*(-((a +
b*ArcSinh[c*x])^2/b^2))^(2*n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x)*sqrt(c^2*d*x^2 + d)*(b*arcsi
nh(c*x) + a)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

3.524 $\int (d+c^2dx^2)^{5/2} (a+b\sinh^{-1}(cx))^n \, dx$

Optimal. Leaf size=632

$$\frac{5d^2\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-}}{c\sqrt{1+c^2x^2}}$$

```
[Out] 5/16*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)
^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*arcsinh
(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+3*2^(-7-2*n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsi
nh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)+15*2^(-7-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(
c^2*x^2+1)^(1/2)-15*2^(-7-n)*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,
2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)-3*2^(-7-2*n)*d^2*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,
4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arcsinh(c*x))^n*GAMMA
(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^
n)/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5791, 3393, 3388, 2212}

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(16*b*c*(1 + n)*Sq
rt[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c*E^((6*a)/b)*Sqr
t[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3*2^(-7 - 2*n)*d^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/
b])/(c*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (15*2
^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(
a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh
[c*x])/b))^n) - (15*2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) - (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/
```

```
(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d
^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(
a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^n \, dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right) \int \left(1+c^2x^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^n \, dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right) \operatorname{Subst}\left(\int (a+bx)^n \cosh^6(x) \, dx, x, \sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right) \operatorname{Subst}\left(\int \left(\frac{5}{16}(a+bx)^n + \frac{15}{32}(a+bx)^n \cosh(2x) \right.\right.}{c\sqrt{1}}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right) \operatorname{Subst}}{}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right) \operatorname{Subst}}{6}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+}}{}$$

Mathematica [A]
time = 3.73, size = 529, normalized size = 0.84

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2^n*
b*(1 + n)*(a/b + ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1
+ n, (-6*(a + b*ArcSinh[c*x]))/b] + 3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b +
ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x
])^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]
))/b] - E^((6*a)/b)*(5*2^n*3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-((a + b*ArcSin
h[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a + b*ArcS
inh[c*x]))/b] + 3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((
a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^
n*(-5*2^(3 + n)*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^
2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[
c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b]))))/(b*c*E^((6*a)/
b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

3.525 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^n}{x}\,dx$

Optimal. Leaf size=756

$$\frac{5^{-1-n}d^3e^{-\frac{5a}{b}}\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}}-\frac{5\,3^{-1-n}d^3e^{-\frac{3a}{b}}}{}$$

```
[Out] 1/32*5^(-1-n)*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(
c^2*x^2+1)^(1/2)/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)
-5/32*3^(-1-n)*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*
(c^2*x^2+1)^(1/2)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2
)+1/8*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+
1)^(1/2)/(3^n)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1
1/16*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^
(1/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+11/16*d^3*ex
p(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/
2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)-5/32*3^(-1-n)*d^3*exp(3*a
/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2
)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/8*d^3*exp(3*a/b)*(a+b*ar
csinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(3^n)/(((
a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/32*5^(-1-n)*d^3*exp(5*a/b)*(a
+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a
+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d^3*Unintegrable((a+b*arcsinh(c*
x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 1.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] (5^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(
a + b*ArcSinh[c*x]))/b])/(32*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcS
inh[c*x])/b))^n) - (5*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(32*E^((3*a)/b)*Sqrt[d + c^2*
d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(8*3^n*E^((3*a)/b)*Sq
rt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (11*d^3*Sqrt[1 + c^2*x^2
```

```
]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*E^(a/
b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (11*d^3*E^(a/b)*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])
/(16*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (5*3^(-1 - n)*d^3*E^
((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*A
rcSinh[c*x]))/b])/(32*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (d^
3*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a +
 b*ArcSinh[c*x]))/b])/(8*3^n*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n
) + (5^(-1 - n)*d^3*E^((5*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Ga
mma[1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(32*Sqrt[d + c^2*d*x^2]*((a + b*Arc
Sinh[c*x])/b)^n) + d^3*Defer[Int][(a + b*ArcSinh[c*x])^n/(x*Sqrt[d + c^2*d*
x^2]), x]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx=\int \frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
[Out] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2d\,x^2+d)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x, x)
```

3.526 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^n}{x^2}\,dx$

Optimal. Leaf size=455

$$\frac{15cd^3\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^{1+n}}{8b(1+n)\sqrt{d+c^2dx^2}}+\frac{2^{-2(3+n)}cd^3e^{-\frac{4a}{b}}\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma}{\sqrt{d+c^2dx^2}}$$

```
[Out] 15/8*c*d^3*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)
^(1/2)+c*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*x
^2+1)^(1/2)/(2^(6+2*n))/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d
)^(1/2)+2^(-2-n)*c*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))
/b)*(c^2*x^2+1)^(1/2)/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^
(1/2)-2^(-2-n)*c*d^3*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsi
nh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2
)-c*d^3*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(
c^2*x^2+1)^(1/2)/(2^(6+2*n))/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)
+d^3*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\sinh^{-1}(cx))^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] (15*c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*(1 + n)*Sqrt
[d + c^2*d*x^2]) + (c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*E^((4*a)/b)*Sqrt[d + c^2*
d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-2 - n)*c*d^3*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a
)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-2 - n)*c*d^3
*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a +
b*ArcSinh[c*x]))/b])/(Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (c*
d^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a
 + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[
c*x])/b)^n) + d^3*Defer[Int][(a + b*ArcSinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x^2
]), x]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\sinh^{-1}(cx))^n}{x^2}\,dx = \int \frac{(d+c^2dx^2)^{5/2}\,(a+b\sinh^{-1}(cx))^n}{x^2}\,dx$$

Mathematica [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\sinh^{-1}(cx))^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
[Out] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2d\,x^2+d)^{\frac{5}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="fricas
")
```

```
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n/x^2, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

3.527 $\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}},x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
[Out] Defer[Int][(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
[Out] Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

3.528 $\int \frac{x^3 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=113

$$\frac{3^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-3\sinh^{-1}(ax)\right)}{8a^4}-\frac{3\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-\sin\right.}{8a^4}$$

```
[Out] 1/8*3^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/a^4/((-arcsinh(a*x))
^n)-3/8*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^4/((-arcsinh(a*x))^n)-3/8
*GAMMA(1+n,arcsinh(a*x))/a^4+1/8*3^(-1-n)*GAMMA(1+n,3*arcsinh(a*x))/a^4
```

Rubi [A]
time = 0.18, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5819, 3393, 3389, 2212}

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^4}-\frac{3\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)}{8a^4}-\frac{3\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{8a^4}+\frac{3^{-n-1}\text{Gamma}\left(n+1,3\sinh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*Gamma[1 + n, ArcSinh[a*x]])/(8*a^4) + (3^(-1 - n)*Gamma[1 + n
, 3*ArcSinh[a*x]])/(8*a^4)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= \frac{\text{Subst}\left(\int x^n \sinh^3(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^4}\\ &= \frac{i\text{Subst}\left(\int \left(\frac{3}{4}ix^n\sinh(x)-\frac{1}{4}ix^n\sinh(3x)\right)\,dx, x, \sinh^{-1}(ax)\right)}{a^4}\\ &= \frac{\text{Subst}\left(\int x^n \sinh(3x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}-\frac{3\text{Subst}\left(\int x^n \sinh(x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}\\ &= -\frac{\text{Subst}\left(\int e^{-3x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{8a^4}+\frac{\text{Subst}\left(\int e^{3x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{8a^4}+\frac{3\text{Subst}}{}\\ &= \frac{3^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-3\sinh^{-1}(ax)\right)}{8a^4}-\frac{3\left(-\sinh^{-1}(ax)\right)^{-n}}{}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 100, normalized size = 0.88

$$\frac{3^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\left(\sinh^{-1}(ax)^n\Gamma\left(1+n,-3\sinh^{-1}(ax)\right)-3^{2+n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-\sinh^{-1}(ax)\right)+\left(-\sinh^{-1}(ax)\right)^n\left(-3^{2+n}\Gamma\left(1+n,\sinh^{-1}(ax)\right)+\Gamma\left(1+n,3\sinh^{-1}(ax)\right)\right)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3^(-1 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]] - 3^(2 + n)*ArcSi
nh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(3^(2 + n)*Gamm
a[1 + n, ArcSinh[a*x]]) + Gamma[1 + n, 3*ArcSinh[a*x]])))/(8*a^4*(-ArcSinh[
a*x])^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**3*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\mathrm{asinh}\left(a\,x\right)}^n}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^3*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

3.529 $\int \frac{x^2 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{2^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-2\sinh^{-1}(ax)\right)}{a^3}-\frac{2^{-3-n}\Gamma\left(1+n,2\sinh^{-1}(ax)\right)}{a^3}$$

```
[Out] -1/2*arcsinh(a*x)^(1+n)/a^3/(1+n)+2^(-3-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcs
inh(a*x))/a^3/((-arcsinh(a*x))^n)-2^(-3-n)*GAMMA(1+n,2*arcsinh(a*x))/a^3
```

Rubi [A]
time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5819, 3393, 3388, 2212}

$$\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2\sinh^{-1}(ax)\right)}{a^3}-\frac{2^{-n-3}\text{Gamma}\left(n+1,2\sinh^{-1}(ax)\right)}{a^3}-\frac{\sinh^{-1}(ax)^{n+1}}{2a^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/2*ArcSinh[a*x]^(1 + n)/(a^3*(1 + n)) + (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[
1 + n, -2*ArcSinh[a*x]])/(a^3*(-ArcSinh[a*x])^n) - (2^(-3 - n)*Gamma[1 + n,
 2*ArcSinh[a*x]])/a^3
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= \frac{\text{Subst}\left(\int x^n\sinh^2(x)\,dx,x,\sinh^{-1}(ax)\right)}{a^3}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{x^n}{2}-\frac{1}{2}x^n\cosh(2x)\right)\,dx,x,\sinh^{-1}(ax)\right)}{a^3}\\ &= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{\text{Subst}\left(\int x^n\cosh(2x)\,dx,x,\sinh^{-1}(ax)\right)}{2a^3}\\ &= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{\text{Subst}\left(\int e^{-2x}x^n\,dx,x,\sinh^{-1}(ax)\right)}{4a^3}+\frac{\text{Subst}\left(\int e^{2x}x^n\,dx,x,\sinh^{-}\right.}{4a^3}\\ &= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{2^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-2\sinh^{-1}(ax)\right)}{a^3}-\end{aligned}$$

Mathematica [A]
time = 0.15, size = 86, normalized size = 1.08

$$\frac{2^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\left((1+n)\sinh^{-1}(ax)^n\Gamma\left(1+n,-2\sinh^{-1}(ax)\right)-\left(-\sinh^{-1}(ax)\right)^n\left(2^{2+n}\sinh^{-1}(ax)^{1+n}+(1+n)\Gamma\left(1+n,2\sinh^{-1}(ax)\right)\right)\right)}{a^3(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2^(-3 - n)*((1 + n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] - (-ArcSi
nh[a*x])^n*(2^(2 + n)*ArcSinh[a*x]^(1 + n) + (1 + n)*Gamma[1 + n, 2*ArcSinh
[a*x]])))/(a^3*(1 + n)*(-ArcSinh[a*x])^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\mathrm{arcsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

[Out] int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)

[Out] Integral(x**2*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\operatorname{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)

[Out] int((x^2*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)

3.530 $\int \frac{x \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\Gamma\left(1+n,\sinh^{-1}(ax)\right)}{2a^2}$$

```
[Out] 1/2*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^2/((-arcsinh(a*x))^n)+1/2*GAM
MA(1+n,arcsinh(a*x))/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5819, 3389, 2212}

$$\frac{\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a^2*(-ArcSinh[a*x])^n) + Ga
mma[1 + n, ArcSinh[a*x]]/(2*a^2)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= \frac{\text{Subst}\left(\int x^n\sinh(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{2a^2}+\frac{\text{Subst}\left(\int e^{x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{2a^2}\\ &= \frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\Gamma\left(1+n,\sinh^{-1}(ax)\right)}{2a^2}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 43, normalized size = 0.88

$$\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-\sinh^{-1}(ax)\right)+\Gamma\left(1+n,\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamma[1 + n, ArcSinh[a*x]])/(2*a^2)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\text{arcsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{asinh}^{n}(ax)}{\sqrt{a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,{\operatorname{asinh}(a\,x)}^{n}}{\sqrt{a^{2}\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

3.531 $\int \frac{\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sinh^{-1}(ax)^{1+n}}{a(1+n)}$$

```
[Out] arcsinh(a*x)^(1+n)/a/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5783}

$$\frac{\sinh^{-1}(ax)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^{1+n}}{a(1+n)}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^{1+n}}{a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

Maple [A]
time = 0.30, size = 18, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^{1+n}}{a(1+n)}$	18
default	$\frac{\operatorname{arcsinh}(ax)^{1+n}}{a(1+n)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(a*x)^(1+n)/a/(1+n)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 1.00

$$\frac{\operatorname{arsinh}(ax)^{n+1}}{a(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(a*x)^(n + 1)/(a*(n + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(17) = 34$.
time = 0.38, size = 83, normalized size = 4.88

$$\frac{\cosh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)\log\left(ax+\sqrt{a^2x^2+1}\right)+\log\left(ax+\sqrt{a^2x^2+1}\right)\sinh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)}{an+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (cosh(n*log(log(a*x + sqrt(a^2*x^2 + 1))))*log(a*x + sqrt(a^2*x^2 + 1)) + l
og(a*x + sqrt(a^2*x^2 + 1))*sinh(n*log(log(a*x + sqrt(a^2*x^2 + 1)))))/(a*n
 + a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.
time = 0.33, size = 34, normalized size = 2.00

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge n = -1 \\ 0^n x & \text{for } a = 0 \\ \frac{\log(\operatorname{asinh}(ax))}{a} & \text{for } n = -1 \\ \frac{\operatorname{asinh}(ax)\operatorname{asinh}^n(ax)}{an+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(n, -1)), (0**n*x, Eq(a, 0)), (log(asinh(a*x
))/a, Eq(n, -1)), (asinh(a*x)*asinh(a*x)**n/(a*n + a), True))
```

Giac [A]
time = 0.40, size = 29, normalized size = 1.71

$$\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^{n+1}}{a(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] log(a*x + sqrt(a^2*x^2 + 1))^(n + 1)/(a*(n + 1))
```

Mupad [B]
time = 0.28, size = 33, normalized size = 1.94

$$\begin{cases} \frac{\ln(\mathrm{asinh}(a\,x))}{a} & \text{if } n=-1 \\ \frac{\mathrm{asinh}(a\,x)^{n+1}}{a\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] piecewise(n == -1, log(asinh(a*x))/a, n ~= -1, asinh(a*x)^(n + 1)/(a*(n + 1
)))
```

3.532 $\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A]
time = 3.96, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^3 + x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^n(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**n/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{asinh}(a\,x)^n}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^n/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.533 $\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
[Out] Defer[Int][ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
[Out] Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^4 + x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^n(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**n/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{asinh}(a\,x)^n}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^n/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.534 $\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=416

$$-\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}}-\frac{2ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}+\frac{bc^3}{}$$

```
[Out] 3/8*d^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2*d^
2*x^3*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+2/3*I*d^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-2/3*I*b*d^
2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*c*d^2*x^2*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d^2*x^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^3*d^2*x^4*(d+
I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/16*d^2*(a+b*arcsinh(c*
x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 5838, 5785, 5783, 30, 5798, 5806, 5812}

$$-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}+\frac{2id^2(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{c^2x^2+1}}-\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{c^2x^2+1}}-\frac{2ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{c^2x^2+1}}+\frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (((-2*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2]
 - (3*b*c*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2]
) - (((2*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 +
c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
 c^2*x^2]) + (3*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/8 - (c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/4 + (((2*I)/3)*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x]))/c + (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*
ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int (d+icdx)^2\sqrt{1+c^2x^2}\left(a\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \left(d^2\sqrt{1+c^2x^2}\left(a+b\sinh\right.\right.}{}$$

$$= \frac{\left(d^2\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}d^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}c^2c$$

$$= -\frac{2ibd^2x\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcd^2x^2\sqrt{d+icdx}}{4\sqrt{1+c^2}}$$

$$= -\frac{2ibd^2x\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{3bcd^2x^2\sqrt{d+icdx}}{16\sqrt{1+c}}$$

Mathematica [A]
time = 0.89, size = 361, normalized size = 0.87

$$\frac{48ad^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}\,(16i+9cx+16ic^2x^2-6c^3x^3)+720ad^{5/2}\sqrt{f}\,\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+144bd^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(2\sinh^{-1}(cx)^2-\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\sinh\left(2\sinh^{-1}(cx)\right)\right)-64ibd^2\sqrt{d+icdx}\,\sqrt{f-icfx}\left(9cx-3\sinh^{-1}(cx)\left(3\sqrt{1+c^2x^2}+\cosh\left(3\sinh^{-1}(cx)\right)\right)+\sinh\left(3\sinh^{-1}(cx)\right)\right)+9bd^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)}{1152c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*(16*I + 9*c
*x + (16*I)*c^2*x^2 - 6*c^3*x^3) + 720*a*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*
Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 144*b*
```

```
d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^2 - Cosh[2*ArcSinh[
c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]) - (64*I)*b*d^2*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*
ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]) + 9*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*
ArcSinh[c*x]]))/(1152*c*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx + d)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(cx)) \sqrt{-icfx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x -
 a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

3.535 $\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=304

$$-\frac{ibdx\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcdx^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}-\frac{ibc^2dx^3\sqrt{d+icdx}\,\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}+\frac{1}{2}dx\sqrt{d+i}$$

```
[Out] 1/2*d*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/3*I*d*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-1/3*I*b*d*
x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*d*x^2*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*I*b*c^2*d*x^3*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*d*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5796, 5838, 5785, 5783, 30, 5798}

$$\frac{d\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{id(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}dx\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcdx^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}-\frac{ibdx\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{c^2x^2+1}}-\frac{ibc^2dx^3\sqrt{d+icdx}\,\sqrt{f-icfx}}{9\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-1/3*I)*b*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b
*c*d*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) - ((I/9
)*b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (d*x
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + ((I/3)*d*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (d
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1
+ c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int (d+icdx)\sqrt{1+c^2x^2}\,(a+}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \left(d\sqrt{1+c^2x^2}\,(a+b\sinh^{-}\right.}{\sqrt{1+c^2x}}$$

$$= \frac{\left(d\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}dx\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{id\sqrt{d}}{}$$

$$= -\frac{ibdx\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcdx^2\sqrt{d+icdx}\,\sqrt{f}}{4\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 1.00, size = 273, normalized size = 0.90

$$\frac{12ad\sqrt{d+icdx}\,\sqrt{f-icfx}\,(2i+3cx+2ic^2x^2)+36ad^{3/2}\sqrt{f}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+\frac{9bd\sqrt{d+icdx}\,\sqrt{f-icfx}\,(2\sinh^{-1}(cx)^2-\cosh(2\sinh^{-1}(cx))+2\sinh^{-1}(cx)\sinh(2\sinh^{-1}(cx)))}{\sqrt{1+c^2x^2}}-\frac{2ibd\sqrt{d+icdx}\,\sqrt{f-icfx}\left(9cx-3\sinh^{-1}(cx)\left(3\sqrt{1+c^2x^2}+\cosh(3\sinh^{-1}(cx))\right)+\sinh(3\sinh^{-1}(cx))\right)}{\sqrt{1+c^2x^2}}}{72c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (12*a*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*I + 3*c*x + (2*I)*c^2*x^2) +
 36*a*d^(3/2)*Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]] + (9*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^
2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1 + c
^2*x^2] - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh
[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]))
/Sqrt[1 + c^2*x^2])/(72*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)\sqrt{-icfx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((I*b*c*d*x + b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) + (I*a*c*d*x + a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (id(cx-i))^{\frac{3}{2}}\sqrt{-if(cx+i)}\,(a+b\operatorname{asinh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeDone
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

3.536 $\int \sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=147

$$-\frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\,(a-}{4bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b*c*x^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*(a+b*arcsinh(c*x))^
2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {5796, 5785, 5783, 30}

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/4*(b*c*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int \sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{\left(\sqrt{d+i\right.}{}$$

$$= -\frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+icdx}\,\sqrt{f-ic}$$

Mathematica [A]
time = 0.31, size = 233, normalized size = 1.59

$$\frac{1}{2}ax\sqrt{id(-i+cx)}\,\sqrt{-if(i+cx)}+\frac{a\sqrt{d}\,\sqrt{f}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{id(-i+cx)}\,\sqrt{-if(i+cx)}\right)}{2c}-\frac{b\sqrt{i(-id+cdx)}\,\sqrt{-i(if+cfx)}\,\sqrt{-df\left(1+c^2x^2\right)}\,\left(\cosh\left(2\sinh^{-1}(cx)\right)-2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{8c\sqrt{-((-id+cdx)(if+cfx))}\,\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*x*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)])/2 + (a*Sqrt[d]*Sqrt[f]*Lo
g[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(
2*c) - (b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 +
 c^2*x^2))]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*A
rcSinh[c*x]])))/(8*c*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x
^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\sqrt{icdx+d}\,\sqrt{-icfx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1)
) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{id\,(cx-i)}\,\sqrt{-if\,(cx+i)}\,(a+b\operatorname{asinh}{(cx)})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeDone
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

3.537 $\int \frac{\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=158

$$\frac{ibfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

```
[Out] -I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I
*b*f*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*f*(a+b*arc
sinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5796, 5838, 5783, 5798, 8}

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{if(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{ibfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] (I*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (I*f*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\,\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}-\frac{icfx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{\left(icf\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(c\cdots\right.}{\sqrt{1+c^2x^2}\cdots}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{ibfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{f\sqrt{1+\cdots}}{2bc\cdots}$$

Mathematica [A]
time = 0.30, size = 227, normalized size = 1.44

$$\frac{2i\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(bcx-a\sqrt{1+c^2x^2}\right)-2ib\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)+b\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2+2a\sqrt{d}\,\sqrt{f}\,\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{2cd\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) -
(2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
+ b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqrt[f
]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/(2*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{-icfx+f}}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] a*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + b*in
tegrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*x + d),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2
+ 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*d*x - I*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-if\,(cx+i)}\,(a+b\,\mathrm{asinh}\,(cx))}{\sqrt{id\,(cx-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

3.538 $\int \frac{\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf^2\left(1+c^2x^2\right)^{3/2}\log(i-}{c(d+icdx)^{3/2}(f-icfx}$$

```
[Out] 2*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c
*f*x)^(3/2)-1/2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)-2*b*f^2*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(
3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 5844, 651, 5837, 12, 641, 31, 5783}

$$-\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf^2\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*I)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*f^2*(1 + c^2*x^2
)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)^2(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{2i(if^2+cf^2x)(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}} - \frac{f^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i(1+c^2x^2)^{3/2}\right)\int \frac{(if^2+cf^2x)(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(f^2(1+c^2x^2)^{3/2}\right.}{(d+icdx)^{3/}}$$

$$= \frac{2if^2(1-icx)(1+c^2x^2)(a+b\sinh^{-1}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+}{2bc(d+icdx)^{3/2}(}$$

$$= \frac{2if^2(1-icx)(1+c^2x^2)(a+b\sinh^{-1}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+}{2bc(d+icdx)^{3/2}(}$$

$$= \frac{2if^2(1-icx)(1+c^2x^2)(a+b\sinh^{-1}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+}{2bc(d+icdx)^{3/2}(}$$

$$= \frac{2if^2(1-icx)(1+c^2x^2)(a+b\sinh^{-1}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+}{2bc(d+icdx)^{3/2}(}$$

Mathematica [A]
time = 0.94, size = 283, normalized size = 1.56

$$-\frac{-\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{-i+cx}+2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+\frac{b\sqrt{d+icdx}\sqrt{f-icfx}\left(\sinh^{-1}(cx)(-4i\cosh(\frac{1}{2}\sinh^{-1}(cx))-4\sinh(\frac{1}{2}\sinh^{-1}(cx)))+\sinh^{-1}(cx)^2(\cosh(\frac{1}{2}\sinh^{-1}(cx))+i\sinh(\frac{1}{2}\sinh^{-1}(cx)))+2\left(4i\mathrm{ArcTan}(\tanh(\frac{1}{2}\sinh^{-1}(cx)))+\log(1+c^2x^2)\right)(\cosh(\frac{1}{2}\sinh^{-1}(cx))+i\sinh(\frac{1}{2}\sinh^{-1}(cx)))\right)}{\sqrt{1+c^2x^2}(\cosh(\frac{1}{2}\sinh^{-1}(cx))+i\sinh(\frac{1}{2}\sinh^{-1}(cx)))}}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] -1/2*((-4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) + 2*a*Sqrt[d]*S
qrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] +
 (b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[
c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c
^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^
2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\sqrt{-icfx+f}}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c*d^
2*sqrt(f/d))) + b*integrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))
/(I*c*d*x + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*x^2 - 2*I*c*d^2*x -
d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-if\left(cx+i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(id\left(cx-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))\ \sqrt{f - c\,f\,x\,1\mathrm{i}}}{(d + c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

3.539 $\int \frac{\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2ibf^3(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{if^3(1-icx)^3\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf^3(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 2/3*I*b*f^3*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+1/3*I*f^3*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b*f^3*(c^2*x^2+1)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5796, 665, 5837, 12, 641, 45}

$$\frac{if^3(1-icx)^3\,(c^2x^2+1)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibf^3(c^2x^2+1)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf^3(c^2x^2+1)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((2*I)/3)*b*f^3*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 - I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*f^3*(1 + c^2*x^2)^(5/
2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)^3(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc(1+c^2x^2)^{5/2}\right)\int \frac{if}{3c}}{(d+icdx)^{5/2}(f-i}$$

$$= \frac{if^3(1-icx)^3\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3(1+c^2x^2)^{5/2}\right)\int}{3(d+icdx)^{5/2}(f-}$$

$$= \frac{if^3(1-icx)^3\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3(1+c^2x^2)^{5/2}\right)\int}{3(d+icdx)^{5/2}(f-}$$

$$= \frac{if^3(1-icx)^3\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3(1+c^2x^2)^{5/2}\right)\int}{3(d+icdx)^{5/}}$$

$$= \frac{2ibf^3(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if^3(1-icx)^3\,(1+c^2x^2)\,(a+}{3c(d+icdx)^{5/2}(f-i}$$

Mathematica [A]
time = 0.27, size = 141, normalized size = 0.75

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(-\left((i+cx)\left(-ib+bcx+a\sqrt{1+c^2x^2}\right)\right)-b(i+cx)\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)+b(-i+cx)^2\log(d+icdx)\right)}{3cd^3(-i+cx)^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-((I + c*x)*((-I)*b + b*c*x + a*Sqrt[
1 + c^2*x^2])) - b*(I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(-I + c*x)^
2*Log[d + I*c*d*x]))/(3*c*d^3*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

Maxima [A]
time = 0.28, size = 219, normalized size = 1.17

$$\frac{1}{3}\,bc\left(\frac{6\,\sqrt{f}}{3i\,c^3d^{\frac{5}{2}}x+3\,c^2d^{\frac{5}{2}}}+\frac{\sqrt{f}\,\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)\operatorname{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/3*b*c*(6*sqrt(f)/(3*I*c^3*d^(5/2)*x + 3*c^2*d^(5/2)) + sqrt(f)*log(c*x -
I)/(c^2*d^(5/2))) - 1/3*b*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I*c
^2*d^3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3))*
arcsinh(c*x) - 1/3*a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I*c^2*d^
3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(142) = 284.
time = 0.45, size = 548, normalized size = 2.93

$$-\frac{4\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,bcx+2\left(bc^2x^2+2i\,bcx-b\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(c^4d^3x^3-i\,c^3d^3x^2+c^2d^3x-i\,cd^3\right)\sqrt{\frac{b^2f}{c^2d^5}}\,\log\left(-\frac{\left(i\,bc^6x^2+2\,bc^5x-2i\,bc^4\right)\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}+\left(i\,c^9d^3x^4+2\,c^8d^3x^3+i\,c^7d^3x^2+2\,c^6d^3x\right)\sqrt{\frac{b^2f}{c^2d^5}}}{8\left(bc^3x^3-i\,bc^2x^2+bcx-i\,b\right)}\right)+\left(c^4d^3x^3-i\,c^3d^3x^2+c^2d^3x-i\,cd^3\right)\sqrt{\frac{b^2f}{c^2d^5}}\,\log\left(-\frac{\left(i\,bc^6x^2+2\,bc^5x-2i\,bc^4\right)\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}+\left(-i\,c^9d^3x^4-2\,c^8d^3x^3-i\,c^7d^3x^2-2\,c^6d^3x\right)\sqrt{\frac{b^2f}{c^2d^5}}}{8\left(bc^3x^3-i\,bc^2x^2+bcx-i\,b\right)}\right)+2\left(ac^2x^2+2i\,acx-a\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}}{6\left(c^4d^3x^3-i\,c^3d^3x^2+c^2d^3x-i\,cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 2*(b
*c^2*x^2 + 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) - (c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x - I*c*d^3)*sqrt
(b^2*f/(c^2*d^5))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*
x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d^3*x^4 + 2*c^8*d^3*
x^3 + I*c^7*d^3*x^2 + 2*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))/(b*c^3*x^3 - I*b*
c^2*x^2 + b*c*x - I*b)) + (c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x - I*c*d^
3)*sqrt(b^2*f/(c^2*d^5))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^4)*sq
rt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (-I*c^9*d^3*x^4 - 2*
c^8*d^3*x^3 - I*c^7*d^3*x^2 - 2*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))/(b*c^3*x^
3 - I*b*c^2*x^2 + b*c*x - I*b)) + 2*(a*c^2*x^2 + 2*I*a*c*x - a)*sqrt(I*c*d*
x + d)*sqrt(-I*c*f*x + f))/(c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x - I*c*d
^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-if\left(cx+i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(id\left(cx-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{{(d+c\,d\,x\,1\mathrm{i})}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

3.540 $\int (d+icdx)^{5/2}(f-icfx)^{3/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=459

$$-\frac{ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/2}}-\frac{5bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}}-\frac{2ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15\left(1+c^2x^2\right)^{3/2}}-$$

```
[Out] -1/5*I*b*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-5/16*b*c
*d*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-2/15*I*b*c^2*d
*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*c^3*d*x^4
*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/25*I*b*c^4*d*x^5*(
d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*d*x*(d+I*c*d*x)^(3
/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+1/5*I*d*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+3/16*d*(d+I*c*d*x)^(3/2)*(f-I*c*
f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A]

time = 0.28, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 5838, 5786, 5785, 5783, 30, 14, 5798, 200}

$$\frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{id\left(c^2x^2+1\right)(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{5c}+\frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{5bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(c^2x^2+1\right)^{3/2}}-\frac{ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(c^2x^2+1\right)^{3/2}}-\frac{2ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15\left(c^2x^2+1\right)^{3/2}}-\frac{ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}}{25\left(c^2x^2+1\right)^{3/2}}-\frac{bc^3dx^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-1/5*I)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2
) - (5*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)
^(3/2)) - (((2*I)/15)*b*c^2*d*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/
(1 + c^2*x^2)^(3/2) - (b*c^3*d*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
/(16*(1 + c^2*x^2)^(3/2)) - ((I/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) + ((I/5)*d*(d + I*c*d*x)^(3/2)*
(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*d*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)
^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (d+icdx)\left(1+c^2x^2\right)^{3/\ldots}}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(d\left(1+c^2x^2\right)^{3/2}\left(a+b\ldots\right.\right.}{(1+\ldots}$$

$$= \frac{\left(d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\,s\ldots\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\ldots$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\ldots$$

$$= -\frac{ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/2}}-\frac{5bcdx^2(d+icdx)\ldots}{16\,(1+\ldots}$$

Mathematica [A]
time = 1.08, size = 683, normalized size = 1.49

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-1200*I)*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (1920*I)*a*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d^2*f*x
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (3840*I)*a*c^2*d^2
```

```
*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^3*d
^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1920*I)*a
*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1800
*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*d^2*f*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(5/2)*f^(3/2
)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]] - (200*I)*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*Ar
cSinh[c*x]] + 60*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(
(10*I)*Cosh[3*ArcSinh[c*x]] + (2*I)*Cosh[5*ArcSinh[c*x]] + 5*((4*I)*Sqrt[1
+ c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) - (24*I)*b*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])/(9600*c*Sqrt[1
 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx + d)^{\frac{5}{2}} (-icfx + f)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((I*b*c^3*d^2*f*x^3 + b*c^2*d^2*f*x^2 + I*b*c*d^2*f*x + b*d^2*f)*sq
rt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^3*
d^2*f*x^3 + a*c^2*d^2*f*x^2 + I*a*c*d^2*f*x + a*d^2*f)*sqrt(I*c*d*x + d)*sq
rt(-I*c*f*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

3.541 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=247

$$-\frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}}-\frac{bc^3x^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}}+\frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh\right.$$

```
[Out] -5/16*b*c*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*
c^3*x^4*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*x*(d+I*c*
d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*x*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+3/16*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5796, 5786, 5785, 5783, 30, 14}

$$\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(c^2x^2+1\right)^{3/2}}-\frac{bc^3x^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3/2
)) - (b*c^3*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^
(3/2)) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/4
 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1
 + c^2*x^2)) + (3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*
x])^2)/(16*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$=\frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{(3}{}$$

$$=\frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3x}{}$$

$$=-\frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}}-\frac{bc^3x^4(d+icdx)}{16\,(1+}$$

Mathematica [A]

time = 0.53, size = 352, normalized size = 1.43

$$\frac{80acdfx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+32ac^3dfx^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+24bdf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2-16bdf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(2\sinh^{-1}(cx)\right)-bdf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(4\sinh^{-1}(cx)\right)+48ad^{3/2}f^{3/2}\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+4bdf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)\left(8\sinh\left(2\sinh^{-1}(cx)\right)+\sinh\left(4\sinh^{-1}(cx)\right)\right)}{128c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (80*a*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32*a*
c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 24*b*d*
f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 16*b*d*f*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - b*d*f*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 48*a*d^(3/2)*f^(3/2)*Sqrt[1 + c^2*x^2
]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 4*b*
d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(8*Sinh[2*ArcSinh[c*x]
] + Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(icdx+d\right)^{\frac{3}{2}} \left(-icfx+f\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((b*c^2*d*f*x^2 + b*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d*f*x^2 + a*d*f)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,{(d+c\,d\,x\,1\mathrm{i})}^{3/2}\,{(f-c\,f\,x\,1\mathrm{i})}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

3.542 $\int \sqrt{d+icdx}\,(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=304

$$\frac{ibfx\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcfx^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{ibc^2fx^3\sqrt{d+icdx}\,\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}+\frac{1}{2}fx\sqrt{d+ic$$

```
[Out] 1/2*f*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/3*I*f*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/3*I*b*f*
x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*f*x^2*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/9*I*b*c^2*f*x^3*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5796, 5838, 5785, 5783, 30, 5798}

$$\frac{f\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{if(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}fx\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcfx^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}+\frac{ibfx\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{c^2x^2+1}}+\frac{ibc^2fx^3\sqrt{d+icdx}\,\sqrt{f-icfx}}{9\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((I/3)*b*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b*c*
f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) + ((I/9)*b
*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (f*x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 - ((I/3)*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c
^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \sqrt{d+icdx}\,(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int (f-icfx)\sqrt{1+c^2x^2}\,(a+}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \left(f\sqrt{1+c^2x^2}\,(a+b\sinh^{-}\right.}{\sqrt{1+c^2x}}$$

$$= \frac{\left(f\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}fx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right) - \frac{if\sqrt{d}}{}$$

$$= \frac{ibfx\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\,\sqrt{f-}}{4\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.97, size = 273, normalized size = 0.90

$$\frac{12af\sqrt{d+icdx}\,\sqrt{f-icfx}\,(-2i+3cx-2ic^2x^2)+36a\sqrt{d}\,f^{3/2}\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+\frac{9bf\sqrt{d+icdx}\,\sqrt{f-icfx}\,(2\sinh^{-1}(cx)^2-\cosh(2\sinh^{-1}(cx))+2\sinh^{-1}(cx)\sinh(2\sinh^{-1}(cx)))}{\sqrt{1+c^2x^2}}+\frac{2ibf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(9cx-3\sinh^{-1}(cx)\left(3\sqrt{1+c^2x^2}+\cosh(3\sinh^{-1}(cx))\right)+\sinh(3\sinh^{-1}(cx))\right)}{\sqrt{1+c^2x^2}}}{72c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (12*a*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*I + 3*c*x - (2*I)*c^2*x^2)
+ 36*a*Sqrt[d]*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]] + (9*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]
^2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1 +
c^2*x^2] + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSin
h[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]])
)/Sqrt[1 + c^2*x^2])/(72*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-icfx+f\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\sqrt{icdx+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((-I*b*c*f*x + b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (-I*a*c*f*x + a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{id\left(cx-i\right)}\left(-if\left(cx+i\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeDone
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

3.543 $\int \frac{(f-icfx)^{3/2}(a+b\sinh^{-1}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=266

$$\frac{2ibf^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{bcf^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{f^2x(1+c^2x^2)\left(a-\right.}{2\sqrt{d+icdx}\,\sqrt{}}$$

```
[Out] -2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)-1/2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)+2*I*b*f^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*
b*c*f^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*f^2*(
a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)
```

Rubi [A]
time = 0.31, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5796, 5838, 5783, 5798, 8, 5812, 30}

$$\frac{3f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{f^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2if^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{bcf^2x^2\sqrt{c^2x^2+1}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2ibf^2x\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*b*f^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
b*c*f^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((
2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]) - (f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]) + (3*f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b
*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int\frac{(f-icfx)^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int\left(\frac{f^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} - \frac{2icf^2x(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} - \frac{c^2f^2x^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(f^2\sqrt{1+c^2x^2}\right)\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{\left(2icf^2\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{f^2x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{2ibf^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2if^2(1+c^2x^2)}{c\sqrt{d+icdx}}$$

Mathematica [A]
time = 0.60, size = 344, normalized size = 1.29

$$\frac{16ibcfx\sqrt{d+icdx}\,\sqrt{f-icfx} - 16iaf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2} - 4acfx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2} - 4bf(4i+cx)\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}\sinh^{-1}(cx) + 6bf\sqrt{d+icdx}\,\sqrt{f-icfx}\sinh^{-1}(cx)^2 + bf\sqrt{d+icdx}\,\sqrt{f-icfx}\cosh\left(2\sinh^{-1}(cx)\right) + 12a\sqrt{d}\,f^{3/2}\sqrt{1+c^2x^2}\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{8cd\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((16*I)*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*f*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*f*(4*I + c*x)*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*f*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Co
sh[2*ArcSinh[c*x]] + 12*a*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + S
qrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*d*Sqrt[1 + c^2*x^
2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
))/(c*d*x - I*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))}{\sqrt{id\,(cx-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

3.544 $\int \frac{(f-icfx)^{3/2}(a+b\sinh^{-1}(cx))}{(d+icdx)^{3/2}} dx$

Optimal. Leaf size=284

$$-\frac{ibf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -I*b*f^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^3*(1
-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)+I*f^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)-3/2*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)-4*b*f^3*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)
```

Rubi [A]

time = 0.32, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5796, 5844, 651, 5837, 12, 641, 31, 5783, 5798, 8}

$$-\frac{3f^3(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{ibf^3x(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4bf^3(c^2x^2+1)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-I)*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) + ((4*I)*f^3*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*
d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x
]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*f^3*(1 + c^2*x^2)^(3/2
)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 (4*b*f^3*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c
*f*x)^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)^3(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{4i(if^3+cf^3x)(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}} - \frac{3f^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{icf}{\cdots}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i(1+c^2x^2)^{3/2}\right)\int \frac{(if^3+cf^3x)(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3f^3(1+c^2x^2)^{3/\cdots}\right.}{(d+icdx)^3\cdots}$$

$$= \frac{4if^3(1-icx)(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2\left(a+b\cdots\right.}{c(d+icdx)^{3/2}(f-\cdots}$$

$$= -\frac{ibf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)(1+c^2x^2)\left(a+b\sinh\cdots\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{ibf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)(1+c^2x^2)\left(a+b\sinh\cdots\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{ibf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)(1+c^2x^2)\left(a+b\sinh\cdots\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A]
time = 1.48, size = 514, normalized size = 1.81

$$\frac{\frac{2af(5+icx)\sqrt{d+icdx}\sqrt{f-icfx}}{d^2(-i+cx)} - \frac{6af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{3/2}} - \frac{bf\sqrt{d+icdx}\sqrt{f-icfx}\left(\sinh^{-1}(cx)\left(-4i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-4\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+\sinh^{-1}(cx)^2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+2\left(4i\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+\log(1+c^2x^2)\right)\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)}{d^2\sqrt{1+c^2x^2}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)} + \frac{2bf\sqrt{d+icdx}\sqrt{f-icfx}\left(-\sinh^{-1}(cx)^2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+\left(cx-4\text{ArcTan}\left(\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)-i\log(1+c^2x^2)\right)\left(-i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+\sinh^{-1}(cx)\left(i\left(2+\sqrt{1+c^2x^2}\right)\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\left(-2+\sqrt{1+c^2x^2}\right)\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)}{d^2\sqrt{1+c^2x^2}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]

[Out] ((2*a*f*(5 + I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^2*(-I + c*x)) -
 (6*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]])/d^(3/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-
4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*
```

```
x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))
)/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) +
 (2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh
[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]]
 - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) +
 ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[
1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]))))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))))/(2*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6*I*
sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c*d^2
*sqrt(f/d))) + b*integrate((-I*c*f*x + f)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1)
)/(I*c*d*x + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c*f*x - b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (I*a*c*f*x - a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2),
 x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(f-c\,f\,x\,1{}\mathrm{i}\right)^{3/2}}{\left(d+c\,d\,x\,1{}\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

3.545 $\int \frac{(f-icfx)^{3/2}(a+b\sinh^{-1}(cx))}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=364

$$\frac{4ibf^4(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bf^4(1+c^2x^2)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\,(1+c^2x^2)\,(a+b\sinh^{-1}$$

```
[Out] 4/3*I*b*f^4*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/2*b*f^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*I*f^4*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-2*I*f^4*(1-I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+f^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*f^4*(c^2*x^2+1)
^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]

time = 0.27, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 683, 667, 221, 5837, 641, 45, 31, 5783}

$$-\frac{2if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4ibf^4\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8bf^4\left(c^2x^2+1\right)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bf^4\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((4*I)/3)*b*f^4*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(1 - I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((2*
I)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^4*(1 + c^
2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
```

```
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx=\frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^4(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=\frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2if^4(1-icx)\left(1+\right.}{c(d+icdx}$$

$$=\frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2if^4(1-icx)\left(1+\right.}{c(d+icdx}$$

$$=-\frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\right.}{3c(d+icdx)^{5/2}(f-icf}$$

$$=-\frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\right.}{3c(d+icdx)^{5/2}(f-icf}$$

$$=\frac{4ibf^4\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(c}{2c(d+icdx)^{5/2}(f-icfx}$$

Mathematica [A]
time = 3.83, size = 706, normalized size = 1.94

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] ((-16*a*f*(-I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^3*(-I + c*x)
^2) + (12*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/d^(5/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I
)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (7*I)*Log[
1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(
8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*Log[1 + c^2*x^2])) + 2*(4 -
(4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
 + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh
[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[Ar
```

```
cSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])^4) + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/
2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] -
 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*
x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log
[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1
 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*(4 + (2 + Sqrt[1 + c^2*x^
2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))}{(icdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*a*(-3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(-3*I*c^4*d^4*x^3 - 9*c^3*d^4*x^2 +
9*I*c^2*d^4*x + 3*c*d^4) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*f/(c^3*d^3*x^2 - 2*I
*c^2*d^3*x - c*d^3) + 21*I*sqrt(c^2*d*f*x^2 + d*f)*f/(3*I*c^2*d^3*x + 3*c*d
^3) - 3*f^2*arcsinh(c*x)/(c*d^3*sqrt(f/d))) + b*integrate((-I*c*f*x + f)^(3
/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

3.546 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=344

$$-\frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\left(1+c^2x^2\right)^{5/2}}-\frac{5bc^3x^4(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\left(1+c^2x^2\right)^{5/2}}-\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2}}{36c}$$

```
[Out] -25/96*b*c*x^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^(5/2)-5/96*b
*c^3*x^4*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^(5/2)+1/6*x*(d+I*c
*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))+5/16*x*(d+I*c*d*x)^(5/2)*(
f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^2+5/24*x*(d+I*c*d*x)^(5/2)*
(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+5/32*(d+I*c*d*x)^(5/2)*(f-
I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(5/2)-1/36*b*(d+I*c*d*x
)^(5/2)*(f-I*c*f*x)^(5/2)*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.20, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5796, 5786, 5785, 5783, 30, 14, 267}

$$\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{16\left(c^2x^2+1\right)^2}+\frac{5(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc\left(c^2x^2+1\right)^{5/2}}+\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\left(c^2x^2+1\right)^{5/2}}-\frac{b\sqrt{c^2x^2+1}(d+icdx)^{5/2}(f-icfx)^{5/2}}{36c}-\frac{5bc^3x^4(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\left(c^2x^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^2)^(5/
2)) - (5*b*c^3*x^4*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^
2)^(5/2)) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*Sqrt[1 + c^2*x^2])/(
36*c) + (x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/6
+ (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16*(1
 + c^2*x^2)^2) + (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSin
h[c*x]))/(24*(1 + c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*(1 + c^2*x^2)^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left((d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int \left(1+c^2x^2\right)^{5/2}\left(a+b\,\text{sinl}\right.}{\left(1+c^2x^2\right)^{5/2}}$$

$$= \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{(5(}{}$$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c}+\frac{1}{6}x(d+ic$$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c}+\frac{1}{6}x(d+ic$$

$$= -\frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\left(1+c^2x^2\right)^{5/2}}-\frac{5bc^3x^4(d+icdx}{96\,(1+}$$

Mathematica [A]
time = 0.75, size = 481, normalized size = 1.40

$$\frac{1584acd^2f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+1248ac^3d^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+384ac^5d^2f^2x^5\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+360bd^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2-270bd^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(2\sinh^{-1}(cx)\right)-27bd^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(4\sinh^{-1}(cx)\right)-2bd^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(6\sinh^{-1}(cx)\right)+720ad^{5/2}f^{5/2}\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+12bd^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)\left(45\sinh\left(2\sinh^{-1}(cx)\right)+9\sinh\left(4\sinh^{-1}(cx)\right)+\sinh\left(6\sinh^{-1}(cx)\right)\right)}{2304c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (1584*a*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] +
 1248*a*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^
2] + 384*a*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + 360*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 -
270*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 27
*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] - 2*b*d
^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSinh[c*x]] + 720*a*d^(
5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]] + 12*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + Sinh[6*ArcS
inh[c*x]]))/(2304*c*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{5}{2}}\left(-icfx+f\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((b*c^4*d^2*f^2*x^4 + 2*b*c^2*d^2*f^2*x^2 + b*d^2*f^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^4*d^2*f^2*x^4
+ 2*a*c^2*d^2*f^2*x^2 + a*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

3.547 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=459

$$\frac{ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/2}}-\frac{5bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}}+\frac{2ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15\left(1+c^2x^2\right)^{3/2}}-$$

```
[Out] 1/5*I*b*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-5/16*b*c*
f*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+2/15*I*b*c^2*f*
x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*c^3*f*x^4*
(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/25*I*b*c^4*f*x^5*(d
+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*f*x*(d+I*c*d*x)^(3/
2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*
x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)-1/5*I*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*
x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+3/16*f*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A]

time = 0.29, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 5838, 5786, 5785, 5783, 30, 14, 5798, 200}

$$\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}-\frac{if\left(c^2x^2+1\right)(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{5c}+\frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{5bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(c^2x^2+1\right)^{3/2}}+\frac{ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(c^2x^2+1\right)^{3/2}}+\frac{2ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15\left(c^2x^2+1\right)^{3/2}}+\frac{ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}}{25\left(c^2x^2+1\right)^{3/2}}-\frac{bc^3fx^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((I/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2) -
 (5*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3
/2)) + (((2*I)/15)*b*c^2*f*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
+ c^2*x^2)^(3/2) - (b*c^3*f*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
6*(1 + c^2*x^2)^(3/2)) + ((I/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)*(a + b*ArcSinh[c*x]))/4 + (3*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) - ((I/5)*f*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*f*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)^(3
/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (f-icfx)\left(1+c^2x^2\right)^{3}\cdots}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(f\left(1+c^2x^2\right)^{3/2}\left(a+\cdots\right.\right.}{\left(1+\cdots\right.}$$

$$= \frac{\left(f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\,\cdots\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\cdots$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\cdots$$

$$= \frac{ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/2}}-\frac{5bcfx^2(d+icdx)^{3/\cdots}}{16\left(1+c^2\cdots\right.}$$

Mathematica [A]
time = 1.03, size = 683, normalized size = 1.49

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((1200*I)*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (1920*I)*a*d*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (3840*I)*a*c^2*d*f^
```

```
2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^3*d*
f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1920*I)*a*
c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1800*
b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*d*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(3/2)*f^(5/2)
*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]] + (200*I)*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*Arc
Sinh[c*x]] + 60*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((
-10*I)*Cosh[3*ArcSinh[c*x]] - (2*I)*Cosh[5*ArcSinh[c*x]] + 5*((-4*I)*Sqrt[1
 + c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + (24*I)*b*d*
f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])/(9600*c*Sqrt[
1 + c^2*x^2])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{3}{2}} (-icfx+f)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((-I*b*c^3*d*f^2*x^3 + b*c^2*d*f^2*x^2 - I*b*c*d*f^2*x + b*d*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^
3*d*f^2*x^3 + a*c^2*d*f^2*x^2 - I*a*c*d*f^2*x + a*d*f^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

3.548 $\int \sqrt{d+icdx}\,(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=416

$$\frac{2ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{3bcf^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}}+\frac{2ibc^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}+\frac{bc^3f^2}{}$$

```
[Out] 3/8*f^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2*f^
2*x^3*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-2/3*I*f^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+2/3*I*b*f^
2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*c*f^2*x^2*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f^2*x^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^3*f^2*x^4*(d+
I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/16*f^2*(a+b*arcsinh(c*
x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 5838, 5785, 5783, 30, 5798, 5806, 5812}

$$-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{5f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{2if^2(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{3}{8}f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{3bcf^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{16\sqrt{c^2x^2+1}}+\frac{2ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{c^2x^2+1}}+\frac{2ibc^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}}{9\sqrt{c^2x^2+1}}+\frac{bc^3f^2x^4\sqrt{d+icdx}\,\sqrt{f-icfx}}{16\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (((2*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] -
 (3*b*c*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c
^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
c^2*x^2]) + (3*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/8 - (c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/4 - (((2*I)/3)*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/c + (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*A
rcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \sqrt{d+icdx}\,(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int (f-icfx)^2\sqrt{1+c^2x^2}\,(a-}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \left(f^2\sqrt{1+c^2x^2}\,(a+b\sinh\right.}{}$$

$$= \frac{\left(f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\,(a+b\sinh^{-}}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}c^2f$$

$$= \frac{2ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+icdx}\,\sqrt{}}{4\sqrt{1+c^2x^2}}$$

$$= \frac{2ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{3bcf^2x^2\sqrt{d+icdx}\,\sqrt{}}{16\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.73, size = 565, normalized size = 1.36

$$\frac{576ibcf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-768iaf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+432acf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-768iac^2f^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-288ac^3f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+360bf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2-144bf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(2\sinh^{-1}(cx)\right)+9bf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(4\sinh^{-1}(cx)\right)+720a\sqrt{d}\,f^{5/2}\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+64ibf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh\left(3\sinh^{-1}(cx)\right)+12bf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)\left(-48i\sqrt{1+c^2x^2}-16i\cosh\left(3\sinh^{-1}(cx)\right)+24\sinh\left(2\sinh^{-1}(cx)\right)-3\sinh\left(4\sinh^{-1}(cx)\right)\right)}{1152c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((576*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (768*I)*a*f^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 432*a*c*f^2*x*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (768*I)*a*c^2*f^2*x^2*Sqrt[d
```

```
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 288*a*c^3*f^2*x^3*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 360*b*f^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 144*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*Cosh[2*ArcSinh[c*x]] + 9*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[4*ArcSinh[c*x]] + 720*a*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x
+ Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (64*I)*b*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 12*b*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-48*I)*Sqrt[1 + c^2*x^2] - (16*I)*Co
sh[3*ArcSinh[c*x]] + 24*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]]))/(11
52*c*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (-icfx+f)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx)) \sqrt{icdx+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x -
 a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

3.549 $\int \frac{(f-icfx)^{5/2}(a+b\sinh^{-1}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=381

$$\frac{11ibf^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{ibc^2f^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{11if^3(1+c^2x^2)\left(a+b\sin\right.}{3c\sqrt{d+icdx}\,\sqrt{f-\right.}}$$

```
[Out] -11/3*I*f^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-3/2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)+1/3*I*c*f^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)+11/3*I*b*f^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+3/4*b*c*f^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)-1/9*I*b*c^2*f^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+5/4*f^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5796, 5838, 5783, 5798, 8, 5812, 30}

$$\frac{5f^3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{icf^3x^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3f^3x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{11if^3(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3bcf^3x^2\sqrt{c^2x^2+1}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{11ibf^3x\sqrt{c^2x^2+1}}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{ibc^2f^3x^3\sqrt{c^2x^2+1}}{9\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] (((11*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) - ((I/9)*b*c^2*f^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} - \frac{3icf^3x(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} - \frac{3c^2f^3x^2(a+b\ldots}{\sqrt{1+\ldots}}\right.}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(f^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{\left(3icf^3\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\ldots}{\sqrt{1-\ldots}}}{\sqrt{d+icdx}\,\sqrt{f-ic\ldots}}$$

$$= -\frac{3if^3(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)\left(a+b\sinh\ldots\right.}{2\sqrt{d+icdx}\,\sqrt{f-ic\ldots}}$$

$$= \frac{3ibf^3x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{ibc^2f^3x\ldots}{9\sqrt{d+ic\ldots}}$$

$$= \frac{11ibf^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{ibc^2f^3\ldots}{9\sqrt{d+i\ldots}}$$

Mathematica [A]
time = 0.99, size = 465, normalized size = 1.22

$$\frac{264ibcf^2x\sqrt{d+icdx}\,\sqrt{f-icfx} - 8ibc^3f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx} - 264iaf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2} - 108acf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2} + 24iac^2f^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2} + 90bf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2 + 27bf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(2\sinh^{-1}(cx)\right) - 6bf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)\left(9(5i+2cx)\sqrt{1+c^2x^2} - i\cosh\left(3\sinh^{-1}(cx)\right)\right) + 180a\sqrt{d}\,f^{5/2}\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{72cd\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((264*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (8*I)*b*c^3*f^2*x^
3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (264*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*Sqrt[1 + c^2*x^2] + (24*I)*a*c^2*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*ArcSinh[c*x]^2 + 27*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcS
inh[c*x]] - 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(9*(5*
I + 2*c*x)*Sqrt[1 + c^2*x^2] - I*Cosh[3*ArcSinh[c*x]]) + 180*a*Sqrt[d]*f^(5
/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]])/(72*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c^2*f^2*x^2 - 2*b*c*f^2*x - I*b*f^2)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*f^2*x^2 - 2*a*c*f^2*x
 - I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

3.550 $\int \frac{(f-icfx)^{5/2}(a+b\sinh^{-1}(cx))}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=518

$$-\frac{3ibf^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{bcf^4x^2(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bf^4(1-icx)^2\,(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15bf^4(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(}$$

```
[Out] -3/2*I*b*f^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+b*c*f^
4*x^2*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+5/4*b*f^4*(1-I*
c*x)^2*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+15/4*b*f^4*(
c^2*x^2+1)^(3/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*f
^4*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*
x)^(3/2)+15/2*I*f^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)+5/2*I*f^4*(1-I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-15/2*f^4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*f^4*(c^2*x^2+1)^(
3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.28, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 683, 685, 655, 221, 5837, 641, 45, 5783}

$$\frac{5if^4(1-icx)\,(c^2x^2+1)^2\,(a+b\sinh^{-1}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15if^4(c^2x^2+1)^2\,(a+b\sinh^{-1}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^4(1-icx)^3\,(c^2x^2+1)\,(a+b\sinh^{-1}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{15f^4(c^2x^2+1)^{3/2}\sinh^{-1}(cx)\,(a+b\sinh^{-1}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{bcf^4x^2(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bf^4(1-icx)^2\,(c^2x^2+1)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{3ibf^4x(c^2x^2+1)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8bf^4(c^2x^2+1)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15bf^4(c^2x^2+1)^{3/2}\sinh^{-1}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] (((-3*I)/2)*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b*c*f^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) + (5*b*f^4*(1 - I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)
/(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((2*I)*f^4*(1 - I*c*x)^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)) + (((15*I)/2)*f^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + (((5*I)/2)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*f^4*
(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^4*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx=\frac{\left(1+c^2x^2\right)^{3/2}\int\frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$=\frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15if^4\left(1+c^2x^2\right)^2(a}{2c(d+icdx)^{3/2}(}$$

$$=-\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2i}{}$$

$$=-\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15}{4}$$

$$=-\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15}{4}$$

$$=-\frac{3ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{bcf^4x^2\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bf}{4c(d}$$

Mathematica [A]
time = 2.52, size = 779, normalized size = 1.50

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((4*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 + (7*I)*c*x + c^2*x^2))/(
d^2*(-I + c*x)) - (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]])/d^(3/2) - (4*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) +
 ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*
Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Si
nh[ArcSinh[c*x]/2])) + (16*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(Arc
Sinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*Arc
Tan[Coth[ArcSinh[c*x]/2]] - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2]
+ Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcS
inh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1
+ c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2]) - (Cosh[2*ArcSinh[c*x]] + 8*((2*I)*c*x + (4*I)*ArcTa
n[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 - 8*Sqrt[1 + c^
2*x^2] + I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((8*I)*(1 + Sqrt[1
+ c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh
[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{5}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x)/
(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a + b*integrate((
-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x -
 a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x -
d^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

3.551 $\int \frac{(f-icfx)^{5/2}(a+b\sinh^{-1}(cx))}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=472

$$\frac{ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8ibf^5(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{5bf^5(1+c^2x^2)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^5(1-}{}$$

```
[Out] I*b*f^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b*f^5
*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5/2*b*f^5*
(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*
I*f^5*(1-I*c*x)^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-10/3*I*f^5*(1-I*c*x)^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*
c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5*I*f^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c/(
d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*f^5*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*b*f^5*(c^2*x^2+1
)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.30, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 683, 655, 221, 5837, 641, 45, 5783}

$$-\frac{5if^5(c^2x^2+1)^3(a+b\sinh^{-1}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{10if^5(1-icx)^2(c^2x^2+1)^2(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^5(1-icx)^4(c^2x^2+1)(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5f^5(c^2x^2+1)^{5/2}\sinh^{-1}(cx)(a+b\sinh^{-1}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf^5x(c^2x^2+1)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8ibf^5(c^2x^2+1)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{28bf^5(c^2x^2+1)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{5bf^5(c^2x^2+1)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (I*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
 (((8*I)/3)*b*f^5*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f
- I*c*f*x)^(5/2)) - (5*b*f^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^5*(1 - I*c*x)^4*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (
((10*I)/3)*f^5*(1 - I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((5*I)*f^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*f^5*(1 + c^2*x
^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (28*b*f^5*(1 + c^2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
```

```
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx=\frac{(1+c^2x^2)^{5/2}\int\frac{(f-icfx)^5\left(a+b\sinh^{-1}(cx)\right)}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=\frac{2if^5(1-icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{10if^5(1-icx)^2\,(1}{3c(d+ic}$$

$$=\frac{5ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^5(1-icx)^4\left(1+c^2x^2\right)\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=\frac{5ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{5bf^5(1+c^2x^2)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2i}{}$$

$$=\frac{5ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{5bf^5(1+c^2x^2)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2i}{}$$

$$=\frac{ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8ibf^5(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1005 vs. 2(472) = 944.
time = 4.91, size = 1005, normalized size = 2.13

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((-4*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 - (34*I)*c*x + 3*c^
2*x^2))/(d^3*(-I + c*x)^2) + (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/d^(5/2) - (2*b*f^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*Ar
cSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[Ar
cSinh[c*x]/2]] + (7*I)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[
Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*L
og[1 + c^2*x^2])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*A
rcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcS
inh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7
```

```
*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x
]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((2*I)*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*Arc
Sinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1
+ c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[C
oth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])
*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I
/2)*(4 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/
(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + (b*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[
c*x]/2])*(2*(4 + (6*I)*c*x - 6*c^2*x^2 + 52*(-I + c*x)*ArcTan[Coth[ArcSinh[
c*x]/2]] + 13*(1 + I*c*x)*Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[
ArcSinh[c*x]/2]) + 18*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2])^3 + ArcSinh[c*x]*((-24*I)*Cosh[ArcSinh[c*x]/2] - (35*I)*Cosh[(3*A
rcSinh[c*x])/2] + (3*I)*Cosh[(5*ArcSinh[c*x])/2] - 24*Sinh[ArcSinh[c*x]/2]
+ 35*Sinh[(3*ArcSinh[c*x])/2] + 3*Sinh[(5*ArcSinh[c*x])/2])))/(d^3*(I + c*x
)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))}{(icdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*(3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*d^5*x^4 - 4*I*c^4*d^5*x^3 - 6*c^3*
d^5*x^2 + 4*I*c^2*d^5*x + c*d^5) - 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*f/(-3*I*c
^4*d^4*x^3 - 9*c^3*d^4*x^2 + 9*I*c^2*d^4*x + 3*c*d^4) + 10*I*sqrt(c^2*d*f*x
^2 + d*f)*f^2/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 105*I*sqrt(c^2*d*f*x^
2 + d*f)*f^2/(3*I*c^2*d^3*x + 3*c*d^3) - 15*f^3*arcsinh(c*x)/(c*d^3*sqrt(f/
d)))*a + b*integrate((-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c
*d*x + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c^2*f^2*x^2 + 2*b*c*f^2*x + I*b*f^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*f^2*x^2 + 2*a*c*f^2
*x + I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*
d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

3.552 $\int \frac{(d+icdx)^{5/2}(a+b\sinh^{-1}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=381

$$-\frac{11ibd^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{ibc^2d^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{11id^3(1+c^2x^2)\left(a+b\sin\right.}{3c\sqrt{d+icdx}\,\sqrt{f-\right.}}$$

```
[Out] 11/3*I*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)-3/2*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)-1/3*I*c*d^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)-11/3*I*b*d^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+3/4*b*c*d^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+1/9*I*b*c^2*d^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)+5/4*d^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5796, 5838, 5783, 5798, 8, 5812, 30}

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{icd^3x^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3d^3x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{11id^3(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3bcd^3x^2\sqrt{c^2x^2+1}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{11ibd^3x\sqrt{c^2x^2+1}}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{ibc^2d^3x^3\sqrt{c^2x^2+1}}{9\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] (((-11*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + ((I/9)*b*c^2*d^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/
(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^3*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{3icd^3x(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} - \frac{3c^2d^3x^2(a+b\sinh}{\sqrt{1+c^2}}\right.}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(d^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{\left(3icd^3\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\sinh}{\sqrt{1+c}}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{3id^3(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)\left(a+b\sinh^{-1}(c\right.}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{3ibd^3x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{ibc^2d^3x}{9\sqrt{d+ic}}$$

$$= -\frac{11ibd^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{ibc^2d^3}{9\sqrt{d+i}}$$

Mathematica [A]
time = 1.01, size = 465, normalized size = 1.22

$$\frac{-264ibcd^2x\sqrt{d+icdx}\,\sqrt{f-icfx}+8ibc^3d^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}+264iad^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-108acd^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-24iac^2d^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+90bd^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2+27bd^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(2\sinh^{-1}(cx)\right)-6bd^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)\left(9(-5i+2cx)\sqrt{1+c^2x^2}+i\cosh\left(3\sinh^{-1}(cx)\right)\right)+180ad^{5/2}\sqrt{f}\,\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{72cf\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-264*I)*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (8*I)*b*c^3*d^2*x
^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (264*I)*a*d^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*Sqrt[1 + c^2*x^2] - (24*I)*a*c^2*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*ArcSinh[c*x]^2 + 27*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*Arc
Sinh[c*x]] - 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(9*(-
5*I + 2*c*x)*Sqrt[1 + c^2*x^2] + I*Cosh[3*ArcSinh[c*x]]) + 180*a*d^(5/2)*Sq
rt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]])/(72*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c^2*d^2*x^2 - 2*b*c*d^2*x + I*b*d^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*d^2*x^2 - 2*a*c*d^2
*x + I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

3.553 $\int \frac{(d+icdx)^{3/2}(a+b\sinh^{-1}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=266

$$-\frac{2ibd^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{d^2x(1+c^2x^2)\left(a\right.}{2\sqrt{d+icdx}}$$

```
[Out] 2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2
)-1/2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)-2*I*b*d^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b
*c*d^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*d^2*(a
+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2
)
```

Rubi [A]

time = 0.30, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5796, 5838, 5783, 5798, 8, 5812, 30}

$$\frac{3d^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{d^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2id^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{c^2x^2+1}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2ibd^2x\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*b*d^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
(b*c*d^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
(2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]) + (3*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{2icd^2x(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} - \frac{c^2d^2x^2(a+b\sin}{\sqrt{1+c}}\right.}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(d^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{\left(2icd^2\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\sin}{\sqrt{1+}}}{\sqrt{d+icdx}\,\sqrt{f-icf}}$$

$$= \frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{d^2x(1+c^2x^2)\left(a+b\sinh^{-1}(c\right.}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{2ibd^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{bcd^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2id^2(1+}{c\sqrt{d}}$$

Mathematica [A]
time = 0.60, size = 344, normalized size = 1.29

$$\frac{-16ibcdx\sqrt{d+icdx}\,\sqrt{f-icfx}+16iad\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-4acdx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-4bd(-4i+cx)\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}\sinh^{-1}(cx)+6bd\sqrt{d+icdx}\,\sqrt{f-icfx}\sinh^{-1}(cx)^2+bd\sqrt{d+icdx}\,\sqrt{f-icfx}\cosh\left(2\sinh^{-1}(cx)\right)+12ad^{3/2}\sqrt{f}\,\sqrt{1+c^2x^2}\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{8cf\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-16*I)*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*d*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*d*(-4*I + c*x)*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*d*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[2*ArcSinh[c*x]] + 12*a*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
 Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*f*Sqrt[1 + c^2*
x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
))/(c*f*x + I*f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))}{\sqrt{-if\,(cx+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))\ (d + c\,d\,x\,1\mathrm{i})^{3/2}}{\sqrt{f - c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

3.554 $\int \frac{\sqrt{d+icdx}\,(a+b\sinh^{-1}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=158

$$-\frac{ibdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

```
[Out] I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*
b*d*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*d*(a+b*arcs
inh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5796, 5838, 5783, 5798, 8}

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{id(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{ibdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-I)*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (I*d
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+icdx}\,\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx &= \frac{\sqrt{1+c^2x^2}\,\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} \\
&= \frac{\sqrt{1+c^2x^2}\,\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{icdx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} \\
&= \frac{\left(d\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{\left(icd\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\ldots}{\sqrt{d+icdx}\,\sqrt{f-icfx}} \\
&= \frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{d\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}\ldots \\
&= -\frac{ibdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{d\sqrt{1\ldots}}{2b\ldots}
\end{aligned}$$

Mathematica [A]

time = 0.30, size = 227, normalized size = 1.44

$$\frac{-2i\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(bcx-a\sqrt{1+c^2x^2}\right)+2ib\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)+b\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2+2a\sqrt{d}\,\sqrt{f}\,\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{2cf\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) +
 (2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
 + b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqrt[
f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]])/(2*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{icdx+d}}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] a*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + b*in
tegrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*x + f),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*f*x + I*f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{id\,(cx-i)}\,(a+b\,\mathrm{asinh}\,(cx))}{\sqrt{-if\,(cx+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}}{\sqrt{f-c\,f\,x\,1{}\mathrm{i}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

3.555 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {5796, 5783}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\,\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$
$$= \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Mathematica [A]
time = 0.26, size = 113, normalized size = 1.92

$$\frac{b\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)^2}{2c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{a\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{c\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + (a*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
)/(c*Sqrt[d]*Sqrt[f])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.54

$$\frac{b\,\mathrm{arsinh}\,(cx)^2}{2\,\sqrt{df}\,c}+\frac{a\,\mathrm{arsinh}\,(cx)}{\sqrt{df}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*b*arcsinh(c*x)^2/(sqrt(d*f)*c) + a*arcsinh(c*x)/(sqrt(d*f)*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1
)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f*x^2 + d*f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{\sqrt{id(cx - i)}\sqrt{-if(cx + i)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{\sqrt{d + c\,d\,x\,1\mathrm{i}}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.556 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=111

$$\frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] f*(c*x+I)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b*f*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5796, 651, 5837, 12, 641, 31}

$$\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*f*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx &= \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{f(i+cx)}{c\left(1+c^2x^2\right)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(bf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{i+cx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(bf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{1}{-i+cx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 113, normalized size = 1.02

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)+b(i-cx)\log(d+icdx)\right)}{cd^2f(-i+cx)\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + b*(I - c*x)*Log[d + I*c*d*x]))/(c*d^2*f*(-I + c*x)*Sqrt
[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{(icdx + d)^{\frac{3}{2}} \sqrt{-icfx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [A]
time = 0.50, size = 98, normalized size = 0.88

$$\frac{i \sqrt{c^2 d f x^2 + d f}\, b \operatorname{arsinh}(cx)}{i c^2 d^2 f x + c d^2 f} + \frac{i \sqrt{c^2 d f x^2 + d f}\, a}{i c^2 d^2 f x + c d^2 f} - \frac{b \log(i cx + 1)}{c d^{\frac{3}{2}} \sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(I*c^2*d^2*f*x + c*d^2*f) + I*sqrt
(c^2*d*f*x^2 + d*f)*a/(I*c^2*d^2*f*x + c*d^2*f) - b*log(I*c*x + 1)/(c*d^(3/
2)*sqrt(f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(89) = 178$.
time = 0.45, size = 443, normalized size = 3.99

$$\frac{2 \sqrt{i cdx + d} \sqrt{-i cfx + f}\, b \log\left(cx + \sqrt{c^2x^2 + 1}\right) + (c^2d^2fx - i cd^2f) \sqrt{\frac{b^2}{c^2d^3f}} \log\left(-\frac{(i bc^6x^2 + 2 bc^5x - 2i bc^4) \sqrt{c^2x^2 + 1} \sqrt{i cdx + d} \sqrt{-i cfx + f} - (i c^9d^2fx^4 + 2 c^8d^2fx^3 + i c^7d^2fx^2 + 2 c^6d^2fx) \sqrt{\frac{b^2}{c^2d^3f}}}{8 (bc^3x^3 - i bc^2x^2 + bcx - i b)}\right) - (c^2d^2fx - i cd^2f) \sqrt{\frac{b^2}{c^2d^3f}} \log\left(-\frac{(i bc^6x^2 + 2 bc^5x - 2i bc^4) \sqrt{c^2x^2 + 1} \sqrt{i cdx + d} \sqrt{-i cfx + f} - (-i c^9d^2fx^4 - 2 c^8d^2fx^3 - i c^7d^2fx^2 - 2 c^6d^2fx) \sqrt{\frac{b^2}{c^2d^3f}}}{8 (bc^3x^3 - i bc^2x^2 + bcx - i b)}\right) + 2 \sqrt{i cdx + d} \sqrt{-i cfx + f}\, a}{2 (c^2d^2fx - i cd^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
+ (c^2*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(-1/8*((I*b*c^6*x^2 +
2*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f) - (I*c^9*d^2*f*x^4 + 2*c^8*d^2*f*x^3 + I*c^7*d^2*f*x^2 + 2*c^6*d^2*f*x)*
sqrt(b^2/(c^2*d^3*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - (c^2*d^2*
f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x -
```

```
 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - (-I*c^
9*d^2*f*x^4 - 2*c^8*d^2*f*x^3 - I*c^7*d^2*f*x^2 - 2*c^6*d^2*f*x)*sqrt(b^2/(
c^2*d^3*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 2*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*f*x - I*c*d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{asinh}\,(cx)}{(id\,(cx - i))^{\frac{3}{2}}\,\sqrt{-if\,(cx + i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asinh}(c\,x)}{{(d + c\,d\,x\,1\mathrm{i})}^{3/2}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.557 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=295

$$\frac{ibf^2(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^2x(1+c^2x^2)^2\left(a+b\,\mathrm{s}\right.}{3(d+icdx)^{5/2}(f-i}$$

```
[Out] 1/3*I*b*f^2*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+2/3*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)+1/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)-1/3*I*b*f^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*f^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.22, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 667, 197, 5837, 641, 46, 209, 266}

$$\frac{f^2x(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{ibf^2(c^2x^2+1)^{5/2}\,\mathrm{ArcTan}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf^2(c^2x^2+1)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bf^2(c^2x^2+1)^{5/2}\log\left(c^2x^2+1\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] ((I/3)*b*f^2*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*b*f
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*f^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)^2(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2\left(a+b\sinh\right.}{3(d+icdx)^{5/2}(f-icf}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2\left(a+b\sinh\right.}{3(d+icdx)^{5/2}(f-icf}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2\left(a+b\sinh\right.}{3(d+icdx)^{5/2}(f-icf}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2\left(a+b\sinh\right.}{3(d+icdx)^{5/2}(f-icf}$$

$$= \frac{ibf^2(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\,\mathrm{si}\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= \frac{ibf^2(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\,\mathrm{si}\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

Mathematica [A]
time = 0.29, size = 143, normalized size = 0.48

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\left((-2i+cx)\left(-ib+bcx+a\sqrt{1+c^2x^2}\right)+b(-2i+cx)\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)-b(-i+cx)^2\log(d+icdx)\right)}{3cd^3f(-i+cx)^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-2*I + c*x)*((-I)*b + b*c*x + a*Sqrt
[1 + c^2*x^2]) + b*(-2*I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*(-I + c*
x)^2*Log[d + I*c*d*x]))/(3*c*d^3*f*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\left(cx\right)}{\left(icdx+d\right)^{\frac{5}{2}}\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)

Maxima [A]
time = 0.49, size = 233, normalized size = 0.79

$$\frac{1}{3}\,bc\left(\frac{3}{3i\,c^{3}d^{\frac{5}{2}}\sqrt{f}\,x+3\,c^{2}d^{\frac{5}{2}}\sqrt{f}}-\frac{\log\left(cx-i\right)}{c^{2}d^{\frac{5}{2}}\sqrt{f}}\right)-\frac{1}{3}\,b\left(\frac{i\sqrt{c^{2}dfx^{2}+df}}{c^{3}d^{3}fx^{2}-2i\,c^{2}d^{3}fx-cd^{3}f}-\frac{3i\sqrt{c^{2}dfx^{2}+df}}{3i\,c^{2}d^{3}fx+3\,cd^{3}f}\right)\mathrm{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{i\sqrt{c^{2}dfx^{2}+df}}{c^{3}d^{3}fx^{2}-2i\,c^{2}d^{3}fx-cd^{3}f}-\frac{3i\sqrt{c^{2}dfx^{2}+df}}{3i\,c^{2}d^{3}fx+3\,cd^{3}f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/3*b*c*(3/(3*I*c^3*d^(5/2)*sqrt(f)*x + 3*c^2*d^(5/2)*sqrt(f)) - log(c*x -
I)/(c^2*d^(5/2)*sqrt(f))) - 1/3*b*(I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*f*x^2
 - 2*I*c^2*d^3*f*x - c*d^3*f) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*f*
x + 3*c*d^3*f))*arcsinh(c*x) - 1/3*a*(I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*f*
x^2 - 2*I*c^2*d^3*f*x - c*d^3*f) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3
*f*x + 3*c*d^3*f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(228) = 456.
time = 0.49, size = 576, normalized size = 1.95

$$-\frac{2\sqrt{c^{2}x^{2}+1}\sqrt{icdx+d}\sqrt{-icfx+f}\,bcx-2\left(bc^{2}x^{2}-ibcx+2b\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(c^{4}d^{3}fx^{3}-ic^{3}d^{3}fx^{2}+c^{2}d^{3}fx-icd^{3}f\right)\sqrt{\frac{b^{2}}{c^{2}d^{5}f}}\log\left(-\frac{\left(ibc^{6}x^{2}+2bc^{5}x-2ibc^{4}\right)\sqrt{c^{2}x^{2}+1}\sqrt{icdx+d}\sqrt{-icfx+f}+\left(ic^{9}d^{3}fx^{4}+2c^{8}d^{3}fx^{3}+ic^{7}d^{3}fx^{2}+2c^{6}d^{3}fx\right)\sqrt{\frac{b^{2}}{c^{2}d^{5}f}}}{8\left(bc^{3}x^{3}-ibc^{2}x^{2}+bcx-ib\right)}\right)-\left(c^{4}d^{3}fx^{3}-ic^{3}d^{3}fx^{2}+c^{2}d^{3}fx-icd^{3}f\right)\sqrt{\frac{b^{2}}{c^{2}d^{5}f}}\log\left(-\frac{\left(ibc^{6}x^{2}+2bc^{5}x-2ibc^{4}\right)\sqrt{c^{2}x^{2}+1}\sqrt{icdx+d}\sqrt{-icfx+f}+\left(-ic^{9}d^{3}fx^{4}-2c^{8}d^{3}fx^{3}-ic^{7}d^{3}fx^{2}-2c^{6}d^{3}fx\right)\sqrt{\frac{b^{2}}{c^{2}d^{5}f}}}{8\left(bc^{3}x^{3}-ibc^{2}x^{2}+bcx-ib\right)}\right)-2\left(ac^{2}x^{2}-iacx+2a\right)\sqrt{icdx+d}\sqrt{-icfx+f}}{6\left(c^{4}d^{3}fx^{3}-ic^{3}d^{3}fx^{2}+c^{2}d^{3}fx-icd^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 2*(b
*c^2*x^2 - I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) + (c^4*d^3*f*x^3 - I*c^3*d^3*f*x^2 + c^2*d^3*f*x - I*c*d^3
*f)*sqrt(b^2/(c^2*d^5*f))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^4)*s
qrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d^3*f*x^4 +
2*c^8*d^3*f*x^3 + I*c^7*d^3*f*x^2 + 2*c^6*d^3*f*x)*sqrt(b^2/(c^2*d^5*f)))/(
b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - (c^4*d^3*f*x^3 - I*c^3*d^3*f*x^2
+ c^2*d^3*f*x - I*c*d^3*f)*sqrt(b^2/(c^2*d^5*f))*log(-1/8*((I*b*c^6*x^2 + 2
*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
) + (-I*c^9*d^3*f*x^4 - 2*c^8*d^3*f*x^3 - I*c^7*d^3*f*x^2 - 2*c^6*d^3*f*x)*
sqrt(b^2/(c^2*d^5*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 2*(a*c^2*
x^2 - I*a*c*x + 2*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4*d^3*f*x^3 -
 I*c^3*d^3*f*x^2 + c^2*d^3*f*x - I*c*d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(cx\right)}{\left(id\left(cx-i\right)\right)^{\frac{5}{2}}\sqrt{-if\left(cx+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)

[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I))), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}\,\sqrt{f-c\,f\,x\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)

[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)

3.558 $\int \frac{(d+icdx)^{5/2}(a+b\sinh^{-1}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=517

$$\frac{3ibd^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{bcd^4x^2(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15bd^4(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f}$$

```
[Out] 3/2*I*b*d^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+b*c*d^4
*x^2*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+5/4*b*d^4*(1+I*c
*x)^2*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+15/4*b*d^4*(c
^2*x^2+1)^(3/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*d^
4*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x
)^(3/2)-15/2*I*d^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-5/2*I*d^4*(1+I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-15/2*d^4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)*(a+b
*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*d^4*(c^2*x^2+1)^(3
/2)*ln(c*x+I)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.28, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 683, 685, 655, 221, 5837, 641, 45, 5783}

$$-\frac{5id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{15id^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{15d^4\left(c^2x^2+1\right)^{3/2}\sinh^{-1}(cx)\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{bcd^4x^2\left(c^2x^2+1\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bd^4(1+icx)^2\left(c^2x^2+1\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{3ibd^4x\left(c^2x^2+1\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8bd^4\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15bd^4\left(c^2x^2+1\right)^{3/2}\sinh^{-1}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] (((3*I)/2)*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) + (b*c*d^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2)) + (5*b*d^4*(1 + I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)/
(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((2*I)*d^4*(1 + I*c*x)^3*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2
)) - (((15*I)/2)*d^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (((5*I)/2)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a
 + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*d^4*(
1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^4*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15id^4\left(1+c^2x^2\right)^2 \left(\ldots\right.}{2c(d+icdx)^{3/2}\ldots}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4\ldots}{\ldots}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd\ldots}{4c(\ldots}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd\ldots}{4c(\ldots}$$

$$= \frac{3ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcd^4x^2\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1\ldots}{4c(d+\ldots}$$

Mathematica [A]
time = 2.58, size = 781, normalized size = 1.51

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((4*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 - (7*I)*c*x + c^2*x^2))/(
f^2*(I + c*x)) - (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]])/f^(3/2) + (4*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*
ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcT
an[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Si
nh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])) + (16*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSi
nh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTa
n[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sin
h[ArcSinh[c*x]/2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSin
h[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 +
c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (b*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*S
inh[ArcSinh[c*x]/2]) + (16*c*x + 32*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Cosh[2
*ArcSinh[c*x]] + (8*I)*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[Arc
Sinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 - 8*Sqrt[1 + c^2*x^
2] - I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((-8*I)*(1 + Sqrt[1 + c
^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x)/
(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a + b*integrate((
I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x -
 a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x -
f^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

3.559 $\int \frac{(d+icdx)^{3/2}(a+b\sinh^{-1}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=283

$$\frac{ibd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\,(1+c^2x^2)\,\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3(1+c^2x^2)^2\,\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] I*b*d^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^3*(1+
I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
-I*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-3/2*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-4*b*d^3*(c^2*x^2+1)^(3/2)*ln(c*x+I)/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)
```

Rubi [A]

time = 0.32, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5796, 5844, 651, 5837, 12, 641, 31, 5783, 5798, 8}

$$-\frac{3d^3(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{ibd^3x(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4bd^3(c^2x^2+1)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] (I*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 ((4*I)*d^3*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*d^3*(1 + c^2*x^2)^(3/2)*(
a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4
*b*d^3*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^3(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{4i(id^3-cd^3x)(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}} - \frac{3d^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} - \ldots\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i(1+c^2x^2)^{3/2}\right)\int \frac{(id^3-cd^3x)(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3d^3(1+c^2x^2)^{\ldots}\right.}{(d+icdx)^{\ldots}}$$

$$= -\frac{4id^3(1+icx)(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(1+c^2x^2)^2(a\ldots}{c(d+icdx)^{3/2}(\ldots}$$

$$= \frac{ibd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)(1+c^2x^2)\left(a+b\sinh^{\ldots}\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{ibd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)(1+c^2x^2)\left(a+b\sinh^{\ldots}\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{ibd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)(1+c^2x^2)\left(a+b\sinh^{\ldots}\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A]
time = 1.55, size = 515, normalized size = 1.82

$$\frac{\frac{2ad(5-icx)\sqrt{d+icdx}\sqrt{f-icfx}}{f^2(i+cx)} - \frac{6ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}} + \frac{bd\sqrt{d+icdx}\sqrt{f-icfx}\left(-\sinh^{-1}(cx)^2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+4\sinh^{-1}(cx)\left(-i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+2\left(4\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+i\log(1+c^2x^2)\right)\left(i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)}{f^2\sqrt{1+c^2x^2}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)} + \frac{2bd\sqrt{d+icdx}\sqrt{f-icfx}\left(-\sinh^{-1}(cx)^2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+\left(cx-4\text{ArcTan}\left(\coth\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+i\log(1+c^2x^2)\right)\left(i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+\sinh^{-1}(cx)\left(-i\left(2+\sqrt{1+c^2x^2}\right)\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-\left(-2+\sqrt{1+c^2x^2}\right)\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)}{f^2\sqrt{1+c^2x^2}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((2*a*d*(5 - I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^2*(I + c*x)) -
(6*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]])/f^(3/2) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cos
h[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2
```

```
]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/
(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (
2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c
*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]] +
 I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + ArcS
inh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1
+ c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))}{(-icfx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 6*I
*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/(c*f
^2*sqrt(d/f))) + b*integrate((I*c*d*x + d)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1
))/(-I*c*f*x + f)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c*d*x - b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a*c*d*x - a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2),
 x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

3.560 $\int \frac{\sqrt{d+icdx}\,(a+b\sinh^{-1}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{2id^2(1+icx)\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{d^2(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd^2(1+c^2x^2)^{3/2}\log(i+}{c(d+icdx)^{3/2}(f-icfx}$$

```
[Out] -2*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-1/2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)
^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d^2*(c^2*x^2+1)^(3/2)*ln(c*x+I)/c/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]

time = 0.26, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 5844, 651, 5837, 12, 641, 31, 5783}

$$-\frac{d^2(c^2x^2+1)^{3/2}\,(a+b\sinh^{-1}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^2(1+icx)\,(c^2x^2+1)\,(a+b\sinh^{-1}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd^2(c^2x^2+1)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((-2*I)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*d^2*(1 + c^2*x^
2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\,\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{2i\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(d^2\left(1+c^2x^2\right)^{3/2}\right)\ldots}{(d+icdx)^{3/2}(\ldots}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+\ldots\right.}{2bc(d+icdx)^{3/2}(f\ldots}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+\ldots\right.}{2bc(d+icdx)^{3/2}(f\ldots}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+\ldots\right.}{2bc(d+icdx)^{3/2}(f\ldots}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+\ldots\right.}{2bc(d+icdx)^{3/2}(f\ldots}$$

Mathematica [A]
time = 0.78, size = 285, normalized size = 1.58

$$\frac{\frac{4a\sqrt{d+icdx}\,\sqrt{f-icfx}}{i+cx}-2a\sqrt{d}\,\sqrt{f}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+\frac{b\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(-\sinh^{-1}(cx)^2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+4\sinh^{-1}(cx)\left(-i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+2\left(4\text{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+i\log\left(1+c^2x^2\right)\right)\left(i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)}{\sqrt{1+c^2x^2}\,\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}}{2cf^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 2*a*Sqrt[d]*Sqrt[f]*
Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (b*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] -
I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh
[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])
*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh
[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c*f^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\sqrt{icdx+d}}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/(c*
f^2*sqrt(d/f))) + b*integrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 + 1)
)/(-I*c*f*x + f)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*f^2*x^2 + 2*I*c*f^2*x -
f^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{id\left(cx-i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(-if\left(cx+i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{(f-c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

3.561 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)-b*d*(c^2*x^2+1)^(3/2)*ln(c*x+I)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5796, 651, 5837, 12, 641, 31}

$$-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] -((d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))) - (b*d*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,(f-icfx)^{3/2}}\,dx &= \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{d(i-cx)}{c(1+c^2x^2)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(bd\left(1+c^2x^2\right)^{3/2}\right)\int \frac{i-cx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(bd\left(1+c^2x^2\right)^{3/2}\right)\int \frac{1}{-i-cx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 94, normalized size = 0.84

$$\frac{\sqrt{f-icfx}\left(a+iacx+(b+ibcx)\sinh^{-1}(cx)-ib\sqrt{1+c^2x^2}\log(d(-1+icx))\right)}{cf^2(i+cx)\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*(a + I*a*c*x + (b + I*b*c*x)*ArcSinh[c*x] - I*b*Sqrt[1 +
 c^2*x^2]*Log[d*(-1 + I*c*x)]))/(c*f^2*(I + c*x)*Sqrt[d + I*c*d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{(-icfx + f)^{\frac{3}{2}} \sqrt{icdx + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [A]
time = 0.49, size = 98, normalized size = 0.88

$$-\frac{i \sqrt{c^2 df x^2 + df}\, b \operatorname{arsinh}(cx)}{-i\, c^2 df^2 x + cdf^2} - \frac{i \sqrt{c^2 df x^2 + df}\, a}{-i\, c^2 df^2 x + cdf^2} - \frac{b \log(i\, cx - 1)}{c\sqrt{d}\, f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(-I*c^2*d*f^2*x + c*d*f^2) - I*sq
rt(c^2*d*f*x^2 + d*f)*a/(-I*c^2*d*f^2*x + c*d*f^2) - b*log(I*c*x - 1)/(c*sq
rt(d)*f^(3/2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(88) = 176.
time = 0.43, size = 443, normalized size = 3.96

$$\frac{2\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right) - (c^2df^2x+i\,cdf^2)\sqrt{\frac{b^2}{c^2df^3}}\log\left(-\frac{(-i\,bc^6x^2+2\,bc^5x+2i\,bc^4)\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f} - (i\,c^9df^2x^4-2\,c^8df^2x^3+i\,c^7df^2x^2-2\,c^6df^2x)\sqrt{\frac{b^2}{c^2df^3}}}{8\,(bc^3x^3+i\,bc^2x^2+bcx+i\,b)}\right) + (c^2df^2x+i\,cdf^2)\sqrt{\frac{b^2}{c^2df^3}}\log\left(-\frac{(-i\,bc^6x^2+2\,bc^5x+2i\,bc^4)\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f} - (-i\,c^9df^2x^4+2\,c^8df^2x^3-i\,c^7df^2x^2+2\,c^6df^2x)\sqrt{\frac{b^2}{c^2df^3}}}{8\,(bc^3x^3+i\,bc^2x^2+bcx+i\,b)}\right) + 2\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,a}{2\,(c^2df^2x+i\,cdf^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
- (c^2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(-1/8*((-I*b*c^6*x^2 +
 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f) - (I*c^9*d*f^2*x^4 - 2*c^8*d*f^2*x^3 + I*c^7*d*f^2*x^2 - 2*c^6*d*f^2*x)
*sqrt(b^2/(c^2*d*f^3)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + (c^2*d*f
^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x
 + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - (-I*
```

```
c^9*d*f^2*x^4 + 2*c^8*d*f^2*x^3 - I*c^7*d*f^2*x^2 + 2*c^6*d*f^2*x)*sqrt(b^2
/(c^2*d*f^3)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + 2*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f^2*x + I*c*d*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{asinh}(cx)}{\sqrt{id(cx-i)}\,(-if(cx+i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{asinh}(c\,x)}{\sqrt{d + c\,d\,x\,1\mathrm{i}}\,(f - c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.562 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b(1+c^2x^2)^{3/2}\log\left(1+c^2x^2\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*
(c^2*x^2+1)^(3/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {5796, 5787, 266}

$$\frac{x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b(c^2x^2+1)^{3/2}\log\left(c^2x^2+1\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) - (b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2*c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{a+b\sinh^{-1}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$= \frac{x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bc(1+c^2x^2)^{3/2}\right)\int \frac{x}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$= \frac{x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b(1+c^2x^2)^{3/2}\log\left(1+c^2x^2\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [**A**]
time = 0.27, size = 118, normalized size = 1.15

$$\frac{i\sqrt{f-icfx}\left(2acx+2bcx\sinh^{-1}(cx)-b\sqrt{1+c^2x^2}\log(d(-1+icx))-b\sqrt{1+c^2x^2}\log(d+icdx)\right)}{2cdf^2(i+cx)\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((I/2)*Sqrt[f - I*c*f*x]*(2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x
^2]*Log[d*(-1 + I*c*x)] - b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(c*d*f^2*(
I + c*x)*Sqrt[d + I*c*d*x])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{3}{2}}\,(-icfx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [**A**]
time = 0.27, size = 82, normalized size = 0.80

$$\frac{bx\,\mathrm{arsinh}\,(cx)}{\sqrt{c^2dfx^2+df}\,df} + \frac{ax}{\sqrt{c^2dfx^2+df}\,df} - \frac{b\sqrt{\frac{1}{df}}\,\log\left(x^2+\frac{1}{c^2}\right)}{2\,cdf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorithm="maxima")

[Out] b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f) - 1/2*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c*d*f)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorithm="fricas")

[Out] 1/4*(4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*x*log(c*x + sqrt(c^2*x^2 + 1)) + 4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*x + (c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^4 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^4 + 2*b*c^2*x^2 + b)) - (c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^4 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^4 + 2*b*c^2*x^2 + b)) - 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)) + 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)) + 4*(c^2*d^2*f^2*x^2 + d^2*f^2)*integral(-sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x/(c^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2*d^2*f^2*x^2 + d^2*f^2)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{\left(id \left(cx - i\right)\right)^{\frac{3}{2}} \left(-if \left(cx + i\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)

[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3/2)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.563 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{ibf(1+c^2x^2)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2fx(1+c^2x^2)^2\left(a+b\sinh^{-1}\right.}{3(d+icdx)^{5/2}(f-icfx}$$

```
[Out] 1/6*I*b*f*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1
/3*f*(c*x+I)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)-1/6*I*b*f*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-1/3*b*f*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.20, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 653, 197, 5837, 641, 46, 209, 266}

$$\frac{2fx(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{ibf(c^2x^2+1)^{5/2}\,\text{ArcTan}(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf(c^2x^2+1)^{5/2}}{6c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bf(c^2x^2+1)^{5/2}\log\left(c^2x^2+1\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((I/6)*b*f*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) + (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/6)*b*f*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*f*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2\left(a+b\sinh^{-1}(\right.}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2\left(a+b\sinh^{-1}(\right.}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2\left(a+b\sinh^{-1}(\right.}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2\left(a+b\sinh^{-1}(\right.}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= \frac{ibf(1+c^2x^2)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= \frac{ibf(1+c^2x^2)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

Mathematica [A]
time = 0.38, size = 201, normalized size = 0.71

$$\frac{\sqrt{f-icfx}\left(4ia+8acx+8iac^2x^2+2b\sqrt{1+c^2x^2}+4b(i+2cx+2ic^2x^2)\sinh^{-1}(cx)+3b(-1-icx)\sqrt{1+c^2x^2}\log(d(-1+icx))-5b\sqrt{1+c^2x^2}\log(d+icdx)-5ibcx\sqrt{1+c^2x^2}\log(d+icdx)\right)}{12d^2f^2\sqrt{d+icdx}\,(c+c^3x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((4*I)*a + 8*a*c*x + (8*I)*a*c^2*x^2 + 2*b*Sqrt[1 + c^2*
x^2] + 4*b*(I + 2*c*x + (2*I)*c^2*x^2)*ArcSinh[c*x] + 3*b*(-1 - I*c*x)*Sqrt
[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] - 5*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]
- (5*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*d^2*f^2*Sqrt[d + I*c
*d*x]*(c + c^3*x^2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\left(cx\right)}{\left(icdx+d\right)^{\frac{5}{2}}\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [A]
time = 0.29, size = 237, normalized size = 0.84

$$\frac{1}{12}bc\left(-\frac{2i\sqrt{d}\sqrt{f}}{c^3d^3f^2x - i\,c^2d^3f^2} - \frac{3\log\left(cx+i\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}} - \frac{5\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}\right) - \frac{1}{3}b\left(-\frac{3i}{3i\sqrt{c^2dfx^2+df}\,c^2d^2fx + 3\sqrt{c^2dfx^2+df}\,cd^2f} - \frac{2x}{\sqrt{c^2dfx^2+df}\,d^2f}\right)\operatorname{arsinh}\left(cx\right) - \frac{1}{3}a\left(-\frac{3i}{3i\sqrt{c^2dfx^2+df}\,c^2d^2fx + 3\sqrt{c^2dfx^2+df}\,cd^2f} - \frac{2x}{\sqrt{c^2dfx^2+df}\,d^2f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*b*c*(-2*I*sqrt(d)*sqrt(f)/(c^3*d^3*f^2*x - I*c^2*d^3*f^2) - 3*log(c*x
+ I)/(c^2*d^(5/2)*f^(3/2)) - 5*log(c*x - I)/(c^2*d^(5/2)*f^(3/2))) - 1/3*b*
(-3*I/(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*f)*
c*d^2*f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))*arcsinh(c*x) - 1/3*a*(-3*I/
(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c*d^2*
f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] -1/24*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 8*(
2*b*c^2*x^2 - 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) - 5*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*
x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + I*b*c^2*x^
3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 3*(c^4*d^3*f^2*x^3 -
I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*lo
g(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt
(b^2/(c^2*d^5*f^3)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x
 + I*b)) + 5*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3
*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + I*b*c^2*x^3 + I*b*x)
/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 3*(c^4*d^3*f^2*x^3 - I*c^3*d^3*
f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(-I*sqr
t(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2
*d^5*f^3)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b))
+ 8*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqr
```

```
t(b^2/(c^2*d^5*f^3))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 + b))
 - 8*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sq
rt(b^2/(c^2*d^5*f^3))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f
*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b
)) - 8*(2*a*c^2*x^2 - 2*I*a*c*x + a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) -
 24*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*int
egral(-1/6*sqrt(c^2*x^2 + 1)*(4*b*c*x + I*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*
x + f)/(c^4*d^3*f^2*x^4 + 2*c^2*d^3*f^2*x^2 + d^3*f^2), x))/(c^4*d^3*f^2*x^
3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.564 $\int \frac{(d+icdx)^{5/2}(a+b\sinh^{-1}(cx))}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=470

$$-\frac{ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8ibd^5(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{5bd^5(1+c^2x^2)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^5(1-}{}$$

```
[Out] -I*b*d^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b*d^
5*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5/2*b*d^5
*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3
*I*d^5*(1+I*c*x)^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)+10/3*I*d^5*(1+I*c*x)^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I
*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*I*d^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c/
(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*d^5*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a
+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*b*d^5*(c^2*x^2+
1)^(5/2)*ln(c*x+I)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]

time = 0.30, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 683, 655, 221, 5837, 641, 45, 5783}

$$\frac{5id^5(c^2x^2+1)^3(a+b\sinh^{-1}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{10id^5(1+icx)^2(c^2x^2+1)^2(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^5(1+icx)^4(c^2x^2+1)(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5d^5(c^2x^2+1)^{5/2}\sinh^{-1}(cx)(a+b\sinh^{-1}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{ibd^5x(c^2x^2+1)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8ibd^5(c^2x^2+1)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{28bd^5(c^2x^2+1)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{5bd^5(c^2x^2+1)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-I)*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)
) + (((8*I)/3)*b*d^5*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (5*b*d^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^5*(1 + I*c*x)^4*(1 +
c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (((10*I)/3)*d^5*(1 + I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((5*I)*d^5*(1 + c^2*x^2)^3*(a + b*
ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*d^5*(1 + c^
2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + (28*b*d^5*(1 + c^2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
```

```
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^5(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^5(1+icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{10id^5(1+icx)^2(1}{3c(d+ic}$$

$$= -\frac{5ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4\left(1+c^2x^2\right)\left(a+b\sinh}{3c(d+icdx)^{5/2}(f-icfx)^{5/}}$$

$$= -\frac{5ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5(1+c^2x^2)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id}{}$$

$$= -\frac{5ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5(1+c^2x^2)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id}{}$$

$$= -\frac{ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8ibd^5(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1083 vs. 2(470) = 940.
time = 6.14, size = 1083, normalized size = 2.30

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((4*I)*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 + (34*I)*c*x + 3*c^2
*x^2))/(f^3*(I + c*x)^2) + (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/f^(5/2) - ((2*I)*b*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(
3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*L
og[1 + c^2*x^2])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[C
oth[ArcSinh[c*x]/2]] + ((3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*ArcSinh[c
*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt[1 + c^2
*x^2]*((2*I)*ArcSinh[c*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^
2*x^2]))/2)*Sinh[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] -
```

```
 I*Sinh[ArcSinh[c*x]/2])^4) + (2*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*(
(14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
- 7*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*Ar
cSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 21*Log[1 + c^2*x^2]) -
 (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I
+ 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2
]))*Sinh[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[
ArcSinh[c*x]/2])^4) - (I*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[Ar
cSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(9 - (3
5*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c*x]/2]]
+ 13*Log[1 + c^2*x^2])) + Cosh[ArcSinh[c*x]/2]*(20 + (24*I)*ArcSinh[c*x] +
27*ArcSinh[c*x]^2 + (156*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 39*Log[1 + c^2*x
^2]) - I*(3*(-I + ArcSinh[c*x])*Cosh[(5*ArcSinh[c*x])/2] + 2*(13 + (7*I)*Ar
cSinh[c*x] + 18*ArcSinh[c*x]^2 + (104*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*
I)*(I + ArcSinh[c*x])*Cosh[2*ArcSinh[c*x]] + 26*Log[1 + c^2*x^2] + Sqrt[1 +
 c^2*x^2]*(6 + (38*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[
ArcSinh[c*x]/2]] + 13*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2])))/(f^3*(-I +
 c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))}{(-icfx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*(-3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*f^5*x^4 + 4*I*c^4*f^5*x^3 - 6*c^3
*f^5*x^2 - 4*I*c^2*f^5*x + c*f^5) + 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*d/(3*I*c
^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9*I*c^2*f^4*x + 3*c*f^4) - 10*I*sqrt(c^2*d*f*x
^2 + d*f)*d^2/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 105*I*sqrt(c^2*d*f*x^
```

```
2 + d*f)*d^2/(-3*I*c^2*f^3*x + 3*c*f^3) - 15*d^3*arcsinh(c*x)/(c*f^3*sqrt(d
/f)))*a + b*integrate((I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*
c*f*x + f)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c^2*d^2*x^2 + 2*b*c*d^2*x - I*b*d^2)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*d^2*x^2 + 2*a*c*d^2*x
 - I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^
3*x^2 - 3*c*f^3*x - I*f^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

3.565 $\int \frac{(d+icdx)^{3/2}(a+b\sinh^{-1}(cx))}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=362

$$\frac{4ibd^4(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd^4(1+c^2x^2)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\,(1+c^2x^2)\,\left(a+b\sinh^{-}\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 4/3*I*b*d^4*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/2*b*d^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)-2/3*I*d^4*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)+2*I*d^4*(1+I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+d^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*d^4*(c^2*x^2+1)
^(5/2)*ln(c*x+I)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]

time = 0.26, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 683, 667, 221, 5837, 641, 45, 31, 5783}

$$\frac{2id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4ibd^4\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8bd^4\left(c^2x^2+1\right)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd^4\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((4*I)/3)*b*d^4*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^4*(1 + I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((2*
I)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^4*(1 + c^
2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
```

```
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx=\frac{\left(1+c^2x^2\right)^{5/2}\int\frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=-\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2id^4(1+icx)\,(1-\ldots}{c(d+icd\ldots}$$

$$=-\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2id^4(1+icx)\,(1-\ldots}{c(d+icd\ldots}$$

$$=-\frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\ldots\right.}{3c(d+icdx)^{5/2}(f-icf\ldots}$$

$$=-\frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\ldots\right.}{3c(d+icdx)^{5/2}(f-icf\ldots}$$

$$=\frac{4ibd^4\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(c\ldots}{2c(d+icdx)^{5/2}(f-icfx)\ldots}$$

Mathematica [A]
time = 3.84, size = 706, normalized size = 1.95

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-16*a*d*(I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^3*(I + c*x)^2
) + (12*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]])/f^(5/2) - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[A
rcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSi
nh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*Log[1 + c^2*x^2])) + Cosh[
ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + ((
3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*ArcSinh[c*x] + (4*I)*ArcTan[Coth[A
rcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt[1 + c^2*x^2]*((2*I)*ArcSinh[c*x]
 + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))/2)*Sinh[ArcSinh[
c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^
```

```
4) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSi
nh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 7*Log[1 + c^2*x^2]) + Cosh[
ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[
Tanh[ArcSinh[c*x]/2]] + 21*Log[1 + c^2*x^2]) - (2*I)*(4 + (4*I)*ArcSinh[c*x
] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2
*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*Ar
cTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(f
^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))}{(-icfx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*a*(3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(3*I*c^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9*
I*c^2*f^4*x + 3*c*f^4) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*d/(c^3*f^3*x^2 + 2*I*c
^2*f^3*x - c*f^3) - 21*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-3*I*c^2*f^3*x + 3*c*f^
3) - 3*d^2*arcsinh(c*x)/(c*f^3*sqrt(d/f))) + b*integrate((I*c*d*x + d)^(3/2
)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I*f^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

3.566 $\int \frac{\sqrt{d+icdx}\,(a+b\sinh^{-1}(cx))}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{2ibd^3(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{id^3(1+icx)^3\,(1+c^2x^2)\,\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^3(1+c^2x^2)^{5/2}\log(i+c$$

```
[Out] 2/3*I*b*d^3*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/3*I*d^3*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b*d^3*(c^2*x^2+1)^(5/2)*ln(c*x+I)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)
```

Rubi [A]

time = 0.20, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5796, 665, 5837, 12, 641, 45}

$$-\frac{id^3(1+icx)^3\,(c^2x^2+1)\,\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibd^3(c^2x^2+1)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^3(c^2x^2+1)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((2*I)/3)*b*d^3*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*d^3*(1 + c^2*x^2)^(5/
2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\,\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^3(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc(1+c^2x^2)^{5/2}\right)\int -}{(d+icdx)^{5/2}(f-}$$

$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3(1+c^2x^2)^{5/2}\right)\int}{3(d+icdx)^{5/2}(f-}$$

$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3(1+c^2x^2)^{5/2}\right)\int}{3(d+icdx)^{5/2}(f-}$$

$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3(1+c^2x^2)^{5/2}\right)\int}{3(d+icdx)^{5/2}}$$

$$= \frac{2ibd^3(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id^3(1+icx)^3\left(1+c^2x^2\right)(a+}{3c(d+icdx)^{5/2}(f-ic}$$

Mathematica [A]
time = 0.26, size = 131, normalized size = 0.71

$$-\frac{id\sqrt{f-icfx}\left((-i+cx)\left(-ia+acx+b\sqrt{1+c^2x^2}\right)+b(-i+cx)^2\sinh^{-1}(cx)-b(i+cx)\sqrt{1+c^2x^2}\,\log(d(-1+icx))\right)}{3cf^3(i+cx)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-1/3*I)*d*Sqrt[f - I*c*f*x]*((-I + c*x)*((-I)*a + a*c*x + b*Sqrt[1 + c^2*
x^2]) + b*(-I + c*x)^2*ArcSinh[c*x] - b*(I + c*x)*Sqrt[1 + c^2*x^2]*Log[d*(
-1 + I*c*x)]))/(c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\sqrt{icdx+d}}{\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [A]
time = 0.29, size = 220, normalized size = 1.19

$$-\frac{1}{3}\,bc\left(\frac{6\,\sqrt{d}}{3i\,c^{3}f^{\frac{5}{2}}x-3\,c^{2}f^{\frac{5}{2}}}-\frac{\sqrt{d}\,\log\left(cx+i\right)}{c^{2}f^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(-\frac{2i\,\sqrt{c^{2}dfx^{2}+df}}{c^{3}f^{3}x^{2}+2i\,c^{2}f^{3}x-cf^{3}}-\frac{3i\,\sqrt{c^{2}dfx^{2}+df}}{-3i\,c^{2}f^{3}x+3\,cf^{3}}\right)\operatorname{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(-\frac{2i\,\sqrt{c^{2}dfx^{2}+df}}{c^{3}f^{3}x^{2}+2i\,c^{2}f^{3}x-cf^{3}}-\frac{3i\,\sqrt{c^{2}dfx^{2}+df}}{-3i\,c^{2}f^{3}x+3\,cf^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*b*c*(6*sqrt(d)/(3*I*c^3*f^(5/2)*x - 3*c^2*f^(5/2)) - sqrt(d)*log(c*x +
 I)/(c^2*f^(5/2))) - 1/3*b*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 + 2*I
*c^2*f^3*x - c*f^3) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*f^3
))*arcsinh(c*x) - 1/3*a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 + 2*I*c^
2*f^3*x - c*f^3) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*f^3))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(141) = 282$.
time = 0.46, size = 548, normalized size = 2.96

$$-\frac{4\,\sqrt{c^{2}x^{2}+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,bcx+2\,(bc^{2}x^{2}-2i\,bcx-b)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+(c^{4}f^{3}x^{3}+i\,c^{3}f^{3}x^{2}+c^{2}f^{3}x+i\,cf^{3})\sqrt{\frac{b^{2}d}{c^{2}f^{5}}}\,\log\left(-\frac{(-i\,bc^{6}x^{2}+2\,bc^{5}x+2i\,bc^{4})\sqrt{c^{2}x^{2}+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}+(i\,c^{9}f^{3}x^{4}-2\,c^{8}f^{3}x^{3}+i\,c^{7}f^{3}x^{2}-2\,c^{6}f^{3}x)\sqrt{\frac{b^{2}d}{c^{2}f^{5}}}}{8\,(bc^{3}x^{3}+i\,bc^{2}x^{2}+bcx+i\,b)}\right)-(c^{4}f^{3}x^{3}+i\,c^{3}f^{3}x^{2}+c^{2}f^{3}x+i\,cf^{3})\sqrt{\frac{b^{2}d}{c^{2}f^{5}}}\,\log\left(-\frac{(-i\,bc^{6}x^{2}+2\,bc^{5}x+2i\,bc^{4})\sqrt{c^{2}x^{2}+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}+(-i\,c^{9}f^{3}x^{4}+2\,c^{8}f^{3}x^{3}-i\,c^{7}f^{3}x^{2}+2\,c^{6}f^{3}x)\sqrt{\frac{b^{2}d}{c^{2}f^{5}}}}{8\,(bc^{3}x^{3}+i\,bc^{2}x^{2}+bcx+i\,b)}\right)+2\,(ac^{2}x^{2}-2i\,acx-a)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}}{6\,(c^{4}f^{3}x^{3}+i\,c^{3}f^{3}x^{2}+c^{2}f^{3}x+i\,cf^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 2*(b
*c^2*x^2 - 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) + (c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x + I*c*f^3)*sqrt
(b^2*d/(c^2*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2
*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*f^3*x^4 - 2*c^8*f^3
*x^3 + I*c^7*f^3*x^2 - 2*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)))/(b*c^3*x^3 + I*b
*c^2*x^2 + b*c*x + I*b)) - (c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x + I*c*f
^3)*sqrt(b^2*d/(c^2*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*
sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (-I*c^9*f^3*x^4 +
2*c^8*f^3*x^3 - I*c^7*f^3*x^2 + 2*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)))/(b*c^3*
x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + 2*(a*c^2*x^2 - 2*I*a*c*x - a)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x + I*c
*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{id\left(cx-i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(-if\left(cx+i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{{(f-c\,f\,x\,1\mathrm{i})}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

3.567 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=294

$$\frac{ibd^2(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^2x(1+c^2x^2)^2\left(a+b\,\mathrm{si}\right.}{3(d+icdx)^{5/2}(f-i}$$

```
[Out] 1/3*I*b*d^2*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-2/3*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)+1/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)+1/3*I*b*d^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*d^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.22, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 667, 197, 5837, 641, 46, 209, 266}

$$\frac{d^2x(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2(c^2x^2+1)^{5/2}\,\mathrm{ArcTan}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2(c^2x^2+1)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd^2(c^2x^2+1)^{5/2}\log\left(c^2x^2+1\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/3)*b*d^2*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*b*d
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*d^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,(f-icfx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^2(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2\,(a+b\sinh}{3(d+icdx)^{5/2}(f-ic}$$

$$= -\frac{2id^2(1+icx)\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2\,(a+b\sinh}{3(d+icdx)^{5/2}(f-ic}$$

$$= -\frac{2id^2(1+icx)\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2\,(a+b\sinh}{3(d+icdx)^{5/2}(f-ic}$$

$$= -\frac{2id^2(1+icx)\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2\,(a+b\sinh}{3(d+icdx)^{5/2}(f-ic}$$

$$= \frac{ibd^2(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)\,(1+c^2x^2)\,(a+b\sin}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= \frac{ibd^2(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)\,(1+c^2x^2)\,(a+b\sin}{3c(d+icdx)^{5/2}(f-icfx)}$$

Mathematica [A]
time = 0.25, size = 139, normalized size = 0.47

$$\frac{\sqrt{f-icfx}\left((2i+cx)\left(a+iacx+ib\sqrt{1+c^2x^2}\right)+ib(2+icx+c^2x^2)\sinh^{-1}(cx)+b(1-icx)\sqrt{1+c^2x^2}\log(d(-1+icx))\right)}{3cf^3(i+cx)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((2*I + c*x)*(a + I*a*c*x + I*b*Sqrt[1 + c^2*x^2]) + I*b
*(2 + I*c*x + c^2*x^2)*ArcSinh[c*x] + b*(1 - I*c*x)*Sqrt[1 + c^2*x^2]*Log[d
*(-1 + I*c*x)]))/(3*c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [A]
time = 0.50, size = 232, normalized size = 0.79

$$-\frac{1}{3}\,bc\left(\frac{3}{3i\,c^3\sqrt{d}\,f^{\frac{5}{2}}x-3\,c^2\sqrt{d}\,f^{\frac{5}{2}}}+\frac{\log\left(cx+i\right)}{c^2\sqrt{d}\,f^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(-\frac{i\,\sqrt{c^2dfx^2+df}}{c^3df^3x^2+2i\,c^2df^3x-cdf^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2df^3x+3\,cdf^3}\right)\mathrm{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(-\frac{i\,\sqrt{c^2dfx^2+df}}{c^3df^3x^2+2i\,c^2df^3x-cdf^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2df^3x+3\,cdf^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*b*c*(3/(3*I*c^3*sqrt(d)*f^(5/2)*x - 3*c^2*sqrt(d)*f^(5/2)) + log(c*x +
 I)/(c^2*sqrt(d)*f^(5/2))) - 1/3*b*(-I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d*f^3*x
^2 + 2*I*c^2*d*f^3*x - c*d*f^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*d*f
^3*x + 3*c*d*f^3))*arcsinh(c*x) - 1/3*a*(-I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d*
f^3*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^
2*d*f^3*x + 3*c*d*f^3))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(228) = 456.
time = 0.46, size = 576, normalized size = 1.96

$$-\frac{2\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,bcx-2\,(bc^2x^2+i\,bcx+2\,b)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)-(c^4df^3x^3+i\,c^3df^3x^2+c^2df^3x+i\,cdf^3)\sqrt{\frac{b^2}{c^2df^5}}\,\log\left(-\frac{(-i\,bc^6x^2+2\,bc^5x+2i\,bc^4)\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}+(i\,c^9df^3x^4-2\,c^8df^3x^3+i\,c^7df^3x^2-2\,c^6df^3x)\sqrt{\frac{b^2}{c^2df^5}}}{8\,(bc^3x^3+i\,bc^2x^2+bcx+i\,b)}\right)+(c^4df^3x^3+i\,c^3df^3x^2+c^2df^3x+i\,cdf^3)\sqrt{\frac{b^2}{c^2df^5}}\,\log\left(-\frac{(-i\,bc^6x^2+2\,bc^5x+2i\,bc^4)\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}+(-i\,c^9df^3x^4+2\,c^8df^3x^3-i\,c^7df^3x^2+2\,c^6df^3x)\sqrt{\frac{b^2}{c^2df^5}}}{8\,(bc^3x^3+i\,bc^2x^2+bcx+i\,b)}\right)-2\,(ac^2x^2+i\,acx+2\,a)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}}{6\,(c^4df^3x^3+i\,c^3df^3x^2+c^2df^3x+i\,cdf^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 2*(b
*c^2*x^2 + I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) - (c^4*d*f^3*x^3 + I*c^3*d*f^3*x^2 + c^2*d*f^3*x + I*c*d*f
^3)*sqrt(b^2/(c^2*d*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*
sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d*f^3*x^4 -
 2*c^8*d*f^3*x^3 + I*c^7*d*f^3*x^2 - 2*c^6*d*f^3*x)*sqrt(b^2/(c^2*d*f^5)))/
(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + (c^4*d*f^3*x^3 + I*c^3*d*f^3*x^2
 + c^2*d*f^3*x + I*c*d*f^3)*sqrt(b^2/(c^2*d*f^5))*log(-1/8*((-I*b*c^6*x^2 +
 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f) + (-I*c^9*d*f^3*x^4 + 2*c^8*d*f^3*x^3 - I*c^7*d*f^3*x^2 + 2*c^6*d*f^3*x
)*sqrt(b^2/(c^2*d*f^5)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) - 2*(a*c^
2*x^2 + I*a*c*x + 2*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4*d*f^3*x^3
 + I*c^3*d*f^3*x^2 + c^2*d*f^3*x + I*c*d*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{asinh}\left(cx\right)}{\sqrt{id\left(cx-i\right)}\,\left(-if\left(cx+i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,{\left(f-c\,f\,x\,1\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.568 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=282

$$\frac{ibd(1+c^2x^2)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{d(i-cx)\,(1+c^2x^2)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2dx(1+c^2x^2)^2\,(a+b\sinh^{-1}(}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

```
[Out] 1/6*I*b*d*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1
/3*d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*d*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+1/6*I*b*d*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-1/3*b*d*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.20, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 653, 197, 5837, 641, 46, 209, 266}

$$\frac{2dx(c^2x^2+1)^2\,(a+b\sinh^{-1}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{d(-cx+i)\,(c^2x^2+1)\,(a+b\sinh^{-1}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd(c^2x^2+1)^{5/2}\,\text{ArcTan}(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd(c^2x^2+1)^{5/2}}{6c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd(c^2x^2+1)^{5/2}\log(c^2x^2+1)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/6)*b*d*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) - (d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/6)*b*d*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*d*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)(a+b\sinh^{-1}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2\left(a+b\sinh^{-1}\right.}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= -\frac{d(i-cx)(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2\left(a+b\sinh^{-1}\right.}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= -\frac{d(i-cx)(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2\left(a+b\sinh^{-1}\right.}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= -\frac{d(i-cx)(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2\left(a+b\sinh^{-1}\right.}{3(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= \frac{ibd(1+c^2x^2)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)(1+c^2x^2)\left(a+b\sinh^{-1}\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/}}$$

$$= \frac{ibd(1+c^2x^2)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)(1+c^2x^2)\left(a+b\sinh^{-1}\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/}}$$

Mathematica [A]
time = 0.36, size = 202, normalized size = 0.72

$$\frac{\sqrt{f-icfx}\left(4ia-8acx+8iac^2x^2-2b\sqrt{1+c^2x^2}+4ib(1+2icx+2c^2x^2)\sinh^{-1}(cx)+5b(1-icx)\sqrt{1+c^2x^2}\log(d(-1+icx))+3b\sqrt{1+c^2x^2}\log(d+icdx)-3ibcx\sqrt{1+c^2x^2}\log(d+icdx)\right)}{12cdf^3(i+cx)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((4*I)*a - 8*a*c*x + (8*I)*a*c^2*x^2 - 2*b*Sqrt[1 + c^2*
x^2] + (4*I)*b*(1 + (2*I)*c*x + 2*c^2*x^2)*ArcSinh[c*x] + 5*b*(1 - I*c*x)*S
qrt[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] + 3*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*
x] - (3*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*c*d*f^3*(I + c*x)
^2*Sqrt[d + I*c*d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{3}{2}}\,(-icfx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [A]
time = 0.30, size = 237, normalized size = 0.84

$$\frac{1}{12}\,bc\left(\frac{2i\,\sqrt{d}\,\sqrt{f}}{c^3d^2f^3x+i\,c^2d^2f^3}-\frac{5\,\log\left(cx+i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}-\frac{3\,\log\left(cx-i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}\,c^2df^2x+3\,\sqrt{c^2dfx^2+df}\,cdf^2}-\frac{2\,x}{\sqrt{c^2dfx^2+df}\,df^2}\right)\operatorname{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}\,c^2df^2x+3\,\sqrt{c^2dfx^2+df}\,cdf^2}-\frac{2\,x}{\sqrt{c^2dfx^2+df}\,df^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*b*c*(2*I*sqrt(d)*sqrt(f)/(c^3*d^2*f^3*x + I*c^2*d^2*f^3) - 5*log(c*x +
 I)/(c^2*d^(3/2)*f^(5/2)) - 3*log(c*x - I)/(c^2*d^(3/2)*f^(5/2))) - 1/3*b*(
3*I/(-3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c
*d*f^2) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))*arcsinh(c*x) - 1/3*a*(3*I/(-
3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c*d*f^2
) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/24*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 8*(
2*b*c^2*x^2 + 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) - 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*
x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + I*b*c^2*x^
3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 5*(c^4*d^2*f^3*x^3 +
I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*lo
g(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt
(b^2/(c^2*d^3*f^5)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x
 + I*b)) + 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2
*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + I*b*c^2*x^3 + I*b*x)
/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 5*(c^4*d^2*f^3*x^3 + I*c^3*d^2*
f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(-I*sqr
t(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2
*d^3*f^5)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b))
+ 8*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqr
```

```
t(b^2/(c^2*d^3*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 + b))
 - 8*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sq
rt(b^2/(c^2*d^3*f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f
*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b
)) - 8*(2*a*c^2*x^2 + 2*I*a*c*x + a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) -
 24*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*int
egral(-1/6*sqrt(c^2*x^2 + 1)*(4*b*c*x - I*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*
x + f)/(c^4*d^2*f^3*x^4 + 2*c^2*d^2*f^3*x^2 + d^2*f^3), x))/(c^4*d^2*f^3*x^
3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.569 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{b(1+c^2x^2)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2x(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b(1+c^2}{3c(d+ic}$$

```
[Out] 1/6*b*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*x*(c^2*x^
2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*x*(c^2*x^2+
1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*(c^2*x^2+
1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.16, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5796, 5788, 5787, 266, 267}

$$\frac{2x(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b(c^2x^2+1)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b(c^2x^2+1)^{5/2}\log\left(c^2x^2+1\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (b*(1 + c^2*x^2)^(3/2))/(6*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (x*
(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) - (b*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
```

```
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx &= \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{(}{}\\
&= \frac{b\left(1+c^2x^2\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+}{3(d-}\\
&= \frac{b\left(1+c^2x^2\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+}{3(d-}
\end{aligned}
$$

Mathematica [A]
time = 0.34, size = 193, normalized size = 0.95

$$
\frac{i\sqrt{f-icfx}\left(6acx+4ac^3x^3+b\sqrt{1+c^2x^2}+2bcx(3+2c^2x^2)\sinh^{-1}(cx)-2b(1+c^2x^2)^{3/2}\log(d(-1+icx))-2b\sqrt{1+c^2x^2}\log(d+icdx)-2bc^2x^2\sqrt{1+c^2x^2}\log(d+icdx)\right)}{6cd^2f^3(-i+cx)(i+cx)^2\sqrt{d+icdx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/6)*Sqrt[f - I*c*f*x]*(6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b
*c*x*(3 + 2*c^2*x^2)*ArcSinh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[d*(-1 + I*c
*x)] - 2*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^
2]*Log[d + I*c*d*x]))/(c*d^2*f^3*(-I + c*x)*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [A]
time = 0.28, size = 159, normalized size = 0.78

$$\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}f^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{x}{(c^2dfx^2+df)^{\frac{3}{2}}df}+\frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f^2}\right)\mathrm{arsinh}\,(cx)+\frac{1}{3}\,a\left(\frac{x}{(c^2dfx^2+df)^{\frac{3}{2}}df}+\frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/6*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^2
+ 1)/(c^2*d^(5/2)*f^(5/2))) + 1/3*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f) + 2*
x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a*(x/((c^2*d*f*x^2
+ d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x^2 - 2*(2
*b*c^2*x^3 + 3*b*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2
*x^2 + 1)) - (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*
d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2
*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^4 + b*x^2)/(b*c^4*x^4 + 2*b*c^2*
x^2 + b)) + (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d
^5*f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2
```

```
*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) - b*c^2*x^4 - b*x^2)/(b*c^4*x^4 + 2*b*c^2*
x^2 + b)) + 2*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2
*d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^
2*f^2*x*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 + b)) - 2*(c^
4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d^5*f^5))*log(-(
sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f^2*x*sqrt(b^2
/(c^2*d^5*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 2*(2*a*c^2*x^3 + 3*a*
x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 6*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^
3*x^2 + d^3*f^3)*integral(-2/3*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*
c*f*x + f)*b*c*x/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3), x))/(c^4*
d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.570 $\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=680

$$\frac{8ib^2d^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}+\frac{15}{64}b^2d^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}+\frac{4ib^2}{}$$

```
[Out] 8/9*I*b^2*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*d^2*x*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*d^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*
f*x)^(1/2)+4/27*I*b^2*d^2*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c
+3/8*d^2*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2
*d^2*x^3*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+2/3*I*d^2
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-15/
64*b^2*d^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^(
1/2)-4/3*I*b*d^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(
c^2*x^2+1)^(1/2)-3/8*b*c*d^2*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*
c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-4/9*I*b*c^2*d^2*x^3*(a+b*arcsinh(c*x))*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/8*b*c^3*d^2*x^4*(a+b*arc
sinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/24*d^2*(
a+b*arcsinh(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/
2)
```

Rubi [A]
time = 0.70, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5838, 5785, 5783, 5776, 327, 221, 5798, 5784, 455, 45, 5806, 5812}

$$-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{3bcd^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{4ibd^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}+\frac{5d^2\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^3}{24bc\sqrt{c^2x^2+1}}+\frac{2id^2(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{3c}-\frac{4ibc^2d^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}+\frac{bc^3d^2x^4\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{3}{8}d^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}+\frac{4ib^2d^2(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}}{27c}-\frac{15b^2d^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)}{64c\sqrt{c^2x^2+1}}+\frac{15}{64}b^2d^2x\sqrt{d+icdx}\,\sqrt{f-icfx}+\frac{8ib^2d^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((8*I)/9)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*d^2*x^3*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x])/32 + (((4*I)/27)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*(1 + c^2*x^2))/c - (15*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Ar
cSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) - (((4*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*d^2*x^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*
x^2]) - (((4*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*d^2*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*d^2*x^3*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 + (((2*I)/3)*d^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c +
```

```
 (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*c
*Sqrt[1 + c^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
```

```
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
```

```
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int (d+icdx)^2\sqrt{1+c^2x^2}\,(a\cdots}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \left(d^2\sqrt{1+c^2x^2}\,(a+b\sinh\cdots\right.}{\cdots}$$

$$= \frac{\left(d^2\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\,(a+b\sinh^{-}\cdots}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}d^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{1}{4}c^2\cdots$$

$$= -\frac{4ibd^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{1}{4}b^2d^2x\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+icd\cdots}$$

$$= \frac{15}{64}b^2d^2x\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+ic\cdots}$$

$$= \frac{8ib^2d^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c} + \frac{15}{64}b^2d^2x\sqrt{d+icdx}\,\sqrt{\cdots}$$

Mathematica [A]
time = 1.54, size = 890, normalized size = 1.31

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-6912*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (4608*I)*a^2*d
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (6912*I)*b^2*d^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (4608*I)*a^2*c^2*d^2
*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c^3*d
^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^2*d^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (256*I)*b^2*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*d^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*d^(5/2)*Sqrt[f]*Sqr
t[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]] + 864*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x
]] - (768*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x
]] - 27*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] +
12*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-576*I)*b*c*x +
 (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] + (192*I)*a*Cosh[
3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*ArcSinh[c*x]] - (
64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) + 72*b*d^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (48*I)*b*Sqrt[1 + c^2*
x^2] + (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh
[4*ArcSinh[c*x]]))/(6912*c*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2 \sqrt{-icfx+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*d^2*x^2 - 2*I
*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) - (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

3.571 $\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=508

$$\frac{4ib^2d\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}+\frac{1}{4}b^2dx\sqrt{d+icdx}\,\sqrt{f-icfx}+\frac{2ib^2d\sqrt{d+icdx}\,\sqrt{f-icfx}\,(1+c^2x^2)}{27c}-\frac{b^2d}{}$$

```
[Out] 4/9*I*b^2*d*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*d*x*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)+2/27*I*b^2*d*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*
x)^(1/2)/c+1/2*d*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)
+1/3*I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/
2)/c-1/4*b^2*d*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+
1)^(1/2)-2/3*I*b*d*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)
/(c^2*x^2+1)^(1/2)-1/2*b*c*d*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*
c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d*x^3*(a+b*arcsinh(c*x))*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*d*(a+b*arcsinh(c*x))^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {5796, 5838, 5785, 5783, 5776, 327, 221, 5798, 5784, 455, 45}

$$-\frac{bcdx^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))}{2\sqrt{c^2x^2+1}}-\frac{2ibdx\sqrt{d+icdx}\,\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))}{3\sqrt{c^2x^2+1}}+\frac{d\sqrt{d+icdx}\,\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{id(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))^2}{3c}-\frac{2ibc^2dx^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))}{9\sqrt{c^2x^2+1}}+\frac{1}{2}dx\sqrt{d+icdx}\,\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))^2+\frac{2ib^2d(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}}{27c}-\frac{b^2d\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)}{4c\sqrt{c^2x^2+1}}+\frac{1}{4}b^2dx\sqrt{d+icdx}\,\sqrt{f-icfx}+\frac{4ib^2d\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((4*I)/9)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*d*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x])/4 + (((2*I)/27)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (((2*I)/3)*b*d*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*d*x^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
- (((2*I)/9)*b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh
[c*x]))/Sqrt[1 + c^2*x^2] + (d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x])^2)/2 + ((I/3)*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/c + (d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a
 + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int (d+icdx)\sqrt{1+c^2x^2}\,(a+}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \left(d\sqrt{1+c^2x^2}\,(a+b\sinh^{-}\right.}{\sqrt{1+c^2}}$$

$$= \frac{\left(d\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\,(a+b\sinh^{-1}}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}dx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{id\sqrt{}}{}$$

$$= -\frac{2ibdx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{bc}{}$$

$$= \frac{1}{4}b^2dx\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{2ibdx\sqrt{d+icdx}\,\sqrt{f}}{3\sqrt{1}}$$

$$= \frac{1}{4}b^2dx\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{b^2d\sqrt{d+icdx}\,\sqrt{f-}}{4c\sqrt{1+c}}$$

$$= \frac{4ib^2d\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c} + \frac{1}{4}b^2dx\sqrt{d+icdx}\,\sqrt{f}$$

Mathematica [A]
time = 1.16, size = 705, normalized size = 1.39

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-108*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (72*I)*a^2*d*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (108*I)*b^2*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*d*x*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*d*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*d*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cos
h[3*ArcSinh[c*x]] + 108*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
 Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*d*Sqrt[d + I
```

```
*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a + (3*I)*b*Sqrt[1 + c^2*x^2] + I*b*Cosh
[3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*d*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*d*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((-9*I)*b*c*x + (9*I
)*a*Sqrt[1 + c^2*x^2] + (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sinh[2*ArcSinh[c
*x]] - I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))^2 \sqrt{-icfx+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
[Out] integral((I*b^2*c*d*x + b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*d*x - a*b*d)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*d*x + a^2*d)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (id(cx-i))^{\frac{3}{2}} \sqrt{-if(cx+i)}\, (a+b \operatorname{asinh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2,
 x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeDone
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

3.572 $\int \sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=244

$$\frac{1}{4}b^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{b^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}}-\frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,(a+b\sin$$

```
[Out] 1/4*b^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f
-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(a+b*arcsinh(c*x))*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*(a+b*arcsinh(c*x))^3*(d+I
*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {5796, 5785, 5783, 5776, 327, 221}

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{b^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)}{4c\sqrt{c^2x^2+1}}+\frac{1}{4}b^2x\sqrt{d+icdx}\,\sqrt{f-icfx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (b^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int \sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\left(\sqrt{d+\right.}{}$$

$$= -\frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{}$$

$$= \frac{1}{4}b^2x\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-ic}}{2\sqrt{1+}}$$

$$= \frac{1}{4}b^2x\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{b^2\sqrt{d+icdx}\,\sqrt{f-icf}}{4c\sqrt{1+c^2x^2}}$$

Mathematica [A]
time = 0.65, size = 352, normalized size = 1.44

$$\frac{12a^2cx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+4b^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^3-6ab\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(2\sinh^{-1}(cx)\right)+12a^2\sqrt{d}\,\sqrt{f}\,\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+3b^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh\left(2\sinh^{-1}(cx)\right)-6b\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)\left(b\cosh\left(2\sinh^{-1}(cx)\right)-2a\sinh\left(2\sinh^{-1}(cx)\right)\right)+6b\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2\left(2a+b\sinh\left(2\sinh^{-1}(cx)\right)\right)}{24c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (12*a^2*c*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 6*a*b*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 12*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c
^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
+ 3*b^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - 6*b*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(b*Cosh[2*ArcSinh[c*x]] - 2*a*
Sinh[2*ArcSinh[c*x]]) + 6*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x
]^2*(2*a + b*Sinh[2*ArcSinh[c*x]]))/(24*c*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}\sqrt{icdx+d}\,\sqrt{-icfx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 +
 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{id\left(cx-i\right)}\sqrt{-if\left(cx+i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeDone
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,\sqrt{f-c\,f\,x\,1{}\mathrm{i}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

3.573 $\int \frac{\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=259

$$\frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2ib^2f(1+c^2x^2)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2ib^2fx\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{if(1+c^2x^2)\,(a+b\sin}{c\sqrt{d+icdx}\,\sqrt{f-}}$$

```
[Out] -2*I*b^2*f*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*f*(c^2*x^2+1
)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*a*b*f*x*(c
^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*b^2*f*x*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*f*(a+b*arcsinh(
c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {5796, 5838, 5783, 5798, 5772, 267}

$$\frac{2iabfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{f\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{if(c^2x^2+1)\,(a+b\sinh^{-1}(cx))^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2ib^2f(c^2x^2+1)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2ib^2fx\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*a*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (
(2*I)*b^2*f*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)
*b^2*f*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) - (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) + (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\,\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{\left(icf\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\sinh}{\sqrt{1+}}}{\sqrt{d+icdx}\,\sqrt{f-ic}}$$

$$= -\frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx\right.}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{f\sqrt{}}{3}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{if(1+}{c\sqrt{}}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2ib^2f(1+c^2x^2)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1}}{\sqrt{d+ic}}$$

Mathematica [A]
time = 0.60, size = 315, normalized size = 1.22

$$\frac{-3i\sqrt{d+icdx}\,\sqrt{f-icfx}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}\right)+6ib\sqrt{d+icdx}\,\sqrt{f-icfx}\left(bcx-a\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)+3b\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a-ib\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2+b^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^3+3a^2\sqrt{d}\,\sqrt{f}\,\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{3cd\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((-3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*
x^2] + 2*b^2*Sqrt[1 + c^2*x^2]) + (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a - I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c^2*
x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(3
*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2\sqrt{-icfx+f}}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + in
tegrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(I*c*d*x
+ d) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*x +
 d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2 - 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2
*x^2 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*d*x - I*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-if\,(cx+i)}\,(a+b\operatorname{asinh}{(cx)})^2}{\sqrt{id\,(cx-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}{\sqrt{d + c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

3.574 $\int \frac{\sqrt{f-icfx}\,(a+b\sinh^{-1}(cx))^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=544

$$\frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2f^2x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2f^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] 2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)+2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)+2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-1/3*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ar
ctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b*f^2*(
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*f^2*(c^2*
x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-2*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))
^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]

time = 0.65, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783}

$$-\frac{8ibf^2(c^2x^2+1)^{3/2}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{f^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2f^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2f^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4bf^2(c^2x^2+1)^{3/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2f^2(c^2x^2+1)^{3/2}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4b^2f^2(c^2x^2+1)^{3/2}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2f^2(c^2x^2+1)^{3/2}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) + (2*f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSi
nh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2
)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*f^2*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-
I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*f^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*
(f - I*c*f*x)^(3/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
```

```
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
```

```
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)^2(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{2i(if^2+cf^2x)(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}} - \frac{f^2(a+b\sinh^{-1}(cx))^2}{\sqrt{1+c^2x^2}}\right) d\ldots}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i(1+c^2x^2)^{3/2}\right)\int \frac{(if^2+cf^2x)(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(f^2(1+c^2x^2)^{3\ldots}\right.}{(d+icd\ldots}$$

$$= -\frac{f^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2i(1+c^2x^2)^{3/2}\right)\int\left(\frac{if^2(a\ldots}{(\ldots}\right.}{(d+ic\ldots}$$

$$= -\frac{f^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2f^2(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\ldots}{(1\ldots}}{(d+icdx)^{3/2}(f-i\ldots}$$

$$= \frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\left(a+b\sinh^{-1}\ldots\right.}{(d+icdx)^{3/2}(f-icfx)\ldots}$$

$$= \frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\left(a+b\sinh^{-1}\ldots\right.}{(d+icdx)^{3/2}(f-icfx)\ldots}$$

$$= \frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\left(a+b\sinh^{-1}\ldots\right.}{(d+icdx)^{3/2}(f-icfx)\ldots}$$

$$= \frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\left(a+b\sinh^{-1}\ldots\right.}{(d+icdx)^{3/2}(f-icfx)\ldots}$$

$$= \frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\left(a+b\sinh^{-1}\ldots\right.}{(d+icdx)^{3/2}(f-icfx)\ldots}$$

$$= \frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\left(a+b\sinh^{-1}\ldots\right.}{(d+icdx)^{3/2}(f-icfx)\ldots}$$

Mathematica [A]
time = 2.69, size = 594, normalized size = 1.09

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) - 3*a^2*Sqrt[d]*Sqr
t[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - (
3*a*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSin
h[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2]
+ I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 +
 c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*
x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*((-6 + 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Si
nh[ArcSinh[c*x]/2]) - ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2]) + (12*I)*Pi*(Log[1 - I/E^ArcSinh[c*x]] + 2*Log[1 + E^ArcSinh[c*x]
] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 24*PolyLog[2, I/E^ArcSinh[c
*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 6*ArcSinh[c*x]*(Pi -
 (4*I)*Log[1 - I/E^ArcSinh[c*x]])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh
[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])))/(3*c*d^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c*
d^2*sqrt(f/d))) + integrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/(I*c*d*x + d)^(3/2) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^
2 + 1))/(I*c*d*x + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d^2*x^2 - 2*I*c*d^2
*x - d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-if\,(cx+i)}\,(a+b\,\mathrm{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(3/2),
 x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{(d+c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

3.575 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=518

$$-\frac{f^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{if^3(1+c^2x^2)^{5/2}\left(a+b\mathrm{s}\right.}{3c(d+ic}$$

```
[Out] -1/3*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)-4/3*I*b^2*f^3*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsi
nh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(
5/2)+2/3*b*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsin
h(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I*f^3*(c^2*x^2+1)^(5/2)
*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsi
nh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*b*f^3*(c^2*x^2+1)^(5/2
)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)+4/3*b^2*f^3*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.76, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {5796, 5844, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$-\frac{f^3(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bf^3(c^2x^2+1)^{5/2}\log\left(1+ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{if^3(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2bf^3(c^2x^2+1)^{5/2}\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{if^3(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4b^2f^3(c^2x^2+1)^{5/2}\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4ib^2f^3(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] -1/3*(f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b^2*f^3*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 +
(I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*f
^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]
])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*f^3*(1 + c^2*x^2)^(5/
2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSin
h[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/
(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*f^3*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) + (4*b^2*f^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(-\frac{2f^3\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)^2\sqrt{1+c^2x^2}} + \frac{if^3\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(if^3(1+c^2x^2)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2f^3(1+c^2x^2)^{5/2}\right)\int}{(d+icdx)^{5/}}$$

$$= \frac{\left(if^3(1+c^2x^2)^{5/2}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^2}{-ic+c\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2cf\right.}{}$$

$$= -\frac{\left(f^3(1+c^2x^2)^{5/2}\right)\mathrm{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx, x, \sinh^{-1}(\right.}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{if^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2}{}$$

$$= -\frac{f^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^3(1+c^2x^2)^{5/2}\left(a+b\sin\right.}{3c(d+icd}$$

$$= -\frac{f^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}\right.}{3c(d+icdx)^{5/2}(f}$$

$$= -\frac{f^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}\right.}{3c(d+icdx)^{5/2}(f}$$

$$= -\frac{f^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}\right.}{3c(d+icdx)^{5/2}(f}$$

$$= -\frac{f^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}\right.}{3c(d+icdx)^{5/2}(f}$$

Mathematica [A]
time = 7.30, size = 783, normalized size = 1.51

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-2*I)/3)*a^2)/(d^3*(-I + c
*x)^2) - a^2/(3*d^3*(-I + c*x))))/c + ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*S
```

```
qrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] -
I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*Ar
cTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/
2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqr
t[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSi
nh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Co
th[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*
d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2]
 + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x
)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*
x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I
)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 +
E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSi
nh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[Arc
Sinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + A
rcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2
]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*((b^2*c*x + I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1))^2 - 3*(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)*integral(1/3*(3*
I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f)*b^2 + 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3
*x + I*d^3), x))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

3.576 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=774

$$\frac{8ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c}+\frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2}+\frac{16ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{75c\left(1+c^2x^2\right)}+\frac{15b^2d}{}$$

[Out] 8/225*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c+1/32*b^2*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)+16/75*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c/(c^2*x^2+1)+15/64*b^2*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)+2/125*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)/c-9/64*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)-2/5*I*b*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)-3/8*b*c*d*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)-4/15*I*b*c^2*d*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)-2/25*I*b*c^4*d*x^5*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+1/5*I*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/8*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c

Rubi [A]
time = 0.54, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {5796, 5838, 5786, 5785, 5783, 5776, 327, 221, 5798, 201, 200, 5784, 12, 1261, 712}

Antiderivative was successfully verified.

[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]

[Out] (((8*I)/225)*b^2*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (((16*I)/75)*b^2*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) + (((2*I)/125)*b^2*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (((2*I)/5)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (3*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) - (((4*I)/15)*b*c^2*d*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2)

```
 - (((2*I)/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*A
rcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((I/5)*
d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/c + (d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3)
/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right) \int (d+icdx) \left(1+c^2x^2\right)^{3/}}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right) \int \left(d(1+c^2x^2)^{3/2} \left(a+b\right.\right.}{(1+}$$

$$= \frac{\left(d(d+icdx)^{3/2}(f-icfx)^{3/2}\right) \int \left(1+c^2x^2\right)^{3/2} \left(a+b\,\mathrm{si}\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 +$$

$$= -\frac{2ibdx(d+icdx)^{3/2}(f-icfx)^{3/2} \left(a+b\sinh^{-1}(cx)\right)}{5\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{2ibdx(d+icdx)^{3}}{}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2dx(d+icdx)}{64\,(1+}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2dx(d+icdx)}{64\,(1+}$$

$$= \frac{8ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c} + \frac{1}{32}b^2dx(d+icdx)^{3}$$

Mathematica [A]
time = 2.11, size = 1084, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-72000*I)*a*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (57600*I)*a
^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72000*I)*
b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*a^
2*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (115200
*I)*a^2*c^2*d^2*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
 + 72000*a^2*c^3*d^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
```

```
*x^2] + (57600*I)*a^2*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqr
t[1 + c^2*x^2] + 36000*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSin
h[c*x]^3 - 72000*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSi
nh[c*x]] + (4000*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*Ar
cSinh[c*x]] - 4500*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*Arc
Sinh[c*x]] + (288*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[5*A
rcSinh[c*x]] + 108000*a^2*d^(5/2)*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + S
qrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^2*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - (12000*I)*a*b*d^2*f*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1125*b^2*d^2*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1800*b*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (20*I)*b*Sqrt[1 + c^2
*x^2] + (10*I)*b*Cosh[3*ArcSinh[c*x]] + (2*I)*b*Cosh[5*ArcSinh[c*x]] + 40*b
*Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) - (1440*I)*a*b*d^2*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]] + 60*b*d^2*f*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-1200*I)*b*c*x + (1200*I)*a*Sqrt[
1 + c^2*x^2] - 1200*b*Cosh[2*ArcSinh[c*x]] + (600*I)*a*Cosh[3*ArcSinh[c*x]]
 - 75*b*Cosh[4*ArcSinh[c*x]] + (120*I)*a*Cosh[5*ArcSinh[c*x]] + 2400*a*Sinh
[2*ArcSinh[c*x]] - (200*I)*b*Sinh[3*ArcSinh[c*x]] + 300*a*Sinh[4*ArcSinh[c*
x]] - (24*I)*b*Sinh[5*ArcSinh[c*x]]))/(288000*c*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{5}{2}} (-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((I*b^2*c^3*d^2*f*x^3 + b^2*c^2*d^2*f*x^2 + I*b^2*c*d^2*f*x + b^2*d
^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 -
 2*(-I*a*b*c^3*d^2*f*x^3 - a*b*c^2*d^2*f*x^2 - I*a*b*c*d^2*f*x - a*b*d^2*f)
*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2
*c^3*d^2*f*x^3 + a^2*c^2*d^2*f*x^2 + I*a^2*c*d^2*f*x + a^2*d^2*f)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}\,(f-c\,f\,x\,1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

3.577 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=396

$$\frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}+\frac{15b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\,(1+c^2x^2)}-\frac{9b^2(d+icdx)^{3/2}(f-icfx)^{3/2}\sinh^{-1}(cx)}{64c\,(1+c^2x^2)^{3/2}}$$

```
[Out] 1/32*b^2*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)+15/64*b^2*x*(d+I*c*d*x)^(3/2
)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)-9/64*b^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2
)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)-3/8*b*c*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x
)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*x*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*
(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+1/8*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a
+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*(d+I*c*d*x)^(3/2)*(f-I*c*f*x
)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.31, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {5796, 5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\,(c^2x^2+1)^{3/2}}+\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{8\,(c^2x^2+1)}-\frac{b\sqrt{c^2x^2+1}\,(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\,(c^2x^2+1)^{3/2}}+\frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{15b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\,(c^2x^2+1)}-\frac{9b^2(d+icdx)^{3/2}(f-icfx)^{3/2}\sinh^{-1}(cx)}{64c\,(c^2x^2+1)^{3/2}}+\frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (15*b^2*x*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (9*b^2*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (3*b*c*x^2
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*
x^2)^(3/2)) - (b*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x]))/(8*c) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x])^2)/4 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*(a + b*ArcSinh[c*x])^3)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\,\mathrm{s}\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 +$$

$$= -\frac{b(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{8c}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{3bcx^2(d+icdx)^{3}}{}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2x(d+icdx)^{3}}{64\,(1+}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2x(d+icdx)^{3}}{64\,(1+}$$

Mathematica [A]
time = 1.11, size = 524, normalized size = 1.32

$$\frac{160a^2cdfx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+64a^2c^3dfx^3\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+32b^2df\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)^3-64abdf\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(2\sinh^{-1}(cx)\right)-4abdf\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(4\sinh^{-1}(cx)\right)+96a^2d^{3/2}f^{3/2}\sqrt{1+c^2x^2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+32b^2df\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(2\sinh^{-1}(cx)\right)+b^2df\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(4\sinh^{-1}(cx)\right)+8bdf\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)^2\left(12a+8b\sinh\left(2\sinh^{-1}(cx)\right)+b\sinh\left(4\sinh^{-1}(cx)\right)\right)-4bdf\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)\left(16b\cosh\left(2\sinh^{-1}(cx)\right)+b\cosh\left(4\sinh^{-1}(cx)\right)-4a\left(8\sinh\left(2\sinh^{-1}(cx)\right)+\sinh\left(4\sinh^{-1}(cx)\right)\right)\right)}{256c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (160*a^2*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 64
*a^2*c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32
```

```
*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 64*a*b*d*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 4*a*b*d*f*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 96*a^2*d^(3/2)*f^(3/2)*S
qrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]] + 32*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*
x]] + b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 8*
b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(12*a + 8*b*Sinh[2
*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) - 4*b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*ArcSinh[c*x]*(16*b*Cosh[2*ArcSinh[c*x]] + b*Cosh[4*ArcSinh[c*x]
] - 4*a*(8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(256*c*Sqrt[1 + c
^2*x^2])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{3}{2}} (-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((b^2*c^2*d*f*x^2 + b^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d*f*x^2 + a*b*d*f)*sqrt(I*c*d*x
```

```
+ d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^2*d*f*x^2 + a
^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}\,(f-c\,f\,x\,1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

3.578 $\int \sqrt{d+icdx}\,(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=508

$$-\frac{4ib^2f\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}+\frac{1}{4}b^2fx\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{2ib^2f\sqrt{d+icdx}\,\sqrt{f-icfx}\,(1+c^2x^2)}{27c}-\frac{b^2}{}$$

```
[Out] -4/9*I*b^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*f*x*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)-2/27*I*b^2*f*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f
*x)^(1/2)/c+1/2*f*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)-1/3*I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1
/2)/c-1/4*b^2*f*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2
+1)^(1/2)+2/3*I*b*f*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)/(c^2*x^2+1)^(1/2)-1/2*b*c*f*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I
*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f*x^3*(a+b*arcsinh(c*x))*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*f*(a+b*arcsinh(c*x))^3*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {5796, 5838, 5785, 5783, 5776, 327, 221, 5798, 5784, 455, 45}

$$-\frac{bcfx^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{2ibfx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}+\frac{f\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}-\frac{if(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3c}+\frac{2ibc^2fx^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}+\frac{1}{2}fx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2ib^2f(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}}{27c}-\frac{b^2f\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)}{4c\sqrt{c^2x^2+1}}+\frac{1}{4}b^2fx\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{4ib^2f\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-4*I)/9)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*f*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (((2*I)/27)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) + (((2*I)/3)*b*f*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*f*x^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSin
h[c*x]))/Sqrt[1 + c^2*x^2] + (f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x])^2)/2 - ((I/3)*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/c + (f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(
a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \sqrt{d+icdx}\,(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int(f-icfx)\sqrt{1+c^2x^2}\,(a}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\left(f\sqrt{1+c^2x^2}\,(a+b\sin\right.}{\sqrt{1+}}$$

$$= \frac{\left(f\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\,(a+b\sinh}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}fx\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{if}{}$$

$$= \frac{2ibfx\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{bc}{}$$

$$= \frac{1}{4}b^2fx\sqrt{d+icdx}\,\sqrt{f-icfx} + \frac{2ibfx\sqrt{d+icdx}\,\sqrt{}}{3}$$

$$= \frac{1}{4}b^2fx\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{b^2f\sqrt{d+icdx}\,\sqrt{f}}{4c\sqrt{1+}}$$

$$= -\frac{4ib^2f\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c} + \frac{1}{4}b^2fx\sqrt{d+icdx}$$

Mathematica [A]
time = 1.14, size = 705, normalized size = 1.39

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((108*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (72*I)*a^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (108*I)*b^2*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*f*x*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*f*x^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh
[3*ArcSinh[c*x]] + 108*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*f*Sqrt[d + I*
```

```
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a - (3*I)*b*Sqrt[1 + c^2*x^2] - I*b*Cosh[
3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*f*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*f*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((9*I)*b*c*x - (9*I)*
a*Sqrt[1 + c^2*x^2] - (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sinh[2*ArcSinh[c*x
]] + I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2 \sqrt{icdx+d}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((-I*b^2*c*f*x + b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*f*x - a*b*f)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*f*x + a^2*f)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{id\,(cx-i)}\,(-if(cx+i))^{\frac{3}{2}} (a+b\,\mathrm{asinh}\,(cx))^2\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2,
 x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeDone
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d + c\,d\,x\,1\mathrm{i}}\,\left(f - c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

3.579 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=436

$$-\frac{4ib^2f^2(1+c^2x^2)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{b^2f^2x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{b^2f^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{4c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{4ibf^2x\sqrt{1+c^2x^2}\,(a+ \ldots}{\sqrt{d+icdx}\,\sqrt{f- \ldots}}$$

```
[Out] -4*I*b^2*f^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*f^2*
x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*f^2*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*f^2*x*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*f^2*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*I*b*f^2*x
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1
/2*b*c*f^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*
c*f*x)^(1/2)+1/2*f^2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)
^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {5796, 5843, 3398, 3377, 2718, 3392, 32, 2715, 8}

$$\frac{f^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2if^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{f^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{bcf^2x^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{4ibf^2x\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{4ib^2f^2(c^2x^2+1)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{b^2f^2x(c^2x^2+1)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{b^2f^2\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{4c\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((-4*I)*b^2*f^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b
^2*f^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*f^2*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 ((4*I)*b*f^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]) + (b*c*f^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I)*f^2*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (f^2*x*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (f^2*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx &= \frac{\sqrt{1+c^2x^2}\int\frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} \\
&= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int(a+bx)^2(cf-icf\sinh(x))^2\,dx,x,\sinh^{-1}(cx)\right)}{c^3\sqrt{d+icdx}\,\sqrt{f-icfx}} \\
&= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int\left(c^2f^2(a+bx)^2-2ic^2f^2(a+bx)^2\sinh(x)-c^2\right.\right.}{c^3\sqrt{d+icdx}\,\sqrt{f-icfx}} \\
&= \frac{f^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{\left(2if^2\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int(a\right.}{c\sqrt{d+icd}} \\
&= \frac{bcf^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2if^2(1+c^2x^2)\left(a+b\sinh\right.}{c\sqrt{d+icdx}\,\sqrt{f-ic}} \\
&= -\frac{b^2f^2x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{4ibf^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{\sqrt{d+icdx}\,\sqrt{f-icfx}} \\
&= -\frac{4ib^2f^2(1+c^2x^2)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{b^2f^2x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{b^2f^2\sqrt{1}}{4c\sqrt{d-}}
\end{aligned}
$$

Mathematica [A]
time = 1.35, size = 532, normalized size = 1.22

$$
\frac{32iabcfx\sqrt{d+icdx}\,\sqrt{f-icfx}-16ia^2f\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-32ib^2f\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-4a^2cfx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+4b^2f\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^3+2abf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(2\sinh^{-1}(cx)\right)+2bf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)\left(4i\left(4bcx+a(-4+icx)\sqrt{1+c^2x^2}\right)+b\cosh\left(2\sinh^{-1}(cx)\right)\right)+12a^2\sqrt{d}\,f^{3/2}\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)-b^2f\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh\left(2\sinh^{-1}(cx)\right)+2bf\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2\left(6a-8ib\sqrt{1+c^2x^2}-b\sinh\left(2\sinh^{-1}(cx)\right)\right)}{8cd\sqrt{1+c^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((32*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a^2*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (32*I)*b^2*f*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*f*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*ArcSinh[c*x]^3 + 2*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*A
```

```
rcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((4*I
)*(4*b*c*x + a*(-4 + I*c*x)*Sqrt[1 + c^2*x^2]) + b*Cosh[2*ArcSinh[c*x]]) +
12*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*Sinh[2*ArcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[
c*x]^2*(6*a - (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]))/(8*c*d*S
qrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
[Out] integral(-((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{id\,(cx-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,{(f-c\,f\,x\,1\mathrm{i})}^{3/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

3.580 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=752

$$-\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2ib^2f^3x(1+c^2x^2)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1+c^2x}{c(d+icd}$$

```
[Out] -2*I*a*b*f^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*b^
2*f^3*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*b^2*f^3*x*(c^
2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^3*(c^
2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*f^3*x
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*f^3
*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)+I*f^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)-f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-16*I*b*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(
c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*f^3*(c^2*x^2+1)
^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/
2)/(f-I*c*f*x)^(3/2)-8*b^2*f^3*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*b^2*f^3*(c^2*x^2+1)^(3/
2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-4*b^2*f^3*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.73, antiderivative size = 752, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783, 5772, 267}

$$-\frac{16ibf^3(c^2x^2+1)^{3/2}\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2iabf^3x(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{f^3(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4f^3(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4f^3x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8bf^3(c^2x^2+1)^{3/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8b^2f^3(c^2x^2+1)^{3/2}\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8b^2f^3(c^2x^2+1)^{3/2}\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2f^3(c^2x^2+1)^{3/2}\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2ib^2f^3(c^2x^2+1)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2ib^2f^3x(c^2x^2+1)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-2*I)*a*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((2*I)*b^2*f^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) - ((2*I)*b^2*f^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*x*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*(
1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*
```

```
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((16*I)*b*f^3*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*
x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) - (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
 I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyL
og[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b
^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}} dx = \frac{(1+c^2x^2)^{3/2} \int \frac{(f-icfx)^3 (a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}} dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2} \int \left(-\frac{4i(if^3+cf^3x)(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}} - \frac{3f^3(a+b\sinh^{-1}(cx))^2}{\sqrt{1+c^2x^2}} \right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i(1+c^2x^2)^{3/2}\right) \int \frac{(if^3+cf^3x)(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}} dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3f^3(1+c^2x\right.}{(d+ic}$$

$$= \frac{if^3(1+c^2x^2)^2 \left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3(1+c^2x^2)^{3/2} \left(a+b\sinh\right.}{bc(d+icdx)^{3/2}(f-icf}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2 \left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x(1+c^2x^2)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2i}{}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2i}{}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2i}{}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2i}{}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2i}{}$$

Mathematica [A]
time = 5.87, size = 1174, normalized size = 1.56

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),
x]
```

```
[Out] ((I/3)*f*(-3*a^2*(-5*I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 9*a^2*Sqrt[d]*Sq
rt[f]*(-I + c*x)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2
]) + 6*a*b*(I - c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]
/2]*(-(c*x) + (2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*A
rcTan[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2]) + I*(-(c*x) + (-2 + Sqrt[
1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/
2]] + I*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + (3*I)*a*b*(I - c*x)*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x]*(-4*I +
ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*Log[1 + c^2*x^2]) +
I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] +
 2*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + I*b^2*(I - c*x)*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*((6 - 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sin
h[ArcSinh[c*x]/2]) + ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2]) + 6*ArcSinh[c*x]*(I*Pi + 4*Log[1 - I/E^ArcSinh[c*x]])*(Cosh[ArcSin
h[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - 24*PolyLog[2, I/E^ArcSinh[c*x]]*(Cosh
[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 12*Pi*(Log[1 - I/E^ArcSinh[c*x
]] + 2*Log[1 + E^ArcSinh[c*x]] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi
+ (2*I)*ArcSinh[c*x])/4]])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2
])) + b^2*(I - c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((2*I)*ArcSinh[c*x]
^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - 6*ArcSinh[c*x]*(Pi + c
*x - (4*I)*Log[1 - I/E^ArcSinh[c*x]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSin
h[c*x]/2]) + 6*(Sqrt[1 + c^2*x^2] + 2*Pi*Log[1 - I/E^ArcSinh[c*x]] + 4*Pi*L
og[1 + E^ArcSinh[c*x]] - 4*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 2*Pi*Log[Sin[(Pi
+ (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])
+ 24*PolyLog[2, I/E^ArcSinh[c*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh
[c*x]/2]) + 3*ArcSinh[c*x]^2*(((2 + 2*I) + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[
c*x]/2] + I*((-2 + 2*I) + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]))))/(c*d^
2*(I - c*x)*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2
]))
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{\left(icdx+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6*
I*sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c*d
^2*sqrt(f/d))) + integrate((-I*c*f*x + f)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2/(I*c*d*x + d)^(3/2) + 2*(-I*c*f*x + f)^(3/2)*a*b*log(c*x + sqrt(c^2
*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c*f*x - b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*f*x + a*b*f)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*f*x - a^2*f)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{3/2}}{(d+c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

3.581 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=580

$$-\frac{8f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8ib^2f^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}\right.}{3c(d+icdx)^{5/2}(f-ic}$$

[Out] -8/3*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-8/3*I*b^2*f^4*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-8/3*I*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*b*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*I*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+32/3*b*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+32/3*b^2*f^4*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)

Rubi [A]
time = 0.80, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5844, 5783, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\frac{f^4(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8f^4(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{32bf^4(c^2x^2+1)^{5/2}\log\left(1+ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8if^4(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bf^4(c^2x^2+1)^{5/2}\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{32b^2f^4(c^2x^2+1)^{5/2}\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8ib^2f^4(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]

[Out] (-8*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((8*I)/3)*b^2*f^4*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((8*I)/3)*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b^2*f^4*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)^4(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{f^4(a+b\sinh^{-1}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{4f^4(a+b\sinh^{-1}(cx))^2}{(-i+cx)^2\sqrt{1+c^2x^2}} + \frac{4if^4(a+b\mathrm{s}}{(-i+cx)\sqrt{}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(4if^4(1+c^2x^2)^{5/2}\right)\int \frac{(a+b\sinh^{-1}(cx))^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^4(1+c^2x^2)^{5/2}\right)}{(d+icdx)^{5/2}(}$$

$$= \frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4if^4(1+c^2x^2)^{5/2}\right)\mathrm{Subst}\Big(}{(d+icdx)^{5}}$$

$$= \frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^4(1+c^2x^2)^{5/2}\right)\mathrm{Subst}\left(\int(\right.}{c(d+i}$$

$$= \frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4if^4(1+c^2x^2)^{5/2}\left(a+b\sinh\right.}{c(d+icdx)}$$

$$= -\frac{4f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sin\right.}{3bc(d+icdx)^{5/2}(f-}$$

$$= -\frac{8f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sin\right.}{3bc(d+icdx)^{5/2}(f-}$$

$$= -\frac{8f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sin\right.}{3bc(d+icdx)^{5/2}(f-}$$

$$= -\frac{8f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sin\right.}{3bc(d+icdx)^{5/2}(f-}$$

$$= -\frac{8f^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}\left(a+b\sin\right.}{3bc(d+icdx)^{5/2}(f-}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1609 vs. $2(580) = 1160$.
time = 8.68, size = 1609, normalized size = 2.77

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-4*I)/3)*a^2*f)/(d^3*(-I +
 c*x)^2) - (8*a^2*f)/(3*d^3*(-I + c*x))))/c + (a^2*f^(3/2)*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*d^(5/2)) + (
(I/3)*a*b*f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1
 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3
*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sq
rt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*Ar
cTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^
2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]
]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)
*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) - (a*b*
f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/
2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2
]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[
ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 42*Log[Sq
rt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2
]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/
2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(6*c*d^3*(I + c*x)
*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^4) + ((I/3)*b^2*f*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I
)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*x]^2 - (2*A
rcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I)*ArcSinh[c
*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c
*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]
]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2
])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]
^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))
/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[Arc
Sinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2) + (b^2*f*(I + c*x)*Sqrt[I*((-I)*d
 + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(7*Pi*ArcSin
h[c*x] - (7 + 7*I)*ArcSinh[c*x]^2 - I*ArcSinh[c*x]^3 + (2*ArcSinh[c*x]*(-2*
I + ArcSinh[c*x]))/(1 + I*c*x) - 14*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^A
rcSinh[c*x]] - 28*Pi*Log[1 + E^ArcSinh[c*x]] + 28*Pi*Log[Cosh[ArcSinh[c*x]/
2]] + 14*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (28*I)*PolyLog[2, I/E^A
rcSinh[c*x]] - ((4*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*
x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[
c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*d^3*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx + f)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx))^2}{(icdx + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I
*d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{3/2}}{(d+c\,d\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

3.582 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=548

$$\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{245b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1152\left(1+c^2x^2\right)^2}+\frac{65b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1728\left(1+c^2x^2\right)}-\frac{115b^2}{}$$

```
[Out] 1/108*b^2*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)+245/1152*b^2*x*(d+I*c*d*x)^
(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^2+65/1728*b^2*x*(d+I*c*d*x)^(5/2)*(f-I*
c*f*x)^(5/2)/(c^2*x^2+1)-115/1152*b^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*a
rcsinh(c*x)/c/(c^2*x^2+1)^(5/2)-5/16*b*c*x^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^
(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(5/2)+1/6*x*(d+I*c*d*x)^(5/2)*(f-I*c*f
*x)^(5/2)*(a+b*arcsinh(c*x))^2+5/16*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(
a+b*arcsinh(c*x))^2/(c^2*x^2+1)^2+5/24*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2
)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+5/48*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)
*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(5/2)-5/48*b*(d+I*c*d*x)^(5/2)*(f-I*c
*f*x)^(5/2)*(a+b*arcsinh(c*x))/c/(c^2*x^2+1)^(1/2)-1/18*b*(d+I*c*d*x)^(5/2)
*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.39, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {5796, 5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\frac{5(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\left(c^2x^2+1\right)^{5/2}}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16\left(c^2x^2+1\right)^2}-\frac{b\sqrt{c^2x^2+1}(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5b(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{48c\sqrt{c^2x^2+1}}-\frac{5bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{16\left(c^2x^2+1\right)^{5/2}}+\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{65b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1728\left(c^2x^2+1\right)}+\frac{245b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1152\left(c^2x^2+1\right)^2}-\frac{115b^2(d+icdx)^{5/2}(f-icfx)^{5/2}\sinh^{-1}(cx)}{1152c\left(c^2x^2+1\right)^{5/2}}+\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/108 + (245*b^2*x*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1152*(1 + c^2*x^2)^2) + (65*b^2*x*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1728*(1 + c^2*x^2)) - (115*b^2*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)*ArcSinh[c*x])/(1152*c*(1 + c^2*x^2)^(5/2)) - (
5*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16
*(1 + c^2*x^2)^(5/2)) - (5*b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b
*ArcSinh[c*x]))/(48*c*Sqrt[1 + c^2*x^2]) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(18*c) + (x*(d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*x*(d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(16*(1 + c^2*x^2)^2) + (5
*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(24*(1 +
 c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]
)^3)/(48*b*c*(1 + c^2*x^2)^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
```

```
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left((d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sin\right.}{\left(1+c^2x^2\right)^{5/2}}$$

$$=\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{(}{}$$

$$=-\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\right.}{18c}$$

$$=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}-\frac{5b(d+icdx)^{5/2}(}{}$$

$$=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{65b^2x(d+icdx)^{5}}{1728\,(1-}$$

$$=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{245b^2x(d+icdx)}{1152\,(1-}$$

$$=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{245b^2x(d+icdx)}{1152\,(1-}$$

Mathematica [**A**]
time = 1.46, size = 735, normalized size = 1.34

$$\frac{9504a^2cd^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+7488a^2c^3d^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+2304a^2c^5d^2f^2x^5\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+1440b^2d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)^3-3240abd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(2\sinh^{-1}(cx)\right)-324abd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(4\sinh^{-1}(cx)\right)-24abd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(6\sinh^{-1}(cx)\right)+4320a^2d^{5/2}f^{5/2}\sqrt{1+c^2x^2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+1620b^2d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(2\sinh^{-1}(cx)\right)+81b^2d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(4\sinh^{-1}(cx)\right)+4b^2d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(6\sinh^{-1}(cx)\right)-12bd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)\left(270b\cosh\left(2\sinh^{-1}(cx)\right)+27b\cosh\left(4\sinh^{-1}(cx)\right)+2b\cosh\left(6\sinh^{-1}(cx)\right)-540a\sinh\left(2\sinh^{-1}(cx)\right)-108a\sinh\left(4\sinh^{-1}(cx)\right)-12a\sinh\left(6\sinh^{-1}(cx)\right)\right)+72bd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)^2\left(60a+45b\sinh\left(2\sinh^{-1}(cx)\right)+9b\sinh\left(4\sinh^{-1}(cx)\right)+b\sinh\left(6\sinh^{-1}(cx)\right)\right)}{13824c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (9504*a^2*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
 + 7488*a^2*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^
2*x^2] + 2304*a^2*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[
1 + c^2*x^2] + 1440*b^2*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^3 - 3240*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSi
nh[c*x]] - 324*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSi
nh[c*x]] - 24*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSin
h[c*x]] + 4320*a^2*d^(5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*
Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 1620*b^2*d^2*f^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 81*b^2*d^2*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d^2*f^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*Sinh[6*ArcSinh[c*x]] - 12*b*d^2*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] + 27*b*Cosh[4*A
rcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSinh[c*x]] - 108*
a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d^2*f^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]
] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c*Sqrt[1 + c
^2*x^2])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(icdx+d\right)^{\frac{5}{2}} \left(-icfx+f\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((b^2*c^4*d^2*f^2*x^4 + 2*b^2*c^2*d^2*f^2*x^2 + b^2*d^2*f^2)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^4*
d^2*f^2*x^4 + 2*a*b*c^2*d^2*f^2*x^2 + a*b*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^4*d^2*f^2*x^4 + 2*a^2*c^
2*d^2*f^2*x^2 + a^2*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

3.583 $\int (d+icdx)^{3/2}(f-icfx)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=774

$$-\frac{8ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c}+\frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}-\frac{16ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{75c\,(1+c^2x^2)}+15$$

```
[Out] -8/225*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c+1/32*b^2*f*x*(d+I*c*d*
x)^(3/2)*(f-I*c*f*x)^(3/2)-16/75*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2
)/c/(c^2*x^2+1)+15/64*b^2*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+
1)-2/125*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)/c-9/64*b^2
*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)+2/5
*I*b*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)
^(3/2)-3/8*b*c*f*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))
/(c^2*x^2+1)^(3/2)+4/15*I*b*c^2*f*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(
a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+2/25*I*b*c^4*f*x^5*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*f*x*(d+I*c*d*x)^(3
/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*f*x*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)-1/5*I*f*(d+I*c*d*x)^(3/2)*(f-I
*c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/8*f*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*f*(d+I*c*d
*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.56, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {5796, 5838, 5786, 5785, 5783, 5776, 327, 221, 5798, 201, 200, 5784, 12, 1261, 712}

$$-\frac{3bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)^{3/2}}+\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{8\left(c^2x^2+1\right)}+\frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{5\left(c^2x^2+1\right)^{3/2}}+\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\left(c^2x^2+1\right)^{3/2}}-\frac{if\left(c^2x^2+1\right)(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{5c}-\frac{bf\sqrt{c^2x^2+1}\,(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}+\frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15\left(c^2x^2+1\right)^{3/2}}+\frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{25\left(c^2x^2+1\right)^{3/2}}+\frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{15b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\left(c^2x^2+1\right)}-\frac{2ib^2f\left(c^2x^2+1\right)(d+icdx)^{3/2}(f-icfx)^{3/2}}{125c}-\frac{16ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{75c\left(c^2x^2+1\right)}-\frac{9b^2f(d+icdx)^{3/2}(f-icfx)^{3/2}\sinh^{-1}(cx)}{64c\left(c^2x^2+1\right)^{3/2}}+\frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}-\frac{8ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-8*I)/225)*b^2*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*f*x*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 - (((16*I)/75)*b^2*f*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*f*x*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (((2*I)/125)*b^2*f*(d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*f*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) + (((2*I
)/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
 + c^2*x^2)^(3/2) - (3*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) + (((4*I)/15)*b*c^2*f*x^3*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2
```

```
) + (((2*I)/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*
ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) - ((I/5)
*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/c + (f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3
)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (f-icfx)\left(1+c^2x^2\right)}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(f(1+c^2x^2)^{3/2}\left(a+\right.\right.}{(1}$$

$$= \frac{\left(f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 -$$

$$= \frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{5\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{2ibfx(d+icdx}{}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2fx(d+icd}{64\,(1}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2fx(d+icd}{64\,(1}$$

$$= -\frac{8ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c} + \frac{1}{32}b^2fx(d+icc$$

Mathematica [A]
time = 2.04, size = 1084, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((72000*I)*a*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (57600*I)*a^
2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72000*I)*b
^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*a^2
*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (115200*
I)*a^2*c^2*d*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
+ 72000*a^2*c^3*d*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*
```

```
x^2] - (57600*I)*a^2*c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt
[1 + c^2*x^2] + 36000*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^3 - 72000*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSin
h[c*x]] - (4000*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*Arc
Sinh[c*x]] - 4500*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcS
inh[c*x]] - (288*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[5*Ar
cSinh[c*x]] + 108000*a^2*d^(3/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^2*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + (12000*I)*a*b*d*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1125*b^2*d*f^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1800*b*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (20*I)*b*Sqrt[1 + c^2*
x^2] - (10*I)*b*Cosh[3*ArcSinh[c*x]] - (2*I)*b*Cosh[5*ArcSinh[c*x]] + 40*b*
Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) + (1440*I)*a*b*d*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]] + 60*b*d*f^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((1200*I)*b*c*x - (1200*I)*a*Sqrt[1
+ c^2*x^2] - 1200*b*Cosh[2*ArcSinh[c*x]] - (600*I)*a*Cosh[3*ArcSinh[c*x]] -
 75*b*Cosh[4*ArcSinh[c*x]] - (120*I)*a*Cosh[5*ArcSinh[c*x]] + 2400*a*Sinh[2
*ArcSinh[c*x]] + (200*I)*b*Sinh[3*ArcSinh[c*x]] + 300*a*Sinh[4*ArcSinh[c*x]
] + (24*I)*b*Sinh[5*ArcSinh[c*x]]))/(288000*c*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (icdx + d)^{\frac{3}{2}} (-icfx + f)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((-I*b^2*c^3*d*f^2*x^3 + b^2*c^2*d*f^2*x^2 - I*b^2*c*d*f^2*x + b^2*
d*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2
- 2*(I*a*b*c^3*d*f^2*x^3 - a*b*c^2*d*f^2*x^2 + I*a*b*c*d*f^2*x - a*b*d*f^2)
*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^
2*c^3*d*f^2*x^3 + a^2*c^2*d*f^2*x^2 - I*a^2*c*d*f^2*x + a^2*d*f^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

3.584 $\int \sqrt{d+icdx}\,(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=680

$$-\frac{8ib^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}+\frac{15}{64}b^2f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{4ib}{}$$

```
[Out] -8/9*I*b^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*f^2*x*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c
*f*x)^(1/2)-4/27*I*b^2*f^2*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/
c+3/8*f^2*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^
2*f^2*x^3*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-2/3*I*f^
2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-15
/64*b^2*f^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^
(1/2)+4/3*I*b*f^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/
(c^2*x^2+1)^(1/2)-3/8*b*c*f^2*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I
*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+4/9*I*b*c^2*f^2*x^3*(a+b*arcsinh(c*x))*(d+I
*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/8*b*c^3*f^2*x^4*(a+b*ar
csinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/24*f^2*
(a+b*arcsinh(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1
/2)
```

Rubi [A]
time = 0.71, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5838, 5785, 5783, 5776, 327, 221, 5798, 5784, 455, 45, 5806, 5812}

$$-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{3bcf^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{4ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}+\frac{5f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^3}{24bc\sqrt{c^2x^2+1}}-\frac{2if^2(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{3c}+\frac{4ibc^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}+\frac{bc^3f^2x^4\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{3}{8}f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{4ib^2f^2(c^2x^2+1)\sqrt{d+icdx}\,\sqrt{f-icfx}}{27c}-\frac{15b^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\sinh^{-1}(cx)}{64c\sqrt{c^2x^2+1}}+\frac{15}{64}b^2f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{8ib^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-8*I)/9)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*f^2*x^3*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x])/32 - (((4*I)/27)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*(1 + c^2*x^2))/c - (15*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) + (((4*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*f^2*x^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2
*x^2]) + (((4*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*f^2*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*f^2*x^3*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 - (((2*I)/3)*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c
```

```
+ (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*
c*Sqrt[1 + c^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
```

```
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
```

```
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \sqrt{d+icdx}\,(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int (f-icfx)^2\sqrt{1+c^2x^2}\,(}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\left(f^2\sqrt{1+c^2x^2}\,(a+b\sin\right.}{}$$

$$= \frac{\left(f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\,(a+b\sinh}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{1}{4}$$

$$= \frac{4ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{b}{}$$

$$= \frac{1}{4}b^2f^2x\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{1}{32}b^2c^2f^2x^3\sqrt{d+ic}$$

$$= \frac{15}{64}b^2f^2x\sqrt{d+icdx}\,\sqrt{f-icfx} - \frac{1}{32}b^2c^2f^2x^3\sqrt{d+}$$

$$= -\frac{8ib^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c} + \frac{15}{64}b^2f^2x\sqrt{d+icd}$$

Mathematica [A]
time = 1.47, size = 890, normalized size = 1.31

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]

[Out] ((6912*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (4608*I)*a^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (6912*I)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (4608*I)*a^2*c^2*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c^3*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (256*I)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 864*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + (768*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] - 27*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 12*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((576*I)*b*c*x - (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] - (192*I)*a*Cosh[3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*ArcSinh[c*x]] + (64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) + 72*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (48*I)*b*Sqrt[1 + c^2*x^2] - (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4*ArcSinh[c*x]]))/(6912*c*Sqrt[1 + c^2*x^2])

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-icfx+f\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2} \sqrt{icdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)

[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algorithm="maxima")

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*f^2*x^2 + 2*I
*a*b*c*f^2*x - a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) - (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

3.585 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=615

$$-\frac{68ib^2f^3(1+c^2x^2)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2ib^2f^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3b^2f^3\sqrt{1+c^2x^2}\,\sinh}{4c\sqrt{d+icdx}\,\sqrt{f-}}$$

```
[Out] -68/9*I*b^2*f^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*f
^3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2/27*I*b^2*f^3*(c^2*x^
2+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-11/3*I*f^3*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*f^3*x*(c^2*x^2+1)*(
a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*I*c*f^3*x^2*(c^
2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b^2*f
^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+22/
3*I*b*f^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+3/2*b*c*f^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)-2/9*I*b*c^2*f^3*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*f^3*(a+b*arcsinh(c*x))^3*(c^2
*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {5796, 5843, 3398, 3377, 2718, 3392, 32, 2715, 8, 2713}

$$\frac{5f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{icf^3x^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3f^3x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{11if^3(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{3c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3bcf^3x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{22ibf^3x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2ibc^2f^3x^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2ib^2f^3(c^2x^2+1)^2}{27c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3b^2f^3x(c^2x^2+1)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{68ib^2f^3(c^2x^2+1)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3b^2f^3\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{4c\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] (((-68*I)/9)*b^2*f^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
 - (3*b^2*f^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((
(2*I)/27)*b^2*f^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
+ (3*b^2*f^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (((22*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((2*I)/9)*b*c
^2*f^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[
c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f
^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)^2(cf-icf\sinh(x))^3\,dx, x, \sinh^{-1}(cx)\right)}{c^4\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int \left(c^3f^3(a+bx)^2-3ic^3f^3(a+bx)^2\sinh(x)-3c\right.\right.}{c^4\sqrt{d+icdx}}$$

$$= \frac{f^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{\left(if^3\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int (a\right.}{c\sqrt{d+icd}}$$

$$= \frac{3bcf^3x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}\,(a+}{9\sqrt{d+icdx}\,\sqrt{f-}}$$

$$= -\frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{6ibf^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{56ib^2f^3(1+c^2x^2)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2}{27c\sqrt{d}}$$

$$= -\frac{68ib^2f^3(1+c^2x^2)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2}{27c\sqrt{d}}$$

Mathematica [A]
time = 2.11, size = 723, normalized size = 1.18

$$\frac{1620iabcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-792ia^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}-1620ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}-324a^2cf^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+72ia^2c^2f^2x^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+180b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)^3+162abf^2\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(2\sinh^{-1}(cx)\right)+4ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(3\sinh^{-1}(cx)\right)+6bf^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)\left(27b\cosh\left(2\sinh^{-1}(cx)\right)+2i\left(-4bcx\left(-33+c^2x^2\right)+27a(-5+2icx)\sqrt{1+c^2x^2}+3a\cosh\left(3\sinh^{-1}(cx)\right)\right)\right)+540a^2\sqrt{d}f^{5/2}\sqrt{1+c^2x^2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)-81b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(2\sinh^{-1}(cx)\right)+18bf^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)^2\left(30a-45ib\sqrt{1+c^2x^2}+ib\cosh\left(3\sinh^{-1}(cx)\right)-9b\sinh\left(2\sinh^{-1}(cx)\right)\right)-12iabf^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(3\sinh^{-1}(cx)\right)}{216cd\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((1620*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (792*I)*a^2*f^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1620*I)*b^2*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*f^2*x*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*f^2*x^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*f^2*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*f^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(-4*b*c*x*(-33 + c
^2*x^2) + 27*a*(-5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] + 3*a*Cosh[3*ArcSinh[c*x]
])) + 540*a^2*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[
f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^2*(30*a - (45*I)*b*Sqrt[1 + c^2*x^2] + I*b*Cosh[3*ArcSi
nh[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{5}{2}}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c^2*f^2*x^2 - 2*b^2*c*f^2*x - I*b^2*f^2)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c^2*f^2*x^2
+ 2*a*b*c*f^2*x + I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*f^2*x^2 - 2*a^2*c*f^2*x - I*a^2*f^2)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

3.586 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=972

$$-\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2f^4(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(\ldots}$$

```
[Out] 8*I*f^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)+8*I*b^2*f^4*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/4*b^2
*f^4*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/4*b^2*f^4*(c^2*x
^2+1)^(3/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^4*(c^2
*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*
c*f^4*x^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x
)^(3/2)-8*I*a*b*f^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+8*f^4*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)+8*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*
f*x)^(3/2)-32*I*b*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*
x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/2*f^4*x*(c^2*x^2+1)^2
*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-5/2*f^4*(c^2*x^2+
1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b
^2*f^4*x*(c^2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
-16*b*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-16*b^2*f^4*(c^2*x^2+1)^(3/2)*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+16*
b^2*f^4*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*f^4*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*
x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.89, antiderivative size = 972, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 19, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783, 5772, 267, 5812, 5776, 327, 221}

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-8*I)*a*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b^2*f^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) + (b^2*f^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) - (b^2*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/
```

```
2)*(f - I*c*f*x)^(3/2)) - ((8*I)*b^2*f^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]
)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*f^4*x^2*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((8
*I)*f^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (8*f^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^4*(1 + c^2*
x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
+ (f^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (5*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b
*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((32*I)*b*f^4*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) - (16*b*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16
*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
 I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*f^
4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)^4(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{8i(if^4+cf^4x)(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}} - \frac{7f^4(a+b\sinh^{-1}(cx))^2}{\sqrt{1+c^2x^2}} + \right.}{(d+icdx)^{3/2}(f-icfx}$$

$$= -\frac{\left(8i(1+c^2x^2)^{3/2}\right)\int \frac{(if^4+cf^4x)(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(7f^4(1+c^2x^2\right.}{(d+icd}$$

$$= \frac{4if^4(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f^4x(1+c^2x^2)^2\left(a+b\sinh^{-}\right.}{2(d+icdx)^{3/2}(f-icfx}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(\right.}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib}{(}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{}{4(d}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{}{4(d}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{}{4(d}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{}{4(d}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{}{4(d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2492 vs. $2(972) = 1944$.
time = 9.62, size = 2492, normalized size = 2.56

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((4*I)*a^2*f^2)/d^2 + (a^2*c*
f^2*x)/(2*d^2) + (8*a^2*f^2)/(d^2*(-I + c*x))))/c - (15*a^2*f^(5/2)*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*c*d
^(3/2)) + ((4*I)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*
Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] +
 Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[
c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]]) + I*(-(c*x) - 2*ArcSinh[c*x] + Sqr
t[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]
/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-((
(-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2])) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
+ c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x]*(-
4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c
^2*x^2]]) + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSin
h[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-
(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] +
 I*Sinh[ArcSinh[c*x]/2])) - (b^2*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*
f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*((6*I)*Pi*ArcS
inh[c*x] + (6 - 6*I)*ArcSinh[c*x]^2 + ArcSinh[c*x]^3 + 12*((-I)*Pi + 2*ArcS
inh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - (24*I)*Pi*Log[1 + E^ArcSinh[c*x]] + (
24*I)*Pi*Log[Cosh[ArcSinh[c*x]/2]] + (12*I)*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[
c*x])/4]]) - 24*PolyLog[2, I/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2]) + (-6*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcS
inh[c*x]^3 + 12*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi
*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[
(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSinh[c*x]/2]))/(3*c*d^2*Sqrt[-(((-I)
*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])) + (((2*I)/3)*b^2*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)
*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-6*Pi*Arc
Sinh[c*x] - 6*c*x*ArcSinh[c*x] + (6 + 6*I)*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c
*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 12*Pi*Log[1 - I/E^ArcSin
h[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^A
rcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*
ArcSinh[c*x])/4]]) + I*(-6*Pi*ArcSinh[c*x] - 6*c*x*ArcSinh[c*x] - (6 - 6*I)
*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c
*x]^2) + 12*Pi*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]
/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSinh[c*x]/2] +
24*PolyLog[2, I/E^ArcSinh[c*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c
*x]/2])))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*
(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*f^2*Sqrt[I*((-I)*d
+ c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(96*PolyLog[2
, I/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + Sinh[
ArcSinh[c*x]/2]*(24*Pi*ArcSinh[c*x] + 48*c*x*ArcSinh[c*x] + (24 - 24*I)*Arc
Sinh[c*x]^2 - (10*I)*ArcSinh[c*x]^3 + (3*I)*Sqrt[1 + c^2*x^2]*(c*x + (8*I)*
```

```
(2 + ArcSinh[c*x]^2)) - (3*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - 48*Pi*Log
[1 - I/E^ArcSinh[c*x]] - (96*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - 96
*Pi*Log[1 + E^ArcSinh[c*x]] + 96*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 48*Pi*Log[S
in[(Pi + (2*I)*ArcSinh[c*x])/4]] + (3*I)*ArcSinh[c*x]^2*Sinh[2*ArcSinh[c*x]
]) + Cosh[ArcSinh[c*x]/2]*(3*Sqrt[1 + c^2*x^2]*(c*x + (8*I)*(2 + ArcSinh[c*
x]^2)) - 3*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - I*(24*Pi*ArcSinh[c*x] + 48*c
*x*ArcSinh[c*x] - (24 + 24*I)*ArcSinh[c*x]^2 - (10*I)*ArcSinh[c*x]^3 - 48*P
i*Log[1 - I/E^ArcSinh[c*x]] - (96*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]]
 - 96*Pi*Log[1 + E^ArcSinh[c*x]] + 96*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 48*Pi*
Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (3*I)*ArcSinh[c*x]^2*Sinh[2*ArcSinh
[c*x]]))))/(12*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x
^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (a*b*f^2*Sqrt[I*((-I
)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-(Sinh[A
rcSinh[c*x]/2]*((-16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + Cosh[2*ArcSinh[c*x
]] + 2*((8*I)*c*x + (8*I)*ArcSinh[c*x] + 5*ArcSinh[c*x]^2 + (16*I)*ArcTan[T
anh[ArcSinh[c*x]/2]] + 8*Log[Sqrt[1 + c^2*x^2]] - ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]]))) + Cosh[ArcSinh[c*x]/2]*(16*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*(
Cosh[2*ArcSinh[c*x]] + 2*((8*I)*c*x - (8*I)*ArcSinh[c*x] + 5*ArcSinh[c*x]^2
 + (16*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 8*Log[Sqrt[1 + c^2*x^2]] - ArcSinh
[c*x]*Sinh[2*ArcSinh[c*x]]))))))/(4*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f
*x))]*Sqrt[1 + c^2*x^2]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x)/
(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a^2 + integrate((
```

```
-I*c*f*x + f)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(I*c*d*x + d)^(3/2)
+ 2*(-I*c*f*x + f)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(3/
2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*f^2*x^2 + 2*I
*a*b*c*f^2*x - a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) + (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,{(f-c\,f\,x\,1{}\mathrm{i})}^{5/2}}{{(d+c\,d\,x\,1{}\mathrm{i})}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

3.587 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=790

$$\frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2f^5x(1+c^2x^2)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{28f^5(1+c^2x^2)}{3c(d+icd}$$

```
[Out] 2*I*a*b*f^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*b^2
*f^5*(c^2*x^2+1)^3/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*b^2*f^5*x*(c^2
*x^2+1)^(5/2)*arcsinh(c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-28/3*f^5*(c^
2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-I
*f^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)+5/3*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)-16/3*I*b^2*f^5*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arcsinh(c*
x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-28/3*I*f^5*(c^2*x^2+1)^(5/2)*(a+b
*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)+8/3*b*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*
arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*f^5*(c^2*x^2+1)
^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I
*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+112/3*b*f^5*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+112/3*b^2*f^5*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(
c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.94, antiderivative size = 790, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {5796, 5844, 5783, 5798, 5772, 267, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\frac{2iabf^5x(c^2x^2+1)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5f^5(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{if^5(c^2x^2+1)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{28f^5(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{112bf^5(c^2x^2+1)^{5/2}\log\left(1+ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{28if^5(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8bf^5(c^2x^2+1)^{5/2}\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4if^5(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{112b^2f^5(c^2x^2+1)^{5/2}\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2f^5(c^2x^2+1)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2f^5x(c^2x^2+1)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{16ib^2f^5(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] ((2*I)*a*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - ((2*I)*b^2*f^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)
^(5/2)) + ((2*I)*b^2*f^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(5/2)*(f - I*c*f*x)^(5/2)) - (28*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*
x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (I*f^5*(1 + c^2*x^2)
^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5
*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (((16*I)/3)*b^2*f^5*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 +
```

```
(I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((28*I)
/3)*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh
[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^5*(1 + c^2*x^2
)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*f^5*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[
c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b*f^5*(1 + c^2*
x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b^2*f^5*(1 + c^2*x^2)^(5/2)*PolyLog[2,
 (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)^5\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{5f^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{icf^5x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{8f^5(a+}{(-i+cx)}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(12if^5(1+c^2x^2)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(5f^5(1+c^2x^2)^{5/}\right.}{(d+icdx)}$$

$$= -\frac{if^5(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2}\left(a+b\,\mathrm{si}\right.}{3bc(d+icdx)^{5/2}(f-}$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^5(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2}{(}$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2}{(}$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2}{(}$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2}{(}$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2}{(}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2622 vs. 2(790) = 1580.
time = 10.76, size = 2622, normalized size = 3.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*f^2)/d^3 - (((8*I)/
3)*a^2*f^2)/(d^3*(-I + c*x)^2) - (28*a^2*f^2)/(3*d^3*(-I + c*x))))/c + (5*a
^2*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(
I + c*x)]])/(c*d^(5/2)) + ((I/3)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)
*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[A
rcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Cot
h[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 +
(3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^
2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/
2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSi
nh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I +
 c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])^4) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c
*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x]
- 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[A
rcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x]
 + 9*ArcSinh[c*x]^2 + 42*Log[Sqrt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*
x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1
 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (
28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSi
nh[c*x]/2]))/(3*c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Co
sh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*f^2*(I + c*x)*
Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))
]*((-1 + I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c
*x) + (2*I)*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(Arc
Sinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log
[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*A
rcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*
x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f
*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
- ((I/3)*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)
]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] +
((13 - 13*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (3*ArcSinh[c*x]^3)/Sqrt[1
+ c^2*x^2] + (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/((-I + c*x)*Sqrt[1 + c^
2*x^2]) - (3*I)*(2 + ArcSinh[c*x]^2) + ((13*I)*(-2*(Pi + (2*I)*ArcSinh[c*x]
)*Log[1 - I/E^ArcSinh[c*x]] + Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]]
+ 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) +
(4*I)*PolyLog[2, I/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (4*ArcSinh[c*x]^2*
Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^3) - (2*(4 + 13*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Sqrt[1
 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))))/(c*d^3*Sqrt[
-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c
*x]/2])^2) + (2*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
```

```
 c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(7*Pi*ArcSinh[c*x] - (7 + 7*I)*ArcSinh[
c*x]^2 - I*ArcSinh[c*x]^3 + (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(1 + I*c
*x) - 14*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - 28*Pi*Log[1
+ E^ArcSinh[c*x]] + 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 14*Pi*Log[Sin[(Pi + (
2*I)*ArcSinh[c*x])/4]] + (28*I)*PolyLog[2, I/E^ArcSinh[c*x]] - ((4*I)*ArcSi
nh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])^3 + (2*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/((-I)*Cosh[ArcSinh
[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f +
c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^
2) + ((I/6)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[
-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(-3*C
osh[(5*ArcSinh[c*x])/2] + (3*I)*ArcSinh[c*x]*Cosh[(5*ArcSinh[c*x])/2] - Cos
h[(3*ArcSinh[c*x])/2]*(9 + (35*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (52*I)*
ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*
x]/2]*(20 - (24*I)*ArcSinh[c*x] + 27*ArcSinh[c*x]^2 - (156*I)*ArcTan[Coth[A
rcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*x^2]]) + (20*I)*Sinh[ArcSinh[c*x]/2]
- 24*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2] + (27*I)*ArcSinh[c*x]^2*Sinh[ArcSinh
[c*x]/2] + 156*ArcTan[Coth[ArcSinh[c*x]/2]]*Sinh[ArcSinh[c*x]/2] + (78*I)*L
og[Sqrt[1 + c^2*x^2]]*Sinh[ArcSinh[c*x]/2] + (9*I)*Sinh[(3*ArcSinh[c*x])/2]
 + 35*ArcSinh[c*x]*Sinh[(3*ArcSinh[c*x])/2] + (9*I)*ArcSinh[c*x]^2*Sinh[(3*
ArcSinh[c*x])/2] + 52*ArcTan[Coth[ArcSinh[c*x]/2]]*Sinh[(3*ArcSinh[c*x])/2]
 + (26*I)*Log[Sqrt[1 + c^2*x^2]]*Sinh[(3*ArcSin...
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c^2*f^2*x^2 + 2*b^2*c*f^2*x + I*b^2*f^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c^2*f^2*x^2
- 2*a*b*c*f^2*x - I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*f^2*x^2 + 2*a^2*c*f^2*x + I*a^2*f^2)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*
x + I*d^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}{(d+c\,d\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

3.588 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=615

$$\frac{68ib^2d^3(1+c^2x^2)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2ib^2d^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3b^2d^3\sqrt{1+c^2x^2}\,\sinh}{4c\sqrt{d+icdx}\,\sqrt{f-}}$$

```
[Out] 68/9*I*b^2*d^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*d^
3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2/27*I*b^2*d^3*(c^2*x^2
+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+11/3*I*d^3*(c^2*x^2+1)*(a+b*arc
sinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*d^3*x*(c^2*x^2+1)*(a
+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*I*c*d^3*x^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b^2*d^
3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-22/3
*I*b*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+3/2*b*c*d^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(
1/2)/(f-I*c*f*x)^(1/2)+2/9*I*b*c^2*d^3*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(
1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*d^3*(a+b*arcsinh(c*x))^3*(c^2*
x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {5796, 5843, 3398, 3377, 2718, 3392, 32, 2715, 8, 2713}

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{icd^3x^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3d^3x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{11id^3(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{3c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3bcd^3x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{22ibd^3x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2ibc^2d^3x^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2ib^2d^3(c^2x^2+1)^2}{27c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3b^2d^3x(c^2x^2+1)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{68ib^2d^3(c^2x^2+1)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{3b^2d^3\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{4c\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] (((68*I)/9)*b^2*d^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
- (3*b^2*d^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((
2*I)/27)*b^2*d^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (3*b^2*d^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (((22*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (((2*I)/9)*b*c^
2*d^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c
*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^
3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2(cd+icd\sinh(x))^3\,dx, x, \sinh^{-1}(cx)\right.}{c^4\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \left(c^3d^3(a+bx)^2+3ic^3d^3(a+bx)^2\sinh(x)-3\right.\right.}{c^4\sqrt{d+icd}}$$

$$= \frac{d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{\left(id^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a\right.}{c\sqrt{d+icd}}$$

$$= \frac{3bcd^3x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}\,(a}{9\sqrt{d+icdx}\,\sqrt{f}}$$

$$= -\frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{6ibd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{56ib^2d^3(1+c^2x^2)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2i}{27c\sqrt{d}}$$

$$= \frac{68ib^2d^3(1+c^2x^2)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2i}{27c\sqrt{d}}$$

Mathematica [A]
time = 2.01, size = 723, normalized size = 1.18

$$\frac{-1620iabcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+792ia^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+1620ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}-324a^2cd^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}-72ia^2c^2d^2x^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+180b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)^3+162abd^2\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(2\sinh^{-1}(cx)\right)-4ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\cosh\left(3\sinh^{-1}(cx)\right)+6bd^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)\left(27b\cosh\left(2\sinh^{-1}(cx)\right)+2i\left(4bcx(-33+c^2x^2)+27a(5+2icx)\sqrt{1+c^2x^2}-3a\cosh\left(3\sinh^{-1}(cx)\right)\right)\right)+540a^2d^{5/2}\sqrt{f}\sqrt{1+c^2x^2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)-81b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(2\sinh^{-1}(cx)\right)+18bd^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)^2\left(30a+45ib\sqrt{1+c^2x^2}-ib\cosh\left(3\sinh^{-1}(cx)\right)-9b\sinh\left(2\sinh^{-1}(cx)\right)\right)+12iabd^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh\left(3\sinh^{-1}(cx)\right)}{216cf\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-1620*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (792*I)*a^2*d^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1620*I)*b^2*d^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*d^2*x*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*d^2*x^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*d^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*d^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(4*b*c*x*(-33 + c
^2*x^2) + 27*a*(5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] - 3*a*Cosh[3*ArcSinh[c*x]]
)) + 540*a^2*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f
]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*ArcSinh[c*x]^2*(30*a + (45*I)*b*Sqrt[1 + c^2*x^2] - I*b*Cosh[3*ArcSin
h[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c^2*d^2*x^2 - 2*b^2*c*d^2*x + I*b^2*d^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c^2*d^2*x^2
+ 2*a*b*c*d^2*x - I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*d^2*x^2 - 2*a^2*c*d^2*x + I*a^2*d^2)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

3.589 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=436

$$\frac{4ib^2d^2(1+c^2x^2)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{b^2d^2x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{b^2d^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{4c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{4ibd^2x\sqrt{1+c^2x^2}\,(a+b\text{s}}{\sqrt{d+icdx}\,\sqrt{f-}}$$

```
[Out] 4*I*b^2*d^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*d^2*x
*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*d^2*(c^2*x^2+1)*(a+b*a
rcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*d^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*d^2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4*I*b*d^2*x*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/
2*b*c*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c
*f*x)^(1/2)+1/2*d^2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {5796, 5843, 3398, 3377, 2718, 3392, 32, 2715, 8}

$$\frac{d^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2id^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{d^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{4ibd^2x\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{4ib^2d^2(c^2x^2+1)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{b^2d^2x(c^2x^2+1)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{b^2d^2\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{4c\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((4*I)*b^2*d^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b^
2*d^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*d^2*S
qrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) -
((4*I)*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]) + (b*c*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)*d^2*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (d^2*x*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (d^2*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)^2(cd+icd\sinh(x))^2\,dx, x, \sinh^{-1}(cx)\right)}{c^3\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int \left(c^2d^2(a+bx)^2+2ic^2d^2(a+bx)^2\sinh(x)-c^2c\right.\right.}{c^3\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{\left(2id^2\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int (a\right.}{c\sqrt{d+icdx}}$$

$$= \frac{bcd^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-}\right.}{c\sqrt{d+icdx}\,\sqrt{f-ic}}$$

$$= -\frac{b^2d^2x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{4ibd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{4ib^2d^2(1+c^2x^2)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{b^2d^2x(1+c^2x^2)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{b^2d^2\sqrt{1+}}{4c\sqrt{d+i}}$$

Mathematica [A]
time = 1.24, size = 529, normalized size = 1.21

$$\frac{-32iabcdx\sqrt{d+icdx}\,\sqrt{f-icfx}+16ia^2d\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+32ib^2d\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}-4a^2cdx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sqrt{1+c^2x^2}+4b^2d\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^3+2abd\sqrt{d+icdx}\,\sqrt{f-icfx}\,\cosh\left(2\sinh^{-1}(cx)\right)+2bd\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)\left(-16ibcx-4a(-4i+cx)\sqrt{1+c^2x^2}+b\cosh\left(2\sinh^{-1}(cx)\right)\right)+12a^2d^{3/2}\sqrt{f}\,\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)-b^2d\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh\left(2\sinh^{-1}(cx)\right)+2bd\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^2\left(6a+8ib\sqrt{1+c^2x^2}-b\sinh\left(2\sinh^{-1}(cx)\right)\right)}{8cf\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-32*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a^2*d*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (32*I)*b^2*d*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*d*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^3 + 2*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*
```

```
ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-1
6*I)*b*c*x - 4*a*(-4*I + c*x)*Sqrt[1 + c^2*x^2] + b*Cosh[2*ArcSinh[c*x]]) +
 12*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Sinh[2*ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^2*(6*a + (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]))/(8*c*f*
Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b\operatorname{arcsinh}(cx))^2}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
[Out] integral(-((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{-if\,(cx+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

3.590 $\int \frac{\sqrt{d+icdx}\,(a+b\sinh^{-1}(cx))^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=259

$$-\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2ib^2d(1+c^2x^2)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2ib^2dx\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{id(1+c^2x^2)\,(a+b\,\mathrm{si}}{c\sqrt{d+icdx}\,\sqrt{f}}$$

```
[Out] 2*I*b^2*d*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I*d*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*a*b*d*x*(c^
2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*b^2*d*x*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*d*(a+b*arcsinh(c
*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {5796, 5838, 5783, 5798, 5772, 267}

$$-\frac{2iabdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{d\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{id(c^2x^2+1)\,(a+b\sinh^{-1}(cx))^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{2ib^2d(c^2x^2+1)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{2ib^2dx\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*a*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
((2*I)*b^2*d*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I
)*b^2*d*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (I*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}+\frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(d\sqrt{1+c^2x^2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{\left(icd\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\sinh}{\sqrt{1+c}}}{\sqrt{d+icdx}\,\sqrt{f-icf}}$$

$$= \frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{d\sqrt{}}{}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{id(1}{c}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2ib^2d(1+c^2x^2)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1}}{\sqrt{d+i}}$$

Mathematica [A]
time = 0.58, size = 315, normalized size = 1.22

$$\frac{3i\sqrt{d+icdx}\,\sqrt{f-icfx}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}\right)-6ib\sqrt{d+icdx}\,\sqrt{f-icfx}\left(bcx-a\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)+3b\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+ib\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)^2+b^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx)^3+3a^2\sqrt{d}\,\sqrt{f}\,\sqrt{1+c^2x^2}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{3cf\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x
^2] + 2*b^2*Sqrt[1 + c^2*x^2]) - (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c^2*x
^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(3*
c*f*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2\sqrt{icdx+d}}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + in
tegrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(-I*c*f*x
+ f) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*x +
 f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*
x^2 + 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*f*x + I*f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{id\left(cx-i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{\sqrt{-if\left(cx+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

3.591 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {5796, 5783}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{d+icdx}\,\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\,\int \frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$
$$= \frac{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 168 vs. $2(59) = 118$.
time = 0.46, size = 168, normalized size = 2.85

$$\frac{ab\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)^3}{3c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{a^2\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{c\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^3)/(3*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (a^2*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]])/(c*Sqrt[d]*Sqrt[f])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{\sqrt{icdx+d}\,\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [**A**]
time = 0.29, size = 53, normalized size = 0.90

$$\frac{b^2\,\mathrm{arsinh}\,(cx)^3}{3\,\sqrt{df}\,c} + \frac{ab\,\mathrm{arsinh}\,(cx)^2}{\sqrt{df}\,c} + \frac{a^2\,\mathrm{arsinh}\,(cx)}{\sqrt{df}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/3*b^2*arcsinh(c*x)^3/(sqrt(d*f)*c) + a*b*arcsinh(c*x)^2/(sqrt(d*f)*c) + a
^2*arcsinh(c*x)/(sqrt(d*f)*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
+ 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d*f*x^2 + d*f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}\left(cx\right)\right)^{2}}{\sqrt{id\left(cx - i\right)}\,\sqrt{-if\left(cx + i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,\sqrt{f-c\,f\,x\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.592 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=464

$$\frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{fx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{f(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-4$$

```
[Out] I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+
f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+f*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-4*I*b*f*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*f*(c^2*x^2+1)^(3/2)*(a+b*arcsin
h(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c
*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c
^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*f*(c^2*x^2+1)^(
3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)
```

Rubi [A]
time = 0.46, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {5796, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265}

$$-\frac{4ibf(c^2x^2+1)^{3/2}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{f(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{fx(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf(c^2x^2+1)^{3/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2f(c^2x^2+1)^{3/2}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2b^2f(c^2x^2+1)^{3/2}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2f(c^2x^2+1)^{3/2}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (f*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) + (f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*b*f*(1 + c^2*x^2)^(3/2)*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) - (2*b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(
2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b^2*f*(1
 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2)) + (2*b^2*f*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c
*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*f*(1 + c^2*x^2)^(3
/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(f(1+c^2x^2)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(icf(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\sinh^{-1}(cx}{(1+c^2x^2)^{3/2}}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{(2}{}$$

$$= \frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{(2}{}$$

$$= \frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(}{}$$

$$= \frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(}{}$$

$$= \frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(}{}$$

$$= \frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(}{}$$

Mathematica [A]
time = 1.43, size = 508, normalized size = 1.09

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\left(\left((-1+i)b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2\left(\cosh\left(\tfrac12\sinh^{-1}(cx)\right)-\sinh\left(\tfrac12\sinh^{-1}(cx)\right)\right)+\left(ia^2+a^2cx-4iab\sqrt{1+c^2x^2}\,\mathrm{ArcTan}\left(\tanh\left(\tfrac12\sinh^{-1}(cx)\right)\right)+2ib^2\pi\sqrt{1+c^2x^2}\log\left(1-ie^{-\sinh^{-1}(cx)}\right)+4ib^2\pi\sqrt{1+c^2x^2}\log\left(1+e^{\sinh^{-1}(cx)}\right)-ab\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)-4ib^2\pi\sqrt{1+c^2x^2}\log\left(\cosh\left(\tfrac12\sinh^{-1}(cx)\right)\right)-2ib^2\pi\sqrt{1+c^2x^2}\log\left(\sin\left(\tfrac14\left(\pi+2i\sinh^{-1}(cx)\right)\right)\right)\right)\left(\cosh\left(\tfrac12\sinh^{-1}(cx)\right)+i\sinh\left(\tfrac12\sinh^{-1}(cx)\right)\right)+4b^2\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\left(\cosh\left(\tfrac12\sinh^{-1}(cx)\right)+i\sinh\left(\tfrac12\sinh^{-1}(cx)\right)\right)+b\sqrt{1+c^2x^2}\sinh^{-1}(cx)\left(i\cosh\left(\tfrac12\sinh^{-1}(cx)\right)\left(2a-b\pi+4ib\log\left(1-ie^{-\sinh^{-1}(cx)}\right)\right)+\left(2a+b\pi-4ib\log\left(1-ie^{-\sinh^{-1}(cx)}\right)\right)\sinh\left(\tfrac12\sinh^{-1}(cx)\right)\right)\right)}{cd^2f(-i+cx)(i+cx)\left(\cosh\left(\tfrac12\sinh^{-1}(cx)\right)+i\sinh\left(\tfrac12\sinh^{-1}(cx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 + I)*b^2*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + (I*a^2 + a^2*c*x -
 (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Pi*Sq
rt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*
Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - (4*I)*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi*Sqrt[1 + c^
2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sin
```

```
h[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]]*(
Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]*(I*Cosh[ArcSinh[c*x]/2]*(2*a - b*Pi + (4*I)*b*Log[1 - I/E^ArcSinh[c*
x]]) + (2*a + b*Pi - (4*I)*b*Log[1 - I/E^ArcSinh[c*x]])*Sinh[ArcSinh[c*x]/2
])))/(c*d^2*f*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c
*x]/2]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{(icdx + d)^{\frac{3}{2}} \sqrt{-icfx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*sqrt(-I*c
*f*x + f)), x) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(I*c^2*d^2*f*
x + c*d^2*f) + I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(I*c^2*d^2*f*x + c*d^2*f) - 2*
a*b*log(I*c*x + 1)/(c*d^(3/2)*sqrt(f))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
(c^2*d^2*f*x - I*c*d^2*f)*integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 + I*sq
rt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*
```

```
d^2*f*x^3 - I*c^2*d^2*f*x^2 + c*d^2*f*x - I*d^2*f), x))/(c^2*d^2*f*x - I*c*
d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{(id(cx - i))^{\frac{3}{2}}\sqrt{-if(cx + i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{(d + c\,d\,x\,1\mathrm{i})^{3/2}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.593 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=942

$$-\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2f^2(1+c^2x^2)^{5/2}\sinh^{-1}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf^2(1+c^2x^2)}{3c(d+icd}$$

```
[Out] -2/3*I*b^2*f^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*
f^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*f^2*(c^2*x^
2+1)^(5/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*f^2*(c^
2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3
*I*b*f^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)-1/3*b*c*f^2*x^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-4/3*I*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arc
tan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*f^2*x*
(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*c^
2*f^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+2/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+1/3*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2
)/(f-I*c*f*x)^(5/2)+2/3*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x
)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln
(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2
*f^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+2/3*b^2*f^2*(c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+(c^2
*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*f^2*(c^2*x^2+
1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)
```

Rubi [A]
time = 0.87, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {5796, 5838, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5789, 4265, 267, 5800, 5810, 294, 221}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (((-2*I)/3)*b^2*f^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (2*b^2*f^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^
(5/2)) + (b^2*f^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
```

```
3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f^2*x*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (b*c*f^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*f^
2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (2*f^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b*f^2*(
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*
x)^(5/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])
/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b^2*f^2*(1 + c^2*x^2)^(
5/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
```

```
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
```

```
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{f^2\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}} - \frac{2icf^2x\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}} - \frac{c^2f^2x^2\left(a+b\sinh^{-1}(c\right.}{(1+c^2x^2)^{5/2}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(f^2(1+c^2x^2)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2icf^2(1+c^2x^2)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}\right.}{(1+c^2x^2}}{(d+icdx)^{5/2}(f-icfx)^{5/}}$$

$$= \frac{2if^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= \frac{bf^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf^2x(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(c\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2(1+}{3c(d-}$$

$$= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2(1+}{3c(d+ic}$$

$$= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2(1+}{3c(d+ic}$$

$$= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2(1+}{3c(d+ic}$$

$$= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2(1+}{3c(d+ic}$$

Mathematica [A]
time = 4.62, size = 524, normalized size = 0.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(-2*I + c*x))/(-I + c*x)^2 - (a*
b*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]
/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 - (3*I)*ArcSinh[c
*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((-1
```

```
 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[
ArcSinh[c*x]/2]] + (I/2)*(-2 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*S
inh[ArcSinh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^3) - (b^2*((1 - I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(-2*I + Arc
Sinh[c*x]))/(-I + c*x) + 2*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c
*x]] + I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[
c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) - 4*PolyLog[2, I/E^ArcS
inh[c*x]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] +
 I*Sinh[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])
/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]))/(3*c
*d^3*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((b^2*c*x - 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 3*(c^3*d^3*f*x^2 - 2*I*c^2*d^3*f*x - c*d^3*f)*integral(
-1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d*x + d)*s
```

```
qrt(-I*c*f*x + f)*a*b + (b^2*c*x - 2*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^3*f*x^4 - 2*I
*c^3*d^3*f*x^3 - 2*I*c*d^3*f*x - d^3*f), x))/(c^3*d^3*f*x^2 - 2*I*c^2*d^3*f
*x - c*d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{(id(cx - i))^{\frac{5}{2}}\sqrt{-if(cx + i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I))
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{(d + c\,d\,x\,1\mathrm{i})^{5/2}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.594 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=972

$$\frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2d^4(1+c^2x^2)^{3/2}\,\mathrm{s}}{4c(d+icdx)^{3/2}(f}$$

```
[Out] 8*I*a*b*d^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b^2
*d^4*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/4*b^2*d^4*x*(c^2
*x^2+1)^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/4*b^2*d^4*(c^2*x^2+1)^(3/2)
*arcsinh(c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*d^4*(c^2*x^2+1)*(a+
b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*c*d^4*x^2*(c^
2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+32*I*
b*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/
(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*d^4*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^
2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(
c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^4*(c^2*x^2+1)^2*(a+b*ar
csinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/2*d^4*x*(c^2*x^2+1)^2
*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-5/2*d^4*(c^2*x^2+
1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b
^2*d^4*x*(c^2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
-16*b*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+16*b^2*d^4*(c^2*x^2+1)^(3/2)*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-16*
b^2*d^4*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*d^4*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*
x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.90, antiderivative size = 972, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 19, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783, 5772, 267, 5812, 5776, 327, 221}

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((8*I)*a*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b^2*d^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b^2*d^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) - (b^2*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/2
```

```
)*(f - I*c*f*x)^(3/2)) + ((8*I)*b^2*d^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])
/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*d^4*x^2*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((8*
I)*d^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) + (8*d^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^4*(1 + c^2*x
^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) +
 (d^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (5*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b*
c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((32*I)*b*d^4*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (16*b*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[
1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*
b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16*b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^4
*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{8i(id^4-cd^4x)\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}} - \frac{7d^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \cdots\right.}{(d+icdx)^{3/2}(f-icfx)\cdots}$$

$$= -\frac{\left(8i(1+c^2x^2)^{3/2}\right)\int \frac{(id^4-cd^4x)\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(7d^4(1+c^2x^2)\cdots\right.}{(d+icdx\cdots}$$

$$= -\frac{4id^4(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d^4x(1+c^2x^2)^2\left(a+b\sinh\cdots\right.}{2(d+icdx)^{3/2}(f-icf\cdots}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx\cdots\right.}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2d\cdots}{(d\cdots}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\cdots}{4(d+\cdots}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\cdots}{4(d+\cdots}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\cdots}{4(d+\cdots}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\cdots}{4(d+\cdots}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\cdots}{4(d+\cdots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2143 vs. $2(972) = 1944$.
time = 11.57, size = 2143, normalized size = 2.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-4*I)*a^2*d^2)/f^2 + (a^2*c
*d^2*x)/(2*f^2) + (8*a^2*d^2)/(f^2*(I + c*x))))/c - (15*a^2*d^(5/2)*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*c*f
^(3/2)) - ((4*I)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*
Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] +
 Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[
c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x^2]]) - ((-I)*c*x - (2*I)*ArcSinh[c*x] +
 I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 + (4*I)*ArcTan[Coth[ArcS
inh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])) - (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(
I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(8*ArcTan[Ta
nh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + 4*Log[Sqrt[1 +
 c^2*x^2]])) + (ArcSinh[c*x]*(-4*I + ArcSinh[c*x]) - (8*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(I*Cosh[ArcSinh[c*x]/
2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*
Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-18*Pi*ArcSinh[c*x] -
(6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*
Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[
(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I)*Po
lyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]
)/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2*Sqrt[-(((-I)*d
 + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[
ArcSinh[c*x]/2])^2) - (((2*I)/3)*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)
]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((-6*I)*c*x*ArcSinh[
c*x])/Sqrt[1 + c^2*x^2] + ((6 + 6*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2
*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (3*I)*(2 + ArcSinh[c*x]^2) + ((6*I)*(2
*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] -
 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*L
og[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1
+ c^2*x^2] - (12*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(C
osh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(c*f^2*Sqrt[-(((-I)*d + c*
d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (
b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-
(d*f*(1 + c^2*x^2))]*((-96*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ((48 - 48*
I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] - ((20*I)*ArcSinh[c*x]^3)/Sqrt[1 + c^2
*x^2] + 48*(2 + ArcSinh[c*x]^2) + (6*I)*c*x*(1 + 2*ArcSinh[c*x]^2) - ((6*I)
*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (48*(2*(Pi - (2*I)*
ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^
ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSi
nh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] +
((96*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(24*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(
I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (a*b*d^
```

```
2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Sinh[ArcSinh[c*x]/2]*(-16*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*Cosh[2*Ar
cSinh[c*x]] + 2*(8*c*x + 8*ArcSinh[c*x] + (5*I)*ArcSinh[c*x]^2 + 16*ArcTan[
Tanh[ArcSinh[c*x]/2]] + (8*I)*Log[Sqrt[1 + c^2*x^2]] - I*ArcSinh[c*x]*Sinh[
2*ArcSinh[c*x]])) - Cosh[ArcSinh[c*x]/2]*((16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + Cosh[2*ArcSinh[c*x]] - 2*((8*I)*c*x - (8*I)*ArcSinh[c*x] - 5*ArcSinh
[c*x]^2 + (16*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 8*Log[Sqrt[1 + c^2*x^2]] +
ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))))/(4*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f
 + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2
]))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(icdx+d\right)^{\frac{5}{2}} \left(a+b \,\mathrm{arcsinh}\left(c x\right)\right)^{2}}{\left(-icfx+f\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x)/
(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a^2 + integrate((
I*c*d*x + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(-I*c*f*x + f)^(3/2)
+ 2*(I*c*d*x + d)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/
2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d^2*x^2 - 2*I
*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) + (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}}{(f-c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

3.595 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=752

$$\frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2ib^2d^3x(1+c^2x^2)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+c^2x^2)\,(}{c(d+icdx)^{3/}}$$

```
[Out] 2*I*a*b*d^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*b^2
*d^3*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*b^2*d^3*x*(c^2
*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^3*(c^2
*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*d^3*x*
(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*d^3*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-I*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)-d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)+16*I*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c
^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*d^3*(c^2*x^2+1)^
(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2
)/(f-I*c*f*x)^(3/2)+8*b^2*d^3*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+
1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*d^3*(c^2*x^2+1)^(3/2
)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-4*b^2*d^3*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.78, antiderivative size = 752, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783, 5772, 267}

$$\frac{16ibd^3(c^2x^2+1)^{3/2}\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2iabd^3x(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{d^3(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4d^3(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4d^3x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8bd^3(c^2x^2+1)^{3/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8b^2d^3(c^2x^2+1)^{3/2}\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8b^2d^3(c^2x^2+1)^{3/2}\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2d^3(c^2x^2+1)^{3/2}\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2ib^2d^3(c^2x^2+1)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2ib^2d^3x(c^2x^2+1)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((2*I)*a*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((2*I)*b^2*d^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((2*I)*b^2*d^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])
^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*(1
 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x
```

```
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((16*I)*b*d^3*(1 + c^2
*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
 + (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^
2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^3(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{4i(id^3-cd^3x)(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}} - \frac{3d^3(a+b\sinh^{-1}(cx))^2}{\sqrt{1+c^2x^2}} - \right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i(1+c^2x^2)^{3/2}\right)\int \frac{(id^3-cd^3x)(a+b\sinh^{-1}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3d^3(1+c^2x^2)\right.}{(d+icdx}$$

$$= -\frac{id^3(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}\left(a+b\sinh\right.}{bc(d+icdx)^{3/2}(f-icf}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(1+c^2x^2)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c^2x^2)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4i}{}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3}{(d}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3}{(d}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3}{(d}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3}{(d}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3}{(d}$$

Mathematica [A]
time = 8.85, size = 1084, normalized size = 1.44

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),
x]
```

```
[Out] ((3*a^2*d*(5 - I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^2*(I + c*x))
- (9*a^2*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]])/f^(3/2) - (b^2*d*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(
-18*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi
- (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c
*x]] + 12*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSin
h[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^
2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(
f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) -
(I*b^2*d*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(((-6*I)*c*x*ArcSin
h[c*x])/Sqrt[1 + c^2*x^2] + ((6 + 6*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] +
(2*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (3*I)*(2 + ArcSinh[c*x]^2) + ((6*I)*
(2*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x]
 - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4
*Log[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[
1 + c^2*x^2] - (12*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(f^2*(Cosh[ArcSinh[c*x]/
2] + I*Sinh[ArcSinh[c*x]/2])^2) + (3*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*
ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcT
an[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Si
nh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])) + (6*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan
[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh
[ArcSinh[c*x]/2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh
[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c
^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(3*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 6
*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/(c
*f^2*sqrt(d/f))) + integrate((I*c*d*x + d)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2/(-I*c*f*x + f)^(3/2) + 2*(I*c*d*x + d)^(3/2)*a*b*log(c*x + sqrt(c^
2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c*d*x - b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*d*x + a*b*d)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*d*x - a^2*d)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}}{(f-c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

3.596 $\int \frac{\sqrt{d+icdx}\,(a+b\sinh^{-1}(cx))^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=544

$$-\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2d^2x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2d^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)+2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)+2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-1/3*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*a
rctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b*d^2*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2*d^2*(c^2
*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-2*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2)
)^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]

time = 0.64, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783}

$$\frac{8ibd^2(c^2x^2+1)^{3/2}\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{d^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2d^2(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2d^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4bd^2(c^2x^2+1)^{3/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4b^2d^2(c^2x^2+1)^{3/2}\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2d^2(c^2x^2+1)^{3/2}\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2d^2(c^2x^2+1)^{3/2}\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((-2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) + (2*d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^
2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcS
inh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*d^2*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (
-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*d
^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)
*(f - I*c*f*x)^(3/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
```

```
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
```

```
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\,\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{2i(id^2-cd^2x)\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}} - \frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i(1+c^2x^2)^{3/2}\right)\int \frac{(id^2-cd^2x)\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(d^2(1+c^2x^2)^{3/\ldots}\right.}{(d+icdx)\ldots}$$

$$= -\frac{d^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2i(1+c^2x^2)^{3/2}\right)\int\left(\frac{id^2(a+\ldots}{(1\ldots}\right.}{(d+icd\ldots}$$

$$= -\frac{d^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2d^2(1+c^2x^2)^{3/2}\right)\int\frac{(a+b\,\mathrm{si}\ldots}{(1+\ldots}}{(d+icdx)^{3/2}(f-ic\ldots}$$

$$= -\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)\left(a+b\sinh^{-\ldots}\right.}{(d+icdx)^{3/2}(f-icfx\ldots}$$

$$= -\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)\left(a+b\sinh^{-\ldots}\right.}{(d+icdx)^{3/2}(f-icfx\ldots}$$

$$= -\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)\left(a+b\sinh^{-\ldots}\right.}{(d+icdx)^{3/2}(f-icfx\ldots}$$

$$= -\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)\left(a+b\sinh^{-\ldots}\right.}{(d+icdx)^{3/2}(f-icfx\ldots}$$

$$= -\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)\left(a+b\sinh^{-\ldots}\right.}{(d+icdx)^{3/2}(f-icfx\ldots}$$

$$= -\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)\left(a+b\sinh^{-\ldots}\right.}{(d+icdx)^{3/2}(f-icfx\ldots}$$

Mathematica [A]
time = 3.58, size = 530, normalized size = 0.97

$$\frac{\frac{6a^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{i+cx} - 3a^2\sqrt{d}\,\sqrt{f}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right) - \frac{b^2(-i+cx)\sqrt{d+icdx}\,\sqrt{f-icfx}\left(-18\pi\sinh^{-1}(cx)-(6-6i)\sinh^{-1}(cx)^2+i\sinh^{-1}(cx)^3-12\left(\pi-2i\sinh^{-1}(cx)\right)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)+24\pi\log\left(1+e^{\sinh^{-1}(cx)}\right)+12\pi\log\left(-\cos\left(\frac{1}{4}\left(\pi+2i\sinh^{-1}(cx)\right)\right)\right)-24\pi\log\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)-24i\,\mathrm{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-\frac{12i\sinh^{-1}(cx)^2\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)}{\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)}\right)}{\sqrt{1+c^2x^2}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)^2} + \frac{3ab\sqrt{d+icdx}\,\sqrt{f-icfx}\left(-\sinh^{-1}(cx)^2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+4\sinh^{-1}(cx)\left(-i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+2\left(4\,\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+i\log\left(1+c^2x^2\right)\right)\left(i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right)}{\sqrt{1+c^2x^2}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)}}{3cf^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 3*a^2*Sqrt[d]*Sqrt
[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - (b
^2*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-18*Pi*ArcSinh[c*x] - (6
 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*Lo
g[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[(P
i + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I)*Poly
Log[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (3*a*b*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x
]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])
+ 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c
*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2 \sqrt{icdx+d}}{(-icfx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/(
c*f^2*sqrt(d/f))) + integrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2/(-I*c*f*x + f)^(3/2) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^2*x
^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*f^2*x^2 + 2*I*c*f^2
*x - f^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{id\left(cx-i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{\left(-if\left(cx+i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(3/2),
 x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}}{{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

3.597 $\int \frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{d+icdx}\,(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=464

$$-\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\ldots$$

```
[Out] -I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+d
*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)+4*I*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2
))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)+2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(
c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*d*(c^2*x^2+1)^
(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)
```

Rubi [A]
time = 0.45, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {5796, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265}

$$\frac{4ibd(c^2x^2+1)^{3/2}\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd(c^2x^2+1)^{3/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2b^2d(c^2x^2+1)^{3/2}\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2d(c^2x^2+1)^{3/2}\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2d(c^2x^2+1)^{3/2}\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((-I)*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*b*d*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 +
E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*b^2*d
*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) - (2*b^2*d*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*d*(1 + c^2*x^2)
^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}} + \frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(d(1+c^2x^2)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(icd(1+c^2x^2)^{3/2}\right)\int \frac{x\left(a+b\sinh^{-1}(c\right.}{(1+c^2x^2)^{3/2}}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} +$$

$$= -\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} +$$

$$= -\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} +$$

$$= -\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} +$$

$$= -\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} +$$

$$= -\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} +$$

Mathematica [A]
time = 1.46, size = 511, normalized size = 1.10

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]

[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 - I)*b^2*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + ((-I)*a^2 + a^2*c*
x + (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi
*Sqrt[1 + c^2*x^2]*Log[1 + I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^
2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + (2*I)
*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]])*(Cosh[ArcSinh[c*x]/2] - I
```

```
*Sinh[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[
c*x]]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2]*(2*a + 3*b*Pi - (4*I)*b*Log[1 + I/
E^ArcSinh[c*x]]) + (2*a - 3*b*Pi + (4*I)*b*Log[1 + I/E^ArcSinh[c*x]])*Sinh[
ArcSinh[c*x]/2])))/(c*d*f^2*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] - I*
Sinh[ArcSinh[c*x]/2]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arcsinh}\,(cx))^2}{(-icfx + f)^{\frac{3}{2}}\,\sqrt{icdx + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(I*c*d*x + d)*(-I*c*f*x +
 f)^(3/2)), x) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(-I*c^2*d*f^2
*x + c*d*f^2) - I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(-I*c^2*d*f^2*x + c*d*f^2) -
2*a*b*log(I*c*x - 1)/(c*sqrt(d)*f^(3/2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
(c^2*d*f^2*x + I*c*d*f^2)*integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - I*sqr
t(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*d
```

```
*f^2*x^3 + I*c^2*d*f^2*x^2 + c*d*f^2*x + I*d*f^2), x))/(c^2*d*f^2*x + I*c*d
*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx-i)}\,\left(-if\,(cx+i)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\text{asinh}(c\,x))^2}{\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.598 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/}}$$

```
[Out] x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+(c^2
*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*
b*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d
+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A]
time = 0.28, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {5796, 5787, 5797, 3799, 2221, 2317, 2438}

$$\frac{(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b(c^2x^2+1)^{3/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2(c^2x^2+1)^{3/2}\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - (2*b*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2b\left(1+c^2x^2\right)^{3/2}\right)\text{Subst}\left(\int (a+bx)\tan\right.}{c(d+icdx)^{3/2}(f-ic}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{(4}{}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b}{}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b}{}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 488 vs. $2(224) = 448$.
time = 0.82, size = 488, normalized size = 2.18

$$\frac{a^2cx+2abcx\sinh^{-1}(cx)-2ib^2\pi\sqrt{1+c^2x^2}\sinh^{-1}(cx)+b^2cx\sinh^{-1}(cx)^2-b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2+ib^2\pi\sqrt{1+c^2x^2}\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-2b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-ib^2\pi\sqrt{1+c^2x^2}\log\left(1+ie^{-\sinh^{-1}(cx)}\right)-2b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)+4ib^2\pi\sqrt{1+c^2x^2}\log\left(1+e^{\sinh^{-1}(cx)}\right)-ab\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)+ib^2\pi\sqrt{1+c^2x^2}\log\left(-\cos\left(\frac{1}{4}\left(\pi+2i\sinh^{-1}(cx)\right)\right)\right)-4ib^2\pi\sqrt{1+c^2x^2}\log\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)-ib^2\pi\sqrt{1+c^2x^2}\log\left(\sin\left(\frac{1}{4}\left(\pi+2i\sinh^{-1}(cx)\right)\right)\right)+2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)}{cdf\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),
x]

[Out] (a^2*c*x + 2*a*b*c*x*ArcSinh[c*x] - (2*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + b^2*c*x*ArcSinh[c*x]^2 - b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + I*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 +
 I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSi
nh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqr
t[1 + c^2*x^2]*Log[1 + c^2*x^2] + I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi +
 (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[
c*x]/2]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]
 + 2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + 2*b^2*Sqrt[1 +
```

```
 c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(c x\right)\right)^{2}}{\left(i c d x+d\right)^{\frac{3}{2}} \left(-i c f x+f\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x
 + f)^(3/2)), x) + 2*a*b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a^2
*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f) - a*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c*d*
f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2
+ (c^2*d^2*f^2*x^2 + d^2*f^2)*integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*c*x -
 sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c
^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2*d^2*f^2*x^2 + d^2*f
^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}{\left(i d \left(c x - i\right)\right)^{\frac{3}{2}} \left(- i f \left(c x + i\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3
/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2))
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.599 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=743

$$-\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{ibfx(1 \cdots}{3(d \cdots}$$

```
[Out] -1/3*I*b^2*f*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*f*
x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*f*(c^2*x^2+1)^(3/
2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*b*f*x*(c^
2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I
*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1
/3*f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+2/3*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(
5/2)+2/3*f*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)-2/3*I*b*f*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2
*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b*f*(c^2*x^2+1)^(5
/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)-1/3*b^2*f*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*f*(c^2*x^2+1)^(5/2)*
polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-
2/3*b^2*f*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.61, antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {5796, 5838, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5789, 4265, 267}

$$-\frac{2ibf(c^2x^2+1)^{5/2}\,\text{ArcTan}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2f(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2fx(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{ibfx(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{if(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{fx(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4bf(c^2x^2+1)^{5/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2f(c^2x^2+1)^{5/2}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2f(c^2x^2+1)^{5/2}\,\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f(c^2x^2+1)^{5/2}\,\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{ib^2f(c^2x^2+1)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2fx(c^2x^2+1)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((-1/3*I)*b^2*f*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)
) - (b^2*f*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
 (b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) - ((I/3)*b*f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))
/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f*(1 + c^2*x^2)*(a + b*
ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f*x*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f
```

```
- I*c*f*x)^(5/2)) + (2*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f*(1 + c^2*x^2)^(5/2
)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) - (4*b*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E
^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b^2*f*
(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/
2)*(f - I*c*f*x)^(5/2)) + (b^2*f*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh
[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*f*(1 + c^2*x
^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(f(1+c^2x^2)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(icf(1+c^2x^2)^{5/2}\right)\int \frac{x(a+b\sinh^{-1}}{(1+c^2x^2)}}{(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= \frac{if(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= \frac{bf(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+}{3c(}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+}{3c(}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+}{3c(}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+}{3c(}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+}{3c(}$$

<u>Mathematica</u> [A]
time = 7.51, size = 754, normalized size = 1.01

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-1/6*I)*a^2)/(d^3*f^2*(-I +
 c*x)^2) + (5*a^2)/(12*d^3*f^2*(-I + c*x)) + a^2/(4*d^3*f^2*(I + c*x))))/c
+ ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSi
```

```
nh[c*x] + (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 -
(2*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] -
(2*I)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d^2*f*(-I +
c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]) +
((I/6)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^2*x
^2]*(7*Pi*ArcSinh[c*x] + ((2 + I*ArcSinh[c*x])*ArcSinh[c*x])/(-I + c*x) - (
1 + 4*I)*ArcSinh[c*x]^2 - 5*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c
*x]] + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - 16*Pi*Log[1
+ E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 16*Pi*Log
[Cosh[ArcSinh[c*x]/2]] + 5*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (6*I)
*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (10*I)*PolyLog[2, I/E^ArcSinh[c*x]] + ((
3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[Ar
cSinh[c*x]/2]) + ((2*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + ((-4 + 5*ArcSinh[c*x]^2)*Sinh[ArcSinh
[c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(c*d^2*f*Sqr
t[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{\left(icdx+d\right)^{\frac{5}{2}}\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*b^2*c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2
+ c^2*d^3*f^2*x - I*c*d^3*f^2)*integral(1/3*(-3*I*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*a^2 - 2*(3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*b^2*
c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*
f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^5*d^3*f^2*x^5 - I*c^4*d^3*f^2*x^
4 + 2*c^3*d^3*f^2*x^3 - 2*I*c^2*d^3*f^2*x^2 + c*d^3*f^2*x - I*d^3*f^2), x))
/(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^{5/2}\,(f-c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.600 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=794

$$-\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2d^5x(1+c^2x^2)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{28d^5(1+c^2x^2)}{3c(d+icd}$$

```
[Out] -2*I*a*b*d^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*b^
2*d^5*(c^2*x^2+1)^3/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*b^2*d^5*x*(c^
2*x^2+1)^(5/2)*arcsinh(c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*d^5*(c
^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+
I*d^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+5/3*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)+112/3*b*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I/(c*
x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-112/3*b^2*d^5*(
c^2*x^2+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)+8/3*b*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi
+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+16/3*I*b^2*d^5
*(c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)+28/3*I*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1
/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*I*d^5*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+
1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.93, antiderivative size = 794, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {5796, 5844, 5783, 5798, 5772, 267, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-2*I)*a*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + ((2*I)*b^2*d^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) - ((2*I)*b^2*d^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)) + (28*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (I*d^5*(1 + c^2*x^2
)^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
5*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (112*b*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]
```

```
)*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (112*b^2*d^5*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^5*(1 + c^2*x^2)^(5/2)*(a +
b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) + (((16*I)/3)*b^2*d^5*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (
I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((28*I)/
3)*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[
c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*d^5*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^5\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{5d^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{icd^5x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{8d^5(a+b\sin}{(i+cx)^2\sqrt{}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(12id^5(1+c^2x^2)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(5d^5(1+c^2x^2)^{5/2}\right.}{(d+icdx)^{5/}}$$

$$= \frac{id^5(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}\left(a+b\sinh^{-}\right.}{3bc(d+icdx)^{5/2}(f-icfx}$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^5(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2d^5x(1+c^2x^2)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2}{(}$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2}{(}$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2}{(}$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2}{(}$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2}{(}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2552 vs. 2(794) = 1588.
time = 11.05, size = 2552, normalized size = 3.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a^2*d^2)/f^3 + (((8*I)/3)*
a^2*d^2)/(f^3*(I + c*x)^2) - (28*a^2*d^2)/(3*f^3*(I + c*x))))/c + (5*a^2*d^
(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c
*x)]])/(c*f^(5/2)) - ((I/3)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
 + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSin
h[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSin
h[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*Arc
Sinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]])
+ 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]]
 + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcS
inh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 +
I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Si
nh[ArcSinh[c*x]/2])^4) + (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
 c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] +
(28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[Arc
Sinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tan
h[ArcSinh[c*x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[
c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt
[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) -
(28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcS
inh[c*x]/2]))/(3*c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*d^2*(-I + c
*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x
^2))]*((-1 - I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I +
 c*x) - (2*I)*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3
*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[
c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]
] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[
ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[Ar
cSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I
*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])^2) + ((I/3)*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
 + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((-6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c
^2*x^2] + ((13 + 13*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (3*ArcSinh[c*x]^
3)/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/((I + c*x)*Sqr
t[1 + c^2*x^2]) + (3*I)*(2 + ArcSinh[c*x]^2) + ((13*I)*(2*(Pi - (2*I)*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSi
nh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*
x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (4*Ar
cSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])^3) - (2*(4 + 13*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x
]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))
/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])^2) + (2*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sq
```

```
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-21*Pi*ArcSinh[c*x] - (7
 - 7*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + ((2*I)*ArcSinh[c*x]*(2*I + ArcS
inh[c*x]))/(I + c*x) - 14*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x
]] + 28*Pi*Log[1 + E^ArcSinh[c*x]] + 14*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x
])/4]] - 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (28*I)*PolyLog[2, (-I)/E^ArcSinh
[c*x]] - ((2*I)*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[
c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])
/(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])^3))/(3*c*f^3*Sqrt[-(((-I)*
d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2])^2) - ((I/6)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I
*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcS
inh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(9 - (35*I)*ArcSinh[c*x] + 9*ArcSi
nh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]
)) + Cosh[ArcSinh[c*x]/2]*(20 + (24*I)*ArcSinh[c*x] + 27*ArcSinh[c*x]^2 + (
156*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*x^2]]) - I*(3*(-I
 + ArcSinh[c*x])*Cosh[(5*ArcSinh[c*x])/2] + 2*(13 + (7*I)*ArcSinh[c*x] + 18
*ArcSinh[c*x]^2 + (104*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*(I + ArcSinh
[c*x])*Cosh[2*ArcSinh[c*x]] + 52*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]
*(6 + (38*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c
*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2])))/(c*f^3*(-I +
c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])^4)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx))^2}{(-icfx+f)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c^2*d^2*x^2 + 2*b^2*c*d^2*x - I*b^2*d^2)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c^2*d^2*x^2
- 2*a*b*c*d^2*x + I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x - I*a^2*d^2)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x
 - I*f^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}}{{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

3.601 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=584

$$\frac{8d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{32bd^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(\right.}{3c(d+icdx)^{5/2}(f}$$

```
[Out] 8/3*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+1/3*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+32/3*b*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+
I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-32/3*b^2*d
^4*(c^2*x^2+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+4/3*b*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/
4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b^2*
d^4*(c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)+8/3*I*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi
+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*d^4*(c^2*x
^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*P
i+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]

time = 0.79, antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5844, 5783, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\frac{d^4(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8d^4(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{32bd^4(c^2x^2+1)^{5/2}\log\left(1+ie^{-\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bd^4(c^2x^2+1)^{5/2}\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8id^4(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(c^2x^2+1)^{5/2}\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{32b^2d^4(c^2x^2+1)^{5/2}\text{Li}_2\left(-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8ib^2d^4(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (8*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3
*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b*d^4*(1 + c^2*x^2)^(5/
2)*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (32*b^2*d^4*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^A
rcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*d^4*(1 +
 c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((8*I)/3)*b^2*d^4*(1 + c^2*x^2
)^(5/2)*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (((8*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((
2*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*Arc
Sinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2} \int \frac{(d+icdx)^4 \left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2} \int \left(\frac{d^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{4d^4\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)^2\sqrt{1+c^2x^2}} - \frac{4id^4\left(a+b\mathrm{si}\right.}{(i+cx)\sqrt{1}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(4id^4(1+c^2x^2)^{5/2}\right) \int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^4(1+c^2x^2)^{5/2}\right)}{(d+icdx)^{5/}}$$

$$= \frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4id^4(1+c^2x^2)^{5/2}\right)\mathrm{Subst}(}{(d+icdx)^5}$$

$$= \frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^4(1+c^2x^2)^{5/2}\right)\mathrm{Subst}(\int}{c(d+}$$

$$= \frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4bd^4(1+c^2x^2)^{5/2}\left(a+b\sin\right.}{3c(d+icd}$$

$$= \frac{4d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-ic}$$

$$= \frac{8d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-ic}$$

$$= \frac{8d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-ic}$$

$$= \frac{8d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-ic}$$

$$= \frac{8d^4(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-ic}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1617 vs. 2(584) = 1168.
time = 8.61, size = 1617, normalized size = 2.77

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((4*I)/3)*a^2*d)/(f^3*(I + c
*x)^2) - (8*a^2*d)/(3*f^3*(I + c*x))))/c + (a^2*d^(3/2)*Log[c*d*f*x + Sqrt[
d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(5/2)) - ((I/
3)*a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 +
c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSi
nh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
 c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[A
rcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*A
rcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]])
+ 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 +
 c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d
*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (a
*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*
x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x
])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSi
nh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 42*Log[S
qrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56
*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2
*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(6*c*f^3*(1 + I
*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sin
h[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*d*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sq
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSinh[c*x]^2
- (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2*I)*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^Arc
Sinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[
c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcS
inh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + Ar
cSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[
c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]
*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (b^2*d*(-I + c*x)*Sqr
t[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(
-21*Pi*ArcSinh[c*x] - (7 - 7*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + ((2*I)*
ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - 14*(Pi - (2*I)*ArcSinh[c*x])
*Log[1 + I/E^ArcSinh[c*x]] + 28*Pi*Log[1 + E^ArcSinh[c*x]] + 14*Pi*Log[-Cos
[(Pi + (2*I)*ArcSinh[c*x])/4]] - 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (28*I)*P
olyLog[2, (-I)/E^ArcSinh[c*x]] - ((2*I)*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh
[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (4*ArcSinh[c*x]
^2*Sinh[ArcSinh[c*x]/2])/(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])^3)
)/(3*c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}}(a+b\operatorname{arcsinh}(cx))^2}{(-icfx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I
*f^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}}{(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

3.602 $\int \frac{\sqrt{d+icdx}\,(a+b\sinh^{-1}(cx))^2}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=522

$$\frac{d^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bd^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4b^2d^3(1+}{3}$$

```
[Out] 1/3*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+4/3*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I/(c*x+(c^2*x
^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b^2*d^3*(c^2*x^2+1)
^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x
)^(5/2)+2/3*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi+1/2*I*arc
sinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*b^2*d^3*(c^2*x^2+1
)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)+1/3*I*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsinh
(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*d^3*(c^2*x^2+1)^(5/2)*(a
+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsin
h(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.76, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {5796, 5844, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\frac{d^3(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bd^3(c^2x^2+1)^{5/2}\log\left(1+ie^{-\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2bd^3(c^2x^2+1)^{5/2}\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{id^3(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{id^3(c^2x^2+1)^{5/2}\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4b^2d^3(c^2x^2+1)^{5/2}\text{Li}_2\left(-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4ib^2d^3(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(
f - I*c*f*x)^(5/2)) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log
[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*
b^2*d^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (((4*I)/3)*b^2*d^3*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (I/2)*Arc
Sinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*d^3*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\,\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(-\frac{2d^3\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)^2\sqrt{1+c^2x^2}} - \frac{id^3\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(id^3(1+c^2x^2)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2d^3(1+c^2x^2)^{5/2}\right)\int}{(d+icdx)^{5/2}(}$$

$$= -\frac{\left(id^3(1+c^2x^2)^{5/2}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^2}{ic+c\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2cd^3(\right.}{}$$

$$= -\frac{\left(d^3(1+c^2x^2)^{5/2}\right)\mathrm{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx, x, \sinh^{-1}(cx\right.}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2bd^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^3(}{}$$

$$= \frac{d^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bd^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-}\right.}{3c(d+icdx)^{5/}}$$

$$= \frac{d^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}\right.}{3c(d+icdx)^{5/2}(f-ic}$$

$$= \frac{d^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}\right.}{3c(d+icdx)^{5/2}(f-ic}$$

$$= \frac{d^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}\right.}{3c(d+icdx)^{5/2}(f-ic}$$

$$= \frac{d^3(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}\right.}{3c(d+icdx)^{5/2}(f-ic}$$

Mathematica [A]
time = 7.36, size = 788, normalized size = 1.51

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((2*I)/3)*a^2)/(f^3*(I + c*x
)^2) - a^2/(3*f^3*(I + c*x))))/c - ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt
```

```
[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[C
oth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4
*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c
^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSin
h[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan
[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c
*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*(-I + c*x)*Sqrt[I*((-I)*d + c
*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSin
h[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2
*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[
1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh
[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*
Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (
2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 +
 c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2\,\sqrt{icdx+d}}{(-icfx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*((b^2*c*x - I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1))^2 - 3*(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3)*integral(1/3*(-3
*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^
3*x - I*f^3), x))/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}}{{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

3.603 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=942

$$\frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2d^2(1+c^2x^2)^{5/2}\sinh^{-1}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^2(1+c^2x^2)^{3/}}{3c(d+icdx)}$$

```
[Out] 4/3*I*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/
2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*d^2*x*(c^2*x^2+1)^2/(d+I*
c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*d^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*d^2*(c^2*x^2+1)*(a+b*a
rcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*c*d^2*x^2*(c^2*x
^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*I*b*
d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+1/3*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x
)^(5/2)-1/3*c^2*d^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)+2/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+1/3*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d
+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*I*b^2*d^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*b^2*
d^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*d^2*(c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+(c^2*
x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d^2*(c^2*x^2+1
)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)
```

Rubi [A]
time = 0.86, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {5796, 5838, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5789, 4265, 267, 5800, 5810, 294, 221}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (((2*I)/3)*b^2*d^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (2*b^2*d^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + (b^2*d^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3
```

```
*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d^2*x*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b*c*d^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5
/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*d^2
*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)) + (2*d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*b*d^2*(1
 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (2*b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(5
/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
```

```
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
```

```
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}} + \frac{2icd^2x\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}} - \frac{c^2d^2x^2\left(a+b\sinh^{-1}(cx\right.}{(1+c^2x^2)^{5/2}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(d^2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2icd^2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}\right.}{(1+c^2x^2)}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^2(1+c^2x}{3c(d+i}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2(1+c^2}{3c(d+icdx}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2(1+c^2}{3c(d+icdx}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2(1+c^2}{3c(d+icdx}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2(1+c^2}{3c(d+icdx}$$

Mathematica [A]
time = 4.67, size = 528, normalized size = 0.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(2*I + c*x))/(I + c*x)^2 - (a*b*
(I*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]]
+ (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 + (3*I)*ArcSinh[c*x] +
 (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*(I + (-1
```

```
+ Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[A
rcSinh[c*x]/2]] - (I/2)*(2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2])*Sinh[ArcS
inh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x
]/2])^3) - (b^2*((1 + I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(2*I + ArcSinh[c*x]
))/(I + c*x) + 2*(I*Pi + 2*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + I*Pi*(
3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh
[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) - 4*PolyLog[2, (-I)/E^ArcSinh[c*x
]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[
ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]))/(3*c*d*f^3)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(c x\right)\right)^{2}}{\left(-icfx+f\right)^{\frac{5}{2}} \sqrt{icdx+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((b^2*c*x + 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 3*(c^3*d*f^3*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3)*integral(
-1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*a*b + (b^2*c*x + 2*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
```

```
+ d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^4*d*f^3*x^4 + 2*I
*c^3*d*f^3*x^3 + 2*I*c*d*f^3*x - d*f^3), x))/(c^3*d*f^3*x^2 + 2*I*c^2*d*f^3
*x - c*d*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{asinh}(cx))^2}{\sqrt{id(cx-i)}\,(-if(cx+i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.604 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=743

$$\frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibdx(1+c}{3(d+}$$

```
[Out] 1/3*I*b^2*d*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d*x
*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*d*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I*b*d*x*(c^2
*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*
d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/
3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+
2/3*d*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+2/3*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)+2/3*I*b*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*
x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b*d*(c^2*x^2+1)^(5/
2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b^2*d*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d*(c^2*x^2+1)^(5/2)*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2
/3*b^2*d*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.60, antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {5796, 5838, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5789, 4265, 267}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/3)*b^2*d*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b^2*d*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b
*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) + ((I/3)*b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I
```

```
*c*f*x)^(5/2)) + (2*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d +
 I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d*(1 + c^2*x^2)^(5/2)*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (4*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2
*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b^2*d*(1
+ c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (b^2*d*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh[c*
x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d*(1 + c^2*x^2)
^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}+\frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(d(1+c^2x^2)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(icd(1+c^2x^2)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}\right.}{(1+c^2x^2)^{\cdots}}}{(d+icdx)^{5/2}(f-icfx)^{5\cdots}}$$

$$= -\frac{id(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \cdots$$

$$= \frac{bd(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx\cdots\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c\cdots}{3c(d\cdots}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c\cdots}{3c(d\cdots}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c\cdots}{3c(d\cdots}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c\cdots}{3c(d\cdots}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c\cdots}{3c(d\cdots}$$

Mathematica [A]
time = 7.55, size = 757, normalized size = 1.02

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(a^2/(4*d^2*f^3*(-I + c*x)) +
((I/6)*a^2)/(d^2*f^3*(I + c*x)^2) + (5*a^2)/(12*d^2*f^3*(I + c*x))))/c - ((
I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSinh[c
```

```
*x] - (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 + (2*I
)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I
)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d*f^2*(I + c*x)*
Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]) - ((I/6
)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^2*x^2]*(
-9*Pi*ArcSinh[c*x] + ((2 - I*ArcSinh[c*x])*ArcSinh[c*x])/(I + c*x) - (1 - 4
*I)*ArcSinh[c*x]^2 + 3*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]]
- 5*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 16*Pi*Log[1 + E^A
rcSinh[c*x]] + 5*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 16*Pi*Log[Cosh
[ArcSinh[c*x]/2]] - 3*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] - (10*I)*Pol
yLog[2, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[2, I/E^ArcSinh[c*x]] - ((2*I)*
ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh
[c*x]/2])^3 + (I*(4 - 5*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) - ((3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x
]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d*f^2*Sqrt[-(((-
I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}\,(-icfx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2
+ c^2*d^2*f^3*x + I*c*d^2*f^3)*integral(1/3*(3*I*sqrt(I*c*d*x + d)*sqrt(-I*
c*f*x + f)*a^2 - 2*(-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*b^2*
c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*
f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^5*d^2*f^3*x^5 + I*c^4*d^2*f^3*x^
4 + 2*c^3*d^2*f^3*x^3 + 2*I*c^2*d^2*f^3*x^2 + c*d^2*f^3*x + I*d^2*f^3), x))
/(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^{3/2}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.605 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=386

$$-\frac{b^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2x(1+c}{3(d+}$$

```
[Out] -1/3*b^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*(c^2*x^2
+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*x*(c
^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*x*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*(c
^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-
4/3*b*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/
c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*(c^2*x^2+1)^(5/2)*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A]
time = 0.38, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {5796, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197}

$$\frac{2(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2x(c^2x^2+1)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4b(c^2x^2+1)^{5/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2(c^2x^2+1)^{5/2}\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2x(c^2x^2+1)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] -1/3*(b^2*x*(1 + c^2*x^2)^2)/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b
*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*(1 + c^2*x^2)^(5/2)*(a + b*A
rcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*b*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I
*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*(1 + c^2*x^2)^(5/2)*PolyLog[2,
-E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x}{}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1}{3(}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1}{3(}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1}{3(}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1}{3(}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1}{3(}$$

Mathematica [A]

time = 5.22, size = 642, normalized size = 1.66

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)), x]

[Out] (4*a^2*c*x*(3 + 2*c^2*x^2) - b^2*(c*x - 6*c*x*ArcSinh[c*x]^2 + (4*I)*Pi*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] + 2*ArcSinh[c*x]^2*Cosh[3*ArcSinh[c*x]] - (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 + I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]*Log[1 + I/E^ArcSinh[c*x]] - (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 + E^ArcSinh[c*x]] - (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[Cosh[ArcSinh[c*x]/2]] + (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + 2*Sqrt[1 + c^2*x^2]*(((-3*I)*Pi + 6*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + I*((2*I)*ArcSinh[c*x] + 6*Pi*ArcSinh[c*x] - (3*I)*ArcSinh[c*x]^2 + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - 12*Pi*Log[1 + E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 12*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 3*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])) - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]] + Sinh[3*ArcSinh[c*x]] - 2*ArcSinh[c*x]^2*Sinh[3*ArcSinh[c*x]]) + 2*a*b*(Sqrt[1 + c^2*x^2]*(2 - 3*Log[1 + c^2*x^2]) - Cosh[3*ArcSinh[c*x]]*Log[1 + c^2*x^2] + 2*ArcSinh[c*x]*(3*c*x + Sinh[3*ArcSinh[c*x]])))/(12*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(c + c^3*x^2))

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)

[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^
2 + 1)/(c^2*d^(5/2)*f^(5/2))) + 2/3*a*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f)
+ 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a^2*(x/((c^2*d*
f*x^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2)) + b^2*inte
grate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(5/2)*(-I*c*f*x + f)^(5
/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*b^2*c^2*x^3 + 3*b^2*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
*integral(1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f)*a*b - (2*b^2*c^3*x^3 + 3*b^2*c*x)*sqrt(c^2*x^2 +
 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^
6*d^3*f^3*x^6 + 3*c^4*d^3*f^3*x^4 + 3*c^2*d^3*f^3*x^2 + d^3*f^3), x))/(c^4*
d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2}{(d + c\,d\,x\,1\mathrm{i})^{5/2}\,(f - c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.606 $\int (d+ex^2)^4 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=312

$$-\frac{b(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4)\sqrt{1+c^2x^2}}{315c^9}-\frac{4be(105c^6d^3-189c^4d^2e+135c^2de^2-3}{945c^9}$$

```
[Out] -4/945*b*e*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*(c^2*x^2+1)^(3/
2)/c^9-2/525*b*e^2*(63*c^4*d^2-90*c^2*d*e+35*e^2)*(c^2*x^2+1)^(5/2)/c^9-4/4
41*b*(9*c^2*d-7*e)*e^3*(c^2*x^2+1)^(7/2)/c^9-1/81*b*e^4*(c^2*x^2+1)^(9/2)/c
^9+d^4*x*(a+b*arcsinh(c*x))+4/3*d^3*e*x^3*(a+b*arcsinh(c*x))+6/5*d^2*e^2*x^
5*(a+b*arcsinh(c*x))+4/7*d*e^3*x^7*(a+b*arcsinh(c*x))+1/9*e^4*x^9*(a+b*arcs
inh(c*x))-1/315*b*(315*c^8*d^4-420*c^6*d^3*e+378*c^4*d^2*e^2-180*c^2*d*e^3+
35*e^4)*(c^2*x^2+1)^(1/2)/c^9
```

Rubi [A]
time = 0.24, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {200, 5792, 12, 1813, 1864}

$$d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}e^4x^9\left(a+b\sinh^{-1}(cx)\right)-\frac{4be^3\left(c^2x^2+1\right)^{7/2}\left(9c^2d-7e\right)}{441c^9}-\frac{be^4\left(c^2x^2+1\right)^{9/2}}{81c^9}-\frac{2be^2\left(c^2x^2+1\right)^{5/2}\left(63c^4d^2-90c^2de+35e^2\right)}{525c^9}-\frac{4be\left(c^2x^2+1\right)^{3/2}\left(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3\right)}{945c^9}-\frac{b\sqrt{c^2x^2+1}\left(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4\right)}{315c^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/315*(b*(315*c^8*d^4 - 420*c^6*d^3*e + 378*c^4*d^2*e^2 - 180*c^2*d*e^3 +
35*e^4)*Sqrt[1 + c^2*x^2])/c^9 - (4*b*e*(105*c^6*d^3 - 189*c^4*d^2*e + 135*
c^2*d*e^2 - 35*e^3)*(1 + c^2*x^2)^(3/2))/(945*c^9) - (2*b*e^2*(63*c^4*d^2 -
 90*c^2*d*e + 35*e^2)*(1 + c^2*x^2)^(5/2))/(525*c^9) - (4*b*(9*c^2*d - 7*e)
*e^3*(1 + c^2*x^2)^(7/2))/(441*c^9) - (b*e^4*(1 + c^2*x^2)^(9/2))/(81*c^9)
+ d^4*x*(a + b*ArcSinh[c*x]) + (4*d^3*e*x^3*(a + b*ArcSinh[c*x]))/3 + (6*d^
2*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (4*d*e^3*x^7*(a + b*ArcSinh[c*x]))/7 +
(e^4*x^9*(a + b*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^4 \left(a+b\sinh^{-1}(cx)\right)\,dx = d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= -\frac{b(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4)\sqrt{1+c^2x^2}}{315c^9}$$

Mathematica [A]
time = 0.19, size = 260, normalized size = 0.83

$$\frac{315ax(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)-\frac{b\sqrt{1+c^2x^2}\,(4480e^4-320c^2e^3(81d+7ex^2)+48c^4e^2(1323d^2+270dex^2+35e^2x^4)-8c^6e(11025d^3+3969d^2ex^2+1215de^2x^4+175e^3x^6)+c^8(99225d^4+44100d^3ex^2+23814d^2e^2x^4+8100de^3x^6+1225e^4x^8))}{c^9}+315bx(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)\sinh^{-1}(cx)}{99225}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^
4*x^8) - (b*Sqrt[1 + c^2*x^2]*(4480*e^4 - 320*c^2*e^3*(81*d + 7*e*x^2) + 48
*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) - 8*c^6*e*(11025*d^3 + 3969*
```

```
d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 44100*d^3*e*x^
2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 315*b*x*(315
*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*x^8)*ArcSin
h[c*x])/99225
```

Maple [A]
time = 0.64, size = 451, normalized size = 1.45

method	result
derivativedivides	$\frac{\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\operatorname{arcsinh}(cx)d^4c^9x+\frac{4\operatorname{arcsinh}(cx)d^3c^9e\,x^3}{3}+\frac{6\operatorname{arcsinh}(cx)d^2c^9e^2x^5}{5}+4\right.}{}}{}$
default	$\frac{\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\operatorname{arcsinh}(cx)d^4c^9x+\frac{4\operatorname{arcsinh}(cx)d^3c^9e\,x^3}{3}+\frac{6\operatorname{arcsinh}(cx)d^2c^9e^2x^5}{5}+4\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^8*(d^4*c^9*x+4/3*d^3*c^9*e*x^3+6/5*d^2*c^9*e^2*x^5+4/7*d*c^9*e^3*x
^7+1/9*e^4*c^9*x^9)+b/c^8*(arcsinh(c*x)*d^4*c^9*x+4/3*arcsinh(c*x)*d^3*c^9*
e*x^3+6/5*arcsinh(c*x)*d^2*c^9*e^2*x^5+4/7*arcsinh(c*x)*d*c^9*e^3*x^7+1/9*a
rcsinh(c*x)*e^4*c^9*x^9-d^4*c^8*(c^2*x^2+1)^(1/2)-4/3*d^3*c^6*e*(1/3*c^2*x^
2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-6/5*d^2*c^4*e^2*(1/5*(c^2*x^2+1)
^(1/2)*c^4*x^4-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-4/7*d
*c^2*e^3*(1/7*(c^2*x^2+1)^(1/2)*c^6*x^6-6/35*(c^2*x^2+1)^(1/2)*c^4*x^4+8/35
*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*x^2+1)^(1/2))-1/9*e^4*(1/9*(c^2*x^2+1
)^(1/2)*c^8*x^8-8/63*(c^2*x^2+1)^(1/2)*c^6*x^6+16/105*(c^2*x^2+1)^(1/2)*c^4
*x^4-64/315*c^2*x^2*(c^2*x^2+1)^(1/2)+128/315*(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.29, size = 411, normalized size = 1.32

$$\frac{1}{9}ax^9e^4+\frac{4}{7}adx^7e^3+\frac{6}{5}ad^2x^5e^2+\frac{4}{3}ad^3x^3e+ad^4x+\frac{4}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^3e+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd^4}{c}+\frac{2}{25}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^2e^2+\frac{4}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bde^3+\frac{1}{2835}\left(315x^9\operatorname{arsinh}(cx)-\left(\frac{35\sqrt{c^2x^2+1}\,x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}\,x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2+1}}{c^{10}}\right)c\right)be^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9*e^4 + 4/7*a*d*x^7*e^3 + 6/5*a*d^2*x^5*e^2 + 4/3*a*d^3*x^3*e + a*d
^4*x + 4/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*
x^2 + 1)/c^4))*b*d^3*e + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^4/c + 2
/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 +
```

```
1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^2*e^2 + 4/245*(35*x^7*arcsinh(
c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(
c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*d*e^3 + 1/2835*(315*x
^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/
c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqr
t(c^2*x^2 + 1)/c^10)*c)*b*e^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1039 vs. $2(278) = 556$.
time = 0.38, size = 1039, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*x^9*cosh(1)^4 + 11025*a*c^9*x^9*sinh(1)^4 + 56700*a*c^
9*d*x^7*cosh(1)^3 + 119070*a*c^9*d^2*x^5*cosh(1)^2 + 132300*a*c^9*d^3*x^3*c
osh(1) + 99225*a*c^9*d^4*x + 6300*(7*a*c^9*x^9*cosh(1) + 9*a*c^9*d*x^7)*sin
h(1)^3 + 1890*(35*a*c^9*x^9*cosh(1)^2 + 90*a*c^9*d*x^7*cosh(1) + 63*a*c^9*d
^2*x^5)*sinh(1)^2 + 315*(35*b*c^9*x^9*cosh(1)^4 + 35*b*c^9*x^9*sinh(1)^4 +
180*b*c^9*d*x^7*cosh(1)^3 + 378*b*c^9*d^2*x^5*cosh(1)^2 + 420*b*c^9*d^3*x^3
*cosh(1) + 315*b*c^9*d^4*x + 20*(7*b*c^9*x^9*cosh(1) + 9*b*c^9*d*x^7)*sinh(
1)^3 + 6*(35*b*c^9*x^9*cosh(1)^2 + 90*b*c^9*d*x^7*cosh(1) + 63*b*c^9*d^2*x^
5)*sinh(1)^2 + 4*(35*b*c^9*x^9*cosh(1)^3 + 135*b*c^9*d*x^7*cosh(1)^2 + 189*
b*c^9*d^2*x^5*cosh(1) + 105*b*c^9*d^3*x^3)*sinh(1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 1260*(35*a*c^9*x^9*cosh(1)^3 + 135*a*c^9*d*x^7*cosh(1)^2 + 189*a*c^
9*d^2*x^5*cosh(1) + 105*a*c^9*d^3*x^3)*sinh(1) - (99225*b*c^8*d^4 + 35*(35*
b*c^8*x^8 - 40*b*c^6*x^6 + 48*b*c^4*x^4 - 64*b*c^2*x^2 + 128*b)*cosh(1)^4 +
 35*(35*b*c^8*x^8 - 40*b*c^6*x^6 + 48*b*c^4*x^4 - 64*b*c^2*x^2 + 128*b)*sin
h(1)^4 + 1620*(5*b*c^8*d*x^6 - 6*b*c^6*d*x^4 + 8*b*c^4*d*x^2 - 16*b*c^2*d)*
cosh(1)^3 + 20*(405*b*c^8*d*x^6 - 486*b*c^6*d*x^4 + 648*b*c^4*d*x^2 - 1296*
b*c^2*d + 7*(35*b*c^8*x^8 - 40*b*c^6*x^6 + 48*b*c^4*x^4 - 64*b*c^2*x^2 + 12
8*b)*cosh(1))*sinh(1)^3 + 7938*(3*b*c^8*d^2*x^4 - 4*b*c^6*d^2*x^2 + 8*b*c^4
*d^2)*cosh(1)^2 + 6*(3969*b*c^8*d^2*x^4 - 5292*b*c^6*d^2*x^2 + 10584*b*c^4*
d^2 + 35*(35*b*c^8*x^8 - 40*b*c^6*x^6 + 48*b*c^4*x^4 - 64*b*c^2*x^2 + 128*b
)*cosh(1)^2 + 810*(5*b*c^8*d*x^6 - 6*b*c^6*d*x^4 + 8*b*c^4*d*x^2 - 16*b*c^2
*d)*cosh(1))*sinh(1)^2 + 44100*(b*c^8*d^3*x^2 - 2*b*c^6*d^3)*cosh(1) + 4*(1
1025*b*c^8*d^3*x^2 - 22050*b*c^6*d^3 + 35*(35*b*c^8*x^8 - 40*b*c^6*x^6 + 48
*b*c^4*x^4 - 64*b*c^2*x^2 + 128*b)*cosh(1)^3 + 1215*(5*b*c^8*d*x^6 - 6*b*c^
6*d*x^4 + 8*b*c^4*d*x^2 - 16*b*c^2*d)*cosh(1)^2 + 3969*(3*b*c^8*d^2*x^4 - 4
*b*c^6*d^2*x^2 + 8*b*c^4*d^2)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))/c^9
```

Sympy [A]
time = 2.00, size = 593, normalized size = 1.90

$$\begin{cases} ad^4x + \frac{4ad^3ex^3}{3} + \frac{6ad^2e^2x^5}{5} + \frac{4ade^3x^7}{7} + \frac{ae^4x^9}{9} + bd^4x\operatorname{asinh}(cx) + \frac{4bd^3ex^3\operatorname{asinh}(cx)}{3} + \frac{6bd^2e^2x^5\operatorname{asinh}(cx)}{5} + \frac{4bde^3x^7\operatorname{asinh}(cx)}{7} + \frac{be^4x^9\operatorname{asinh}(cx)}{9} - \frac{bd^4\sqrt{c^2x^2+1}}{c} - \frac{4bd^3ex^2\sqrt{c^2x^2+1}}{9c} - \frac{6bd^2e^2x^4\sqrt{c^2x^2+1}}{25c} - \frac{4bde^3x^6\sqrt{c^2x^2+1}}{49c} - \frac{be^4x^8\sqrt{c^2x^2+1}}{81c} + \frac{8bd^3e\sqrt{c^2x^2+1}}{9c^3} + \frac{8bd^2e^2x^2\sqrt{c^2x^2+1}}{25c^3} + \frac{24bde^3x^4\sqrt{c^2x^2+1}}{245c^3} + \frac{8be^4x^6\sqrt{c^2x^2+1}}{567c^3} - \frac{16bd^2e^2\sqrt{c^2x^2+1}}{25c^5} - \frac{32bde^3x^2\sqrt{c^2x^2+1}}{245c^5} - \frac{16be^4x^4\sqrt{c^2x^2+1}}{945c^5} + \frac{64bde^3\sqrt{c^2x^2+1}}{245c^7} + \frac{64be^4x^2\sqrt{c^2x^2+1}}{2835c^7} - \frac{128be^4\sqrt{c^2x^2+1}}{2835c^9} & \text{for}\: c \neq 0 \\ a\left(d^4x + \frac{4d^3ex^3}{3} + \frac{6d^2e^2x^5}{5} + \frac{4de^3x^7}{7} + \frac{e^4x^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**3
*x**7/7 + a*e**4*x**9/9 + b*d**4*x*asinh(c*x) + 4*b*d**3*e*x**3*asinh(c*x)/
3 + 6*b*d**2*e**2*x**5*asinh(c*x)/5 + 4*b*d*e**3*x**7*asinh(c*x)/7 + b*e**4
*x**9*asinh(c*x)/9 - b*d**4*sqrt(c**2*x**2 + 1)/c - 4*b*d**3*e*x**2*sqrt(c*
*2*x**2 + 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 4*b*d*
e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) - b*e**4*x**8*sqrt(c**2*x**2 + 1)/(81*
c) + 8*b*d**3*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 8*b*d**2*e**2*x**2*sqrt(c**2
*x**2 + 1)/(25*c**3) + 24*b*d*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c**3) + 8*
b*e**4*x**6*sqrt(c**2*x**2 + 1)/(567*c**3) - 16*b*d**2*e**2*sqrt(c**2*x**2
+ 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) - 16*b*e**
4*x**4*sqrt(c**2*x**2 + 1)/(945*c**5) + 64*b*d*e**3*sqrt(c**2*x**2 + 1)/(24
5*c**7) + 64*b*e**4*x**2*sqrt(c**2*x**2 + 1)/(2835*c**7) - 128*b*e**4*sqrt(
c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), (a*(d**4*x + 4*d**3*e*x**3/3 + 6*d**
2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x**9/9), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(e\,x^2+d\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^4,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^4, x)
```

3.607 $\int (d+ex^2)^3 \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=221

$$-\frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\sqrt{1+c^2x^2}}{35c^7}-\frac{be(35c^4d^2-42c^2de+15e^2)\,(1+c^2x^2)^{3/2}}{105c^7}-\frac{3b(7c^2d-5}{}$$

```
[Out] -1/105*b*e*(35*c^4*d^2-42*c^2*d*e+15*e^2)*(c^2*x^2+1)^(3/2)/c^7-3/175*b*(7*
c^2*d-5*e)*e^2*(c^2*x^2+1)^(5/2)/c^7-1/49*b*e^3*(c^2*x^2+1)^(7/2)/c^7+d^3*x
*(a+b*arcsinh(c*x))+d^2*e*x^3*(a+b*arcsinh(c*x))+3/5*d*e^2*x^5*(a+b*arcsinh
(c*x))+1/7*e^3*x^7*(a+b*arcsinh(c*x))-1/35*b*(35*c^6*d^3-35*c^4*d^2*e+21*c^
2*d*e^2-5*e^3)*(c^2*x^2+1)^(1/2)/c^7
```

Rubi [A]
time = 0.18, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {200, 5792, 12, 1813, 1864}

$$d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\sinh^{-1}(cx)\right)-\frac{3be^2(c^2x^2+1)^{5/2}(7c^2d-5e)}{175c^7}-\frac{be^3(c^2x^2+1)^{7/2}}{49c^7}-\frac{be(c^2x^2+1)^{3/2}(35c^4d^2-42c^2de+15e^2)}{105c^7}-\frac{b\sqrt{c^2x^2+1}\,(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)}{35c^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/35*(b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Sqrt[1 + c^2*x^
2])/c^7 - (b*e*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*(1 + c^2*x^2)^(3/2))/(105
*c^7) - (3*b*(7*c^2*d - 5*e)*e^2*(1 + c^2*x^2)^(5/2))/(175*c^7) - (b*e^3*(1
 + c^2*x^2)^(7/2))/(49*c^7) + d^3*x*(a + b*ArcSinh[c*x]) + d^2*e*x^3*(a + b
*ArcSinh[c*x]) + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (e^3*x^7*(a + b*Arc
Sinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx = d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh\right.$$

$$= -\frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\sqrt{1+c^2x^2}}{35c^7}-\frac{be(35c^4d^2-42c^2}{}$$

Mathematica [A]
time = 0.18, size = 187, normalized size = 0.85

$$a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)-\frac{b\sqrt{1+c^2x^2}\left(-240e^3+24c^2e^2(49d+5ex^2)-2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2ex^2+441de^2x^4+75e^3x^6)\right)}{3675c^7}+b\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[1 + c^2*x^2
]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2
 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^
6)))/(3675*c^7) + b*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7)*Arc
Sinh[c*x]
```

Maple [A]
time = 0.65, size = 316, normalized size = 1.43

method	result
derivativedivides	$\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\operatorname{arcsinh}(cx)d^3c^7x+\operatorname{arcsinh}(cx)d^2c^7e\,x^3+\frac{3\operatorname{arcsinh}(cx)d\,c^7e^2x^5}{5}+\frac{\operatorname{arcsinh}(cx)e^3c^7x^7}{7}\right.}{}$
default	$\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\operatorname{arcsinh}(cx)d^3c^7x+\operatorname{arcsinh}(cx)d^2c^7e\,x^3+\frac{3\operatorname{arcsinh}(cx)d\,c^7e^2x^5}{5}+\frac{\operatorname{arcsinh}(cx)e^3c^7x^7}{7}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+b/c^
6*(arcsinh(c*x)*d^3*c^7*x+arcsinh(c*x)*d^2*c^7*e*x^3+3/5*arcsinh(c*x)*d*c^7
*e^2*x^5+1/7*arcsinh(c*x)*e^3*c^7*x^7-d^3*c^6*(c^2*x^2+1)^(1/2)-d^2*c^4*e*(
1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-3/5*d*c^2*e^2*(1/5*(c^
2*x^2+1)^(1/2)*c^4*x^4-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2
))-1/7*e^3*(1/7*(c^2*x^2+1)^(1/2)*c^6*x^6-6/35*(c^2*x^2+1)^(1/2)*c^4*x^4+8/
35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.27, size = 285, normalized size = 1.29

$$\frac{1}{7}ax^7e^3+\frac{3}{5}adx^5e^2+ad^2x^3e+ad^3x+\frac{1}{3}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^2e+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd^3}{c}+\frac{1}{25}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bde^2+\frac{1}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e^3 + 3/5*a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x + 1/3*(3*x^3*arcsin
h(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2*e +
 (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c + 1/25*(15*x^5*arcsinh(c*x)
 - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*
x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1
)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*
sqrt(c^2*x^2 + 1)/c^8)*c)*b*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(199) = 398$.
time = 0.35, size = 611, normalized size = 2.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*x^7*cosh(1)^3 + 525*a*c^7*x^7*sinh(1)^3 + 2205*a*c^7*d*x^
5*cosh(1)^2 + 3675*a*c^7*d^2*x^3*cosh(1) + 3675*a*c^7*d^3*x + 315*(5*a*c^7*
x^7*cosh(1) + 7*a*c^7*d*x^5)*sinh(1)^2 + 105*(5*b*c^7*x^7*cosh(1)^3 + 5*b*c
^7*x^7*sinh(1)^3 + 21*b*c^7*d*x^5*cosh(1)^2 + 35*b*c^7*d^2*x^3*cosh(1) + 35
*b*c^7*d^3*x + 3*(5*b*c^7*x^7*cosh(1) + 7*b*c^7*d*x^5)*sinh(1)^2 + (15*b*c^
7*x^7*cosh(1)^2 + 42*b*c^7*d*x^5*cosh(1) + 35*b*c^7*d^2*x^3)*sinh(1))*log(c
*x + sqrt(c^2*x^2 + 1)) + 105*(15*a*c^7*x^7*cosh(1)^2 + 42*a*c^7*d*x^5*cosh
(1) + 35*a*c^7*d^2*x^3)*sinh(1) - (3675*b*c^6*d^3 + 15*(5*b*c^6*x^6 - 6*b*c
^4*x^4 + 8*b*c^2*x^2 - 16*b)*cosh(1)^3 + 15*(5*b*c^6*x^6 - 6*b*c^4*x^4 + 8*
b*c^2*x^2 - 16*b)*sinh(1)^3 + 147*(3*b*c^6*d*x^4 - 4*b*c^4*d*x^2 + 8*b*c^2*
d)*cosh(1)^2 + 3*(147*b*c^6*d*x^4 - 196*b*c^4*d*x^2 + 392*b*c^2*d + 15*(5*b
*c^6*x^6 - 6*b*c^4*x^4 + 8*b*c^2*x^2 - 16*b)*cosh(1))*sinh(1)^2 + 1225*(b*c
^6*d^2*x^2 - 2*b*c^4*d^2)*cosh(1) + (1225*b*c^6*d^2*x^2 - 2450*b*c^4*d^2 +
45*(5*b*c^6*x^6 - 6*b*c^4*x^4 + 8*b*c^2*x^2 - 16*b)*cosh(1)^2 + 294*(3*b*c^
6*d*x^4 - 4*b*c^4*d*x^2 + 8*b*c^2*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))/c
^7
```

Sympy [A]
time = 0.76, size = 389, normalized size = 1.76

$$\begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{asinh}(cx) + bd^2ex^3\operatorname{asinh}(cx) + \frac{3bde^2x^5\operatorname{asinh}(cx)}{5} + \frac{be^3x^7\operatorname{asinh}(cx)}{7} - \frac{bd^3\sqrt{c^2x^2+1}}{c} - \frac{bd^2ex^2\sqrt{c^2x^2+1}}{3c} - \frac{3bde^2x^4\sqrt{c^2x^2+1}}{25c} - \frac{be^3x^6\sqrt{c^2x^2+1}}{49c} + \frac{2bd^2e\sqrt{c^2x^2+1}}{3c^3} + \frac{4bde^2x^2\sqrt{c^2x^2+1}}{25c^3} + \frac{6be^3x^4\sqrt{c^2x^2+1}}{245c^3} - \frac{8bde^2\sqrt{c^2x^2+1}}{25c^5} - \frac{8be^3x^2\sqrt{c^2x^2+1}}{245c^5} + \frac{16be^3\sqrt{c^2x^2+1}}{245c^7} & \text{for } c \neq 0 \\ a\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*asinh(c*x) + b*d**2*e*x**3*asinh(c*x) + 3*b*d*e**2*x**5*asinh(c*x)/
5 + b*e**3*x**7*asinh(c*x)/7 - b*d**3*sqrt(c**2*x**2 + 1)/c - b*d**2*e*x**2
*sqrt(c**2*x**2 + 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - b
*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) + 2*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c
**3) + 4*b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(25*c**3) + 6*b*e**3*x**4*sqrt(c
**2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 8*b*e
**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) + 16*b*e**3*sqrt(c**2*x**2 + 1)/(24
5*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/
7), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^3, x)
```

3.608 $\int (d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=147

$$-\frac{b(15c^4d^2-10c^2de+3e^2)\sqrt{1+c^2x^2}}{15c^5}-\frac{2b(5c^2d-3e)\,e(1+c^2x^2)^{3/2}}{45c^5}-\frac{be^2(1+c^2x^2)^{5/2}}{25c^5}+d^2x\left(a+b\sinh^{-1}(cx\right.$$

```
[Out] -2/45*b*(5*c^2*d-3*e)*e*(c^2*x^2+1)^(3/2)/c^5-1/25*b*e^2*(c^2*x^2+1)^(5/2)/
c^5+d^2*x*(a+b*arcsinh(c*x))+2/3*d*e*x^3*(a+b*arcsinh(c*x))+1/5*e^2*x^5*(a+
b*arcsinh(c*x))-1/15*b*(15*c^4*d^2-10*c^2*d*e+3*e^2)*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A]
time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {200, 5792, 12, 1261, 712}

$$d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{2be(c^2x^2+1)^{3/2}(5c^2d-3e)}{45c^5}-\frac{be^2(c^2x^2+1)^{5/2}}{25c^5}-\frac{b\sqrt{c^2x^2+1}\,(15c^4d^2-10c^2de+3e^2)}{15c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/15*(b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Sqrt[1 + c^2*x^2])/c^5 - (2*b*(5
*c^2*d - 3*e)*e*(1 + c^2*x^2)^(3/2))/(45*c^5) - (b*e^2*(1 + c^2*x^2)^(5/2))
/(25*c^5) + d^2*x*(a + b*ArcSinh[c*x]) + (2*d*e*x^3*(a + b*ArcSinh[c*x]))/3
 + (e^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int (d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right) dx = d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh\right.$$

$$= -\frac{b(15c^4d^2-10c^2de+3e^2)\sqrt{1+c^2x^2}}{15c^5}-\frac{2b(5c^2d-3e)\,e(1+c^2x^2)^3}{45c^5}$$

Mathematica [A]
time = 0.10, size = 125, normalized size = 0.85

$$\frac{1}{225}\left(15ax(15d^2+10dex^2+3e^2x^4)-\frac{b\sqrt{1+c^2x^2}\,(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))}{c^5}+15bx(15d^2+10dex^2+3e^2x^4)\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[1 + c^2*x^2]*(24*e^2 -
4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 + 1
5*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcSinh[c*x])/225
```

Maple [A]
time = 0.64, size = 204, normalized size = 1.39

method	result

derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5ex^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\operatorname{arcsinh}(cx)d^2c^5x+\frac{2\operatorname{arcsinh}(cx)dc^5ex^3}{3}+\frac{\operatorname{arcsinh}(cx)e^2c^5x^5}{5}-d^2c^4\sqrt{c^2x^2+1}-\frac{2dc^2e(}{}\right.}{}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5ex^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\operatorname{arcsinh}(cx)d^2c^5x+\frac{2\operatorname{arcsinh}(cx)dc^5ex^3}{3}+\frac{\operatorname{arcsinh}(cx)e^2c^5x^5}{5}-d^2c^4\sqrt{c^2x^2+1}-\frac{2dc^2e(}{}\right.}{}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+b/c^4*(arcsinh(c*x)*
d^2*c^5*x+2/3*arcsinh(c*x)*d*c^5*e*x^3+1/5*arcsinh(c*x)*e^2*c^5*x^5-d^2*c^4
*(c^2*x^2+1)^(1/2)-2/3*d*c^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+
1)^(1/2))-1/5*e^2*(1/5*(c^2*x^2+1)^(1/2)*c^4*x^4-4/15*c^2*x^2*(c^2*x^2+1)^(
1/2)+8/15*(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.27, size = 180, normalized size = 1.22

$$\frac{1}{5}ax^5e^2+\frac{2}{3}adx^3e+ad^2x+\frac{2}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bde+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd^2}{c}+\frac{1}{75}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e^2 + 2/3*a*d*x^3*e + a*d^2*x + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt
(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e + (c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*d^2/c + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2
 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b
*e^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(132) = 264$.
time = 0.40, size = 321, normalized size = 2.18

$$\frac{45ac^5x^5\cosh(1)^2+45ac^5x^5\sinh(1)^2+150ac^5dx^3\cosh(1)+225ac^5d^2x+15\left(3bc^5x^5\cosh(1)^2+3bc^5x^5\sinh(1)^2+10bc^5dx^3\cosh(1)+15bc^5d^2x+2\left(3bc^5x^5\cosh(1)+5bc^5dx^3\right)\sinh(1)\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+30\left(3ac^5x^5\cosh(1)+5ac^5dx^3\right)\sinh(1)-\left(225bc^4d^2+3\left(3bc^4x^4-4bc^2x^2+8b\right)\cosh(1)^2+3\left(3bc^4x^4-4bc^2x^2+8b\right)\sinh(1)^2+50\left(bc^4dx^2-2bc^2d\right)\cosh(1)+2\left(25bc^4dx^2-50bc^2d+3\left(3bc^4x^4-4bc^2x^2+8b\right)\cosh(1)\right)\sinh(1)\right)\sqrt{c^2x^2+1}}{225c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*x^5*cosh(1)^2 + 45*a*c^5*x^5*sinh(1)^2 + 150*a*c^5*d*x^3*co
sh(1) + 225*a*c^5*d^2*x + 15*(3*b*c^5*x^5*cosh(1)^2 + 3*b*c^5*x^5*sinh(1)^2
```

```
 + 10*b*c^5*d*x^3*cosh(1) + 15*b*c^5*d^2*x + 2*(3*b*c^5*x^5*cosh(1) + 5*b*c
^5*d*x^3)*sinh(1))*log(c*x + sqrt(c^2*x^2 + 1)) + 30*(3*a*c^5*x^5*cosh(1) +
 5*a*c^5*d*x^3)*sinh(1) - (225*b*c^4*d^2 + 3*(3*b*c^4*x^4 - 4*b*c^2*x^2 + 8
*b)*cosh(1)^2 + 3*(3*b*c^4*x^4 - 4*b*c^2*x^2 + 8*b)*sinh(1)^2 + 50*(b*c^4*d
*x^2 - 2*b*c^2*d)*cosh(1) + 2*(25*b*c^4*d*x^2 - 50*b*c^2*d + 3*(3*b*c^4*x^4
 - 4*b*c^2*x^2 + 8*b)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A]
time = 0.36, size = 240, normalized size = 1.63

$$\begin{cases} ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\operatorname{asinh}(cx) + \frac{2bdex^3\operatorname{asinh}(cx)}{3} + \frac{be^2x^5\operatorname{asinh}(cx)}{5} - \frac{bd^2\sqrt{c^2x^2+1}}{c} - \frac{2bdex^2\sqrt{c^2x^2+1}}{9c} - \frac{be^2x^4\sqrt{c^2x^2+1}}{25c} + \frac{4bde\sqrt{c^2x^2+1}}{9c^3} + \frac{4be^2x^2\sqrt{c^2x^2+1}}{75c^3} - \frac{8be^2\sqrt{c^2x^2+1}}{75c^5} & \text{for}\ c \neq 0 \\ a\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asinh(c*x)
+ 2*b*d*e*x**3*asinh(c*x)/3 + b*e**2*x**5*asinh(c*x)/5 - b*d**2*sqrt(c**2*x
**2 + 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) - b*e**2*x**4*sqrt(c**2
*x**2 + 1)/(25*c) + 4*b*d*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 4*b*e**2*x**2*sq
rt(c**2*x**2 + 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 + 1)/(75*c**5), Ne(c,
 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\operatorname{asinh}(c\,x))\,\left(e\,x^2 + d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^2, x)
```

3.609 $\int (d+ex^2) \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=81

$$-\frac{b(3c^2d-e)\sqrt{1+c^2x^2}}{3c^3}-\frac{be(1+c^2x^2)^{3/2}}{9c^3}+dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] -1/9*b*e*(c^2*x^2+1)^(3/2)/c^3+d*x*(a+b*arcsinh(c*x))+1/3*e*x^3*(a+b*arcsin
h(c*x))-1/3*b*(3*c^2*d-e)*(c^2*x^2+1)^(1/2)/c^3
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5792, 455, 45}

$$dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{b\sqrt{c^2x^2+1}\,(3c^2d-e)}{3c^3}-\frac{be(c^2x^2+1)^{3/2}}{9c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/3*(b*(3*c^2*d - e)*Sqrt[1 + c^2*x^2])/c^3 - (b*e*(1 + c^2*x^2)^(3/2))/(9
*c^3) + d*x*(a + b*ArcSinh[c*x]) + (e*x^3*(a + b*ArcSinh[c*x]))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int (d+ex^2)\,(a+b\sinh^{-1}(cx))\;dx = dx(a+b\sinh^{-1}(cx)) + \frac{1}{3}ex^3(a+b\sinh^{-1}(cx)) - (bc)\int \frac{x\left(d+\frac{ex^2}{3}\right)}{\sqrt{1+c^2x^2}}$$

$$= dx(a+b\sinh^{-1}(cx)) + \frac{1}{3}ex^3(a+b\sinh^{-1}(cx)) - \frac{1}{2}(bc)\text{Subst}\left(\int \frac{d}{\sqrt{1}}\right.$$

$$= dx(a+b\sinh^{-1}(cx)) + \frac{1}{3}ex^3(a+b\sinh^{-1}(cx)) - \frac{1}{2}(bc)\text{Subst}\left(\int \left(\frac{}{3}\right.\right.$$

$$= -\frac{b(3c^2d-e)\sqrt{1+c^2x^2}}{3c^3} - \frac{be(1+c^2x^2)^{3/2}}{9c^3} + dx(a+b\sinh^{-1}(cx)) +$$

Mathematica [A]
time = 0.05, size = 71, normalized size = 0.88

$$\frac{1}{9}\left(3ax(3d+ex^2) - \frac{b\sqrt{1+c^2x^2}\,(-2e+c^2(9d+ex^2))}{c^3} + 3bx(3d+ex^2)\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*a*x*(3*d + e*x^2) - (b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))/c^3
 + 3*b*x*(3*d + e*x^2)*ArcSinh[c*x])/9
```

Maple [A]
time = 0.60, size = 109, normalized size = 1.35

method	result
derivativedivides	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^3x+\frac{\operatorname{arcsinh}(cx)e\,c^3x^3}{3}-d\,c^2\sqrt{c^2x^2+1}-\frac{e\left(\frac{c^2x^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right.}{3}\right.}{c^2}}{c}$
default	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^3x+\frac{\operatorname{arcsinh}(cx)e\,c^3x^3}{3}-d\,c^2\sqrt{c^2x^2+1}-\frac{e\left(\frac{c^2x^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right.}{3}\right.}{c^2}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^2*(d*c^3*x+1/3*e*c^3*x^3)+b/c^2*(arcsinh(c*x)*d*c^3*x+1/3*arcsinh(
c*x)*e*c^3*x^3-d*c^2*(c^2*x^2+1)^(1/2)-1/3*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)
-2/3*(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.26, size = 93, normalized size = 1.15

$$\frac{1}{3}\,ax^3e + adx + \frac{1}{9}\left(3\,x^3\,\mathrm{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)be + \frac{\left(cx\,\mathrm{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e + a*d*x + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*e + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*
b*d/c
```

Fricas [A]
time = 0.35, size = 134, normalized size = 1.65

$$\frac{3\,ac^3x^3\cosh\,(1) + 3\,ac^3x^3\sinh\,(1) + 9\,ac^3dx + 3\,(bc^3x^3\cosh\,(1) + bc^3x^3\sinh\,(1) + 3\,bc^3dx)\log\left(cx + \sqrt{c^2x^2+1}\right) - (9\,bc^2d + (bc^2x^2 - 2\,b)\cosh\,(1) + (bc^2x^2 - 2\,b)\sinh\,(1))\sqrt{c^2x^2+1}}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*x^3*cosh(1) + 3*a*c^3*x^3*sinh(1) + 9*a*c^3*d*x + 3*(b*c^3*x^3
*cosh(1) + b*c^3*x^3*sinh(1) + 3*b*c^3*d*x)*log(c*x + sqrt(c^2*x^2 + 1)) -
(9*b*c^2*d + (b*c^2*x^2 - 2*b)*cosh(1) + (b*c^2*x^2 - 2*b)*sinh(1))*sqrt(c^
2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.15, size = 109, normalized size = 1.35

$$\begin{cases} adx + \frac{aex^3}{3} + bdx\,\mathrm{asinh}\,(cx) + \frac{bex^3\,\mathrm{asinh}\,(cx)}{3} - \frac{bd\sqrt{c^2x^2+1}}{c} - \frac{bex^2\sqrt{c^2x^2+1}}{9c} + \frac{2be\sqrt{c^2x^2+1}}{9c^3} & \text{for } c \neq 0 \\ a\left(dx + \frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*asinh(c*x) + b*e*x**3*asinh(c*x)/3 -
b*d*sqrt(c**2*x**2 + 1)/c - b*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) + 2*b*e*sqrt
(c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2), x)
```

3.610 $\int \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)$$

[Out] a*x+b*x*arcsinh(c*x)-b*(c^2*x^2+1)^(1/2)/c

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5772, 267}

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b*ArcSinh[c*x],x]

[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sinh^{-1}(cx)\right)\,dx &= ax+b\int \sinh^{-1}(cx)\,dx\\
&= ax+bx\sinh^{-1}(cx)-(bc)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx\\
&= ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c*x],x]
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Maple [A]
time = 0.58, size = 31, normalized size = 1.03

method	result	size
default	$ax+\frac{b\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	31
derivativedivides	$\frac{acx+b\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

Maxima [A]
time = 0.26, size = 30, normalized size = 1.00

$$ax+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
[Out] a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

Fricas [A]
time = 0.38, size = 43, normalized size = 1.43

$$\frac{bcx\log\left(cx+\sqrt{c^2x^2+1}\right)+acx-\sqrt{c^2x^2+1}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.87

$$ax+b\left(\begin{cases}x\operatorname{asinh}(cx)-\frac{\sqrt{c^2x^2+1}}{c} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tru
e))
```

Giac [A]
time = 0.39, size = 41, normalized size = 1.37

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

Mupad [B]
time = 0.24, size = 28, normalized size = 0.93

$$a\,x-\frac{b\,\sqrt{c^2\,x^2+1}}{c}+b\,x\,\mathrm{asinh}(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(c*x),x)
```

```
[Out] a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

3.611 $\int \frac{a+b\sinh^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=485

$$\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\cdots$$

```
[Out] 1/2*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(
-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*
x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1
/2*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-
c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x
^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/
2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+
1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1
/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5793, 5827, 5680, 2221, 2317, 2438}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sq
rt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*S
qrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,\left(a+b\sinh^{-1}(cx)\right)}{2d\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\sinh^{-1}(cx)\right)}{2d\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}\right)\,dx$$

$$= -\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}-\sqrt{e}\,\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}+\sqrt{e}\,\sinh(x)}\,d\right.}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}\,e^x}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{e}{c\sqrt{-d}+\sqrt{}}\right.}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\right.}{2\sqrt{-d}}$$

Mathematica [A]
time = 0.31, size = 434, normalized size = 0.89

$$\frac{2a\sqrt{-d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)-b\sqrt{d}\,\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)+b\sqrt{d}\,\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d+e}}\right)+b\sqrt{d}\,\sinh^{-1}(cx)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)-b\sqrt{d}\,\sinh^{-1}(cx)\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)+b\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)-b\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d+e}}\right)-b\sqrt{d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)+b\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d^2}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

```
[Out] (2*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*Arc
Sinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) +
 e])] + b*Sqrt[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E
^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*PolyLog[2, (S
qrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*Po
```

```
lyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + b
*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.79, size = 233, normalized size = 0.48

method	result
derivativedivides	$\dfrac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{bc^2\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)}\right)}{2}$
default	$\dfrac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{bc^2\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a*c/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*c^2*sum(1/_R1/(_R1^2*e+2
*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-
(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/2*b*c^2
*sum(_R1/(_R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_
R1)+dilog((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)
*_Z^2+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] a*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/(x^2*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{asinh}(cx)}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d),x)
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{asinh}(c\,x)}{e\,x^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2), x)
```

3.612 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=707

$$-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{bc\text{ArcTan}\left(\frac{\sqrt{e}-c^2\sqrt{-d}\,x}{\sqrt{c^2d-e}\,\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\,\sqrt{e}}-\frac{bc\text{ArcTan}\left(\frac{\cdots}{\sqrt{c^2\cdots}}\right)}{4d\sqrt{c^2\cdots}}$$

```
[Out] -1/4*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-
(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2
*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-
1/4*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(
-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*
x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1
/4*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1
/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c
*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)
-1/4*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(
1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*c*arctan((-c^2*x*(-d)^(1/2)+e^(1/2))/(c^2*d
-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c^2*d-e)^(1/2)/e^(1/2)-1/4*b*c*arctan((c^2*
x*(-d)^(1/2)+e^(1/2))/(c^2*d-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c^2*d-e)^(1/2)/
e^(1/2)+1/4*(-a-b*arcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)-x*e^(1/2))+1/4*(a+b*a
rcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)+x*e^(1/2))
```

Rubi [A]

time = 0.78, antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5793, 5828, 739, 210, 5827, 5680, 2221, 2317, 2438}

$$-\frac{(a+b\sinh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\sinh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{(a+b\sinh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\sinh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{bc\text{ArcTan}\left(\frac{\sqrt{e}-c^2\sqrt{-d}\,x}{\sqrt{c^2x^2+1}\,\sqrt{c^2d-e}}\right)}{4d\sqrt{e}\,\sqrt{c^2d-e}}-\frac{bc\text{ArcTan}\left(\frac{c^2\sqrt{-d}\,x+\sqrt{e}}{\sqrt{c^2x^2+1}\,\sqrt{c^2d-e}}\right)}{4d\sqrt{e}\,\sqrt{c^2d-e}}+\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcSinh[c*x])/(d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcSi
nh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTan[(Sqrt[e] - c^2*
Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt
[e]) - (b*c*ArcTan[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2
*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 - (Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt
[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] -
 Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1
```

```
 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/
2)*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqr
t[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqr
t[e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
+ e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2,
(Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*
Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^2}\,dx = \int \left(-\frac{e\left(a+b\sinh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}\,-ex\right)^2}-\frac{e\left(a+b\sinh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}\,+ex\right)^2}-\frac{e\left(a+b\sinh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right) d\cdots$$

$$= -\frac{e\int \frac{a+b\sinh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}\,-ex\right)^2}\,dx}{4d}-\frac{e\int \frac{a+b\sinh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}\,+ex\right)^2}\,dx}{4d}-\frac{e\int \frac{a+b\sinh^{-1}(cx)}{-de-e^2x^2}\,dx}{2d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{(bc)\int \frac{1}{\left(\sqrt{-d}\,\sqrt{e}\,-ex\right)\sqrt{\cdots}}\cdots}{4d}\cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}\,-\sqrt{e}\,x}\,dx}{4(-d)^{3/2}}+\cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}\,-c^2\sqrt{-\cdots}}{\sqrt{c^2d-e}\,\sqrt{1\cdots}}\right.}{4d\sqrt{c^2d-e}\,\sqrt{e}\cdots}\cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}\,-c^2\sqrt{-\cdots}}{\sqrt{c^2d-e}\,\sqrt{1\cdots}}\right.}{4d\sqrt{c^2d-e}\,\sqrt{e}\cdots}\cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}\,-c^2\sqrt{-\cdots}}{\sqrt{c^2d-e}\,\sqrt{1\cdots}}\right.}{4d\sqrt{c^2d-e}\,\sqrt{e}\cdots}\cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}\,-c^2\sqrt{-\cdots}}{\sqrt{c^2d-e}\,\sqrt{1\cdots}}\right.}{4d\sqrt{c^2d-e}\,\sqrt{e}\cdots}\cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}\,-c^2\sqrt{-\cdots}}{\sqrt{c^2d-e}\,\sqrt{1\cdots}}\right.}{4d\sqrt{c^2d-e}\,\sqrt{e}\cdots}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.21, size = 622, normalized size = 0.88

$$\frac{1}{2}\left(\frac{ax}{d^2+dex^2}+\frac{a\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}+\frac{b\left(-2\sqrt{d}\left(-\frac{\sinh^{-1}(cx)}{i\sqrt{d}+\sqrt{e}\,x}+\frac{c\text{ArcTan}\left(\frac{\sqrt{e}-ic^2\sqrt{d}\,x}{\sqrt{c^2d-e}\,\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d-e}}\right)+2i\sqrt{d}\left(\frac{\sinh^{-1}(cx)}{\sqrt{d}+i\sqrt{e}\,x}+\frac{c\tanh^{-1}\left(\frac{i\sqrt{e}-c^2\sqrt{d}\,x}{\sqrt{c^2d-e}\,\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d-e}}\right)+i\left(\sinh^{-1}(cx)\left(-\sinh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d+e}}\right)+\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d+e}}\right)\right)\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d+e}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d+e}}\right)\right)-i\left(\sinh^{-1}(cx)\left(-\sinh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d+e}}\right)+\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d+e}}\right)\right)\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d+e}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d+e}}\right)\right)\right)}{4d^{3/2}\sqrt{e}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*(-2*Sqrt[d]*(-(ArcSinh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*ArcTan[(Sqrt
[e] - I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d - e
]) + (2*I)*Sqrt[d]*(ArcSinh[c*x]/(Sqrt[d] + I*Sqrt[e]*x) + (c*ArcTanh[(I*Sq
rt[e] - c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d - e
]) + I*(ArcSinh[c*x]*(-ArcSinh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(
I*c*Sqrt[d] - Sqrt[-(c^2*d) + e])] + Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(I*c*
Sqrt[d] + Sqrt[-(c^2*d) + e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/((
-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*
x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]))]) - I*(ArcSinh[c*x]*(-ArcSinh[c*x]
+ 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])
] + Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e])]))
+ 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) +
 e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 + e])])))/(4*d^(3/2)*Sqrt[e]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 22.51, size = 1766, normalized size = 2.50

method	result	size
derivativedivides	Expression too large to display	1766
default	Expression too large to display	1766

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*a*c^3*x/d/(c^2*e*x^2+c^2*d)+1/2*a*c/d/(d*e)^(1/2)*arctan(e*x/(d*e)
^(1/2))+1/2*b*c^3*arcsinh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*b*c^2/d*sum(1/_R1/
(_R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog
((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))
+1/4*b*c^2/d*sum(_R1/(_R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2
+1)^(1/2))/_R1)+dilog((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(
4*c^2*d-2*e)*_Z^2+e))+b*c^6*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2
)*arctanh(e*(c*x+(c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)
*e)^(1/2))*d/(c^2*d-e)/e^3+b*c^4*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)
^(1/2)*arctanh(e*(c*x+(c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/
2)+e)*e)^(1/2))/(c^2*d-e)/e^3*(c^2*d*(c^2*d-e))^(1/2)-b*c^4*(-(2*c^2*d-2*(c
^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh(e*(c*x+(c^2*x^2+1)^(1/2))/((-2*c^
2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^(1/2))/(c^2*d-e)/e^2-1/2*b*c^2*(-(2*c^2
*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh(e*(c*x+(c^2*x^2+1)^(1/2))/
((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^(1/2))/d/(c^2*d-e)/e^2*(c^2*d*(c
```

```
^2*d-e))^(1/2)-b*c^4*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arcta
nh(e*(c*x+(c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^(1/
2))/e^3-b*c^2*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh(e*(c
*x+(c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^(1/2))/d/e
^3*(c^2*d*(c^2*d-e))^(1/2)+1/2*b*c^2*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e
)*e)^(1/2)*arctanh(e*(c*x+(c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d-e))
^(1/2)+e)*e)^(1/2))/d/e^2+b*c^6*((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(
1/2)*arctan(e*(c*x+(c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e
)*e)^(1/2))*d/(c^2*d-e)/e^3-b*c^4*((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)
^(1/2)*arctan(e*(c*x+(c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)
-e)*e)^(1/2))/(c^2*d-e)/e^3*(c^2*d*(c^2*d-e))^(1/2)-b*c^4*((2*c^2*d+2*(c^2*
d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan(e*(c*x+(c^2*x^2+1)^(1/2))/((2*c^2*d+2
*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2))/(c^2*d-e)/e^2+1/2*b*c^2*((2*c^2*d+2*(
c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan(e*(c*x+(c^2*x^2+1)^(1/2))/((2*c^2
*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2))/d/(c^2*d-e)/e^2*(c^2*d*(c^2*d-e))
^(1/2)-b*c^4*((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan(e*(c*x+
(c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2))/e^3+b*c
^2*((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan(e*(c*x+(c^2*x^2+1
)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2))/d/e^3*(c^2*d*(c^2
*d-e))^(1/2)+1/2*b*c^2*((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arct
an(e*(c*x+(c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2
))/d/e^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-2.718281828459045>0)', see `a
ssume?`
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{(e\,x^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^2, x)
```

3.613 $\int (d+ex^2)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=559

$$2b^2d^3x-\frac{4b^2d^2ex}{3c^2}+\frac{16b^2de^2x}{25c^4}-\frac{32b^2e^3x}{245c^6}+\frac{2}{9}b^2d^2ex^3-\frac{8b^2de^2x^3}{75c^2}+\frac{16b^2e^3x^3}{735c^4}+\frac{6}{125}b^2de^2x^5-\frac{12b^2e^3x^5}{1225c^2}+\frac{2}{343}b^2e^3x^7$$

```
[Out] 2*b^2*d^3*x-4/3*b^2*d^2*e*x/c^2+16/25*b^2*d*e^2*x/c^4-32/245*b^2*e^3*x/c^6+
2/9*b^2*d^2*e*x^3-8/75*b^2*d*e^2*x^3/c^2+16/735*b^2*e^3*x^3/c^4+6/125*b^2*d
*e^2*x^5-12/1225*b^2*e^3*x^5/c^2+2/343*b^2*e^3*x^7+d^3*x*(a+b*arcsinh(c*x))
^2+d^2*e*x^3*(a+b*arcsinh(c*x))^2+3/5*d*e^2*x^5*(a+b*arcsinh(c*x))^2+1/7*e^
3*x^7*(a+b*arcsinh(c*x))^2-2*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4
/3*b*d^2*e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/25*b*d*e^2*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)/c^5+32/245*b*e^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)
^(1/2)/c^7-2/3*b*d^2*e*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+8/25*b*d*
e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/245*b*e^3*x^2*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)/c^5-6/25*b*d*e^2*x^4*(a+b*arcsinh(c*x))*(c^2*x^
2+1)^(1/2)/c+12/245*b*e^3*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-2/49
*b*e^3*x^6*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.63, antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5793, 5772, 5798, 8, 5776, 5812, 30}

$$-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bd^2ex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{6bde^2x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c}-\frac{2be^3x^6\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{49c}+\frac{32be^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{245c^7}-\frac{16bde^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c^5}-\frac{16be^3x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{245c^5}+\frac{4bd^2e\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c^3}+\frac{8bde^2x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c^3}+\frac{12be^3x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{245c^3}+d^3x\left(a+b\sinh^{-1}(cx)\right)^2+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}e^3x^7\left(a+b\sinh^{-1}(cx)\right)^2-\frac{32b^2e^3x}{245c^6}+\frac{16b^2de^2x}{25c^4}+\frac{16b^2e^3x^3}{735c^4}-\frac{4b^2d^2ex}{3c^2}-\frac{8b^2de^2x^3}{75c^2}-\frac{12b^2e^3x^5}{1225c^2}+2b^2d^3x+\frac{2}{9}b^2d^2ex^3+\frac{6}{125}b^2de^2x^5+\frac{2}{343}b^2e^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) - (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225*c^2
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c + (4*b*d^2*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^3) - (16*b*d*e^
2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c^5) + (32*b*e^3*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(3*c) + (8*b*d*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
45*c^5) - (6*b*d*e^2*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + (
12*b*e^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^3) - (2*b*e^3*x
^6*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(49*c) + d^3*x*(a + b*ArcSinh[c*
x])^2 + d^2*e*x^3*(a + b*ArcSinh[c*x])^2 + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]
)^2)/5 + (e^3*x^7*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \int \left(d^3\left(a+b\sinh^{-1}(cx)\right)^2 + 3d^2ex^2\left(a+b\sinh^{-1}(cx)\right)^2 + 3de^2x^4(a$$

$$= d^3 \int \left(a+b\sinh^{-1}(cx)\right)^2 \, dx + \left(3d^2e\right) \int x^2\left(a+b\sinh^{-1}(cx)\right)^2 \, dx +$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)^2 + d^2ex^3\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{3}{5}de^2x^5(a+b$$

$$= -\frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{2bd^2ex^2\sqrt{1+c^2x^2}\left(a+b\sin\right.}{3c}$$

$$= 2b^2d^3x + \frac{2}{9}b^2d^2ex^3 + \frac{6}{125}b^2de^2x^5 + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{1+c^2x^2}\,(a}{c}$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{6}{125}b^2de^2x^5 - \frac{12b^2e^3x}{1225c^2}$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4}$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} - \frac{32b^2e^3x}{245c^6} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} +$$

Mathematica [**A**]
time = 0.30, size = 443, normalized size = 0.79

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*a
*b*Sqrt[1 + c^2*x^2]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(122
5*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*
e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(-25200*e^3 + 840*c^2*e^2*(147*d + 5*e*x
^2) - 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*d^3 + 428
75*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7*x*(35*d^
3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[1 + c^2*x^2]*(-240*e^
3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2 + 45*e^2*
x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6)))*ArcSi
nh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6
)*ArcSinh[c*x]^2)/(385875*c^7)
```

Maple [A]
time = 2.24, size = 752, normalized size = 1.35

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+1/
c^6*b^2*(3/2000*c^2*d*e^2*(25*arcsinh(c*x)^2*sinh(5*arcsinh(c*x))-10*arcsin
h(c*x)*cosh(5*arcsinh(c*x))+2*sinh(5*arcsinh(c*x)))-1/1600*e^3*(25*arcsinh(
c*x)^2*sinh(5*arcsinh(c*x))-10*arcsinh(c*x)*cosh(5*arcsinh(c*x))+2*sinh(5*a
rcsinh(c*x)))+1/36*c^4*d^2*e*(9*arcsinh(c*x)^2*sinh(3*arcsinh(c*x))-6*arcsi
nh(c*x)*cosh(3*arcsinh(c*x))+2*sinh(3*arcsinh(c*x)))-1/48*c^2*d*e^2*(9*arcs
inh(c*x)^2*sinh(3*arcsinh(c*x))-6*arcsinh(c*x)*cosh(3*arcsinh(c*x))+2*sinh(
3*arcsinh(c*x)))+1/192*e^3*(9*arcsinh(c*x)^2*sinh(3*arcsinh(c*x))-6*arcsinh
(c*x)*cosh(3*arcsinh(c*x))+2*sinh(3*arcsinh(c*x)))+d^3*c^6*(arcsinh(c*x)^2*
c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)-3/4*c^4*d^2*e*(arcsinh(c*x)^2*c
*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+3/8*c^2*d*e^2*(arcsinh(c*x)^2*c*
x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)-5/64*e^3*(arcsinh(c*x)^2*c*x-2*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/21952*e^3*(49*arcsinh(c*x)^2*sinh(7*a
rcsinh(c*x))-14*arcsinh(c*x)*cosh(7*arcsinh(c*x))+2*sinh(7*arcsinh(c*x))))+
2*a*b/c^6*(1/7*arcsinh(c*x)*e^3*x^7*c^7+3/5*arcsinh(c*x)*c^7*d*e^2*x^5+arcs
inh(c*x)*c^7*d^2*e*x^3+arcsinh(c*x)*x*c^7*d^3-1/7*e^3*(1/7*(c^2*x^2+1)^(1/2
)*c^6*x^6-6/35*(c^2*x^2+1)^(1/2)*c^4*x^4+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/
35*(c^2*x^2+1)^(1/2))-3/5*d*c^2*e^2*(1/5*(c^2*x^2+1)^(1/2)*c^4*x^4-4/15*c^2
*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-d^2*c^4*e*(1/3*c^2*x^2*(c^2*
x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-d^3*c^6*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.29, size = 680, normalized size = 1.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7*arcsinh(c*x)^2*e^3 + 3/5*b^2*d*x^5*arcsinh(c*x)^2*e^2 + 1/7*a^2
*x^7*e^3 + b^2*d^2*x^3*arcsinh(c*x)^2*e + 3/5*a^2*d*x^5*e^2 + b^2*d^3*x*arc
sinh(c*x)^2 + a^2*d^2*x^3*e + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)
/c) + a^2*d^3*x + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^2*e + 2
*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c + 2/25*(15*x^5*arcsinh(c*
x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^
```

```
2*x^2 + 1)/c^6)*c)*a*b*d*e^2 - 2/375*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*s
qrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4
*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d*e^2 + 2/245*(35*x^7*arcsinh(c*x) - (5
*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*e^3 - 2/25725*(105*(5*sqrt(c
^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2
/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5
 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(488) = 976$.
time = 0.39, size = 1528, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(385875*(a^2 + 2*b^2)*c^7*d^3*x + 15*(75*(49*a^2 + 2*b^2)*c^7*x^7
- 252*b^2*c^5*x^5 + 560*b^2*c^3*x^3 - 3360*b^2*c*x)*cosh(1)^3 + 15*(75*(49*
a^2 + 2*b^2)*c^7*x^7 - 252*b^2*c^5*x^5 + 560*b^2*c^3*x^3 - 3360*b^2*c*x)*si
nh(1)^3 + 1029*(9*(25*a^2 + 2*b^2)*c^7*d*x^5 - 40*b^2*c^5*d*x^3 + 240*b^2*c
^3*d*x)*cosh(1)^2 + 11025*(5*b^2*c^7*x^7*cosh(1)^3 + 5*b^2*c^7*x^7*sinh(1)^
3 + 21*b^2*c^7*d*x^5*cosh(1)^2 + 35*b^2*c^7*d^2*x^3*cosh(1) + 35*b^2*c^7*d^
3*x + 3*(5*b^2*c^7*x^7*cosh(1) + 7*b^2*c^7*d*x^5)*sinh(1)^2 + (15*b^2*c^7*x
^7*cosh(1)^2 + 42*b^2*c^7*d*x^5*cosh(1) + 35*b^2*c^7*d^2*x^3)*sinh(1))*log(
c*x + sqrt(c^2*x^2 + 1))^2 + 3*(3087*(25*a^2 + 2*b^2)*c^7*d*x^5 - 13720*b^2
*c^5*d*x^3 + 82320*b^2*c^3*d*x + 15*(75*(49*a^2 + 2*b^2)*c^7*x^7 - 252*b^2*
c^5*x^5 + 560*b^2*c^3*x^3 - 3360*b^2*c*x)*cosh(1))*sinh(1)^2 + 42875*((9*a^
2 + 2*b^2)*c^7*d^2*x^3 - 12*b^2*c^5*d^2*x)*cosh(1) + 210*(525*a*b*c^7*x^7*c
osh(1)^3 + 525*a*b*c^7*x^7*sinh(1)^3 + 2205*a*b*c^7*d*x^5*cosh(1)^2 + 3675*
a*b*c^7*d^2*x^3*cosh(1) + 3675*a*b*c^7*d^3*x + 315*(5*a*b*c^7*x^7*cosh(1) +
 7*a*b*c^7*d*x^5)*sinh(1)^2 + 105*(15*a*b*c^7*x^7*cosh(1)^2 + 42*a*b*c^7*d*
x^5*cosh(1) + 35*a*b*c^7*d^2*x^3)*sinh(1) - (3675*b^2*c^6*d^3 + 15*(5*b^2*c
^6*x^6 - 6*b^2*c^4*x^4 + 8*b^2*c^2*x^2 - 16*b^2)*cosh(1)^3 + 15*(5*b^2*c^6*
x^6 - 6*b^2*c^4*x^4 + 8*b^2*c^2*x^2 - 16*b^2)*sinh(1)^3 + 147*(3*b^2*c^6*d*
x^4 - 4*b^2*c^4*d*x^2 + 8*b^2*c^2*d)*cosh(1)^2 + 3*(147*b^2*c^6*d*x^4 - 196
*b^2*c^4*d*x^2 + 392*b^2*c^2*d + 15*(5*b^2*c^6*x^6 - 6*b^2*c^4*x^4 + 8*b^2*
c^2*x^2 - 16*b^2)*cosh(1))*sinh(1)^2 + 1225*(b^2*c^6*d^2*x^2 - 2*b^2*c^4*d^
2)*cosh(1) + (1225*b^2*c^6*d^2*x^2 - 2450*b^2*c^4*d^2 + 45*(5*b^2*c^6*x^6 -
 6*b^2*c^4*x^4 + 8*b^2*c^2*x^2 - 16*b^2)*cosh(1)^2 + 294*(3*b^2*c^6*d*x^4 -
 4*b^2*c^4*d*x^2 + 8*b^2*c^2*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (42875*(9*a^2 + 2*b^2)*c^7*d^2*x^3 - 514500*b^2*c^
5*d^2*x + 45*(75*(49*a^2 + 2*b^2)*c^7*x^7 - 252*b^2*c^5*x^5 + 560*b^2*c^3*x
^3 - 3360*b^2*c*x)*cosh(1)^2 + 2058*(9*(25*a^2 + 2*b^2)*c^7*d*x^5 - 40*b^2*
```

```
c^5*d*x^3 + 240*b^2*c^3*d*x)*cosh(1))*sinh(1) - 210*(3675*a*b*c^6*d^3 + 15*
(5*a*b*c^6*x^6 - 6*a*b*c^4*x^4 + 8*a*b*c^2*x^2 - 16*a*b)*cosh(1)^3 + 15*(5*
a*b*c^6*x^6 - 6*a*b*c^4*x^4 + 8*a*b*c^2*x^2 - 16*a*b)*sinh(1)^3 + 147*(3*a*
b*c^6*d*x^4 - 4*a*b*c^4*d*x^2 + 8*a*b*c^2*d)*cosh(1)^2 + 3*(147*a*b*c^6*d*x
^4 - 196*a*b*c^4*d*x^2 + 392*a*b*c^2*d + 15*(5*a*b*c^6*x^6 - 6*a*b*c^4*x^4
+ 8*a*b*c^2*x^2 - 16*a*b)*cosh(1))*sinh(1)^2 + 1225*(a*b*c^6*d^2*x^2 - 2*a*
b*c^4*d^2)*cosh(1) + (1225*a*b*c^6*d^2*x^2 - 2450*a*b*c^4*d^2 + 45*(5*a*b*c
^6*x^6 - 6*a*b*c^4*x^4 + 8*a*b*c^2*x^2 - 16*a*b)*cosh(1)^2 + 294*(3*a*b*c^6
*d*x^4 - 4*a*b*c^4*d*x^2 + 8*a*b*c^2*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1)
)/c^7
```

Sympy [**A**]
time = 1.22, size = 989, normalized size = 1.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e**
3*x**7/7 + 2*a*b*d**3*x*asinh(c*x) + 2*a*b*d**2*e*x**3*asinh(c*x) + 6*a*b*d
*e**2*x**5*asinh(c*x)/5 + 2*a*b*e**3*x**7*asinh(c*x)/7 - 2*a*b*d**3*sqrt(c*
*2*x**2 + 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 + 1)/(3*c) - 6*a*b*d*e**2
*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*
c) + 4*a*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c**3) + 8*a*b*d*e**2*x**2*sqrt(c**
2*x**2 + 1)/(25*c**3) + 12*a*b*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c**3) - 1
6*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(c**2*x**
2 + 1)/(245*c**5) + 32*a*b*e**3*sqrt(c**2*x**2 + 1)/(245*c**7) + b**2*d**3*
x*asinh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*asinh(c*x)**2 + 2*b**2*d
**2*e*x**3/9 + 3*b**2*d*e**2*x**5*asinh(c*x)**2/5 + 6*b**2*d*e**2*x**5/125
+ b**2*e**3*x**7*asinh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**2*d**3*sqrt(
c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 2*b
**2*e**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/(49*c) - 4*b**2*d**2*e*x/(3*c*
*2) - 8*b**2*d*e**2*x**3/(75*c**2) - 12*b**2*e**3*x**5/(1225*c**2) + 4*b**2
*d**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 8*b**2*d*e**2*x**2*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(25*c**3) + 12*b**2*e**3*x**4*sqrt(c**2*x**2 + 1)*
asinh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*b**2*e**3*x**3/(735
*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c**5) - 16*b**2*
e**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(245*c**5) - 32*b**2*e**3*x/(245*c
**6) + 32*b**2*e**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(245*c**7), Ne(c, 0)), (
a**2*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/7), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,\left(e\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^3,x)

[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^3, x)

3.614 $\int (d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=329

$$2b^2d^2x-\frac{8b^2dex}{9c^2}+\frac{16b^2e^2x}{75c^4}+\frac{4}{27}b^2dex^3-\frac{8b^2e^2x^3}{225c^2}+\frac{2}{125}b^2e^2x^5-\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{8bde\sqrt{1+c}}{}$$

```
[Out] 2*b^2*d^2*x-8/9*b^2*d*e*x/c^2+16/75*b^2*e^2*x/c^4+4/27*b^2*d*e*x^3-8/225*b^
2*e^2*x^3/c^2+2/125*b^2*e^2*x^5+d^2*x*(a+b*arcsinh(c*x))^2+2/3*d*e*x^3*(a+b
*arcsinh(c*x))^2+1/5*e^2*x^5*(a+b*arcsinh(c*x))^2-2*b*d^2*(a+b*arcsinh(c*x)
)*(c^2*x^2+1)^(1/2)/c+8/9*b*d*e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16
/75*b*e^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5-4/9*b*d*e*x^2*(a+b*arcsi
nh(c*x))*(c^2*x^2+1)^(1/2)/c+8/75*b*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/c^3-2/25*b*e^2*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.41, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5793, 5772, 5798, 8, 5776, 5812, 30}

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{4bdex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}-\frac{2be^2x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c}-\frac{16be^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{75c^5}+\frac{8bde\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}+\frac{8be^2x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{75c^3}+d^2x\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)^2+\frac{16b^2e^2x}{75c^4}-\frac{8b^2dex}{9c^2}-\frac{8b^2e^2x^3}{225c^2}+2b^2d^2x+\frac{4}{27}b^2dex^3+\frac{2}{125}b^2e^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (8*b*d*e*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75
*c^5) - (4*b*d*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + (8*b*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + d^2*x*(a + b*ArcSinh[c*x])^2 +
 (2*d*e*x^3*(a + b*ArcSinh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
+ c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \int \left(d^2\left(a+b\sinh^{-1}(cx)\right)^2 + 2dex^2\left(a+b\sinh^{-1}(cx)\right)^2 + e^2x^4\left(a+b\,\mathrm{si}\right.\right.$$

$$= d^2\int \left(a+b\sinh^{-1}(cx)\right)^2 \, dx + (2de)\int x^2\left(a+b\sinh^{-1}(cx)\right)^2 \, dx + e^2$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{5}e^2x^5\left(a+b\,\mathrm{si}\right.$$

$$= -\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{4bdex^2\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{9c}$$

$$= 2b^2d^2x + \frac{4}{27}b^2dex^3 + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}$$

$$= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2}}{}$$

$$= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2}{}$$

Mathematica [A]
time = 0.23, size = 289, normalized size = 0.88

$$\frac{225a^2c^5x(15d^2+10dex^2+3e^2x^4) - 30ab\sqrt{1+c^2x^2}\left(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4)\right) + 2b^2cx\left(360e^2-60c^2e(25d+ex^2)+c^4(3375d^2+250dex^2+27e^2x^4)\right) - 30b\left(-15ac^5x(15d^2+10dex^2+3e^2x^4)+b\sqrt{1+c^2x^2}\left(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4)\right)\right)\sinh^{-1}(cx) + 225b^2c^5x(15d^2+10dex^2+3e^2x^4)\sinh^{-1}(cx)^2}{3375c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]
*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4
)) + 2*b^2*c*x*(360*e^2 - 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^2 + 250*d*e
*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) +
 b*Sqrt[1 + c^2*x^2]*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50
*d*e*x^2 + 9*e^2*x^4)))*ArcSinh[c*x] + 225*b^2*c^5*x*(15*d^2 + 10*d*e*x^2 +
 3*e^2*x^4)*ArcSinh[c*x]^2)/(3375*c^5)
```

Maple [A]
time = 1.75, size = 438, normalized size = 1.33

$$\frac{a^2\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4} + \frac{b^2\left(-\frac{2e^2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{5}{2}}}{25} + \frac{e^2xc\left(25\operatorname{arcsinh}(cx)^2+2\right)\left(c^2x^2+1\right)^2}{125} - \frac{4e\operatorname{arcsinh}(cx)\left(5c^2d-3e\right)\left(c^2x^2+1\right)^{\frac{3}{2}}}{45} + \frac{2\left(1125\,\mathrm{a}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+1/c^4*b^2*(-2/25*e
^2*arcsinh(c*x)*(c^2*x^2+1)^(5/2)+1/125*e^2*x*c*(25*arcsinh(c*x)^2+2)*(c^2*
x^2+1)^2-4/45*e*arcsinh(c*x)*(5*c^2*d-3*e)*(c^2*x^2+1)^(3/2)+2/3375*(1125*a
rcsinh(c*x)^2*c^2*d-675*arcsinh(c*x)^2*e+250*c^2*d-114*e)*e*x*c*(c^2*x^2+1)
-2/15*arcsinh(c*x)*(15*c^4*d^2-10*c^2*d*e+3*e^2)*(c^2*x^2+1)^(1/2)+1/3375*(
3375*arcsinh(c*x)^2*c^4*d^2-2250*arcsinh(c*x)^2*c^2*d*e+6750*c^4*d^2+675*ar
csinh(c*x)^2*e^2-3500*c^2*d*e+894*e^2)*x*c)+2*a*b/c^4*(1/5*arcsinh(c*x)*e^2
*x^5*c^5+2/3*arcsinh(c*x)*c^5*d*e*x^3+arcsinh(c*x)*x*c^5*d^2-1/5*e^2*(1/5*(
c^2*x^2+1)^(1/2)*c^4*x^4-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1
/2))-2/3*d*c^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-d^2*
c^4*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.27, size = 429, normalized size = 1.30

$$\frac{1}{5}b^2x^5\operatorname{arsinh}(cx)^2e^2+\frac{2}{3}b^2dx^3\operatorname{arsinh}(cx)^2e+\frac{1}{5}a^2x^5e^2+b^2d^2x\operatorname{arsinh}(cx)^2+\frac{2}{3}a^2dx^3e+2b^2d^2\left(x-\frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right)+a^2d^2x+\frac{4}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abde-\frac{4}{27}\left(3c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx)-\frac{c^2x^3-6x}{c^2}\right)b^2de+\frac{2\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)abd^2}{c}+\frac{2}{75}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abe^2-\frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\operatorname{arsinh}(cx)-\frac{9c^4x^5-20c^2x^3+120x}{c^4}\right)b^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5*arcsinh(c*x)^2*e^2 + 2/3*b^2*d*x^3*arcsinh(c*x)^2*e + 1/5*a^2*x
^5*e^2 + b^2*d^2*x*arcsinh(c*x)^2 + 2/3*a^2*d*x^3*e + 2*b^2*d^2*(x - sqrt(c
^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 4/9*(3*x^3*arcsinh(c*x) - c*(sqrt
(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e - 4/27*(3*c*(sqrt
(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 -
6*x)/c^2)*b^2*d*e + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c + 2/
75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1
)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^
2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*a
rcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*e^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(291) = 582$.
time = 0.41, size = 767, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(3375*(a^2 + 2*b^2)*c^5*d^2*x + 3*(9*(25*a^2 + 2*b^2)*c^5*x^5 - 40*b
^2*c^3*x^3 + 240*b^2*c*x)*cosh(1)^2 + 225*(3*b^2*c^5*x^5*cosh(1)^2 + 3*b^2*
c^5*x^5*sinh(1)^2 + 10*b^2*c^5*d*x^3*cosh(1) + 15*b^2*c^5*d^2*x + 2*(3*b^2*
c^5*x^5*cosh(1) + 5*b^2*c^5*d*x^3)*sinh(1))*log(c*x + sqrt(c^2*x^2 + 1))^2
```

```
+ 3*(9*(25*a^2 + 2*b^2)*c^5*x^5 - 40*b^2*c^3*x^3 + 240*b^2*c*x)*sinh(1)^2 +
 250*((9*a^2 + 2*b^2)*c^5*d*x^3 - 12*b^2*c^3*d*x)*cosh(1) + 30*(45*a*b*c^5*
x^5*cosh(1)^2 + 45*a*b*c^5*x^5*sinh(1)^2 + 150*a*b*c^5*d*x^3*cosh(1) + 225*
a*b*c^5*d^2*x + 30*(3*a*b*c^5*x^5*cosh(1) + 5*a*b*c^5*d*x^3)*sinh(1) - (225
*b^2*c^4*d^2 + 3*(3*b^2*c^4*x^4 - 4*b^2*c^2*x^2 + 8*b^2)*cosh(1)^2 + 3*(3*b
^2*c^4*x^4 - 4*b^2*c^2*x^2 + 8*b^2)*sinh(1)^2 + 50*(b^2*c^4*d*x^2 - 2*b^2*c
^2*d)*cosh(1) + 2*(25*b^2*c^4*d*x^2 - 50*b^2*c^2*d + 3*(3*b^2*c^4*x^4 - 4*b
^2*c^2*x^2 + 8*b^2)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*(125*(9*a^2 + 2*b^2)*c^5*d*x^3 - 1500*b^2*c^3*d*x + 3*(9*(25
*a^2 + 2*b^2)*c^5*x^5 - 40*b^2*c^3*x^3 + 240*b^2*c*x)*cosh(1))*sinh(1) - 30
*(225*a*b*c^4*d^2 + 3*(3*a*b*c^4*x^4 - 4*a*b*c^2*x^2 + 8*a*b)*cosh(1)^2 + 3
*(3*a*b*c^4*x^4 - 4*a*b*c^2*x^2 + 8*a*b)*sinh(1)^2 + 50*(a*b*c^4*d*x^2 - 2*
a*b*c^2*d)*cosh(1) + 2*(25*a*b*c^4*d*x^2 - 50*a*b*c^2*d + 3*(3*a*b*c^4*x^4
- 4*a*b*c^2*x^2 + 8*a*b)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A]
time = 0.58, size = 595, normalized size = 1.81

$$\begin{cases} a^2d^2x + \frac{2a^2dex^3}{3} + \frac{a^2e^2x^5}{5} + 2abd^2x\operatorname{asinh}(cx) + \frac{4abdex^3\operatorname{asinh}(cx)}{3} + \frac{2abe^2x^5\operatorname{asinh}(cx)}{5} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - \frac{4abdex^2\sqrt{c^2x^2+1}}{9c} - \frac{2abe^2x^4\sqrt{c^2x^2+1}}{25c} + \frac{8abde\sqrt{c^2x^2+1}}{9c^3} + \frac{8abe^2x^2\sqrt{c^2x^2+1}}{75c^3} - \frac{16abe^2\sqrt{c^2x^2+1}}{75c^5} + b^2d^2x\operatorname{asinh}^2(cx) + 2b^2d^2x + \frac{2b^2dex^3\operatorname{asinh}^2(cx)}{3} + \frac{4b^2dex^3}{27} + \frac{b^2e^2x^5\operatorname{asinh}^2(cx)}{5} + \frac{2b^2e^2x^5}{125} - \frac{2b^2d^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} - \frac{4b^2dex^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{9c} - \frac{2b^2e^2x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{25c} - \frac{8b^2dex}{9c^2} - \frac{8b^2e^2x^3}{225c^2} + \frac{8b^2de\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{9c^3} + \frac{8b^2e^2x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{75c^3} + \frac{16b^2e^2x}{75c^4} - \frac{16b^2e^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{75c^5} & \text{for}\: c \neq 0 \\ a^2\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2*
x*asinh(c*x) + 4*a*b*d*e*x**3*asinh(c*x)/3 + 2*a*b*e**2*x**5*asinh(c*x)/5 -
 2*a*b*d**2*sqrt(c**2*x**2 + 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c
) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) + 8*a*b*d*e*sqrt(c**2*x**2 +
 1)/(9*c**3) + 8*a*b*e**2*x**2*sqrt(c**2*x**2 + 1)/(75*c**3) - 16*a*b*e**2*
sqrt(c**2*x**2 + 1)/(75*c**5) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x +
 2*b**2*d*e*x**3*asinh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x**5*asin
h(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(
c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c) - 2*b**2*e**2
*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 8*b**2*d*e*x/(9*c**2) - 8*b**
2*e**2*x**3/(225*c**2) + 8*b**2*d*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3)
 + 8*b**2*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**3) + 16*b**2*e**2
*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**5), Ne(c,
 0)), (a**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^2, x)
```

3.615 $\int (d+ex^2) \left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=153

$$2b^2dx-\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4be\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}-\frac{2bex^2\sqrt{1+c}}{}$$

```
[Out] 2*b^2*d*x-4/9*b^2*e*x/c^2+2/27*b^2*e*x^3+d*x*(a+b*arcsinh(c*x))^2+1/3*e*x^3
*(a+b*arcsinh(c*x))^2-2*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/9*b*e*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-2/9*b*e*x^2*(a+b*arcsinh(c*x))*(c^
2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5793, 5772, 5798, 8, 5776, 5812, 30}

$$-\frac{2bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{4be\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}+dx\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)^2-\frac{4b^2ex}{9c^2}+2b^2dx+\frac{2}{27}b^2ex^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x - (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/c + (4*b*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(9*c^3) - (2*b*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + d*x*(
a + b*ArcSinh[c*x])^2 + (e*x^3*(a + b*ArcSinh[c*x])^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right) \left(a+b\sinh^{-1}(cx)\right)^2 \, dx &= \int \left(d\left(a+b\sinh^{-1}(cx)\right)^2+ex^2\left(a+b\sinh^{-1}(cx)\right)^2\right) \, dx \\ &= d\int \left(a+b\sinh^{-1}(cx)\right)^2 \, dx+e\int x^2\left(a+b\sinh^{-1}(cx)\right)^2 \, dx \\ &= dx\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)^2-(2bcd)\int \frac{x\left(a+b\right.}{\sqrt{1}} \\ &= -\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\right.}{9c} \\ &= 2b^2dx+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4be\sqrt{1+c^2x^2}}{} \\ &= 2b^2dx-\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4be}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 164, normalized size = 1.07

$$\frac{9a^2c^3x(3d+ex^2)-6ab\sqrt{1+c^2x^2}\,(-2e+c^2(9d+ex^2))+2b^2cx(-6e+c^2(27d+ex^2))-6b\left(-3ac^3x(3d+ex^2)+b\sqrt{1+c^2x^2}\,(-2e+c^2(9d+ex^2))\right)\sinh^{-1}(cx)+9b^2c^3x(3d+ex^2)\sinh^{-1}(cx)^2}{27c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x
^2)) + 2*b^2*c*x*(-6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*d + e*x^2
) + b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))*ArcSinh[c*x] + 9*b^2*c^
3*x*(3*d + e*x^2)*ArcSinh[c*x]^2)/(27*c^3)
```

Maple [A]
time = 0.91, size = 206, normalized size = 1.35

$$\frac{\frac{a^2\left(\frac{1}{3}x^3c^3e+x\,c^3d\right)}{c^2}+\frac{b^2\left(\frac{\left(9\operatorname{arcsinh}(cx)^2x^2c^2e+27\operatorname{arcsinh}(cx)^2c^2d+2c^2e\,x^2+54c^2d-12e\right)xc}{27}-\frac{2\operatorname{arcsinh}(cx)\left(c^2e\,x^2+9c^2d-2e\right)\sqrt{c^2x^2+1}}{9}\right)}{c^2}+\frac{2ab\left(\right.}{}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^2*(1/3*x^3*c^3*e+x*c^3*d)+b^2/c^2*(1/27*(9*arcsinh(c*x)^2*x^2*c^
2*e+27*arcsinh(c*x)^2*c^2*d+2*c^2*e*x^2+54*c^2*d-12*e)*x*c-2/9*arcsinh(c*x)
```

```
*(c^2*e*x^2+9*c^2*d-2*e)*(c^2*x^2+1)^(1/2))+2*a*b/c^2*(1/3*arcsinh(c*x)*x^3
*c^3*e+arcsinh(c*x)*x*c^3*d-1/3*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x
^2+1)^(1/2))-d*c^2*(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.27, size = 222, normalized size = 1.45

$$\frac{1}{3}b^2x^3\operatorname{arsinh}(cx)^2e+b^2dx\operatorname{arsinh}(cx)^2+\frac{1}{3}a^2x^3e+2b^2d\left(x-\frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right)+a^2dx+\frac{2}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abe-\frac{2}{27}\left(3c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx)-\frac{c^2x^3-6x}{c^2}\right)b^2e+\frac{2\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)abd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arcsinh(c*x)^2*e + b^2*d*x*arcsinh(c*x)^2 + 1/3*a^2*x^3*e + 2*b
^2*d*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2/9*(3*x^3*arcsinh(
c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*e - 2/2
7*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) -
 (c^2*x^3 - 6*x)/c^2)*b^2*e + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*
d/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(142) = 284.
time = 0.39, size = 304, normalized size = 1.99

$$\frac{27(a^2+2b^2)c^3dx+9(b^2c^3x^3\cosh(1)+b^2c^3x^3\sinh(1)+3b^2c^3dx)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+((9a^2+2b^2)c^3x^3-12b^2cx)\cosh(1)+6\left(3abc^3x^3\cosh(1)+3abc^3x^3\sinh(1)+9abc^3dx-(9b^2c^2d+(b^2c^2x^2-2b^2)\cosh(1)+(b^2c^2x^2-2b^2)\sinh(1))\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+((9a^2+2b^2)c^3x^3-12b^2cx)\sinh(1)-6(9abc^2d+(abc^2x^2-2ab)\cosh(1)+(abc^2x^2-2ab)\sinh(1))\sqrt{c^2x^2+1}}{27c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*(27*(a^2 + 2*b^2)*c^3*d*x + 9*(b^2*c^3*x^3*cosh(1) + b^2*c^3*x^3*sinh(
1) + 3*b^2*c^3*d*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + ((9*a^2 + 2*b^2)*c^3*x
^3 - 12*b^2*c*x)*cosh(1) + 6*(3*a*b*c^3*x^3*cosh(1) + 3*a*b*c^3*x^3*sinh(1)
 + 9*a*b*c^3*d*x - (9*b^2*c^2*d + (b^2*c^2*x^2 - 2*b^2)*cosh(1) + (b^2*c^2*
x^2 - 2*b^2)*sinh(1))*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + ((9
*a^2 + 2*b^2)*c^3*x^3 - 12*b^2*c*x)*sinh(1) - 6*(9*a*b*c^2*d + (a*b*c^2*x^2
 - 2*a*b)*cosh(1) + (a*b*c^2*x^2 - 2*a*b)*sinh(1))*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.26, size = 279, normalized size = 1.82

$$\begin{cases} a^2dx+\frac{a^2ex^3}{3}+2abdx\operatorname{asinh}(cx)+\frac{2abex^3\operatorname{asinh}(cx)}{3}-\frac{2abd\sqrt{c^2x^2+1}}{c}-\frac{2abex^2\sqrt{c^2x^2+1}}{9c}+\frac{4abe\sqrt{c^2x^2+1}}{9c^3}+b^2dx\operatorname{asinh}^2(cx)+2b^2dx+\frac{b^2ex^3\operatorname{asinh}^2(cx)}{3}+\frac{2b^2ex^3}{27}-\frac{2b^2d\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c}-\frac{2b^2ex^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{9c}-\frac{4b^2ex}{9c^2}+\frac{4b^2e\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{9c^3} & \text{for } c\neq 0 \\ a^2\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*asinh(c*x) + 2*a*b*e*x**3*a
sinh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2 +
```

```
 1)/(9*c) + 4*a*b*e*sqrt(c**2*x**2 + 1)/(9*c**3) + b**2*d*x*asinh(c*x)**2 +
 2*b**2*d*x + b**2*e*x**3*asinh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d*sqr
t(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(9*c) - 4*b**2*e*x/(9*c**2) + 4*b**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*
c**3), Ne(c, 0)), (a**2*(d*x + e*x**3/3), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2), x)
```

3.616 $\int \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=46

$$2b^2x-\frac{2b\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2$$

```
[Out] 2*b^2*x+x*(a+b*arcsinh(c*x))^2-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5772, 5798, 8}

$$-\frac{2b\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^2 dx = x\left(a+b\sinh^{-1}(cx)\right)^2 - (2bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx$$
$$= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} + x\left(a+b\sinh^{-1}(cx)\right)^2 + \left(2b^2\right)\int 1\,dx$$
$$= 2b^2x - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} + x\left(a+b\sinh^{-1}(cx)\right)^2$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 1.61

$$\left(a^2+2b^2\right)x - \frac{2ab\sqrt{1+c^2x^2}}{c} + \frac{2b\left(acx-b\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)}{c} + b^2x\sinh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c^
2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

Maple [A]
time = 1.84, size = 72, normalized size = 1.57

method	result	size
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	72
default	$\frac{a^2cx+b^2\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.26, size = 72, normalized size = 1.57

$$b^2x\,\operatorname{arsinh}(cx)^2 + 2\,b^2\left(x - \frac{\sqrt{c^2x^2+1}\,\operatorname{arsinh}(cx)}{c}\right) + a^2x + \frac{2\left(cx\,\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*x
 + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.
time = 0.37, size = 96, normalized size = 2.09

$$\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+(a^2+2\,b^2)cx-2\,\sqrt{c^2x^2+1}\,ab+2\left(abcx-\sqrt{c^2x^2+1}\,b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 + 1
)))/c
```

Sympy [A]
time = 0.09, size = 82, normalized size = 1.78

$$\begin{cases} a^2x+2abx\operatorname{asinh}(cx)-\frac{2ab\sqrt{c^2x^2+1}}{c}+b^2x\operatorname{asinh}^2(cx)+2b^2x-\frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c\neq 0\\ a^2x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**2
*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne(c
, 0)), (a**2*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(44) = 88$.
time = 0.46, size = 111, normalized size = 2.41

$$2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2+1}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2
 + 1))/c^2))*b^2 + a^2*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a + b \operatorname{asinh}(c x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2, x)
```

3.617 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=739

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(
c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/
2)+1/2*(a+b*arcsinh(c*x))^2*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(1+(c*
x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^
(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcsinh(c*x))*polylog
(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1
/2)/e^(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)
/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcsinh(c*x))*p
olylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(
-d)^(1/2)/e^(1/2)+b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c^2*x^2+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b^2*polylo
g(3,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(
-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A]

time = 0.98, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5793, 5827, 5680, 2221, 2611, 2320, 6724}

$$-\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\,\sqrt{e}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\,\sqrt{e}}-\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\,\sqrt{e}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b^2\text{Li}_3\left(-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\,\sqrt{e}}-\frac{b^2\text{Li}_3\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\,\sqrt{e}}+\frac{b^2\text{Li}_3\left(-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\,\sqrt{e}}-\frac{b^2\text{Li}_3\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log[1 + (S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt
[e]) + ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log
[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-
d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])
```

```
/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcSin
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}\,x}dx}{2\sqrt{-d}} - \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}\,x}dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{c\sqrt{-d}-\sqrt{e}\,\sinh(x)}dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{c\sqrt{-d}+\sqrt{e}\,\sinh(x)}\right.}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}\,e^x}dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{}{c\sqrt{-d}+}\right.}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\right.}{2\sqrt{-}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\right.}{2\sqrt{-}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\right.}{2\sqrt{-}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\right.}{2\sqrt{-}}$$

Mathematica [A]
time = 0.54, size = 985, normalized size = 1.33

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

```
[Out] (2*a^2*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Lo
g[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b^2*Sqr
t[d]*ArcSinh[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) + e])] + 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/
```

```
(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + 2*a*b*Sqrt
[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - b^2*Sqrt[d]*ArcSinh
[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])]
 + 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] - Sqrt[-(c^2*d) + e])] - 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[
2, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - 2*a*b*S
qrt[d]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e]))] - 2*b^2*Sqrt[d]*ArcSinh[c*x]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c
*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + 2*a*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^Arc
Sinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + 2*b^2*Sqrt[d]*ArcSinh[c*x]*
PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*
b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
 + e])] + 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d])
+ Sqrt[-(c^2*d) + e])] + 2*b^2*Sqrt[d]*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] - 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E
^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + integrate(b^2*log(c*x + sq
rt(c^2*x^2 + 1))^2/(x^2*e + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(x^2*e
+ d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}\left(cx\right)\right)^2}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2), x)
```

3.618 $\int \frac{(d+ex^2)^3}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=670

$$\frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{3d^2e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^5}-\frac{5e^3\cos}{}$$

[Out] d^3*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-3/4*d^2*e*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^3+3/8*d*e^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^5-5/64*e^3*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^7+3/4*d^2*e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-9/16*d*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^5+9/64*e^3*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^7+3/16*d*e^2*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b/c^5-5/64*e^3*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b/c^7+1/64*e^3*Chi(7*(a+b*arcsinh(c*x))/b)*cosh(7*a/b)/b/c^7-d^3*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+3/4*d^2*e*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^3-3/8*d*e^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^5+5/64*e^3*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^7-3/4*d^2*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3+9/16*d*e^2*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^5-9/64*e^3*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^7-3/16*d*e^2*Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^5+5/64*e^3*Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^7-1/64*e^3*Shi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/b/c^7

Rubi [A]
time = 0.91, antiderivative size = 670, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5793, 5774, 3384, 3379, 3382, 5780, 5556}

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]

[Out] (d^3*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (3*d^2*e*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (3*d*e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) - (5*e^3*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(64*b*c^7) + (3*d^2*e*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (9*d*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (9*e^3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) + (3*d*e^2*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (5*e^3*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) + (e^3*Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) - (d^3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) + (3*d^2*e*Sinh[a/b]*SinhIntegral[(a + b

```
*ArcSinh[c*x])/b])/(4*b*c^3) - (3*d*e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSi
nh[c*x])/b])/(8*b*c^5) + (5*e^3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])
/b])/(64*b*c^7) - (3*d^2*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x
]))/b])/(4*b*c^3) + (9*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b*c^5) - (9*e^3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[
c*x]))/b])/(64*b*c^7) - (3*d*e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSi
nh[c*x]))/b])/(16*b*c^5) + (5*e^3*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcS
inh[c*x]))/b])/(64*b*c^7) - (e^3*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSi
nh[c*x]))/b])/(64*b*c^7)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{a+b\sinh^{-1}(cx)}\,dx = \int \left(\frac{d^3}{a+b\sinh^{-1}(cx)}+\frac{3d^2ex^2}{a+b\sinh^{-1}(cx)}+\frac{3de^2x^4}{a+b\sinh^{-1}(cx)}+\frac{e^3x^6}{a+b\sinh^{-1}(cx)}\right)$$

$$= d^3\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx+\left(3d^2e\right)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx+\left(3de^2\right)\int \frac{x^4}{a+b\sinh^{-}}$$

$$= \frac{d^3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x\right.}{c^3}$$

$$= \frac{\left(3d^2e\right)\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)\,dx,x,\sinh^{-1}(cx)\right)}{c^3}+\frac{\left(3de^2\right)\text{Subst}\left(\int \left(\frac{\cosh(}{8(a+b}\right.\right.}{}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\left(3d^2e\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\left(3d^2e\cosh\left(\frac{a}{b}\right)\right)}{}$$

$$= -\frac{3d^2e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5}-\frac{5e^3}{}$$

Mathematica [A]
time = 0.64, size = 444, normalized size = 0.66

$$\frac{(64c^6d^3-48c^4d^2e+24c^2de^2-5e^3)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+3e(16c^4d^2-12c^2de+3e^2)\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+12c^2de^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-5e^3\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+e^3\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-64c^6d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+48c^4d^2e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-24c^2de^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+5e^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-48c^4d^2e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+36c^2de^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-9e^3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-12c^2de^2\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+5e^3\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-e^3\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{64bc^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((64*c^6*d^3 - 48*c^4*d^2*e + 24*c^2*d*e^2 - 5*e^3)*Cosh[a/b]*CoshIntegral[
a/b + ArcSinh[c*x]] + 3*e*(16*c^4*d^2 - 12*c^2*d*e + 3*e^2)*Cosh[(3*a)/b]*C
oshIntegral[3*(a/b + ArcSinh[c*x])] + 12*c^2*d*e^2*Cosh[(5*a)/b]*CoshIntegr
al[5*(a/b + ArcSinh[c*x])] - 5*e^3*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcS
inh[c*x])] + e^3*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 64*c^
6*d^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 48*c^4*d^2*e*Sinh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e^2*Sinh[a/b]*SinhIntegral[a/b +
```

```
ArcSinh[c*x]] + 5*e^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 48*c^4*
d^2*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 36*c^2*d*e^2*Sin
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 9*e^3*Sinh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])] - 12*c^2*d*e^2*Sinh[(5*a)/b]*SinhIntegral[5
*(a/b + ArcSinh[c*x])] + 5*e^3*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])] - e^3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^7)
```

Maple [A]
time = 11.07, size = 654, normalized size = 0.98 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/128/c^6*e^3/b*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-1/128/c^6*e^3
/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a
/b)*d^3+3/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^2*e-3/16/c^4/b*exp(a/b)
*Ei(1,arcsinh(c*x)+a/b)*d*e^2+5/128/c^6/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e
^3-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2/b*exp(-a/b)*Ei(1,-ar
csinh(c*x)-a/b)*d^2*e-3/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e^2+5/
128/c^6/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^3-3/8/c^2*e/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)*d^2+9/32/c^4*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*
a/b)*d-9/128/c^6*e^3/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3/8/c^2*e/b*ex
p(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*d^2+9/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-
3*arcsinh(c*x)-3*a/b)*d-9/128/c^6*e^3/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b)-3/32/c^4*e^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)*d+5/128/c^6*e^3/b
*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*a
rcsinh(c*x)-5*a/b)*d+5/128/c^6*e^3/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3)/(b*arcsinh(c*x) + a),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^3}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**3/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^3}{a+b\operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*asinh(c*x)),x)
```

```
[Out] int((d + e*x^2)^3/(a + b*asinh(c*x)), x)
```

3.619 $\int \frac{(d+ex^2)^2}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=388

$$\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^5}+\frac{de\cosh\left(\frac{3a}{b}\right)}{}$$

```
[Out] d^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-1/2*d*e*Chi((a+b*arcsinh(c*x))/
b)*cosh(a/b)/b/c^3+1/8*e^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^5+1/2*d*
e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-3/16*e^2*Chi(3*(a+b*arcsinh
(c*x))/b)*cosh(3*a/b)/b/c^5+1/16*e^2*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b
)/b/c^5-d^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+1/2*d*e*Shi((a+b*arcsin
h(c*x))/b)*sinh(a/b)/b/c^3-1/8*e^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^
5-1/2*d*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3+3/16*e^2*Shi(3*(a+b
*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^5-1/16*e^2*Shi(5*(a+b*arcsinh(c*x))/b)*si
nh(5*a/b)/b/c^5
```

Rubi [A]

time = 0.51, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5793, 5774, 3384, 3379, 3382, 5780, 5556}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^5}-\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^5}+\frac{e^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^5}+\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^5}-\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2bc^3}+\frac{de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^3}+\frac{de\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2bc^3}-\frac{de\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{2bc^3}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (d*e*Cosh[a/b]
*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(2*b*c^3) + (e^2*Cosh[a/b]*CoshInteg
ral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) + (d*e*Cosh[(3*a)/b]*CoshIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3) - (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (e^2*Cosh[(5*a)/b]*CoshIntegral[(5*
(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (d^2*Sinh[a/b]*SinhIntegral[(a + b*A
rcSinh[c*x])/b])/(b*c) + (d*e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b
])/(2*b*c^3) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^
5) - (d*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3)
 + (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5
) - (e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{a+b\sinh^{-1}(cx)}\,dx = \int \left(\frac{d^2}{a+b\sinh^{-1}(cx)} + \frac{2dex^2}{a+b\sinh^{-1}(cx)} + \frac{e^2x^4}{a+b\sinh^{-1}(cx)}\right)\,dx$$

$$= d^2\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx + (2de)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx + e^2\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx$$

$$= \frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} + \frac{(2de)\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \text{si}\right.}{c^3}$$

$$= \frac{(2de)\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)\,dx, x, \sinh^{-1}(cx)\right)}{c^3} + \frac{e^2\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)} - \frac{3\,\text{co}}{16(}\right.\right.}{}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{(de)\text{Subst}\left(\int \frac{\cosh(}{a+b}\right.}{}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\left(de\cosh\left(\frac{a}{b}\right)\right)\text{Sub}}{}$$

$$= -\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{2bc^3} + \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5} + \frac{de\cosh\left(\frac{3}{}\right.}{}$$

Mathematica [A]

time = 0.36, size = 253, normalized size = 0.65

$$\frac{2(8c^4d^2-4c^2de+e^2)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+(8c^2d-3e)\,e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+e^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-16c^4d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+8c^2de\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-2e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-8c^2de\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+3e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-e^2\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]]
 + (8*c^2*d - 3*e)*e*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + e
^2*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 16*c^4*d^2*Sinh[a/b
]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*c^2*d*e*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] - 2*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 8*c^2*d
*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 3*e^2*Sinh[(3*a)/b]
*SinhIntegral[3*(a/b + ArcSinh[c*x])] - e^2*Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])])/(16*b*c^5)
```

Maple [A]

time = 8.88, size = 380, normalized size = 0.98

method	result
derivativedivides	$-\frac{e^2 \mathrm{e}^{-\frac{5a}{b}} \operatorname{expIntegral}\left(1,-5\operatorname{arcsinh}(cx)-\frac{5a}{b}\right)}{32c^4b}-\frac{e^2 \mathrm{e}^{\frac{5a}{b}} \operatorname{expIntegral}\left(1,5\operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b}-\frac{\mathrm{e}^{\frac{a}{b}} \operatorname{expIntegral}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}}{}$

default	$-\frac{e^{2} e^{-\frac{5a}{b}} \operatorname{expIntegral}\left(1,-5\operatorname{arcsinh}(cx)-\frac{5a}{b}\right)}{32c^{4}b}-\frac{e^{2} e^{\frac{5a}{b}} \operatorname{expIntegral}\left(1,5\operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^{4}b}-\frac{e^{\frac{a}{b}} \operatorname{expIntegral}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d^{2}}{2b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/32/c^4*e^2/b
*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b
)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d*e-1/16/c^4/b*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^2+1/4/c^2
/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)*e^2-1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d+3/32/c^4
*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)*Ei(1,-
3*arcsinh(c*x)-3*a/b)*d+3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a
/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x^4*e^2 + 2*d*x^2*e + d^2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^{2}\right)^{2}}{a + b \operatorname{asinh}{\left(cx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^2}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x)), x)
```

3.620 $\int \frac{d+ex^2}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=180

$$\frac{d\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{e\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^3}-\frac{d\sinh\left(\frac{a}{b}\right)}{}$$

```
[Out] d*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-1/4*e*Chi((a+b*arcsinh(c*x))/b)*c
osh(a/b)/b/c^3+1/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-d*Shi((a
+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+1/4*e*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c^3-1/4*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3
```

Rubi [A]
time = 0.25, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5793, 5774, 3384, 3379, 3382, 5780, 5556}

$$-\frac{e\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^3}+\frac{e\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4bc^3}+\frac{d\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (e*Cosh[a/b]*Cos
hIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (e*Cosh[(3*a)/b]*CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (d*Sinh[a/b]*SinhIntegral[(a + b
*ArcSinh[c*x])/b])/(b*c) + (e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b
])/(4*b*c^3) - (e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(
4*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{a+b\sinh^{-1}(cx)}\,dx &= \int \left(\frac{d}{a+b\sinh^{-1}(cx)}+\frac{ex^2}{a+b\sinh^{-1}(cx)}\right)dx \\
&= d\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx + e\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx \\
&= \frac{d\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} + \frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}\right.}{c^3} \\
&= \frac{e\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right.}{x}\right.}{bc} \\
&= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{e\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\right.}{4c} \\
&= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\left(e\cosh\left(\frac{a}{b}\right)\right)\text{Subs}}{} \\
&= -\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3} + \frac{d\cosh\left(\right.}{}
\end{aligned}$$

Mathematica [A]

time = 0.16, size = 126, normalized size = 0.70

$$\frac{(4c^2d-e)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4c^2d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
[Out] ((4*c^2*d - e)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e*Cosh[(3*a)/b]
*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c^2*d*Sinh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]] + e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - e*Sinh[(3*
a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

Maple [A]

time = 6.94, size = 178, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\,\text{expIntegral}\left(1,-3\,\text{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,\mathrm{e}^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{\mathrm{e}^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{e}}{}}{c}$
default	$\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\,\text{expIntegral}\left(1,-3\,\text{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,\mathrm{e}^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{\mathrm{e}^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{e}}{}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/8/c^2*e/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/8/c^2*e/b*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d+1/
8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x
)-a/b)*d+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{a + b \operatorname{asinh}{(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*asinh(c*x)),x)
```

```
[Out] int((d + e*x^2)/(a + b*asinh(c*x)), x)
```

3.621 $\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5774, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x\right.}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-1),x]
[Out] Integrate[(a + b*ArcSinh[c*x])^(-1), x]
```

Maple [A]
time = 3.04, size = 56, normalized size = 1.04

method	result	size
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56
default	$\frac{-\frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c
*x)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{asinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x)),x)
[Out] Integral(1/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b \operatorname{asinh}(c\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x)),x)
[Out] int(1/(a + b*asinh(c*x)), x)
```

3.622 $\int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((x^2*e + d)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*x^2*e + a*d + (b*x^2*e + b*d)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{asinh}{(cx)})\,(d + ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)), x)
```

3.623 $\int \frac{1}{(d+ex^2)^2(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))}\,dx=\int \frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A]
time = 2.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^2\,(a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^2*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*
d^2)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asinh}(cx)) (d + ex^2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^2), x)
```

3.624 $\int \frac{(d+ex^2)^2}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=495

$$-\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{d^2\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{de\mathrm{C}}{}$$

```
[Out] d^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/2*d*e*cosh(a/b)*Shi((a+b*ar
csinh(c*x))/b)/b^2/c^3+1/8*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^5+
3/2*d*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-9/16*e^2*cosh(3*a/b
)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^5+5/16*e^2*cosh(5*a/b)*Shi(5*(a+b*arcsi
nh(c*x))/b)/b^2/c^5-d^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/2*d*e*C
hi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^3-1/8*e^2*Chi((a+b*arcsinh(c*x))/b
)*sinh(a/b)/b^2/c^5-3/2*d*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3
+9/16*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^5-5/16*e^2*Chi(5*(a
+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^5-d^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcs
inh(c*x))-2*d*e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e^2*x^4*(c^2*x
^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A]
time = 0.55, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5793, 5773, 5819, 3384, 3379, 3382, 5778}

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^5}+\frac{9e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\sinh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^5}+\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^5}-\frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^5}+\frac{5e^2\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^5}+\frac{de\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2b^2c^3}-\frac{3de\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{2b^2c^3}-\frac{de\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2b^2c^3}+\frac{3de\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{2b^2c^3}-\frac{d^2\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d^2\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^4*Sqrt[1 + c^2*x^2])/(b*c*(a
 + b*ArcSinh[c*x])) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/
(b^2*c) + (d*e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(2*b^2*c^3)
- (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b^2*c^5) - (3*d*e
*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(2*b^2*c^3) + (9*e
^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c^5) - (
5*e^2*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^5)
+ (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (d*e*Cosh[
a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(2*b^2*c^3) + (e^2*Cosh[a/b]*Sin
hIntegral[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*SinhI
ntegral[(3*(a + b*ArcSinh[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Cosh[(3*a)/b]*Sin
hIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*
SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d^2}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{2dex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{e^2x^4}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx$$

$$= d^2\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+(2de)\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+e^2\int \frac{}{(a+b\mathrm{s}}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{(cd^2}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{d^2\mathrm{S}}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{(d^2}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{d^2\mathrm{C}}{}$$

Mathematica [A]
time = 1.35, size = 356, normalized size = 0.72

$$-\frac{\frac{16bc^4d^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}+\frac{32bc^4dex^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}+\frac{16bc^4e^2x^4\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}+2(8c^4d^2-4c^2de+e^2)\,\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+3(8c^2d-3e)e\,\mathrm{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+5e^2\mathrm{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)-16c^4d^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+8c^2de\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-2e^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-24c^2de\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+9e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-5e^2\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{16b^2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*((16*b*c^4*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (32*b*c^4*d*
e*x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (16*b*c^4*e^2*x^4*Sqrt[1 +
c^2*x^2])/(a + b*ArcSinh[c*x]) + 2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*CoshIntegr
al[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*(8*c^2*d - 3*e)*e*CoshIntegral[3*(a/b
+ ArcSinh[c*x])]*Sinh[(3*a)/b] + 5*e^2*CoshIntegral[5*(a/b + ArcSinh[c*x])]
*Sinh[(5*a)/b] - 16*c^4*d^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*
c^2*d*e*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 2*e^2*Cosh[a/b]*SinhIn
tegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcSinh[c*x])] + 9*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])]
- 5*e^2*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])])/(b^2*c^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(469) = 938$.
time = 9.45, size = 1036, normalized size = 2.09

method	result	size
derivativedivides	Expression too large to display	1036
default	Expression too large to display	1036

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/32*(-16*(c^2*x^2+1)^(1/2)*c^4*x^4+16*c^5*x^5-12*c^2*x^2*(c^2*x^2+1)^
(1/2)+20*c^3*x^3-(c^2*x^2+1)^(1/2)+5*c*x)*e^2/c^4/b/(a+b*arcsinh(c*x))+5/32
*e^2/c^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/b*e^2/c^4*(16*c^5*x
^5+20*c^3*x^3+16*(c^2*x^2+1)^(1/2)*c^4*x^4+5*c*x+12*c^2*x^2*(c^2*x^2+1)^(1/
2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-5/32/b^2*e^2/c^4*exp(-5*a/b)*Ei(1,
-5*arcsinh(c*x)-5*a/b)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d^2/b/(a+b*arcsinh(c*x)
)+1/2*d^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/4*(-(c^2*x^2+1)^(1/2)+c*x)*
d*e/c^2/b/(a+b*arcsinh(c*x))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b
)+1/16*(-(c^2*x^2+1)^(1/2)+c*x)*e^2/c^4/b/(a+b*arcsinh(c*x))+1/16/c^4*e^2/b
^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*a
rcsinh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4/c^2/b*d*e*(c
*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/4/c^2/b^2*d*e*exp(-a/b)*Ei(1,-ar
csinh(c*x)-a/b)-1/16/c^4/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1
/16/c^4/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4*(-4*c^2*x^2*(c^2*x^2+
1)^(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)*d*e/c^2/b/(a+b*arcsinh(c*x))-3/
32*(-4*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)*e^2/c^4
/b/(a+b*arcsinh(c*x))+3/4*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d
-9/32*e^2/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*(4*c^3*
x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))
*d+3/32/c^4*e^2/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^
(1/2))/(a+b*arcsinh(c*x))-3/4/c^2*e/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b)*d+9/32/c^4*e^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^7*e^2 + (2*c^3*d*e + c*e^2)*x^5 + c*d^2*x + (c^3*d^2 + 2*c*d*e)*x^3
 + (c^2*x^6*e^2 + (2*c^2*d*e + e^2)*x^4 + (c^2*d^2 + 2*d*e)*x^2 + d^2)*sqrt
(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^
```

```
3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)))
+ integrate((5*c^5*x^8*e^2 + 2*(3*c^5*d*e + 5*c^3*e^2)*x^6 + (c^5*d^2 + 12*
c^3*d*e + 5*c*e^2)*x^4 + c*d^2 + 2*(c^3*d^2 + 3*c*d*e)*x^2 + (5*c^3*x^6*e^2
 + 3*(2*c^3*d*e + c*e^2)*x^4 - c*d^2 + (c^3*d^2 + 2*c*d*e)*x^2)*(c^2*x^2 +
1) + (10*c^4*x^7*e^2 + (12*c^4*d*e + 13*c^2*e^2)*x^5 + 2*(c^4*d^2 + 7*c^2*d
*e + 2*e^2)*x^3 + (c^2*d^2 + 4*d*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c
^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 +
 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sq
rt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((x^4*e^2 + 2*d*x^2*e + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*
x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^2}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
[Out] Integral((d + e*x**2)**2/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^2}{(a+b\operatorname{asinh}(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x))^2,x)
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x))^2, x)
```

3.625 $\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=247

$$-\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{d\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{e\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3}$$

```
[Out] d*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/4*e*cosh(a/b)*Shi((a+b*arcsin
h(c*x))/b)/b^2/c^3+3/4*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-d*
Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e*Chi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^3-3/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d*(c
^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arc
sinh(c*x))
```

Rubi [A]

time = 0.29, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5793, 5773, 5819, 3384, 3379, 3382, 5778}

$$\frac{e\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{d\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{ex^2\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x^2*Sqrt[1 + c^2*x
^2])/(b*c*(a + b*ArcSinh[c*x])) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]*S
inh[a/b])/(b^2*c) + (e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b
^2*c^3) - (3*e*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b
^2*c^3) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (e*C
osh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) + (3*e*Cosh[(3*a
)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx$$

$$= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx + e\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(cd)\int \frac{x}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(}}{b}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{d\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-}\right.}{bc}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\right.}{a+ba}\right.}{bc}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{d\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\right.}{b^2c}$$

Mathematica [A]

time = 0.59, size = 190, normalized size = 0.77

$$-\frac{\frac{4bc^2d\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}+\frac{4bc^2ex^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}+(4c^2d-e)\,\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+3e\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-4c^2d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-3e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{4b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/4*((4*b*c^2*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e*x^2*S
qrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*c^2*d - e)*CoshIntegral[a/b + A
rcSinh[c*x]]*Sinh[a/b] + 3*e*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a
)/b] - 4*c^2*d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e*Cosh[a/b]*Sin
hIntegral[a/b + ArcSinh[c*x]] - 3*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])])/(b^2*c^3)
```

Maple [A]

time = 7.01, size = 438, normalized size = 1.77

method	result
derivativedivides	$\frac{\frac{\left(-4c^2x^2\sqrt{c^2x^2+1}+4c^3x^3-\sqrt{c^2x^2+1}+3cx\right)e}{8c^2b(a+b\,\text{arcsinh}(cx))}+\frac{3e\,\mathrm{e}^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4c^3x^3+3cx+4c^2x^2\sqrt{\right.}{8b\,c^2(}}{}$

default	$\frac{\left(-4c^2x^2\sqrt{c^2x^2+1}+4c^3x^3-\sqrt{c^2x^2+1}+3cx\right)e}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{3e\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{expIntegral}\left(1,3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4c^3x^3+3cx+4c^2x^2\sqrt{c\right.}}{8b\,c^2(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/8*(-4*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)*e
/c^2/b/(a+b*arcsinh(c*x))+3/8*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/
b)-1/8/b*e/c^2*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/
2))/(a+b*arcsinh(c*x))-3/8/b^2*e/c^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b
)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d/b/(a+b*arcsinh(c*x))+1/2*d/b^2*exp(a/b)*Ei
(1,arcsinh(c*x)+a/b)-1/8*(-(c^2*x^2+1)^(1/2)+c*x)*e/c^2/b/(a+b*arcsinh(c*x)
)-1/8/c^2*e/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d*(c*x+(c^2*x^2+1)^(1
/2))/(a+b*arcsinh(c*x))-1/2/b^2*d*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/8/c^2
/b*e*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/8/c^2/b^2*e*exp(-a/b)*Ei(
1,-arcsinh(c*x)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^5*e + (c^3*d + c*e)*x^3 + c*d*x + (c^2*x^4*e + (c^2*d + e)*x^2 + d)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate((3*c^5*x^6*e + (c^5*d + 6*c^3*e)*x^4 + (2*c^3*d + 3*c*e)*x
^2 + (3*c^3*x^4*e + (c^3*d + c*e)*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (6*c^4*x
^5*e + (2*c^4*d + 7*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c
^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 +
 (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c
^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*
b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{(a + b\operatorname{asinh}(cx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**2,x)
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{{\left(a + b\operatorname{asinh}\left(c\,x\right)\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^2,x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^2, x)
```

3.626 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b^2/c-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A]
time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5773, 5819, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(a + b*ArcS
inh[c*x])/b]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{c\int \frac{x}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx}{b}\\ &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}\\ &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}-\\ &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\right.}{b^2c}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 71, normalized size = 0.84

$$\frac{-\frac{b\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}-\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSinh
[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

Maple [A]
time = 3.17, size = 118, normalized size = 1.39

method	result	size
derivativedivides	$\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$	118
default	$\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^
2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b
*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*
x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)*(c
^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b
^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2
+ 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^2,x)
```

```
[Out] int(1/(a + b*asinh(c*x))^2, x)
```

3.627 $\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 7.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^4*e + a*b*c*d + (a*b*c^3*
d + a*b*c*e)*x^2 + (b^2*c^3*x^4*e + b^2*c*d + (b^2*c^3*d + b^2*c*e)*x^2 + (
b^2*c^2*x^3*e + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^2*x^3*e + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^6*e
 - (c^5*d - 2*c^3*e)*x^4 - (2*c^3*d - c*e)*x^2 + (c^3*x^4*e - (c^3*d - 3*c*
e)*x^2 + c*d)*(c^2*x^2 + 1) - c*d + (2*c^4*x^5*e - (2*c^4*d - 5*c^2*e)*x^3
- (c^2*d - 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8*e^2 + 2*(a*b*c^5*d*e + a
*b*c^3*e^2)*x^6 + a*b*c*d^2 + (a*b*c^5*d^2 + 4*a*b*c^3*d*e + a*b*c*e^2)*x^4
 + 2*(a*b*c^3*d^2 + a*b*c*d*e)*x^2 + (a*b*c^3*x^6*e^2 + 2*a*b*c^3*d*x^4*e +
 a*b*c^3*d^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^8*e^2 + 2*(b^2*c^5*d*e + b^2*c
^3*e^2)*x^6 + b^2*c*d^2 + (b^2*c^5*d^2 + 4*b^2*c^3*d*e + b^2*c*e^2)*x^4 + 2
*(b^2*c^3*d^2 + b^2*c*d*e)*x^2 + (b^2*c^3*x^6*e^2 + 2*b^2*c^3*d*x^4*e + b^2
*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*x^7*e^2 + b^2*c^2*d^2*x + (2*b^2*c
^4*d*e + b^2*c^2*e^2)*x^5 + (b^2*c^4*d^2 + 2*b^2*c^2*d*e)*x^3)*sqrt(c^2*x^2
 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7*e^2 + a*b*c^2*d^2*x +
(2*a*b*c^4*d*e + a*b*c^2*e^2)*x^5 + (a*b*c^4*d^2 + 2*a*b*c^2*d*e)*x^3)*sqrt
(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x^2*e + a^2*d + (b^2*x^2*e + b^2*d)*arcsinh(c*x)^2 + 2*(a*b
*x^2*e + a*b*d)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2} \left(d + ex^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)), x)
```

3.628 $\int \frac{1}{(d+ex^2)^2(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Mathematica [A]
time = 20.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^2\,(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^6*e^2 + a*b*c*d^2 + (2*a*
b*c^3*d*e + a*b*c*e^2)*x^4 + (a*b*c^3*d^2 + 2*a*b*c*d*e)*x^2 + (b^2*c^3*x^6
*e^2 + b^2*c*d^2 + (2*b^2*c^3*d*e + b^2*c*e^2)*x^4 + (b^2*c^3*d^2 + 2*b^2*c
*d*e)*x^2 + (b^2*c^2*x^5*e^2 + 2*b^2*c^2*d*x^3*e + b^2*c^2*d^2*x)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*x^5*e^2 + 2*a*b*c^2*d*x^3
*e + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^6*e - (c^5*d -
6*c^3*e)*x^4 - (2*c^3*d - 3*c*e)*x^2 + (3*c^3*x^4*e - (c^3*d - 5*c*e)*x^2 +
 c*d)*(c^2*x^2 + 1) - c*d + (6*c^4*x^5*e - (2*c^4*d - 11*c^2*e)*x^3 - (c^2*
d - 4*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^10*e^3 + (3*a*b*c^5*d*e^2 + 2*a*b
*c^3*e^3)*x^8 + (3*a*b*c^5*d^2*e + 6*a*b*c^3*d*e^2 + a*b*c*e^3)*x^6 + a*b*c
*d^3 + (a*b*c^5*d^3 + 6*a*b*c^3*d^2*e + 3*a*b*c*d*e^2)*x^4 + (2*a*b*c^3*d^3
 + 3*a*b*c*d^2*e)*x^2 + (a*b*c^3*x^8*e^3 + 3*a*b*c^3*d*x^6*e^2 + 3*a*b*c^3*
d^2*x^4*e + a*b*c^3*d^3*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^10*e^3 + (3*b^2*c^5
*d*e^2 + 2*b^2*c^3*e^3)*x^8 + (3*b^2*c^5*d^2*e + 6*b^2*c^3*d*e^2 + b^2*c*e^
3)*x^6 + b^2*c*d^3 + (b^2*c^5*d^3 + 6*b^2*c^3*d^2*e + 3*b^2*c*d*e^2)*x^4 +
(2*b^2*c^3*d^3 + 3*b^2*c*d^2*e)*x^2 + (b^2*c^3*x^8*e^3 + 3*b^2*c^3*d*x^6*e^
2 + 3*b^2*c^3*d^2*x^4*e + b^2*c^3*d^3*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*x^9*e
^3 + b^2*c^2*d^3*x + (3*b^2*c^4*d*e^2 + b^2*c^2*e^3)*x^7 + 3*(b^2*c^4*d^2*e
 + b^2*c^2*d*e^2)*x^5 + (b^2*c^4*d^3 + 3*b^2*c^2*d^2*e)*x^3)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^9*e^3 + a*b*c^2*d^3*x + (3
*a*b*c^4*d*e^2 + a*b*c^2*e^3)*x^7 + 3*(a*b*c^4*d^2*e + a*b*c^2*d*e^2)*x^5 +
 (a*b*c^4*d^3 + 3*a*b*c^2*d^2*e)*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x^4*e^2 + 2*a^2*d*x^2*e + a^2*d^2 + (b^2*x^4*e^2 + 2*b^2*d*
x^2*e + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*x^4*e^2 + 2*a*b*d*x^2*e + a*b*d^2)
*arcsinh(c*x)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2), x)
```

3.629 $\int (d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}\, dx$

Optimal. Leaf size=672

$$d^2x\sqrt{a+b\sinh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}\,d^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\,\mathrm{s}}}{}\right.}{4c}$$

```
[Out] 1/1600*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)
*5^(1/2)*Pi^(1/2)/c^5-1/1600*e^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)+1/72*d*e*exp(3*a/b)*erf(3^(1/2
)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^3-1/192*e^2*
exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi
^(1/2)/c^5-1/72*d*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*
3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+1/192*e^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)+1/4*d^2*exp(a/b)*erf(
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/8*d*e*exp(a/b)*erf((
a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3+1/32*e^2*exp(a/b)*erf
((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5-1/4*d^2*erfi((a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+1/8*d*e*erfi((a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)-1/32*e^2*erfi((a+
b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5/exp(a/b)+d^2*x*(a+b*arc
sinh(c*x))^(1/2)+2/3*d*e*x^3*(a+b*arcsinh(c*x))^(1/2)+1/5*e^2*x^5*(a+b*arcs
inh(c*x))^(1/2)
```

Rubi [A]

time = 1.20, antiderivative size = 672, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5793, 5772, 5819, 3389, 2211, 2236, 2235, 5777, 3393}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] d^2*x*Sqrt[a + b*ArcSinh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcSinh[c*x]])/5 + (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3) + (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5) + (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(64*c^5) + (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
```

```
*ArcSinh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) + (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) + (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
```

```
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}\,dx = \int \left(d^2\sqrt{a+b\sinh^{-1}(cx)} + 2dex^2\sqrt{a+b\sinh^{-1}(cx)} + e^2x^4\sqrt{a+\right.$$

$$= d^2\int \sqrt{a+b\sinh^{-1}(cx)}\,dx + (2de)\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx + \epsilon$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+}$$

Mathematica [A]
time = 4.17, size = 535, normalized size = 0.80

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -1/7200*(b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcSinh[c*x]] + 8*b*c^4
*d^2*ArcSinh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + b*(4*c^2*d - e)*e*Sqrt[-((a +
b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, a/b + A
rcSinh[c*x]] + 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSi
```

```
nh[c*x])^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcSinh[c*x]))/b] + 25*Sqrt[3]*b*(8*
c^2*d - 3*e)*e*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSinh[c
*x])^2/b^2)]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] + 450*E^((4*a)/b)*(8*a
*c^4*d^2*Sqrt[-((a + b*ArcSinh[c*x])/b)] + 8*b*c^4*d^2*ArcSinh[c*x]*Sqrt[-(
(a + b*ArcSinh[c*x])/b)] + b*e*(-4*c^2*d + e)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt
[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)] - b*
e*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2
/b^2)]*(25*Sqrt[3]*(8*c^2*d - 3*e)*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b] +
 9*Sqrt[5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcSinh[c*x]))/b])))/(c^5*E^(
(5*a)/b)*(a + b*ArcSinh[c*x])^(3/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right)^{2} \sqrt{a+b\,\mathrm{arcsinh}\left(c x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{asinh}\left(cx\right)}\left(d+ex^{2}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,\left(e\,x^2+d\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2, x)
```

3.630 $\int (d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=322

$$dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}\,ee^{a/b}\sqrt{\pi}\,\mathrm{E}}{}$$

```
[Out] 1/144*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^
(1/2)*Pi^(1/2)/c^3-1/144*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b
^(1/2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+1/4*d*exp(a/b)*erf((a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/16*e*exp(a/b)*erf((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3-1/4*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(
1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+1/16*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1
/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)+d*x*(a+b*arcsinh(c*x))^(1/2)+1/3*e*x^3*(
a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5793, 5772, 5819, 3389, 2211, 2236, 2235, 5777, 3393}

$$-\frac{\sqrt{\pi}\sqrt{b}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,ee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] d*x*Sqrt[a + b*ArcSinh[c*x]] + (e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) + (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
```

```
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(d+ex^2\right)\sqrt{a+b\sinh^{-1}(cx)}\,dx &= \int \left(d\sqrt{a+b\sinh^{-1}(cx)}+ex^2\sqrt{a+b\sinh^{-1}(cx)}\right)dx\\
&= d\int \sqrt{a+b\sinh^{-1}(cx)}\,dx+e\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx\\
&= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{1}{2}(bcd)\int \frac{}{\sqrt{1+c}}\\
&= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{(bd)\text{Subst}\left(\int \frac{\text{si}}{\sqrt{a}}\right.}{}\\
&= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{(bd)\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{}\\
&= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{d\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\right.}{}\\
&= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}\left(\right.}{}\\
&= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}\left(\right.}{}\\
&= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}\left(\right.}{}
\end{aligned}$$

Mathematica [A]
time = 1.97, size = 319, normalized size = 0.99

$$\frac{de^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}}\right)}{2c}+\frac{ee^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-9e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)}{72c^3\sqrt{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcSinh[c*x]]*(9*E^
((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]] +
Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] -
9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)
] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(a +
b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2
)])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \sqrt{a +b \,\mathrm{arcsinh}\left(c x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \left(d + e x^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2), x)
```

3.631 $\int \sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$x\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/4*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+x*(a+b*ar
csinh(c*x))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5772, 5819, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\sqrt{b}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\sinh^{-1}(cx)}\,dx &= x\sqrt{a+b\sinh^{-1}(cx)}-\frac{1}{2}(bc)\int \frac{x}{\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}\,dx\\
&= x\sqrt{a+b\sinh^{-1}(cx)}-\frac{b\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{2c}\\
&= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{b\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{4c}-\frac{b\text{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{}\\
&= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{2c}-\frac{\text{Subst}\left(\int\right.}{}\\
&= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\text{er}}{}
\end{aligned}
$$

Mathematica [**A**]

time = 0.09, size = 101, normalized size = 0.99

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/S
qrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((a
+ b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{asinh}{\left(cx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^(1/2), x)
```

3.632 $\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
[Out] Defer[Int][Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Mathematica [A]
time = 5.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
[Out] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}\,(cx)}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(x^2*e + d), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \operatorname{asinh}\left(cx\right)}}{d + ex^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2), x)
```

3.633 $\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Mathematica [A]
time = 10.91, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

```
[Out] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}\,(cx)}}{(e\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(x^2*e + d)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \operatorname{asinh}(cx)}}{(d + ex^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2, x)
```

3.634 $\int (d+ex^2) \left(a+b\sinh^{-1}(cx)\right)^{3/2} dx$

Optimal. Leaf size=427

$$-\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3}-\frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c}$$

```
[Out] d*x*(a+b*arcsinh(c*x))^(3/2)+1/3*e*x^3*(a+b*arcsinh(c*x))^(3/2)+1/288*b^(3/
2)*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/c^3+1/288*b^(3/2)*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/c^3/exp(3*a/b)+3/8*b^(3/2)*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*Pi^(1/2)/c-3/32*b^(3/2)*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c^3+3/8*b^(3/2)*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/c/exp(a/b)-3/32*b^(3/2)*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c^3/exp(a/b)-3/2*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/
c+1/3*b*e*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c^3-1/6*b*e*x^2*(c^2*x
^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

Rubi [A]
time = 0.81, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5793, 5772, 5798, 5774, 3388, 2211, 2236, 2235, 5777, 5812, 5780, 5556}

$$-\frac{3\sqrt{\pi}b^{3/2}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}b^{3/2}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}b^{3/2}de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3bd\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}-\frac{bex^2\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{6c}+\frac{be\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3}+dx\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + (b*e*Sqrt[1 + c
^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*e*x^2*Sqrt[1 + c^2*x^2]*Sqrt
[a + b*ArcSinh[c*x]])/(6*c) + d*x*(a + b*ArcSinh[c*x])^(3/2) + (e*x^3*(a +
b*ArcSinh[c*x])^(3/2))/3 + (3*b^(3/2)*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*A
rcSinh[c*x]]/Sqrt[b]])/(32*c^3) + (b^(3/2)*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c*E^(a/b)) - (3*b^(3/2)*e*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sq
rt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)
/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(d+ex^2\right) \left(a+b\sinh^{-1}(cx)\right)^{3/2} \, dx = \int \left(d\left(a+b\sinh^{-1}(cx)\right)^{3/2}+ex^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}\right) \, dx$$

$$= d\int \left(a+b\sinh^{-1}(cx)\right)^{3/2} \, dx + e\int x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2} \, dx$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bcd)\int$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}-\frac{bex^2\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh}}{6c}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}}}{3c^3}$$

<u>Mathematica</u> [A]
time = 2.91, size = 770, normalized size = 1.80

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]
])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcSinh[c*x]]*(
9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]
] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b
] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x]
)/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(
a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2
/b^2)]) + (Sqrt[b]*d*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x
^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*
x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*c) + (Sqrt[b]*e*(-9*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x
]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b]
 - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]
*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (-2*a + b)*Sqrt[3*
Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(
3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-Cosh[3*ArcSinh[c*x]] + 2*A
rcSinh[c*x]*Sinh[3*ArcSinh[c*x]])))/(288*c^3)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
[Out] int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{asinh}(cx))^{\frac{3}{2}}\left(d+ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**(3/2),x)

[Out] Integral((a + b*asinh(c*x))**(3/2)*(d + e*x**2), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))^{3/2}\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asinh(c*x))^(3/2)*(d + e*x^2),x)

[Out] int((a + b*asinh(c*x))^(3/2)*(d + e*x^2), x)

3.635 $\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=135

$$-\frac{3b\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3b^{3}}{}$$

```
[Out] x*(a+b*arcsinh(c*x))^(3/2)+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2
)/b^(1/2))*Pi^(1/2)/c+3/8*b^(3/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c/exp(a/b)-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

Rubi [A]
time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5772, 5798, 5774, 3388, 2211, 2236, 2235}

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx = x\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{3b\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{1}{4}\left(3b^2\right)\int \frac{}{\sqrt{}}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{(3b)\text{Subst}\Big(\int}{}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{(3b)\text{Subst}\Big(\int}{}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{(3b)\text{Subst}\Big(\int}{}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}}{}$$

Mathematica [A]
time = 0.37, size = 251, normalized size = 1.86

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}}\right)}{2c}+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\sinh^{-1}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\sinh^{-1}(cx)\right)+(2a+3b)\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)+(-2a+3b)\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*Ar
cSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a
+ 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
])))/(8*c)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^(3/2), x)
```

3.636 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
[Out] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(x^2*e + d), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{asinh}\,(cx))^{\frac{3}{2}}}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d),x)
[Out] Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2), x)
```

3.637 $\int \frac{(a+b\sinh^{-1}(cx))^{3/2}}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A]
time = 5.88, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^{\frac{3}{2}}}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(x^2*e + d)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}{(cx)})^{\frac{3}{2}}}{(d + ex^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^{3/2}}{(e\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2, x)
```

3.638 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=608

$$\frac{d^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{dee^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}+\frac{e^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{16\sqrt{b}\,c^5}$$

```
[Out] 1/160*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*
Pi^(1/2)/c^5/b^(1/2)+1/160*e^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)/b^(1/2)+1/12*d*e*exp(3*a/b)*erf(3^(1/2)*
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/b^(1/2)+1/12*d*e*erf
i(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)
/b^(1/2)+1/2*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/
b^(1/2)-1/4*d*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3
/b^(1/2)+1/16*e^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c
^5/b^(1/2)+1/2*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/
b)/b^(1/2)-1/4*d*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp(
a/b)/b^(1/2)+1/16*e^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^5/e
xp(a/b)/b^(1/2)-1/32*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/c^5/b^(1/2)-1/32*e^2*erfi(3^(1/2)*(a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)/b^(1/2)
```

Rubi [A]
time = 0.80, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5793, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$\frac{\sqrt{\pi}e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{5}}e^2e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}-\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{5}}e^2e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}-\frac{\sqrt{\pi}dee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\pi}dee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\pi}d^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}d^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
- (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c
^3) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt
[b]*c^5) + (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (e^2*E^((5*a)/b)*Sqrt[P
i/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (d
^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b)) -
 (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^(a/
```

```
b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*c^5
*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(4*Sqrt[b]*c^3*E^((3*a)/b)) - (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*Erfi
[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
```

```
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \left(\frac{d^2}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2dex^2}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{e^2x^4}{\sqrt{a+b\sinh^{-1}(cx)}}\right)dx$$

$$= d^2\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx+(2de)\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx+e^2\int \frac{}{\sqrt{a+b}}$$

$$= \frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{(2de)\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx,\right.}{c^3}$$

$$= \frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{2bc}+\frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+\right.}{2bc}$$

$$= \frac{d^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{bc}+\frac{d^2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a-}\right.}{bc}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{dee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}$$

Mathematica [A]

time = 0.76, size = 530, normalized size = 0.87

$$\frac{e^{-\frac{5a}{b}}\left(-30(8c^4d^2-4c^2de+e^2)e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+3\sqrt{5}\,e^2\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\sinh^{-1}(cx))}{b}\right)+40\sqrt{3}\,c^2dee^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)-15\sqrt{3}\,e^2e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+240c^4d^2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-120c^2dee^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)+30e^2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-40\sqrt{3}\,c^2dee^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+15\sqrt{3}\,e^2e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(cx))}{b}\right)-3\sqrt{5}\,e^2e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\sinh^{-1}(cx))}{b}\right)\right)}{480c^5\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-30*(8*c^4*d^2 - 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gam
ma[1/2, a/b + ArcSinh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]
*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b)*S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - 15
*Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a
+ b*ArcSinh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])
/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - 120*c^2*d*e*E^((4*a)/b)*Sqrt[-
((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 30*e^2*E^
((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])
/b)] - 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (
3*(a + b*ArcSinh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b + ArcSinh[
c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*e^2*E^((10*a)/b)*S
qrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b])/(480*c^5*E^
((5*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a+b\operatorname{arcsinh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\operatorname{asinh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^2}{\sqrt{a+b\operatorname{asinh}(c\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2), x)
```

3.639 $\int \frac{d+ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=287

$$\frac{de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{ee^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}$$

```
[Out] 1/24*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/c^3/b^(1/2)+1/24*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/c^3/exp(3*a/b)/b^(1/2)+1/2*d*exp(a/b)*erf((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)-1/8*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c^3/b^(1/2)+1/2*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/c/exp(a/b)/b^(1/2)-1/8*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))
*Pi^(1/2)/c^3/exp(a/b)/b^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5793, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$-\frac{\sqrt{\pi}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
(e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3)
+ (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]
)/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*
Sqrt[b]*c*E^(a/b)) - (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8
*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx &= \int \left(\frac{d}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\right)dx \\
&= d\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx+e\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx \\
&= \frac{d\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\text{s}\right.}{c^3} \\
&= \frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{2bc}+\frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+\right.}{2bc} \\
&= \frac{d\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{bc}+\frac{d\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\right.}}{bc} \\
&= \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} \\
&= \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} \\
&= \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{ee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.43, size = 218, normalized size = 0.76

$$
\frac{e^{-\frac{3a}{b}}\left(-3(4c^2d-e)\,e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\,e\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+3(4c^2d-e)\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-\sqrt{3}\,ee^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\right)}{24c^3\sqrt{a+b\sinh^{-1}(cx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-3*(4*c^2*d - e)*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + Arc
Sinh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c*x]))/b] + 3*(4*c^2*d - e)*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x
```

```
])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*e*E^((6*a)/b)*Sqrt[a
/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^((3*a)
/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2+d}{\sqrt{a+b\,\mathrm{arcsinh}\left(c x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\sqrt{a+b\,\mathrm{asinh}\left(c x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral((d + e*x**2)/sqrt(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^(1/2),x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^(1/2), x)
```

3.640 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)+1/2*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5774, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\text{s}\right.}{2bc}$$

$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{bc}$$

$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Mathematica [A]
time = 0.08, size = 101, normalized size = 1.15

$$\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)\right)}{2c\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) + S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*c*
E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arcsinh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(1/sqrt(a + b*asinh(c*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(c*x))^(1/2), x)
```

3.641 $\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Defer[Int][1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)\sqrt{a+b\,\mathrm{arcsinh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)`

[Out] `int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)`

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((x^2*e + d)*sqrt(b*arcsinh(c*x) + a)), x)`

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")`

[Out] `Exception raised: TypeError >>  Error detected within library code:   integrate: implementation incomplete (constant residues)`

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \left(d + e x^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)`

[Out] `Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)), x)`

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")`

[Out] `integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)`

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)), x)
```

3.642 $\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}}, x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Defer[Int][1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}} dx = \int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}} dx$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^2 \sqrt{a+b\,\mathrm{arcsinh}\,(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}(cx)}\,(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2), x)
```

3.643 $\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{ee^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+\cdots}}{\cdots}\right)}{4b^{3/2}\cdots}$$

```
[Out] -d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+1/4*e*
exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3+d*erfi(
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*e*erfi((a
+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1/4*e*exp(3*a
/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/
c^3+1/4*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b
^(3/2)/c^3/exp(3*a/b)-2*d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-2*
e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5793, 5773, 5819, 3389, 2211, 2236, 2235, 5778}

$$\frac{\sqrt{\pi}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,ee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2d\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2ex^2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (2*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (e*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(
Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) + (d*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b)) - (e*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b)) + (e*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3*E^((3*a
)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
```

```
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx &= \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}+\frac{ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\right)dx\\
&= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx+e\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx\\
&= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2cd)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+}}}{b}\\
&= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2d)\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx\right.}{bc}\\
&= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{d\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x\right.}{bc}\\
&= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(2d)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\right.}{b^2c}\\
&= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh}}{\sqrt{b}}\right.}{b^{3/2}c}
\end{aligned}
$$

Mathematica [A]
time = 0.94, size = 303, normalized size = 0.87

$$
\frac{e^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left((4c^2d-e)\,e^{\frac{4a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\,ee^{3\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(cx))}{b}\right)+(4c^2d-e)\,e^{\frac{2a}{b}+3\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)+e^{\frac{3a}{b}}\left(-\left(\left(1+e^{2\sinh^{-1}(cx)}\right)\left(4c^2de^{2\sinh^{-1}(cx)}+e\left(-1+e^{2\sinh^{-1}(cx)}\right)^2\right)\right)+\sqrt{3}\,ee^{3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(cx))}{b}\right)\right)\right)}{4bc^3\sqrt{a+b\sinh^{-1}(cx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] ((4*c^2*d - e)*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*e*E^(3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSi
```

```
nh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + (4*c^2*d - e)*E^((2*
a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*
ArcSinh[c*x])/b)] + E^((3*a)/b)*(-((1 + E^(2*ArcSinh[c*x]))*(4*c^2*d*E^(2*A
rcSinh[c*x]) + e*(-1 + E^(2*ArcSinh[c*x]))^2)) + Sqrt[3]*e*E^(3*(a/b + ArcS
inh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])
)/(4*b*c^3*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2+d}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{(a+b\,\mathrm{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

3.644 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a+
b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c^2*x^2+1)^(1
/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5773, 5819, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{2\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c} + \frac{2S}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}}{}$$

Mathematica [A]
time = 0.17, size = 137, normalized size = 1.18

$$\frac{e^{-\frac{a+b\sinh^{-1}(cx)}{b}}\left(-e^{a/b}\left(1+e^{2\sinh^{-1}(cx)}\right)+e^{\frac{2a}{b}+\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+e^{\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)\right)}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a + b*
ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\text{arcsinh}\,(cx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asinh}(cx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^(3/2),x)
[Out] int(1/(a + b*asinh(c*x))^(3/2), x)
```

3.645 $\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = \int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{\frac{3}{2}} \left(d + e x^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**(3/2)*(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))^{3/2}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)), x)
```

3.646 $\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^{3/2}} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right)^{3/2}} dx = \int \frac{1}{(d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right)^{3/2}} dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^2 \left(a+b\,\mathrm{arcsinh}\,(cx)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}\,{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2), x)
```

3.647 $\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$$

Mathematica [A]
time = 3.08, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\,(cx)\right)\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{asinh}(cx))\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\,\text{asinh}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^(1/2), x)
```

3.648 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + b*integrate(log(c*x + sqrt(c^2*x^2
+ 1))/sqrt(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{\sqrt{d + ex^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asinh}(c\,x)}{\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(1/2), x)
```

3.649 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

[Out] -b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d/e^(1/2)+x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {197, 5792, 12, 455, 65, 223, 212}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]

[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+ex^2}} - (bc)\int \frac{x}{d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}\,dx$$
$$= \frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d}$$
$$= \frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{(bc)\text{Subst}\left(\int \frac{1}{\sqrt{1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2d}$$
$$= \frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1+c^2x^2}\right)}{cd}$$
$$= \frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{b\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{cd}$$
$$= \frac{x(a+b\sinh^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.08, size = 75, normalized size = 1.07

$$\frac{x\left(-bcx\sqrt{1+\frac{ex^2}{d}}\,F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right)+2(a+b\sinh^{-1}(cx))\right)}{2d\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
[Out] (x*(-(b*c*x*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x
^2)/d)]) + 2*(a + b*ArcSinh[c*x])))/(2*d*Sqrt[d + e*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(60) = 120$.
time = 0.36, size = 329, normalized size = 4.70

$$\frac{(bx^2\cosh(1)+bx^2\sinh(1)+bd)\sqrt{\cosh(1)+\sinh(1)}\log\left(c^4d^2+(8c^4x^4+8c^2x^2+1)\cosh(1)^2+(8c^4x^4+8c^2x^2+1)\sinh(1)^2-4(c^3d+(2c^3x^2+c)\cosh(1)+(2c^3x^2+c)\sinh(1))\sqrt{c^2x^2+1}\sqrt{x^2\cosh(1)+x^2\sinh(1)+d}\sqrt{\cosh(1)+\sinh(1)}+2(4c^4dx^2+3c^2d)\cosh(1)+2(4c^4dx^2+3c^2d+(8c^4x^4+8c^2x^2+1)\cosh(1))\sinh(1)\right)+4(bx\cosh(1)+bx\sinh(1))\sqrt{x^2\cosh(1)+x^2\sinh(1)+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+4(ax\cosh(1)+ax\sinh(1))\sqrt{x^2\cosh(1)+x^2\sinh(1)+d}}{4\left(dx^2\cosh(1)^2+dx^2\sinh(1)^2+d^2\cosh(1)+(2dx^2\cosh(1)+d^2)\sinh(1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^2*cosh(1) + b*x^2*sinh(1) + b*d)*sqrt(cosh(1) + sinh(1))*log(c^4*
d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*s
inh(1)^2 - 4*(c^3*d + (2*c^3*x^2 + c)*cosh(1) + (2*c^3*x^2 + c)*sinh(1))*sq
rt(c^2*x^2 + 1)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(cosh(1) + sinh(1))
 + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^
4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 4*(b*x*cosh(1) + b*x*sinh(1))*sqrt(x
^2*cosh(1) + x^2*sinh(1) + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 4*(a*x*cosh(1)
 + a*x*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(d*x^2*cosh(1)^2 + d*x
^2*sinh(1)^2 + d^2*cosh(1) + (2*d*x^2*cosh(1) + d^2)*sinh(1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{asinh}(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(3/2), x)
```

3.650 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{bc\sqrt{1+c^2x^2}}{3d(c^2d-e)\sqrt{d+ex^2}}+\frac{x(a+b\sinh^{-1}(cx))}{3d(d+ex^2)^{3/2}}+\frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(3/2)-2/3*b*arctanh(e^(1/2)*(c^2*x^2+1
)^(1/2)/c/(e*x^2+d)^(1/2))/d^2/e^(1/2)+2/3*x*(a+b*arcsinh(c*x))/d^2/(e*x^2+
d)^(1/2)-1/3*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {198, 197, 5792, 12, 585, 79, 65, 223, 212}

$$\frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\sinh^{-1}(cx))}{3d(d+ex^2)^{3/2}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}-\frac{bc\sqrt{c^2x^2+1}}{3d(c^2d-e)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*Sqrt[1 + c^2*x^2])/(d*(c^2*d - e)*Sqrt[d + e*x^2]) + (x*(a + b*Ar
cSinh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sq
rt[d + e*x^2]) - (2*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2
])])/(3*d^2*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x(a+b\sinh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - (bc)\int \frac{x(3d+2ex^2)}{3d^2\sqrt{1+c^2x^2}\,(d+ex^2)^{3/2}}$$

$$= \frac{x(a+b\sinh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x(3d+2ex^2)}{\sqrt{1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{3d^2}$$

$$= \frac{x(a+b\sinh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\text{Subst}\left(\int \frac{3d+2ex}{\sqrt{1+c^2x}\,(d+ex)^{3/2}}\,dx\right.}{6d^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\,(c^2d-e)\sqrt{d+ex^2}} + \frac{x(a+b\sinh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\text{Su}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\,(c^2d-e)\sqrt{d+ex^2}} + \frac{x(a+b\sinh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(2b)\text{Su}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\,(c^2d-e)\sqrt{d+ex^2}} + \frac{x(a+b\sinh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(2b)\text{Su}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\,(c^2d-e)\sqrt{d+ex^2}} + \frac{x(a+b\sinh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\sinh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{2b\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.22, size = 139, normalized size = 0.95

$$\frac{-\frac{bcd\sqrt{1+c^2x^2}\,(d+ex^2)}{c^2d-e} + ax(3d+2ex^2) - bcx^2(d+ex^2)\sqrt{1+\frac{ex^2}{d}}\,F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right) + bx(3d+2ex^2)\sinh^{-1}(cx)}{3d^2\,(d+ex^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (-((b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^2))/(c^2*d - e)) + a*x*(3*d + 2*e*x^2)
 - b*c*x^2*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x
^2), -((e*x^2)/d)] + b*x*(3*d + 2*e*x^2)*ArcSinh[c*x])/(3*d^2*(d + e*x^2)^(
3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(x^2*e + d)*d^2) + x/((x^2*e + d)^(3/2)*d)) + b*integrate(l
og(c*x + sqrt(c^2*x^2 + 1))/(x^2*e + d)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(125) = 250$.
time = 0.43, size = 970, normalized size = 6.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((b*x^4*cosh(1)^3 + b*x^4*sinh(1)^3 - b*c^2*d^3 - (b*c^2*d*x^4 - 2*b*d*
x^2)*cosh(1)^2 - (b*c^2*d*x^4 - 3*b*x^4*cosh(1) - 2*b*d*x^2)*sinh(1)^2 - (2
*b*c^2*d^2*x^2 - b*d^2)*cosh(1) - (2*b*c^2*d^2*x^2 - 3*b*x^4*cosh(1)^2 - b*
d^2 + 2*(b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(1))
*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x
^2 + 1)*sinh(1)^2 - 4*(c^3*d + (2*c^3*x^2 + c)*cosh(1) + (2*c^3*x^2 + c)*si
nh(1))*sqrt(c^2*x^2 + 1)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(cosh(1) +
 sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d +
(8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) - 2*(3*b*c^2*d^2*x*cosh(1) -
2*b*x^3*cosh(1)^3 - 2*b*x^3*sinh(1)^3 + (2*b*c^2*d*x^3 - 3*b*d*x)*cosh(1)^2
 + (2*b*c^2*d*x^3 - 6*b*x^3*cosh(1) - 3*b*d*x)*sinh(1)^2 + (3*b*c^2*d^2*x -
 6*b*x^3*cosh(1)^2 + 2*(2*b*c^2*d*x^3 - 3*b*d*x)*cosh(1))*sinh(1))*sqrt(x^2
*cosh(1) + x^2*sinh(1) + d)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(3*a*c^2*d^2*x
*cosh(1) - 2*a*x^3*cosh(1)^3 - 2*a*x^3*sinh(1)^3 + (2*a*c^2*d*x^3 - 3*a*d*x
```

```
)*cosh(1)^2 + (2*a*c^2*d*x^3 - 6*a*x^3*cosh(1) - 3*a*d*x)*sinh(1)^2 + (3*a*
c^2*d^2*x - 6*a*x^3*cosh(1)^2 + 2*(2*a*c^2*d*x^3 - 3*a*d*x)*cosh(1))*sinh(1
) - (b*c*d*x^2*cosh(1)^2 + b*c*d*x^2*sinh(1)^2 + b*c*d^2*cosh(1) + (2*b*c*d
*x^2*cosh(1) + b*c*d^2)*sinh(1))*sqrt(c^2*x^2 + 1))*sqrt(x^2*cosh(1) + x^2*
sinh(1) + d))/(d^2*x^4*cosh(1)^4 + d^2*x^4*sinh(1)^4 - c^2*d^5*cosh(1) - (c
^2*d^3*x^4 - 2*d^3*x^2)*cosh(1)^3 - (c^2*d^3*x^4 - 4*d^2*x^4*cosh(1) - 2*d^
3*x^2)*sinh(1)^3 - (2*c^2*d^4*x^2 - d^4)*cosh(1)^2 - (2*c^2*d^4*x^2 - 6*d^2
*x^4*cosh(1)^2 - d^4 + 3*(c^2*d^3*x^4 - 2*d^3*x^2)*cosh(1))*sinh(1)^2 + (4*
d^2*x^4*cosh(1)^3 - c^2*d^5 - 3*(c^2*d^3*x^4 - 2*d^3*x^2)*cosh(1)^2 - 2*(2*
c^2*d^4*x^2 - d^4)*cosh(1))*sinh(1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{(d + ex^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{(e\,x^2 + d)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(5/2), x)
```

3.651 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}-\frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x(a}{15}$$

```
[Out] 1/5*x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arcsinh(c*x))/d^2/(e
*x^2+d)^(3/2)-8/15*b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d
^3/e^(1/2)-1/15*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(3/2)+8/15*x*(a
+b*arcsinh(c*x))/d^3/(e*x^2+d)^(1/2)-2/15*b*c*(3*c^2*d-2*e)*(c^2*x^2+1)^(1/
2)/d^2/(c^2*d-e)^2/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {198, 197, 5792, 12, 6847, 963, 79, 65, 223, 212}

$$\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}-\frac{8b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}-\frac{2bc\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}-\frac{bc\sqrt{c^2x^2+1}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] -1/15*(b*c*Sqrt[1 + c^2*x^2])/(d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (2*b*c*(3
*c^2*d - 2*e)*Sqrt[1 + c^2*x^2])/(15*d^2*(c^2*d - e)^2*Sqrt[d + e*x^2]) + (
x*(a + b*ArcSinh[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcSinh[c*x])
)/(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcSinh[c*x]))/(15*d^3*Sqrt[d +
e*x^2]) - (8*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1
5*d^3*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\,(d+ex^2)^{3/2}}+\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-(bc)\int \frac{x(1}{15d^3}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\,(d+ex^2)^{3/2}}+\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-\frac{(bc)\int \frac{x(15}{\sqrt{1-}}}{}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\,(d+ex^2)^{3/2}}+\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-\frac{(bc)\text{Subst}\Big(}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\,(d+ex^2)^{3/2}}+\frac{8x(a}{15}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}}-\frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}}+$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}}-\frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}}+$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}}-\frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}}+$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}}-\frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.27, size = 191, normalized size = 0.84

$$\frac{ax(15d^2+20dex^2+8e^2x^4)-\frac{bcd\sqrt{1+c^2x^2}\,(d+ex^2)(-e(5d+4ex^2)+c^2d(7d+6ex^2))}{(-c^2d+e)^2}-4bcx^2(d+ex^2)^2\sqrt{1+\frac{ex^2}{d}}\,F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right)+bx(15d^2+20dex^2+8e^2x^4)\sinh^{-1}(cx)}{15d^3\,(d+ex^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) - (b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^
2)*(-(e*(5*d + 4*e*x^2)) + c^2*d*(7*d + 6*e*x^2)))/(-(c^2*d) + e)^2 - 4*b*c
*x^2*(d + e*x^2)^2*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2),
 -((e*x^2)/d)] + b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcSinh[c*x])/(15*d^
3*(d + e*x^2)^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(e\,x^2+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*a*(8*x/(sqrt(x^2*e + d)*d^3) + 4*x/((x^2*e + d)^(3/2)*d^2) + 3*x/((x^2
*e + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(x^2*e + d)^(7
/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2521 vs. $2(200) = 400$.
time = 0.48, size = 2521, normalized size = 11.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")

[Out] 1/15*(2*(b*x^6*cosh(1)^5 + b*x^6*sinh(1)^5 + b*c^4*d^5 - (2*b*c^2*d*x^6 - 3*b*d*x^4)*cosh(1)^4 - (2*b*c^2*d*x^6 - 5*b*x^6*cosh(1) - 3*b*d*x^4)*sinh(1)^4 + (b*c^4*d^2*x^6 - 6*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*cosh(1)^3 + (b*c^4*d^2*x^6 - 6*b*c^2*d^2*x^4 + 10*b*x^6*cosh(1)^2 + 3*b*d^2*x^2 - 4*(2*b*c^2*d*x^6 - 3*b*d*x^4)*cosh(1))*sinh(1)^3 + (3*b*c^4*d^3*x^4 - 6*b*c^2*d^3*x^2 + b*d^3)*cosh(1)^2 + (3*b*c^4*d^3*x^4 + 10*b*x^6*cosh(1)^3 - 6*b*c^2*d^3*x^2 + b*d^3 - 6*(2*b*c^2*d*x^6 - 3*b*d*x^4)*cosh(1)^2 + 3*(b*c^4*d^2*x^6 - 6*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*cosh(1))*sinh(1)^2 + (3*b*c^4*d^4*x^2 - 2*b*c^2*d^4)*cosh(1) + (3*b*c^4*d^4*x^2 + 5*b*x^6*cosh(1)^4 - 2*b*c^2*d^4 - 4*(2*b*c^2*d*x^6 - 3*b*d*x^4)*cosh(1)^3 + 3*(b*c^4*d^2*x^6 - 6*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*cosh(1)^2 + 2*(3*b*c^4*d^3*x^4 - 6*b*c^2*d^3*x^2 + b*d^3)*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^3*d + (2*c^3*x^2 + c)*cosh(1) + (2*c^3*x^2 + c)*sinh(1))*sqrt(c^2*x^2 + 1)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + (15*b*c^4*d^4*x*cosh(1) + 8*b*x^5*cosh(1)^5 + 8*b*x^5*sinh(1)^5 - 4*(4*b*c^2*d*x^5 - 5*b*d*x^3)*cosh(1)^4 - 4*(4*b*c^2*d*x^5 - 10*b*x^5*cosh(1) - 5*b*d*x^3)*sinh(1)^4 + (8*b*c^4*d^2*x^5 - 40*b*c^2*d^2*x^3 + 15*b*d^2*x)*cosh(1)^3 + (8*b*c^4*d^2*x^5 - 40*b*c^2*d^2*x^3 + 80*b*x^5*cosh(1)^2 + 15*b*d^2*x - 16*(4*b*c^2*d*x^5 - 5*b*d*x^3)*cosh(1))*sinh(1)^3 + 10*(2*b*c^4*d^3*x^3 - 3*b*c^2*d^3*x)*cosh(1)^2 + (20*b*c^4*d^3*x^3 + 80*b*x^5*cosh(1)^3 - 30*b*c^2*d^3*x - 24*(4*b*c^2*d*x^5 - 5*b*d*x^3)*cosh(1)^2 + 3*(8*b*c^4*d^2*x^5 - 40*b*c^2*d^2*x^3 + 15*b*d^2*x)*cosh(1))*sinh(1)^2 + (15*b*c^4*d^4*x + 40*b*x^5*cosh(1)^4 - 16*(4*b*c^2*d*x^5 - 5*b*d*x^3)*cosh(1)^3 + 3*(8*b*c^4*d^2*x^5 - 40*b*c^2*d^2*x^3 + 15*b*d^2*x)*cosh(1)^2 + 20*(2*b*c^4*d^3*x^3 - 3*b*c^2*d^3*x)*cosh(1))*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (15*a*c^4*d^4*x*cosh(1) + 8*a*x^5*cosh(1)^5 + 8*a*x^5*sinh(1)^5 - 4*(4*a*c^2*d*x^5 - 5*a*d*x^3)*cosh(1)^4 - 4*(4*a*c^2*d*x^5 - 10*a*x^5*cosh(1) - 5*a*d*x^3)*sinh(1)^4 + (8*a*c^4*d^2*x^5 - 40*a*c^2*d^2*x^3 + 15*a*d^2*x)*cosh(1)^3 + (8*a*c^4*d^2*x^5 - 40*a*c^2*d^2*x^3 + 80*a*x^5*cosh(1)^2 + 15*a*d^2*x - 16*(4*a*c^2*d*x^5 - 5*a*d*x^3)*cosh(1))*sinh(1)^3 + 10*(2*a*c^4*d^3*x^3 - 3*a*c^2*d^3*x)*cosh(1)^2 + (20*a*c^4*d^3*x^3 + 80*a*x^5*cosh(1)^3 - 30*a*c^2*d^3*x - 24*(4*a*c^2*d*x^5 - 5*a*d*x^3)*cosh(1)^2 + 3*(8*a*c^4*d^2*x^5 - 40*a*c^2*d^2*x^3 + 15*a*d^2*x)*cosh(1))*sinh(1)^2 + (15*a*c^4*d^4*x + 40*a*x^5*cosh(1)^4 - 16*(4*a*c^2*d*x^5 - 5*a*d*x^3)*cosh(1)^3 + 3*(8*a*c^4*d^2*x^5 - 40*a*c^2*d^2*x^3 + 15*a*d^2*x)*cosh(1)^2 + 20*(2*a*c^4*d^3*x^3 - 3*a*c^2*d^3*x)*cosh(1))*sinh(1) + (4*b*c*d*x^4*cosh(1)^4 + 4*b*c*d*x^4*sinh(1)^4 - 7*b*c^3*d^4*cosh(1) - 3*(2*b*c^3*d^2*x^4 - 3*b*c*d^2*x^2)*cosh(1)^3 - (6*b*c^3*d^2*x^4 - 16*b*c*d*x^4*cosh(1) - 9*b*c*d^2*x^2)*sinh(1)^3 - (13*b*c^3*d^3*x^2 - 5*b*c*d^3)*cosh(1)^2 - (13*b*c^3*d^3*x^2 - 24*b*c*d*x^4*cosh(1)^2 - 5*b*c*d^3 + 9*(2*b*c^3*d^2*x^4 - 3*b*c*d^2*x^2)*cosh(1))*sinh(1)^2 + (16*b*c*d*x^4*cosh(1)^3 - 7*b*c^3*d^4 - 9*(2*b*c^

```
3*d^2*x^4 - 3*b*c*d^2*x^2)*cosh(1)^2 - 2*(13*b*c^3*d^3*x^2 - 5*b*c*d^3)*cos
h(1))*sinh(1))*sqrt(c^2*x^2 + 1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(d^3
*x^6*cosh(1)^6 + d^3*x^6*sinh(1)^6 + c^4*d^8*cosh(1) - (2*c^2*d^4*x^6 - 3*d
^4*x^4)*cosh(1)^5 - (2*c^2*d^4*x^6 - 6*d^3*x^6*cosh(1) - 3*d^4*x^4)*sinh(1)
^5 + (c^4*d^5*x^6 - 6*c^2*d^5*x^4 + 3*d^5*x^2)*cosh(1)^4 + (c^4*d^5*x^6 - 6
*c^2*d^5*x^4 + 15*d^3*x^6*cosh(1)^2 + 3*d^5*x^2 - 5*(2*c^2*d^4*x^6 - 3*d^4*
x^4)*cosh(1))*sinh(1)^4 + (3*c^4*d^6*x^4 - 6*c^2*d^6*x^2 + d^6)*cosh(1)^3 +
 (3*c^4*d^6*x^4 + 20*d^3*x^6*cosh(1)^3 - 6*c^2*d^6*x^2 + d^6 - 10*(2*c^2*d^
4*x^6 - 3*d^4*x^4)*cosh(1)^2 + 4*(c^4*d^5*x^6 - 6*c^2*d^5*x^4 + 3*d^5*x^2)*
cosh(1))*sinh(1)^3 + (3*c^4*d^7*x^2 - 2*c^2*d^7)*cosh(1)^2 + (3*c^4*d^7*x^2
 + 15*d^3*x^6*cosh(1)^4 - 2*c^2*d^7 - 10*(2*c^2*d^4*x^6 - 3*d^4*x^4)*cosh(1
)^3 + 6*(c^4*d^5*x^6 - 6*c^2*d^5*x^4 + 3*d^5*x^2)*cosh(1)^2 + 3*(3*c^4*d^6*
x^4 - 6*c^2*d^6*x^2 + d^6)*cosh(1))*sinh(1)^2 + (6*d^3*x^6*cosh(1)^5 + c^4*
d^8 - 5*(2*c^2*d^4*x^6 - 3*d^4*x^4)*cosh(1)^4 + 4*(c^4*d^5*x^6 - 6*c^2*d^5*
x^4 + 3*d^5*x^2)*cosh(1)^3 + 3*(3*c^4*d^6*x^4 - 6*c^2*d^6*x^2 + d^6)*cosh(1
)^2 + 2*(3*c^4*d^7*x^2 - 2*c^2*d^7)*cosh(1))*sinh(1))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(e\,x^2+d\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(7/2),x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(7/2), x)
```

3.652 $\int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A]
time = 8.31, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2} \sqrt{e\,x^{2}+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(x^2*e + d), x
)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(cx))^2 \sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2*sqrt(d + e*x**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + b\,\mathrm{asinh}(c\,x))^2\,\sqrt{e\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2), x)
```

3.653 $\int \frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 7.63, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + integrate(b^2*log(c*x + sqrt(c^2*
x^2 + 1))^2/sqrt(x^2*e + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(x^2*e
 + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(x^2*e + d), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2), x)
```

3.654 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [A]
time = 2.13, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(x^2*e + d)/(x
^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2}{(e\,x^2 + d)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2), x)
```

3.655 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

Mathematica [A]
time = 4.40, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
[Out] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(2*x/(sqrt(x^2*e + d)*d^2) + x/((x^2*e + d)^(3/2)*d)) + integrate(b
^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(x^2*e + d)^(5/2) + 2*a*b*log(c*x + sqrt(
c^2*x^2 + 1))/(x^2*e + d)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(x^2*e + d)/(x
^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{(d + ex^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{(e\,x^2+d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2), x)
```

3.656 $\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^2+d}}{a+b\,\mathrm{arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
[Out] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(x^2*e + d)/(b*arcsinh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(x^2*e + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{a + b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)), x)
```

3.657 $\int \frac{1}{\sqrt{d+ex^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2*e + d)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2*e + d)/(a*x^2*e + a*d + (b*x^2*e + b*d)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asinh}(cx))\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(1/((a + b*asinh(c*x))*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)), x)
```

3.658 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^{3/2}\,(a+b\sinh^{-1}(cx))},x\right)$$

`[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)`

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]`

`[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]`

Rubi steps

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx = \int \frac{1}{(d+ex^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]`

`[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]`

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\operatorname{arcsinh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*
b*d*x^2*e + b*d^2)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)), x)
```

3.659 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 2.52, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((x^2*e + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(x^2*e + d)/(a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3
 + (b*x^6*e^3 + 3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)), x)
```

3.660 $\int \frac{\sqrt{d+ex^2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 2.94, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^2+d}}{\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*sqrt(x^2*e + d) + (c^3*x^3 + c*x)*sqrt(x^2*e + d))/(a
*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*
x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c
^3*x^4*e + c^3*d*x^2 - c*d)*(c^2*x^2 + 1)*sqrt(x^2*e + d) + (4*c^4*x^5*e +
2*(c^4*d + 2*c^2*e)*x^3 + (c^2*d + e)*x)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d)
+ (2*c^5*x^6*e + (c^5*d + 4*c^3*e)*x^4 + 2*(c^3*d + c*e)*x^2 + c*d)*sqrt(x^
2*e + d))/(a*b*c^5*x^6*e + (a*b*c^5*d + 2*a*b*c^3*e)*x^4 + a*b*c*d + (2*a*b
*c^3*d + a*b*c*e)*x^2 + (a*b*c^3*x^4*e + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^
2*c^5*x^6*e + (b^2*c^5*d + 2*b^2*c^3*e)*x^4 + b^2*c*d + (2*b^2*c^3*d + b^2*
c*e)*x^2 + (b^2*c^3*x^4*e + b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*x^5*e
 + b^2*c^2*d*x + (b^2*c^4*d + b^2*c^2*e)*x^3)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5*e + a*b*c^2*d*x + (a*b*c^4*d + a*b*c^2*
e)*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{\left(a + b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2, x)
```

3.661 $\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 4.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d)*a
*b*c^2*x + (sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d)*b^2*c^2*x + (b^2*c^3*x^2 + b^
2*c)*sqrt(x^2*e + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*
sqrt(x^2*e + d)) - integrate(-(c^5*d*x^4 + 2*c^3*d*x^2 + (c^2*x^2 + 1)*((c^
3*d - 2*c*e)*x^2 - c*d) + c*d + sqrt(c^2*x^2 + 1)*(2*(c^4*d - c^2*e)*x^3 +
(c^2*d - e)*x))/((a*b*c^3*x^4*e + a*b*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt(x^2*e +
 d) + 2*(a*b*c^4*x^5*e + a*b*c^2*d*x + (a*b*c^4*d + a*b*c^2*e)*x^3)*sqrt(c^
2*x^2 + 1)*sqrt(x^2*e + d) + ((b^2*c^3*x^4*e + b^2*c^3*d*x^2)*(c^2*x^2 + 1)
*sqrt(x^2*e + d) + 2*(b^2*c^4*x^5*e + b^2*c^2*d*x + (b^2*c^4*d + b^2*c^2*e)
*x^3)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (b^2*c^5*x^6*e + (b^2*c^5*d + 2*b
^2*c^3*e)*x^4 + b^2*c*d + (2*b^2*c^3*d + b^2*c*e)*x^2)*sqrt(x^2*e + d))*log
(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6*e + (a*b*c^5*d + 2*a*b*c^3*e)*x^4
+ a*b*c*d + (2*a*b*c^3*d + a*b*c*e)*x^2)*sqrt(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a^2*x^2*e + a^2*d + (b^2*x^2*e + b^2*d)*arcsinh(c
*x)^2 + 2*(a*b*x^2*e + a*b*d)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2} \sqrt{d + e x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)), x)
```

3.662 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 12.50, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*x^3*e + a*b*c^2*d*x)*sqrt(
c^2*x^2 + 1)*sqrt(x^2*e + d) + ((b^2*c^2*x^3*e + b^2*c^2*d*x)*sqrt(c^2*x^2
+ 1)*sqrt(x^2*e + d) + (b^2*c^3*x^4*e + b^2*c*d + (b^2*c^3*d + b^2*c*e)*x^2
)*sqrt(x^2*e + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^4*e + a*b*c*d
+ (a*b*c^3*d + a*b*c*e)*x^2)*sqrt(x^2*e + d)) - integrate((2*c^5*x^6*e - (c
^5*d - 4*c^3*e)*x^4 - 2*(c^3*d - c*e)*x^2 + (2*c^3*x^4*e - (c^3*d - 4*c*e)*
x^2 + c*d)*(c^2*x^2 + 1) - c*d + (4*c^4*x^5*e - 2*(c^4*d - 4*c^2*e)*x^3 - (
c^2*d - 3*e)*x)*sqrt(c^2*x^2 + 1))/((a*b*c^3*x^6*e^2 + 2*a*b*c^3*d*x^4*e +
a*b*c^3*d^2*x^2)*(c^2*x^2 + 1)*sqrt(x^2*e + d) + 2*(a*b*c^4*x^7*e^2 + a*b*c
^2*d^2*x + (2*a*b*c^4*d*e + a*b*c^2*e^2)*x^5 + (a*b*c^4*d^2 + 2*a*b*c^2*d*e
)*x^3)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + ((b^2*c^3*x^6*e^2 + 2*b^2*c^3*d*
x^4*e + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1)*sqrt(x^2*e + d) + 2*(b^2*c^4*x^7*e^2
 + b^2*c^2*d^2*x + (2*b^2*c^4*d*e + b^2*c^2*e^2)*x^5 + (b^2*c^4*d^2 + 2*b^2
*c^2*d*e)*x^3)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (b^2*c^5*x^8*e^2 + 2*(b^
2*c^5*d*e + b^2*c^3*e^2)*x^6 + b^2*c*d^2 + (b^2*c^5*d^2 + 4*b^2*c^3*d*e + b
^2*c*e^2)*x^4 + 2*(b^2*c^3*d^2 + b^2*c*d*e)*x^2)*sqrt(x^2*e + d))*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^8*e^2 + 2*(a*b*c^5*d*e + a*b*c^3*e^2)*x^6
+ a*b*c*d^2 + (a*b*c^5*d^2 + 4*a*b*c^3*d*e + a*b*c*e^2)*x^4 + 2*(a*b*c^3*d^
2 + a*b*c*d*e)*x^2)*sqrt(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a^2*x^4*e^2 + 2*a^2*d*x^2*e + a^2*d^2 + (b^2*x^4*
e^2 + 2*b^2*d*x^2*e + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*x^4*e^2 + 2*a*b*d*x^
2*e + a*b*d^2)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}\left(d+ex^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)), x)
```

3.663 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{1}{(d+ex^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Mathematica [A]
time = 20.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*x^5*e^2 + 2*a*b*c^2*d*x^3*
e + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + ((b^2*c^2*x^5*e^2 +
2*b^2*c^2*d*x^3*e + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (b^2
*c^3*x^6*e^2 + b^2*c*d^2 + (2*b^2*c^3*d*e + b^2*c*e^2)*x^4 + (b^2*c^3*d^2 +
 2*b^2*c*d*e)*x^2)*sqrt(x^2*e + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3
*x^6*e^2 + a*b*c*d^2 + (2*a*b*c^3*d*e + a*b*c*e^2)*x^4 + (a*b*c^3*d^2 + 2*a
*b*c*d*e)*x^2)*sqrt(x^2*e + d)) - integrate((4*c^5*x^6*e - (c^5*d - 8*c^3*e
)*x^4 - 2*(c^3*d - 2*c*e)*x^2 + (4*c^3*x^4*e - (c^3*d - 6*c*e)*x^2 + c*d)*(
c^2*x^2 + 1) - c*d + (8*c^4*x^5*e - 2*(c^4*d - 7*c^2*e)*x^3 - (c^2*d - 5*e)
*x)*sqrt(c^2*x^2 + 1))/((a*b*c^3*x^8*e^3 + 3*a*b*c^3*d*x^6*e^2 + 3*a*b*c^3*
d^2*x^4*e + a*b*c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(x^2*e + d) + 2*(a*b*c^4*x^9
*e^3 + a*b*c^2*d^3*x + (3*a*b*c^4*d*e^2 + a*b*c^2*e^3)*x^7 + 3*(a*b*c^4*d^2
*e + a*b*c^2*d*e^2)*x^5 + (a*b*c^4*d^3 + 3*a*b*c^2*d^2*e)*x^3)*sqrt(c^2*x^2
 + 1)*sqrt(x^2*e + d) + ((b^2*c^3*x^8*e^3 + 3*b^2*c^3*d*x^6*e^2 + 3*b^2*c^3
*d^2*x^4*e + b^2*c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(x^2*e + d) + 2*(b^2*c^4*x^
9*e^3 + b^2*c^2*d^3*x + (3*b^2*c^4*d*e^2 + b^2*c^2*e^3)*x^7 + 3*(b^2*c^4*d^
2*e + b^2*c^2*d*e^2)*x^5 + (b^2*c^4*d^3 + 3*b^2*c^2*d^2*e)*x^3)*sqrt(c^2*x^
2 + 1)*sqrt(x^2*e + d) + (b^2*c^5*x^10*e^3 + (3*b^2*c^5*d*e^2 + 2*b^2*c^3*e
^3)*x^8 + (3*b^2*c^5*d^2*e + 6*b^2*c^3*d*e^2 + b^2*c*e^3)*x^6 + b^2*c*d^3 +
 (b^2*c^5*d^3 + 6*b^2*c^3*d^2*e + 3*b^2*c*d*e^2)*x^4 + (2*b^2*c^3*d^3 + 3*b
^2*c*d^2*e)*x^2)*sqrt(x^2*e + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x
^10*e^3 + (3*a*b*c^5*d*e^2 + 2*a*b*c^3*e^3)*x^8 + (3*a*b*c^5*d^2*e + 6*a*b*
c^3*d*e^2 + a*b*c*e^3)*x^6 + a*b*c*d^3 + (a*b*c^5*d^3 + 6*a*b*c^3*d^2*e + 3
*a*b*c*d*e^2)*x^4 + (2*a*b*c^3*d^3 + 3*a*b*c*d^2*e)*x^2)*sqrt(x^2*e + d)),
x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a^2*x^6*e^3 + 3*a^2*d*x^4*e^2 + 3*a^2*d^2*x^2*e +
 a^2*d^3 + (b^2*x^6*e^3 + 3*b^2*d*x^4*e^2 + 3*b^2*d^2*x^2*e + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*x^6*e^3 + 3*a*b*d*x^4*e^2 + 3*a*b*d^2*x^2*e + a*b*d^3)*
arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2} \left(d + ex^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```